RELATIVISTIC MOTIONS AROUND A BLACK HOLE

with classical methods

PRESENTER

Rim, Woohyun

MEMBERS

Oh, min ji

Jun, ye jin

Kim, kyeong eun

Lee, hyeong ju

Contents

Introduction

- Quamvis movet! (E pur si muove!)
  - Galilaeo Galilaei
- Quamvis trahitur! (However, attracted)
  - Issaco Newtono
- Quamvis procedit! (However, proceed)
  - Alberto Einsteino

Problem

- $$\int dtL = -mc^2 \int dt \sqrt{f(r) - \frac{\dot{r}^2}{c^2 f(r)} - \frac{r^2}{c^2}\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right)}$$
    - $$f(r) = 1 - \frac{r_c}{r}$$
    - $$f(r) = \left(1 - \frac{r_c}{2r}\right)^2$$

Simplification

$$L = -mc^2 \sqrt{f(r) - \frac{\dot{r}^2}{c^2 f(r)} - \frac{r^2}{c^2}\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right)} = -mc^2 \sqrt{f(r) - \frac{\dot{r}^2}{c^2 f(r)} - \frac{r^2\dot{\phi}^2}{c^2}}$$

$$L' = \sqrt{f(r) - \frac{\dot{r}^2}{c^2 f(r)} - \frac{r^2\dot{\phi}^2}{c^2}}$$

Assumption

$$\theta = \frac{\pi}{2}$$

Discussion

- What this action means?

Discussion

- Schwarzschild Solution
  - Non-rotating non-charged solution
- Reissner-Nordstrom Solution
  - Non-rotating charged solution

Constants of motion

- $$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\phi}}\right] = 0$$
  - $$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = \frac{mr^2\dot{\phi}}{L'}$$
  - $$L' = \sqrt{\frac{f(r) - \frac{\dot{r}^2}{c^2 f(r)}}{1 + \frac{p_\phi^{\ 2}}{m^2 c^2 r^2}}}$$
- $$H = \sum_i p_i \dot{q}_i - L = \frac{mc^2}{L'} f(r)$$

Solving

$$H(r, p_r) = mc^2 f \sqrt{\frac{1 + \frac{p_\phi{}^2}{m^2 c^2 r^2}}{f - \frac{\dot{r}^2}{c^2 f}}}$$

$$\dot{r}^2 = c^2 f^2 - \frac{m^2 c^6 f^3}{H^2}\left(1 + \frac{p_\phi}{m^2 c^2 r^2}\right)$$

t(r)->Schwarzschild

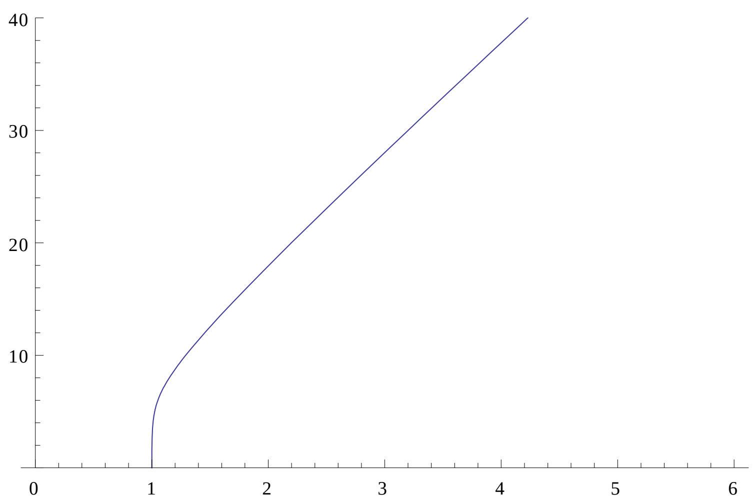
- $$t(r)=\int dt=\int\frac{dt}{dr}dr=\pm\int\frac{dr}{\dot{r}}$$
$$=\int\left[\frac{1+\frac{1}{2}\frac{m^2c^4}{H^2}f\left(1+\frac{p_\phi{}^2}{m^2c^2r^2}\right)+\frac{3}{8}\left[\frac{m^2c^4}{H^2}f\left(1+\frac{p_\phi{}^2}{m^2c^2r^2}\right)\right]^2}{cf}+\cdots\right]dr$$
- Expanding,

