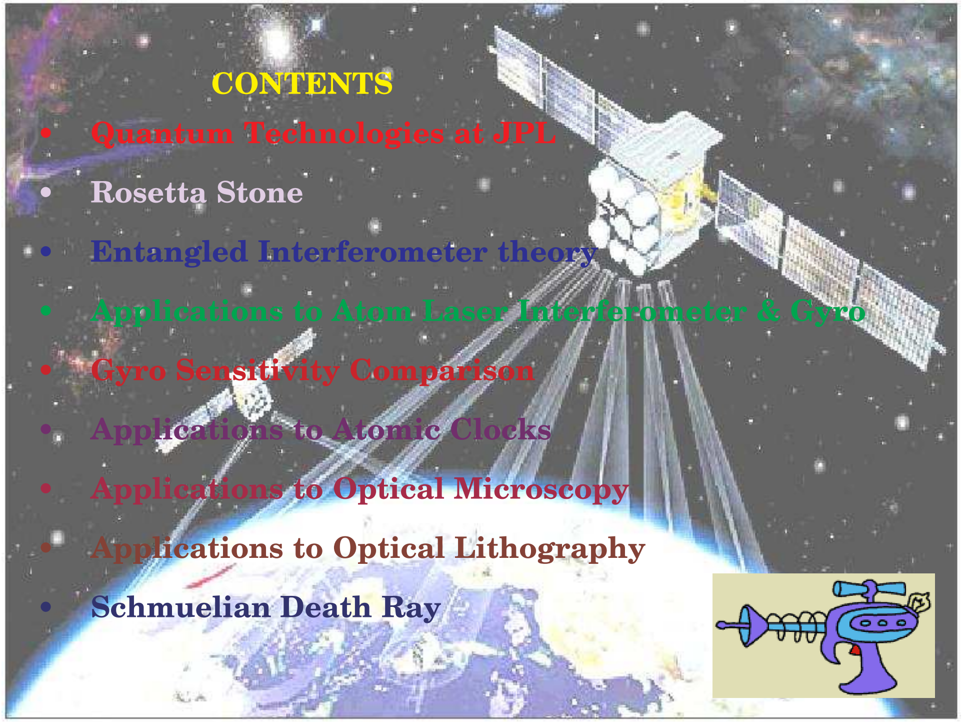
Linear Optics and Projective Measurements*

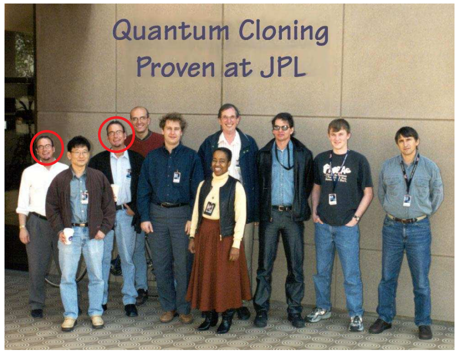
by

Robert M. Gingrich, Pieter Kok, Hwang Lee,
Nicolas J. Cerf, Colin P. Williams, and
Jonathan P. Dowling

sponsored by: NASA



NASA QUANTUM TEAM ALPHA



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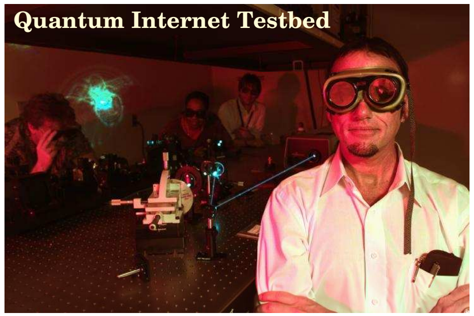
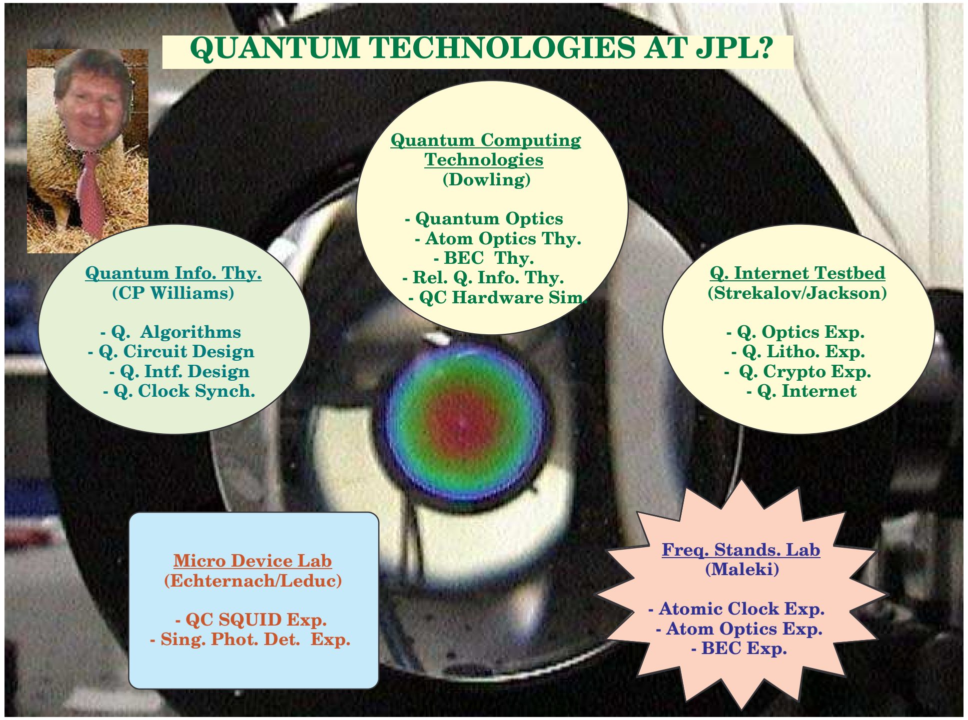
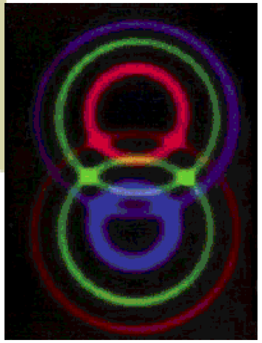




