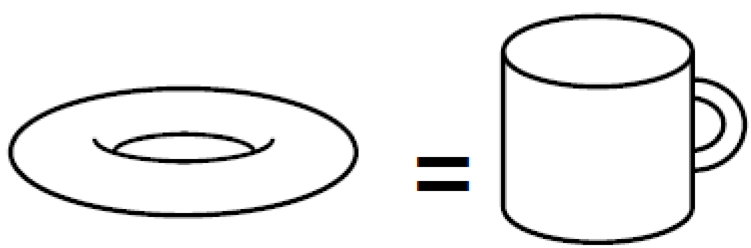
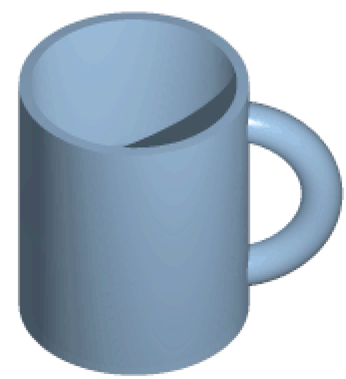
Topological Insulator

오조

Literal meaning of Topology

- Properties that are preserved under continuous deformation.

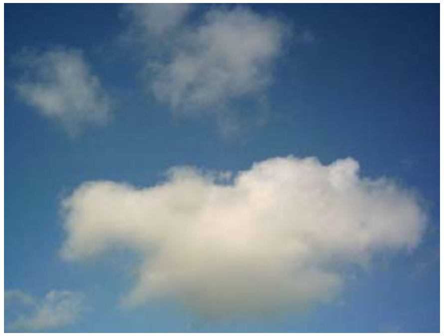




No tearing, no gluing (Mathematical)

Landau symmetry-breaking theory

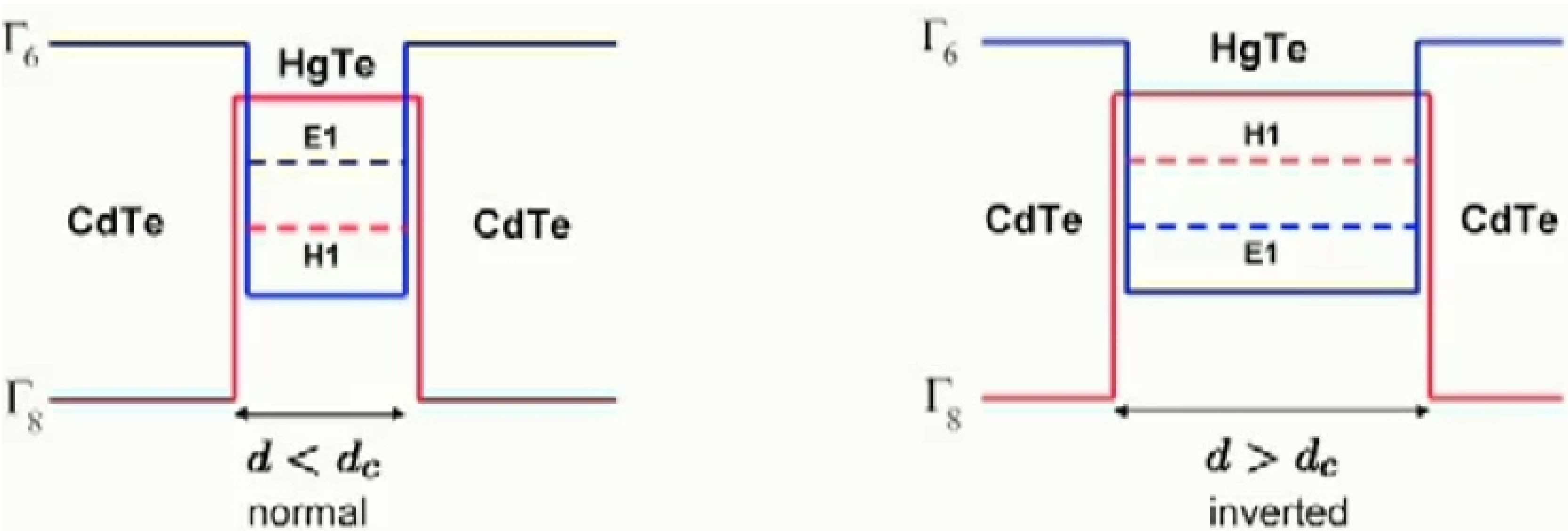




- Same atoms but different properties
- Landau symmetry-breaking theory explained it by symmetry breaking in a way material organizes
- Ex) Water : translational symmetry
  Ice : discrete translational symmetry

