Group Research

Relativistic Motions around a Black Hole

2010 KIAS-SNU Physics Winter Camp

Date : February 7, 2010

Talk : 주부경, 조창우, 김은찬, 서윤지, 고성문, 노대호

Table

- What is space-time ?

- How a particle moves?

  - As a geometry

- The black-hole

- ?

What is space-time ?

Space + Time (additional dim)


성기오빠!
한화리조트
215호에서 만나
215호?
알았어 !!


< 2134ft, N37, E128 >
한화리조트 215호
© 2010 ZENRIN
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Image © 2010 DigitalGlobe
이미지 날짜: 2005/4/23
37°34'44.59" N 128°19'53.24" E 고도 2149 ft
내려다보는 높이 5430 ft


< t, 2134ft, N37, E128 >
Additional Dimension
Pyonggang
속초 Sokcho
춘천
강릉 Gangneung
휘닉스파크가족농장
휘닉스파크CC
Namyangju 남양주
Seoul 서울특별시
Incheon
원주 Wonju
이천시 Icheon
Daebu-do
Hwaseong
Suwon
제천시 Jecheon
Anseong 안성시
Chungju 충주시
천안 Cheonan
Cheongju 청주
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Google
37°34'39.66" N 128°03'31.76" E
167.56 mi

$t_{성기} = t_{연아}$

$t_{성기} \neq t_{연아}$

How a particle moves?

As a Geometry

Action - Euclidian Space

$$\int ds = \int \sqrt{dx^2 + dy^2 + dz^2}$$

$$= \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

Action - Euclidian Space

- Euler – Lagrange Equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0$$

$$\rightarrow \dot{v}=0$$

Action – In special relativity

$$-d\tau^2 = -dt^2 + dx^2 + dy^2 + dz^2 = dS^2$$

$$\tau = \int d\tau = \int \sqrt{-dS^2} = \int \sqrt{-\eta_{\mu\nu} dx^{\mu} dx^{\nu}}$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow \int \sqrt{-\eta_{\mu\nu} \frac{dx^{\mu}}{d\sigma} \frac{dx^{\nu}}{d\sigma}} d\sigma = \int L d\sigma$$

Action - In special relativity

- Euler – Lagrange Equation $\Rightarrow$

$$\frac{d}{d\sigma}\left(\frac{\partial L}{\partial \dot{x}^{\mu}}\right)-\frac{\partial L}{\partial x^{\mu}}=0$$

Action – In general relativity

$$\tau = \int d\tau = \int \sqrt{-dS^2} = \int \sqrt{-g_{\mu\nu} dx^\mu dx^\nu}$$

$$g_{\mu\nu} = (?) \qquad \rightarrow \int \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\sigma} \frac{dx^\nu}{d\sigma}} d\sigma = \int L d\sigma$$

The black hole

Extremely curved space-time

Schwarzschild metric

$$ds^2 = -(1 - \frac{2GM}{c^2 r})c^2 dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2 r}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

Q=0, S=0, Massive

Constant of motion

$$e = -\xi \cdot u = -g_{tt} u^t = (1 - \frac{2M}{r})(\frac{dt}{d\tau})$$

$$l = \eta \cdot u = g_{\phi\phi} u^\phi = r^2 (\frac{d\phi}{d\tau})$$

$$u \cdot u = -1 = g_{tt} (u^t)^2 + g_{rr} (u^r)^2 + g_{\phi\phi} (u^\phi)^2$$

$$\rightarrow \frac{\varepsilon}{m} = \frac{e^2 - 1}{2} = \frac{1}{2} (\frac{dr}{d\tau})^2 - \frac{M}{r} + \frac{l^2}{2r^2} - \frac{Ml^2}{r^3}$$