Part I: Quantum Computing and Metrology

SCIENCE NEWS ONLINE
The Weekly Newsmagazine of Science



Science News

Week of Dec. 8, 2001; Vol. 160, No. 23

Gadgets from the Quantum Spookhouse

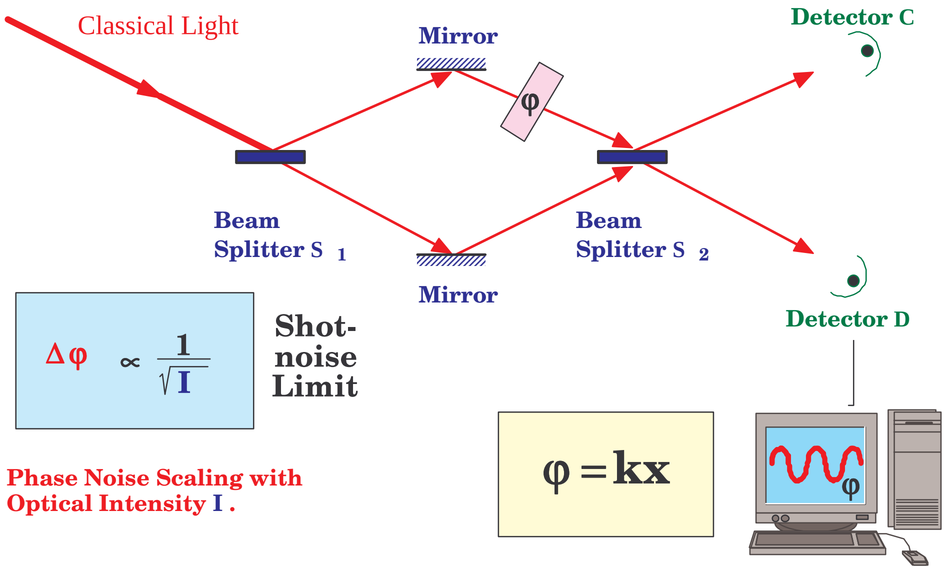
Navigation devices and other technologies may gain from queer quantum effects

Peter Weiss

...The potential payoffs are many. Entanglement and other quantum weirdness may boost the accuracy of radar, Global Positioning System (GPS) receivers, and other navigation devices. It may improve manufacturers' ability to lay down tiny structures on microchips, expedite explorations for oil, improve the vision of telescopes, and increase the accuracy of atomic clocks. Quantum technology might even enable us to see objects without actually looking at them—and without being seen ourselves...

...Light plays a role in the industry because it projects patterns of wires and components onto the surface of a semiconductor wafer—a procedure known as lithography. But light of a given wavelength can define lines no thinner than half that wavelength, at least according to classical physics (SN: 5/5/01, p. 286: http://www.sciencenews.org/20010505/bob18.as p).
There's a quantum way around that law, says Jonathan P. Dowling of JPL. Last year, he and his colleagues at JPL and the University of Wales in Great Britain showed theoretically that a pair of entangled photons of a certain wavelength could "conspire," as he puts it, to act as a single photon of half that wavelength.....

CLASSICAL OPTICAL INTERFEROMETER



Phase Noise Scaling with Optical Intensity I .

Canonical Metrology

Suppose we have an ensemble of N states $|\varphi\rangle = (|0\rangle + e^{i\varphi}|1\rangle)/\sqrt{2}$, and we measure the following observable: $A = |0\rangle\langle 1| + |1\rangle\langle 0|$

The expectation value is given by: $\langle\varphi|A|\varphi\rangle = N\cos\varphi$
and the variance $(\Delta A)^2$ is given by: $N(1-\cos^2\varphi)$

The unknown phase can be estimated with accuracy:

$$\Delta\varphi = \frac{\Delta A}{|d\langle A\rangle/d\varphi|} = \frac{1}{\sqrt{N}}$$

note the square-root

This is the standard shot-noise limit.