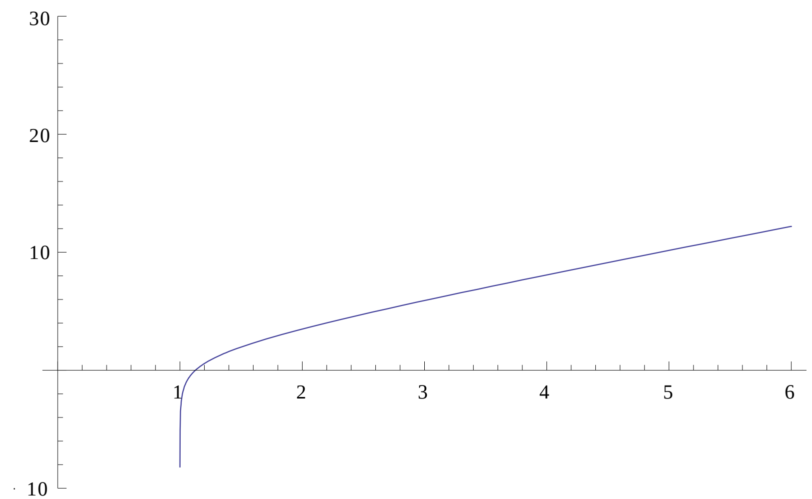
$$\pm t=\left[\frac{r}{c}\left(1+\frac{m^2c^4}{2H^2}+\frac{3m^4c^8}{8H^4}+\ldots\right)+\frac{r_s}{c}\log|r-r_s|-\frac{3r_cm^4c^7\log r}{8H^4}-\frac{c^2p_\phi{}^2}{2H^2r}\left(1+\frac{3m^2c^4}{2H^2}+\cdots\right)+\cdots\right]_{r_0}^{r}$$
$$\approx\left[\frac{r}{c}+\frac{r_s}{c}\log|r-r_s|-\frac{3r_cm^4c^7\log r}{8H^4}-\frac{c}{2H^2r}\left(m^2c^2r^2-p_\phi{}^2\right)\right]_{r_0}^{r}$$