Radial motion

$$l = 0,\ e = 1$$

$$0 = \frac{1}{2}(\frac{dr}{d\tau})^2 - \frac{M}{r}$$

$$u \cdot u = -1 = g_{tt}(u^t)^2 + g_{rr}(u^r)^2$$

Near the horizon

$$\frac{dr}{d\tau} = -\sqrt{\frac{2M}{r}}$$

$$\frac{dt}{d\tau} = (1 - \frac{2M}{r})^{-1}$$

$$\frac{dr}{dt} = \frac{dr}{d\tau} \Big/ \frac{dt}{d\tau} = -(1 - \frac{2M}{r})\sqrt{\frac{2M}{r}}$$

Eddington-Finkelstein coordinates

$$t = \upsilon - r - 2M \log\left|\frac{r}{2M} - 1\right|$$

$$ds^2 = -(1 - \frac{2M}{r})d\upsilon^2 + 2d\upsilon dr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

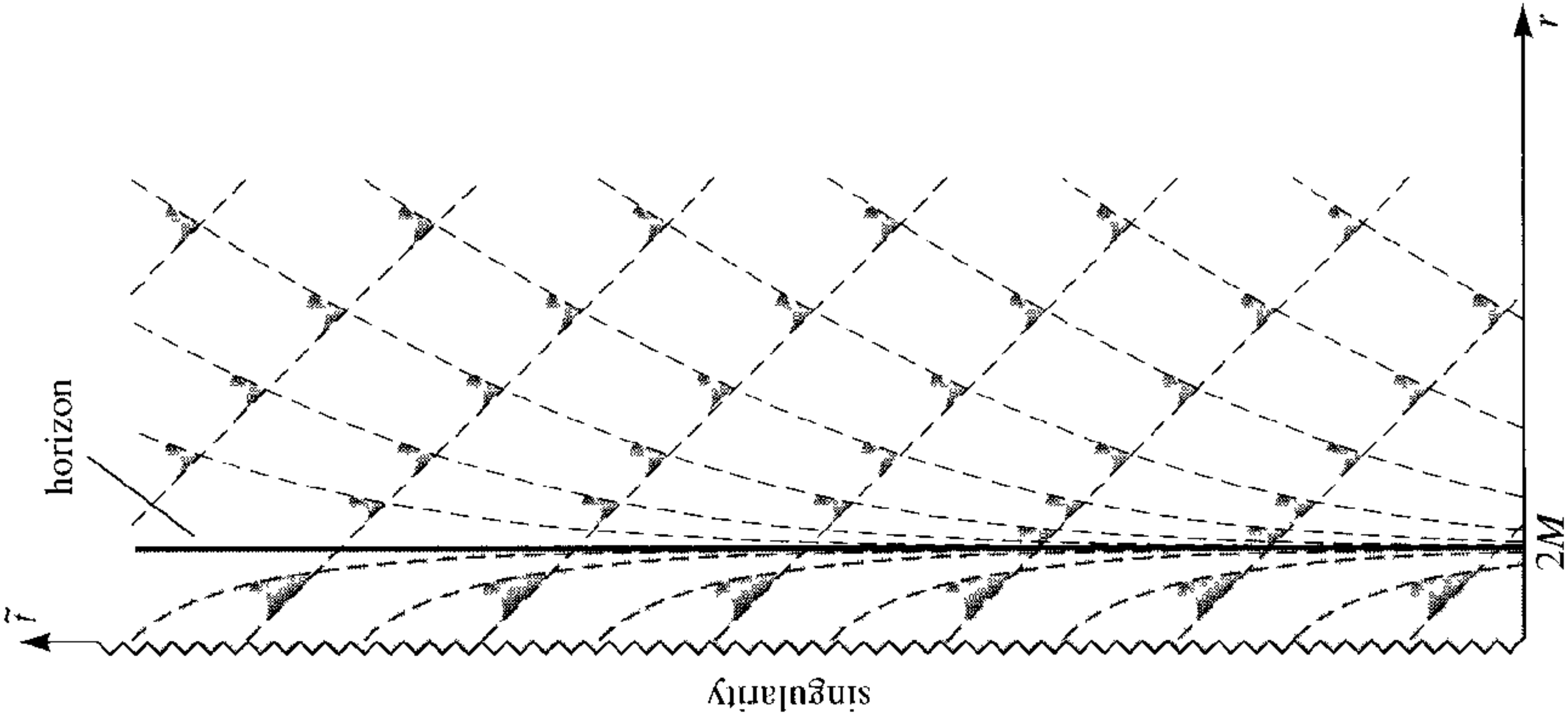
$$-(1 - \frac{2M}{r})d\upsilon^2 + 2d\upsilon dr = 0$$