H. Lee, P. Kok, and J.P. Dowling, quant-ph/0202133

Quantum Lithography & Metrology

Now we consider the state $|\varphi_N\rangle = (|N,0\rangle + e^{iN\varphi}|0,N\rangle)/\sqrt{2}$,

and we measure $A_N = |0,N\rangle\langle N,0| + |N,0\rangle\langle 0,N|$

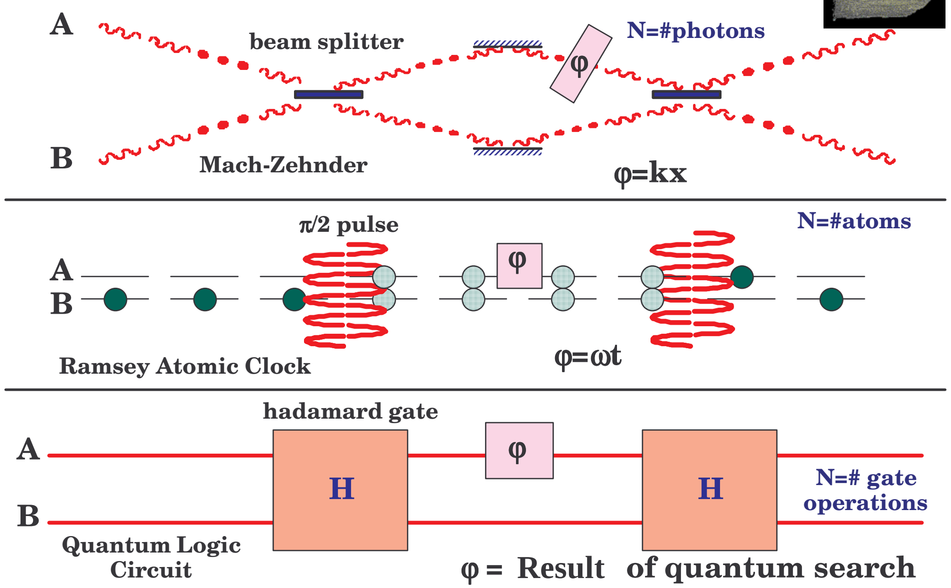
Quantum Lithography*: $\langle\varphi_N|A_N|\varphi_N\rangle = \cos N\varphi$ ← high frequency

Quantum Metrology: $\Delta\varphi_H = \dfrac{\Delta A_N}{|d\langle A_N\rangle/d\varphi|} = \dfrac{1}{N}$ ← no square-root!

*A.N. Boto, P. Kok, D.S. Abrams, S.L. Braunstein, C.P. Williams, and J.P. Dowling, *Phys. Rev. Lett.* **85**, 2733 (2000).
P. Kok, H. Lee, and J.P. Dowling, *Phys. Rev. A* **65**, 0512XX (2002).

THE QUANTUM ROSETTA STONE

Entanglement gives $1/\sqrt{N}$ to 1/N resolution improvement in each case!



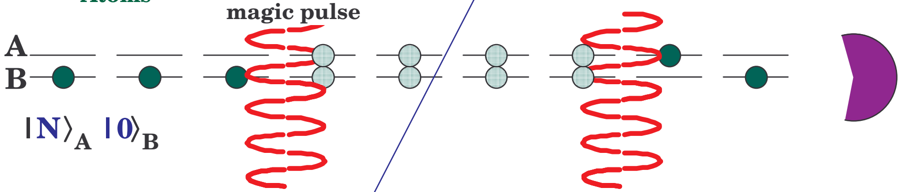
ATOMIC CLOCK INTERFEROMETER

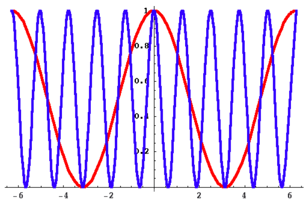
S. F. Huelga, C. Macchiavello, T. Pellizzar, A. K. Ekert, M. B. Plenio, and J. I. Cirac, Phys. Rev. Lett. 79, 3865 1997.

N Two-Level Atoms

$$|N\rangle_A|0\rangle_B + e^{iN\varphi}|0\rangle_A|N\rangle_B$$

N-Atom Detector





Oscillates N times as fast!

$\dfrac{1+\cos\varphi}{2}$ uncorrelated

$\dfrac{1+\cos N\varphi}{2}$ correlated

φ = ωT
Δφ: $1/\sqrt{N} \rightarrow 1/N$

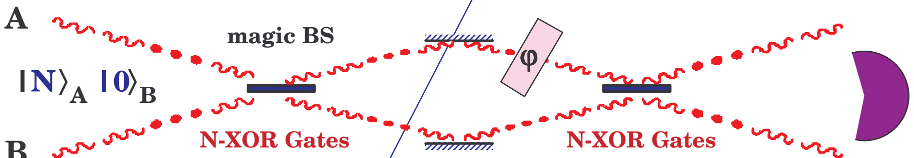
MACH-ZEHNDER INTERFEROMETER

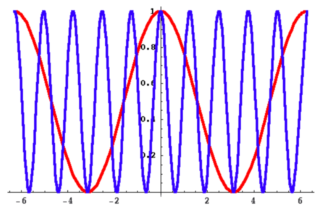
Apply the Quantum Rosetta Stone!