± t(r)-> Schwarzschild

$$m = H = r_c = p_\phi = 1$$

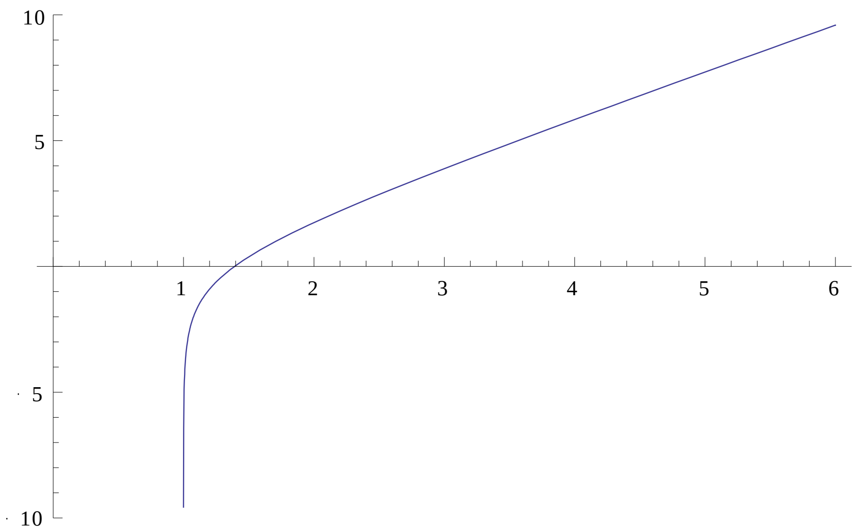
$$m = H = r_c = 1, p_\phi = 0$$

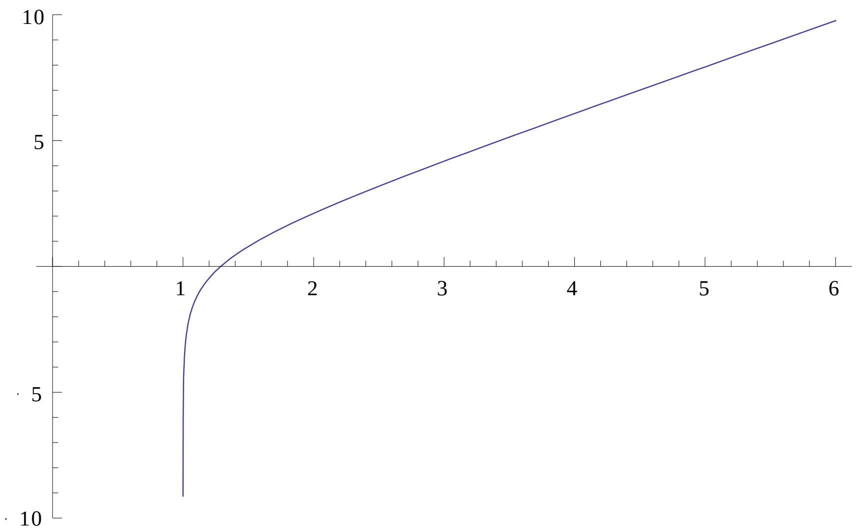




± t(r)-> Reissner maxima

$$m = H = r_c = p_\phi = 1 \qquad m = H = r_c = 1, p_\phi = 0$$



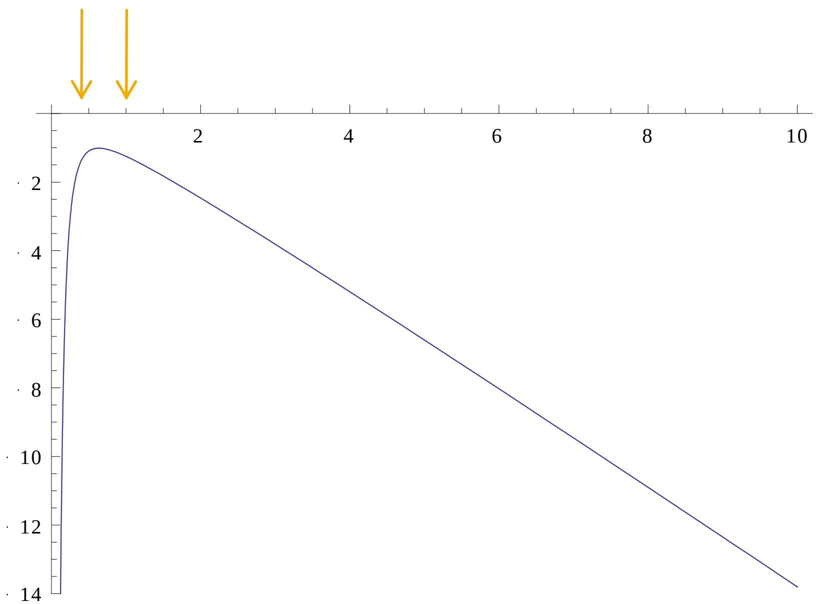


τ(r)

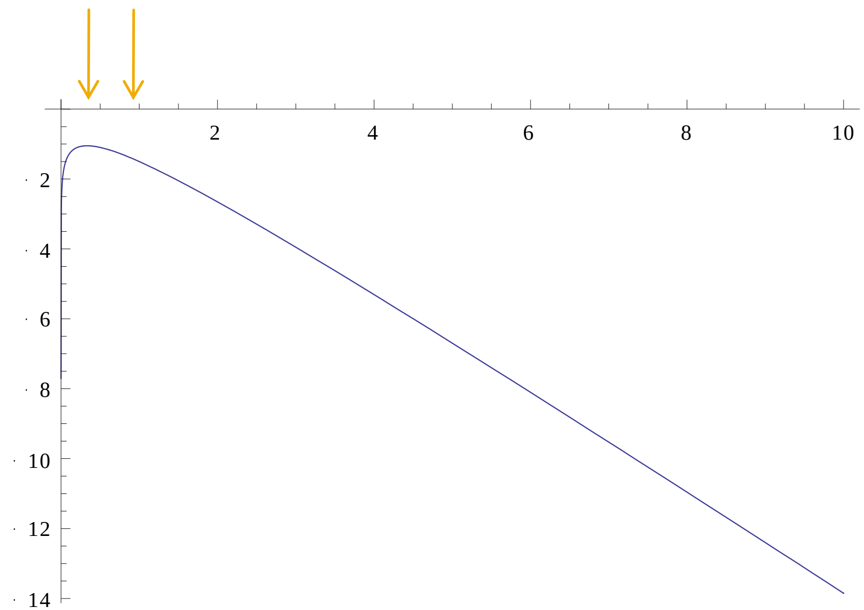
$$d\tau = \int L'\,dt = \int L'(r)\frac{dt}{dr}dr$$