$$\upsilon = const \quad (ingoing\ radial\ light\ rays)$$

$$-(1 - \frac{2M}{r})d\upsilon + 2dr = 0$$

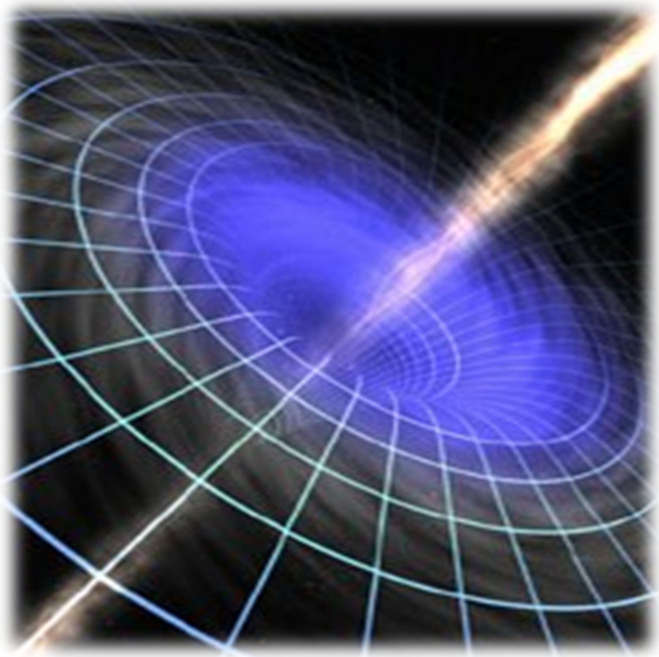
$$\upsilon - 2(r + 2M\log\left|\frac{r}{2M} - 1\right|) = const$$

$$t = t_* + 2M[-\frac{2}{3}(\frac{r}{2M})^{3/2} - 2(\frac{r}{2M})^{1/2} + \log\left|\frac{(r/2M)^{1/2}+1}{(r/2M)^{1/2}-1}\right|]$$



Reissner–Nordström metric

$$ds^2 = -(1-\frac{M}{r})^2 dt^2 + \frac{dr^2}{(1-\frac{M}{r})^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$