N Photons

$$|N\rangle_A|0\rangle_B + e^{iN\varphi}|0\rangle_A|N\rangle_B$$

N-Photon Detector





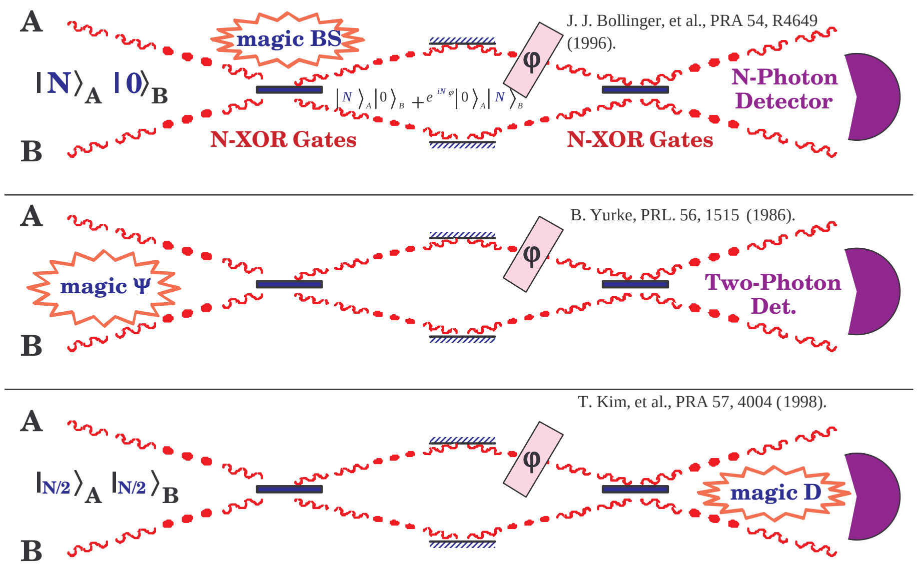
Oscillates N times as fast!

$\dfrac{1+\cos\varphi}{2}$ uncorrelated

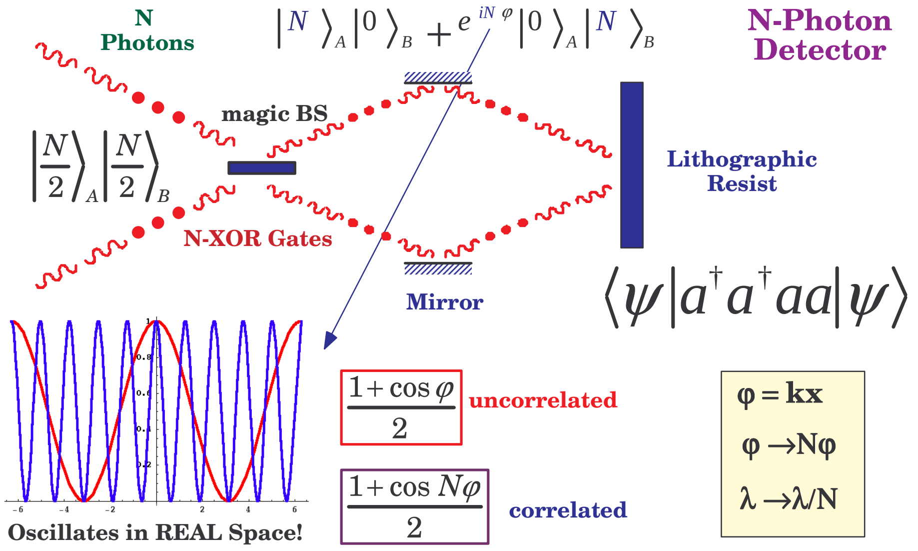
$\dfrac{1+\cos N\varphi}{2}$ correlated

φ = kx
Δφ: $1/\sqrt{N} \rightarrow 1/N$

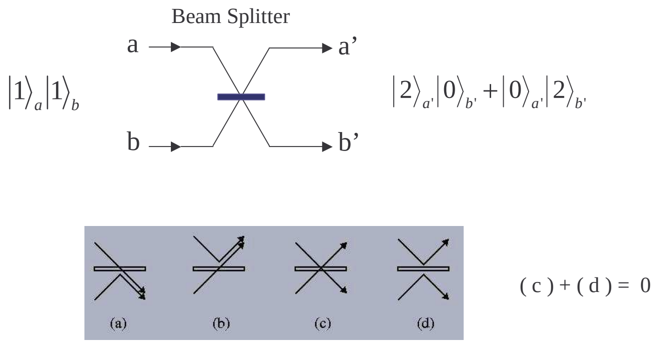
THREE ROADS TO HEISENBERG LIMITED INTERFEROMETRY