Topological insulator

- All different Chiral spin states or Quantum Hall states have the same symmetries



- This inverted gap leads to different states with preserved symmetries

Modified Maxwell equation

$$S = \int d^3x dt [\frac{1}{8\pi}(\epsilon E^2 - \frac{1}{\mu} B^2) + \frac{\theta}{2\pi}\frac{\alpha}{2\pi}\vec{E}\cdot\vec{B}]$$

$$\partial_\mu \left(\frac{\partial L}{\partial(\partial_\mu A_\nu)} \right) - \frac{\partial L}{\partial A_\nu} = 0$$

$$A^\mu = (\phi, \vec{A})$$

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\nabla \cdot \left(\epsilon \vec{E} + \frac{\theta\alpha}{\pi}\vec{B} \right) = 0$$

$$\nabla \times \left(\frac{1}{\mu}\vec{B} - \frac{\theta\alpha}{\pi}\vec{E} \right) = \frac{1}{c}\frac{\partial}{\partial t}\left(\epsilon \vec{E} + \frac{\theta\alpha}{\pi}\vec{B} \right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

If θ is constant

$$\nabla\cdot\left(\varepsilon\vec{E}+\frac{\theta\alpha}{\pi}\vec{B}\right)=0$$

$$\Rightarrow\quad \nabla\cdot\left(\varepsilon\vec{E}\right)+\theta\frac{\alpha}{\pi}\nabla\cdot\vec{B}=0$$

$$\nabla\times\left(\frac{1}{\mu}\vec{B}-\frac{\theta\alpha}{\pi}\vec{E}\right)=\frac{1}{c}\frac{\partial}{\partial t}\left(\varepsilon\vec{E}+\frac{\theta\alpha}{\pi}\vec{B}\right)$$

$$\Rightarrow\quad \nabla\times\left(\frac{1}{\mu}\vec{B}\right)-\frac{\theta\alpha}{\pi}\nabla\times\vec{E}=\frac{1}{c}\frac{\partial}{\partial t}\left(\varepsilon\vec{E}\right)+\frac{\theta\alpha}{c\pi}\frac{\partial}{\partial t}\vec{B}$$

But,

$$\nabla\cdot\vec{B}=0$$

$$\nabla\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

Therefore

$$\nabla\cdot\left(\varepsilon\vec{E}\right)=0$$

$$\nabla\times\left(\frac{1}{\mu}\vec{B}\right)=\frac{1}{c}\frac{\partial}{\partial t}\left(\varepsilon\vec{E}\right)$$