Q≠0, S=0, Massive

Eddington again…

$$\tilde{t} = t + M \ln(r - M)^2 - \frac{M^2}{r - M}$$

$$h \equiv 1 - f$$

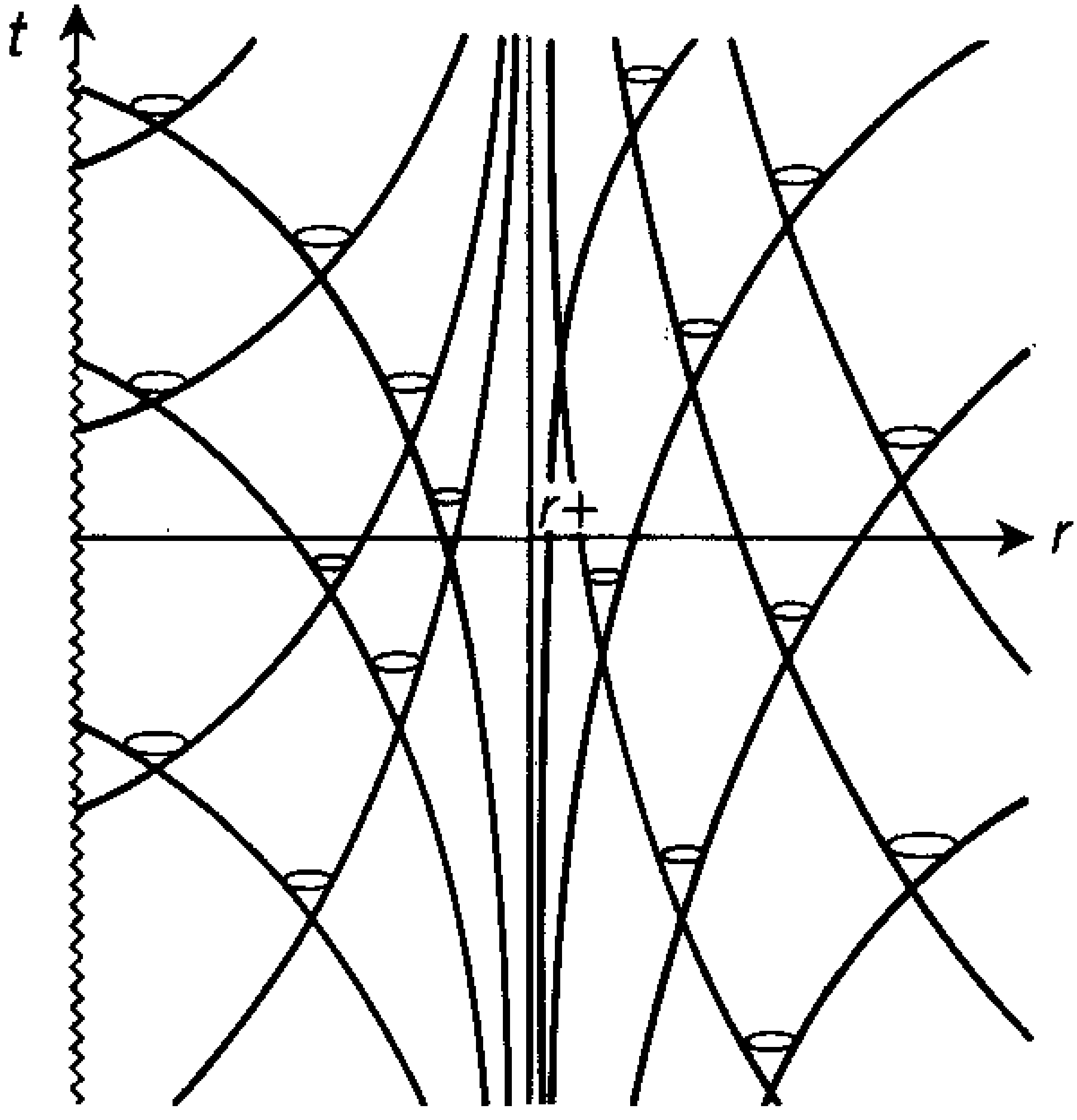
$$ds^2 = -(1-h)d\tilde{t}^2 + 2hd\tilde{t}\,dr + (1+h)dr^2 = 0$$