FROM QUANTUM INTERFEROMETRY TO QUANTUM LITHOGRAPHY



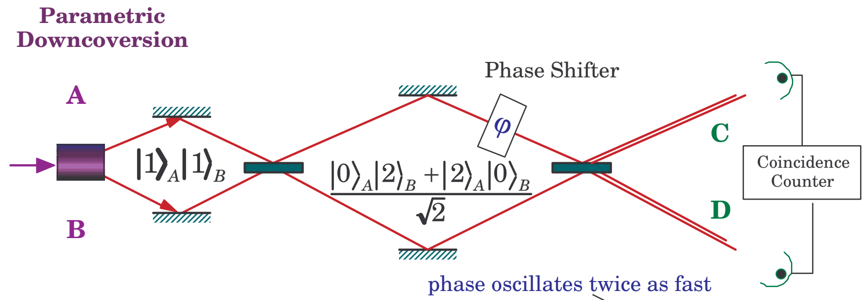
Hong - Ou - Mandel Interferometer



(c) + (d) = 0

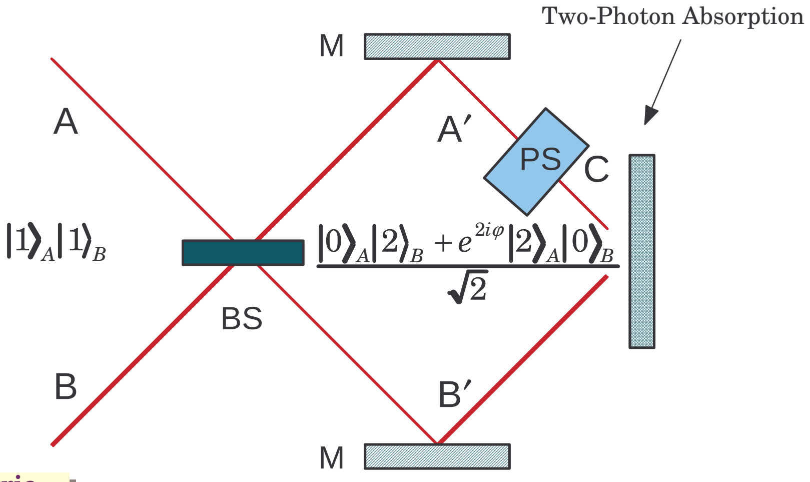
Hong, Ou, Mandel, PRL 59, 2044 (‘87)

ORIGIN OF THE LITHO EFFECT HOMI FOR N=2



$$\frac{|0\rangle_A|2\rangle_B + |2\rangle_A|0\rangle_B}{\sqrt{2}} \rightarrow \frac{|0\rangle_A|2\rangle_B + e^{2i\varphi}|2\rangle_A|0\rangle_B}{\sqrt{2}}$$

EASY (BUT USELESS?) FOR N=2



Parametric Downconversion

Part II: Quantum Interferometers from Quantum Computers

The Importance of CNOT

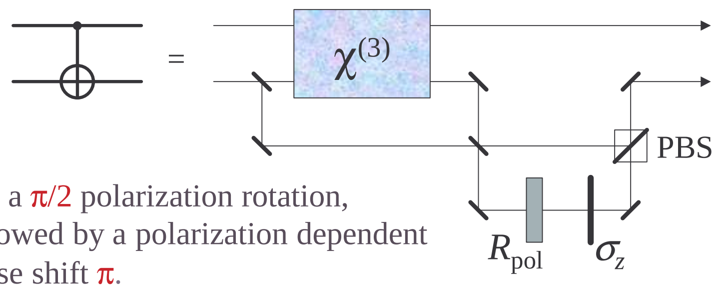
If we want to manipulate quantum systems for communication and computation, we must be able to do logical operations on the quantum bits (or qubits).
In particular, we need the so-called controlled-NOT that acts on two qubits:

$$|0\rangle|0\rangle \to |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \to |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \to |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \to |1\rangle|0\rangle$$

The first stays the same, and the second flips iff the first is a **1.**
This means we need some photon-photon interaction.

Optical CNOT = Kerr?

The controlled-NOT can be implemented using a Kerr medium:



R is a $\pi/2$ polarization rotation, followed by a polarization dependent phase shift π.

Unfortunately, the interaction $\chi^{(3)}$ is extremely weak*: 10^{-16} cm^2 sV^{-2}
This is **not practical**!

*R.W. Boyd, *J. Mod. Opt.* **46**, 367 (1999).

The K.L.M. paper*

Qubits are represented by a photon in two optical modes:

Using path-entanglement, extra optical modes and **projective measurements**, we can do quantum gates, including CNOT.

The big surprise is that we can do this efficiently without Kerr!

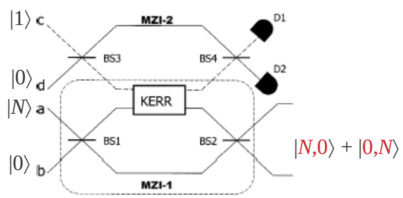
Quantum computing may still be a long shot, but what about quantum **metrology** and quantum **communication**?

*E. Knill, R. Laflamme, and G.J. Milburn, *Nature* **409**, 46 (2001).

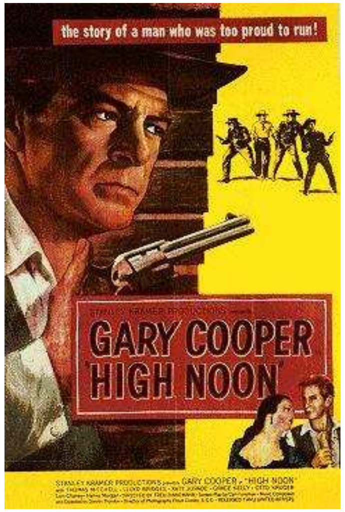
High NOON States

How do we make: $|N,0\rangle + |0,N\rangle$

With a large Kerr non-linearity*:



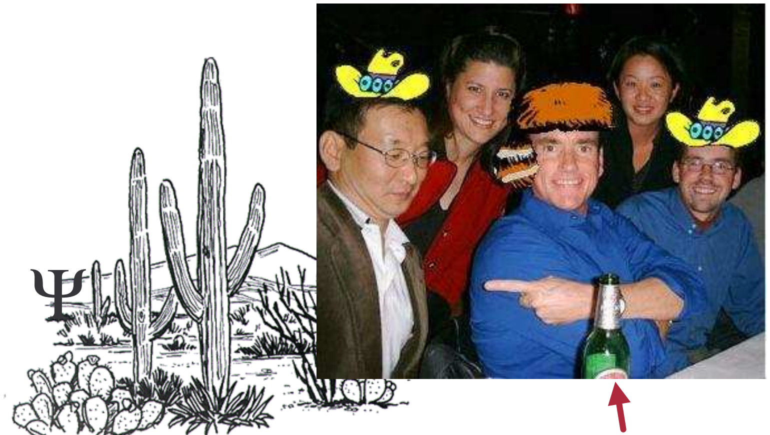
But this is not practical… $\chi_3 = \pi$!



*C. Gerry, and R.A. Campos, *Phys. Rev. A* **64**, 063814 (2001).

Showdown!

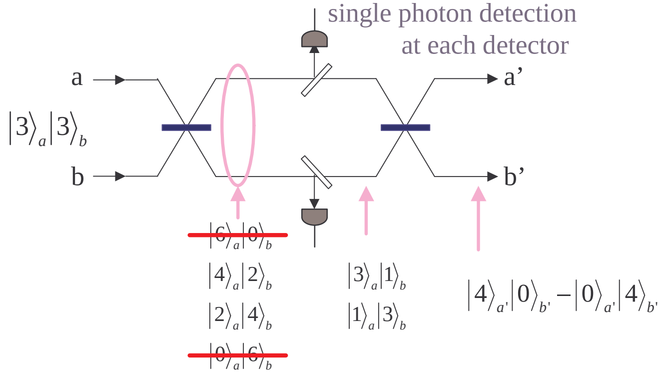
for projective measurements at high N00N…



Note that we are working within the many-beers interpretation.*

*J.P. Dowling, *Physics World* **14**, 64 (2001).

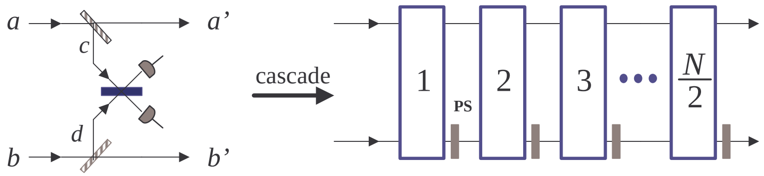
Projective Measurements to the Rescue



Probability of success: (event-ready) $\frac{3}{64}$ Best we found: $\frac{3}{16}$

H. Lee, P. Kok, N.J. Cerf, and J.P. Dowling, *Phys. Rev. A* **65**, R030101 (2002).

Projective Measurements to the Rescue



$|N,N\rangle \rightarrow |N\text{-}2,N\rangle + |N,N\text{-}2\rangle$

$|N,N\rangle \rightarrow |N,0\rangle + |0,N\rangle$

$$p_1 = \frac{1}{2} N\,(N\text{-}1)\, T^{2N\text{-}2} R^2 \xrightarrow[N\to\infty]{} \frac{1}{2e^2}$$

with T = (N–1)/N and R = 1–T

the consecutive phases are given by:

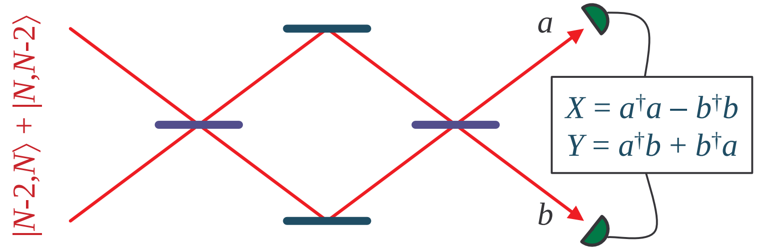
$$\varphi_k = \frac{2\pi k}{N/2}$$

Schemes based on non-detection have been proposed by Fiurásek (quant-ph/0110138) and Zou *et al.* (quant-ph/0110149).

P. Kok, H. Lee, and J.P. Dowling, *Phys. Rev. A* **65**, 0512104 (2002).

Yurke States*

We don't have to run *N*/2 of these circuits in series to get a useful state. We can work wonders with $|N,N\rangle \rightarrow |N\text{-}2,N\rangle + |N,N\text{-}2\rangle$†



Around $\varphi=0$, this leads to: $\Delta\varphi\big|_{\varphi=0} \approx \frac{1}{N}$

i.e., a phase dependent increase in sensitivity

†J.P. Dowling, *Phys. Rev. A.* **57**, 4736 (1998).

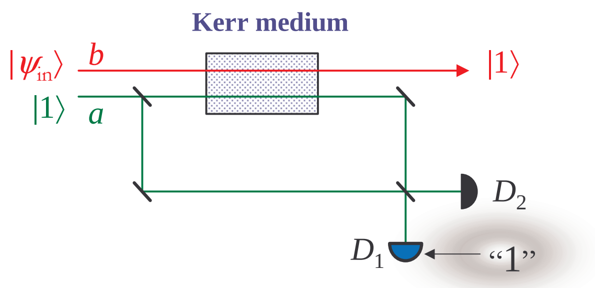
*B. Yurke, *Phys. Rev. Lett.* **56**, 1515 (1985).