± τ(r)-> Schwarzschild

$$m = H = r_c = p_\phi = 1$$

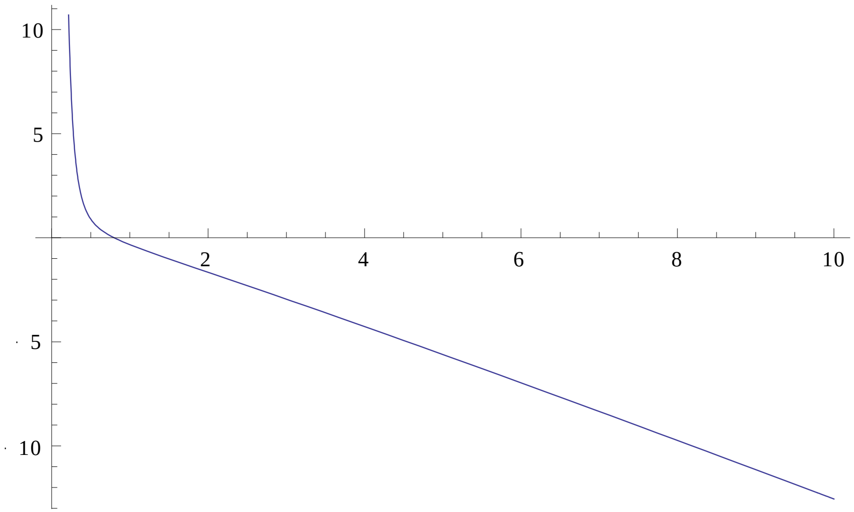


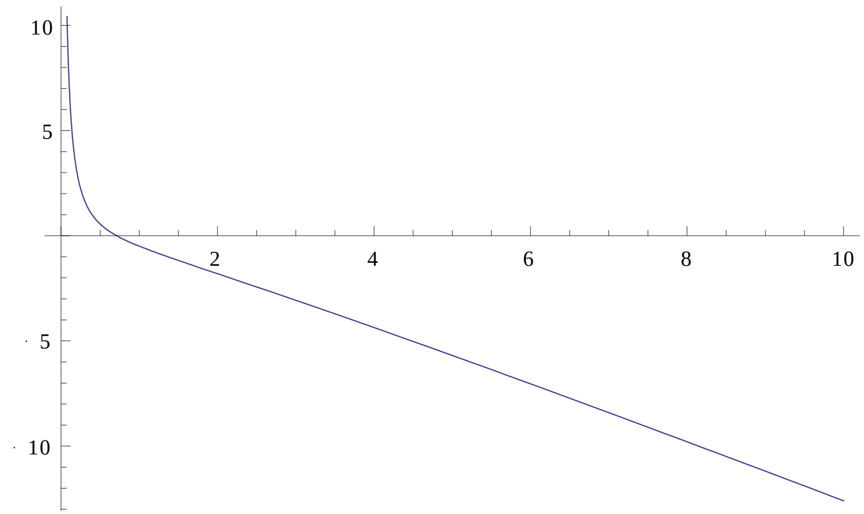
$$m = H = r_c = 1, p_\phi = 0$$



± τ(r)-> Reissner maxima

$$m = H = r_c = p_\phi = 1 \qquad m = H = r_c = 1, p_\phi = 0$$





ϕ(r)

$$\phi = \int \frac{\dot{\phi}}{\dot{r}} dr = \int \frac{p_\phi L'(r)}{mr^2 \dot{r}} dr$$

Visualisation

- See Octave

Classification

$$H = K + V = K_r + V_{r,eff}$$

$$K_r = \int_{\dot{r}=0}^{\dot{r}} \dot{p}\,dr = \int_{\dot{r}=0}^{\dot{r}} \frac{d}{dt}\left[\frac{\partial L}{\partial \dot{r}}\right]\frac{dr}{dt}dt = \left[\dot{r}\frac{\partial L}{\partial \dot{r}} - \int \ddot{r}\frac{\partial L}{\partial \dot{r}}dt\right]_{\dot{r}=0}^{\dot{r}} = \cdots$$

$$V_{r,eff} \approx -\frac{GMm}{r} + \frac{{p_\phi}^2}{3mr^2} - \frac{GM{p_\phi}^2}{mc^2r^3}$$

Complex Angles

- $\cos x + yi = \cos x \cosh y - i \sin x \sinh y$
- See the comparison of the inside orbit with two different complex realisations.

Conclusion

- Apparently, the right choice of the Lagrangian gives the right result, and vice versa.
- At least, from the inside result, the implication of new concept is required.