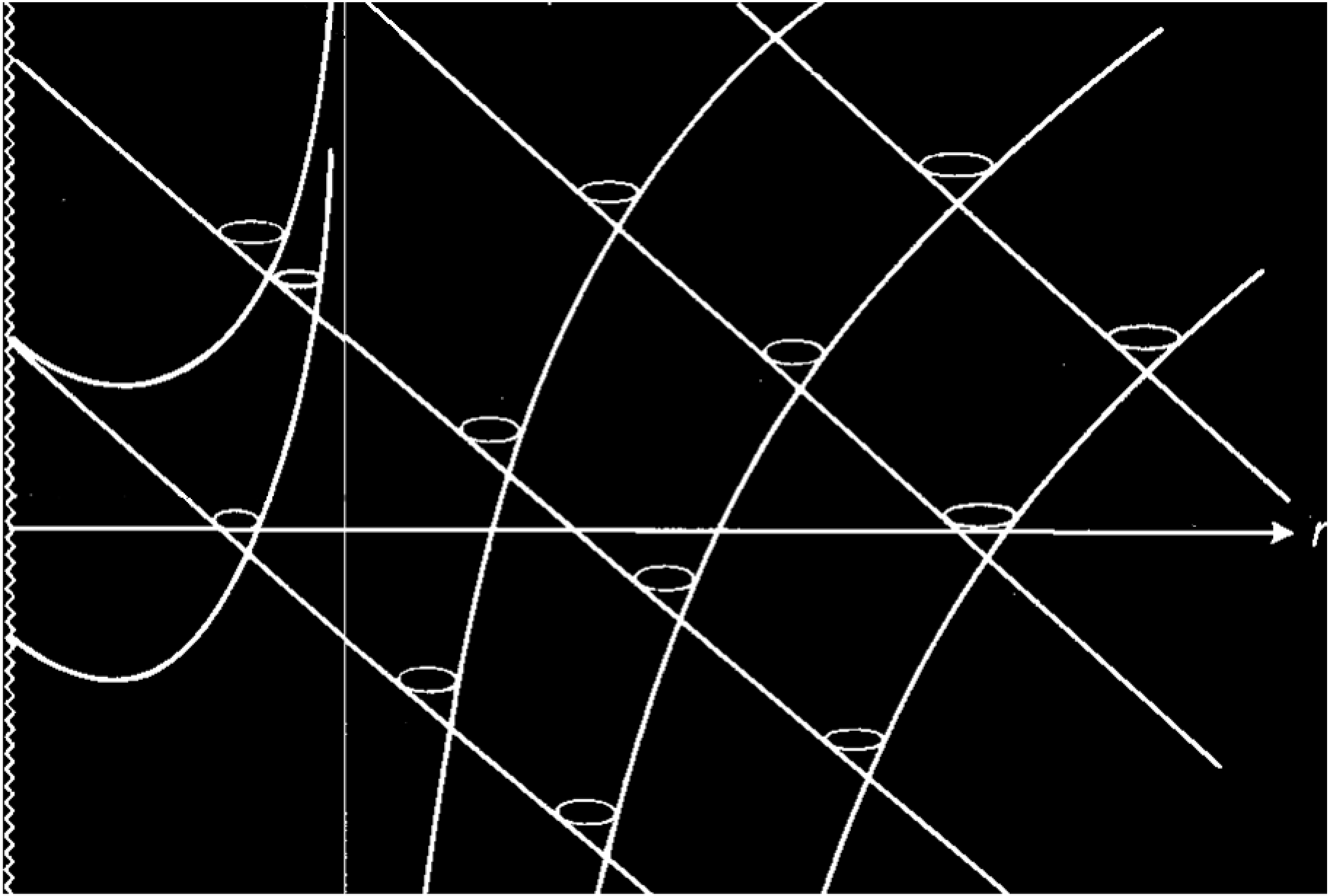
$$\tilde{t} + r = const \rightarrow d\tilde{t} = -dr$$

$$\rightarrow dt = -\frac{M^2}{(r-M)^2}dr = -\frac{dr}{(r - \frac{M}{r})^2}, \quad \frac{dt}{dr} = -\frac{1}{f}$$

$$\frac{d\tilde{t}}{dr} = \frac{1+h}{1-h}$$

$$\frac{dt}{dr} = \frac{1}{f}$$


t
r
r+



We need elevator

Black Hole

Reference

- Wikipedia
  - Key Word
    Black hole, Action Principle, General Relativity, Metric, Lagrangian, etc
- Google
  - Key Word
    Black hole & charged, space-time
- Book
  - Gravity (J.B.Hartle)
  - Introducing Einstein`s Relativity (Ray D’Inverno)