Part III: Applications

Impractical Single-photon Quantum Non-Demolition*

objective: You want to know **how many photons** there are in a light beam **without destroying** them.

Interaction Hamiltonian: $H_{\text{Kerr}} = \kappa\, a^\dagger a\, b^\dagger b$



Again, with a high-Kerr medium, this is possible.

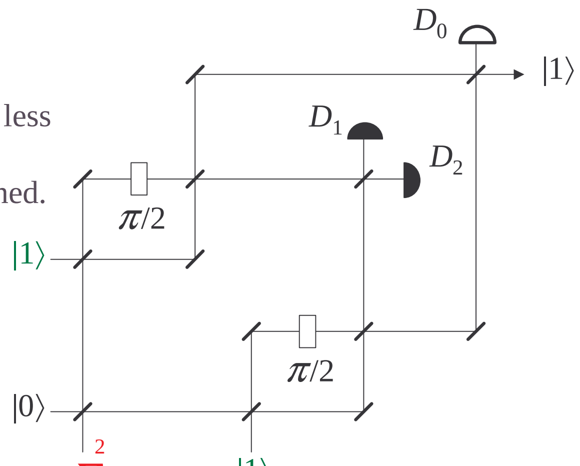
*N. Imoto, H.A. Haus, and Y. Yamamoto, *Phys. Rev. A.* **32**, 2287 (1985).

Practical Single-photon Quantum Non-Demolition

Relaxing conditions:

The success probability is less than 1 (namely 1/8).
The input state is constrained.

Yes! Conditioned on a detector coincidence in D_1 and D_2.



$$|\psi_{\text{in}}\rangle = \sum_{n=0}^{2} c_n |n\rangle$$

P. Kok, H. Lee, and J.P. Dowling, quant-ph/0202046

Quantum Communication

In quantum communication protocols, **ALICE** and **BOB** typically need to share entanglement, e.g., for quantum cryptography.

Alice

D A E D S I L U A P
(using diabolical encryption)

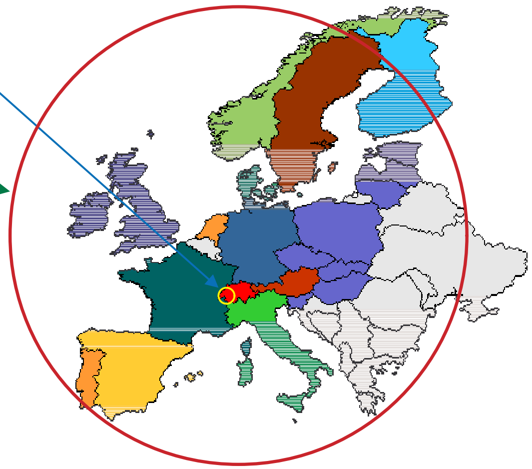
Bob

How do we extend the reach of QKD protocols?

Quantum Repeaters

Without repeaters (in, say, Geneva)

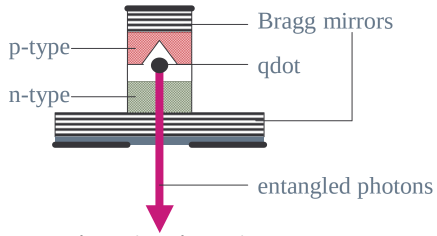
With repeaters



Quantum Repeaters

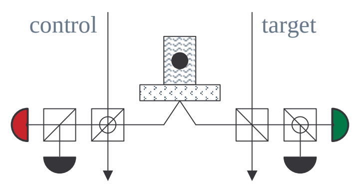
We extend the reach of QKD protocols with a quantum repeater. For this we need two separate resources:

1. Double-photon gun*:



$$|H,V\rangle - |V,H\rangle$$

2. Optical CNOT†:



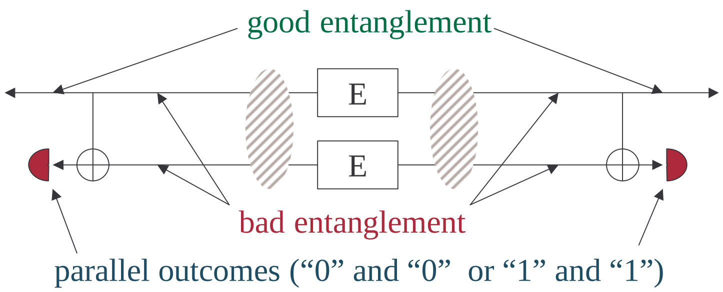
With probability 1/4

*O. Benson, C. Santori, M. Pelton, and Y. Yamamoto, *Phys. Rev. Lett.* **84**, 2513 (2000).

†T.B. Pittman, B.C. Jacobs, and J.D. Franson, *Phys. Rev. A.* **64**, 062311 (2001).

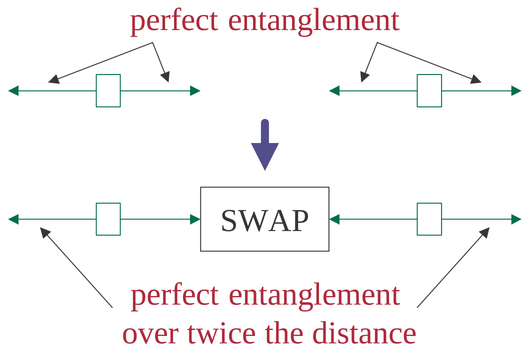
Signal Degradation

Fiber of length L has a loss of $e^{-\gamma L}$, where γ is a property of the fiber. Similarly, the entanglement exhibits decoherence that scales exponentially with the length of the fiber. We therefore need some purification protocol that revives the entanglement.



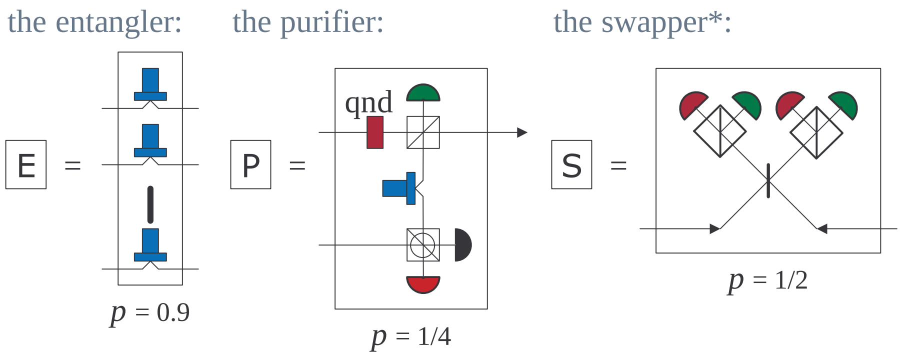
Signal Extension

Now that we can establish maximal entanglement over some distance (rather than some degraded entanglement without purification), we can use teleportation to extend the reach of the signal:



Quantum Repeaters

The components of the quantum repeater are then given by the probabilistic elements:

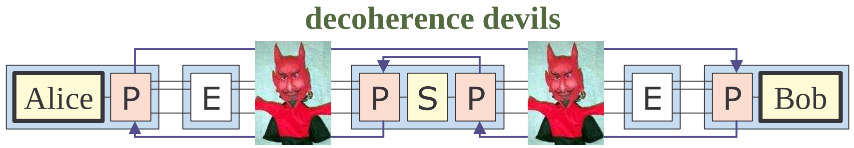


How do we put these together in a repeater?

*S.L. Braunstein and A. Mann, *Phys. Rev. A* **51**, R1727 (1995).

Quantum Repeaters

The assembled quantum repeater would look something like this:



The repeater protocol is now complete, and Alice and Bob share entanglement.

P. Kok, C.P. Williams, and J.P. Dowling, quant-ph/0203134

Quantum Repeaters

The probability of success for a single purification is given by:

$$p_{pur} = p^6_s\, \eta^{10} (1-\gamma)^2\, \zeta^2\, p^2_{\text{CNOT}}\, p_{\text{QND}}$$

source efficiency, detector efficiency, dephasing, photon loss

Due to the tenth power, the detector efficiency will have the largest impact on the repeater efficiency.

P. Kok, C.P. Williams, and J.P. Dowling, quant-ph/0203134

Quantum Repeaters

The lower the efficiency, the more entanglement sources we will have to fire in unison:

η	N_{pur}	N_{swap}	N_{total}
0.3	$3 \cdot 10^7$	20	$\sim 10^9$
0.8	$2 \cdot 10^3$	3	$\sim 10^4$
1	250	2	$\sim 10^3$

By comparison, a Pentium III chip has ~10^7 transistors.

P. Kok, C.P. Williams, and J.P. Dowling, quant-ph/0203134

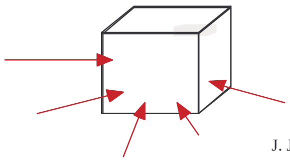
SCALING LAWS

Uncorrelated N-Photon Absorption Probability

$$P \propto I^N$$

Correlated N-Photon Absorption Probability

$$P \propto I$$

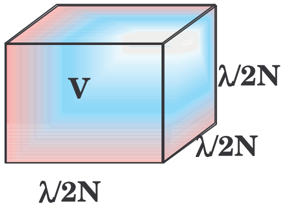


P is the probability of finding N photons in a unit volume per unit time. Hence low intensities for entangled photons will do.

J. Javanainen and P. L. Gould, PRA **41**, 5088 (1990).
J. Perina, Jr., B. E. A. Saleh, and M. C. Teich, PRA **57**, 3972 (1998).

SCHMUELIAN DEATH RAY

How High is High NOON?



Volume: $V = (\lambda/2N)^3$
Energy: $E = N\hbar\omega$
Energy Density: $u = E/V = 64\pi\hbar c N^4/\lambda^4$

N=4	**Atom Ionization**
N=10^3	**Thermonuclear Fusion**
N=10^6	**Big Bang Energies**

Entangled state behaves like a single photon of wavelength $\lambda_{eff} = \lambda/N$ (gamma ray laser).

Summary

- Projective measurements and linear optics can sometimes replace the use of Kerr non-linearities;
- Using this technique, we can create high-N00N states, which are important for quantum metrology and lithography;
- Projective measurements also allow us to build probabilistic elements such as QND devices and CNOT gates;
- We showed that we can make a quantum repeater.
- Finally, we need better detectors!