Boundary Condition

$$\nabla\cdot\left(\epsilon\vec{E}+\frac{\theta\alpha}{\pi}\vec{B}\right)=0$$

$$\nabla\cdot\vec{B}=0$$

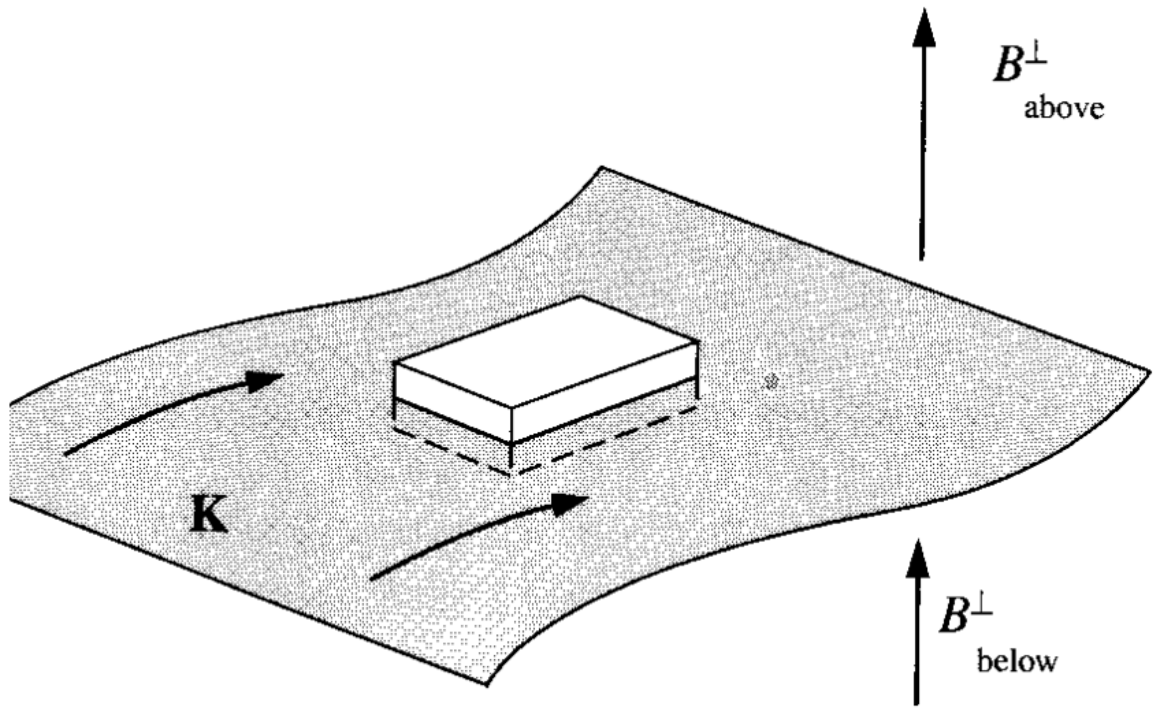
$$\int_V dx^3$$

$$\oint_S\left(\epsilon\vec{E}+\frac{\theta\alpha}{\pi}\vec{B}\right)\cdot d\vec{A}=0$$

$$\epsilon_1 E_{1,\perp}+\frac{\theta_1\alpha}{\pi}B_{1,\perp}=\epsilon_2 E_{2,\perp}+\frac{\theta_2\alpha}{\pi}B_{2,\perp}$$

$$\oint_S\vec{B}\cdot d\vec{A}=0$$

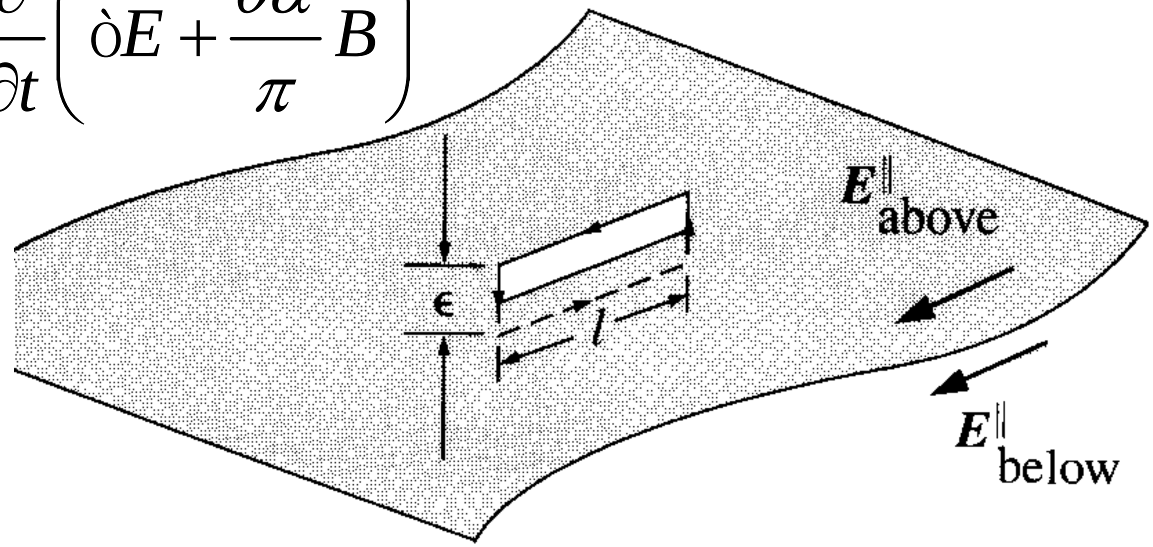
$$B_{1,\perp}=B_{2,\perp}$$



$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \left(\frac{1}{\mu}\vec{B} - \frac{\theta\alpha}{\pi}\vec{E} \right) = \frac{1}{c}\frac{\partial}{\partial t}\left(\epsilon\vec{E} + \frac{\theta\alpha}{\pi}\vec{B} \right)$$

$$\int_S d\vec{A}$$



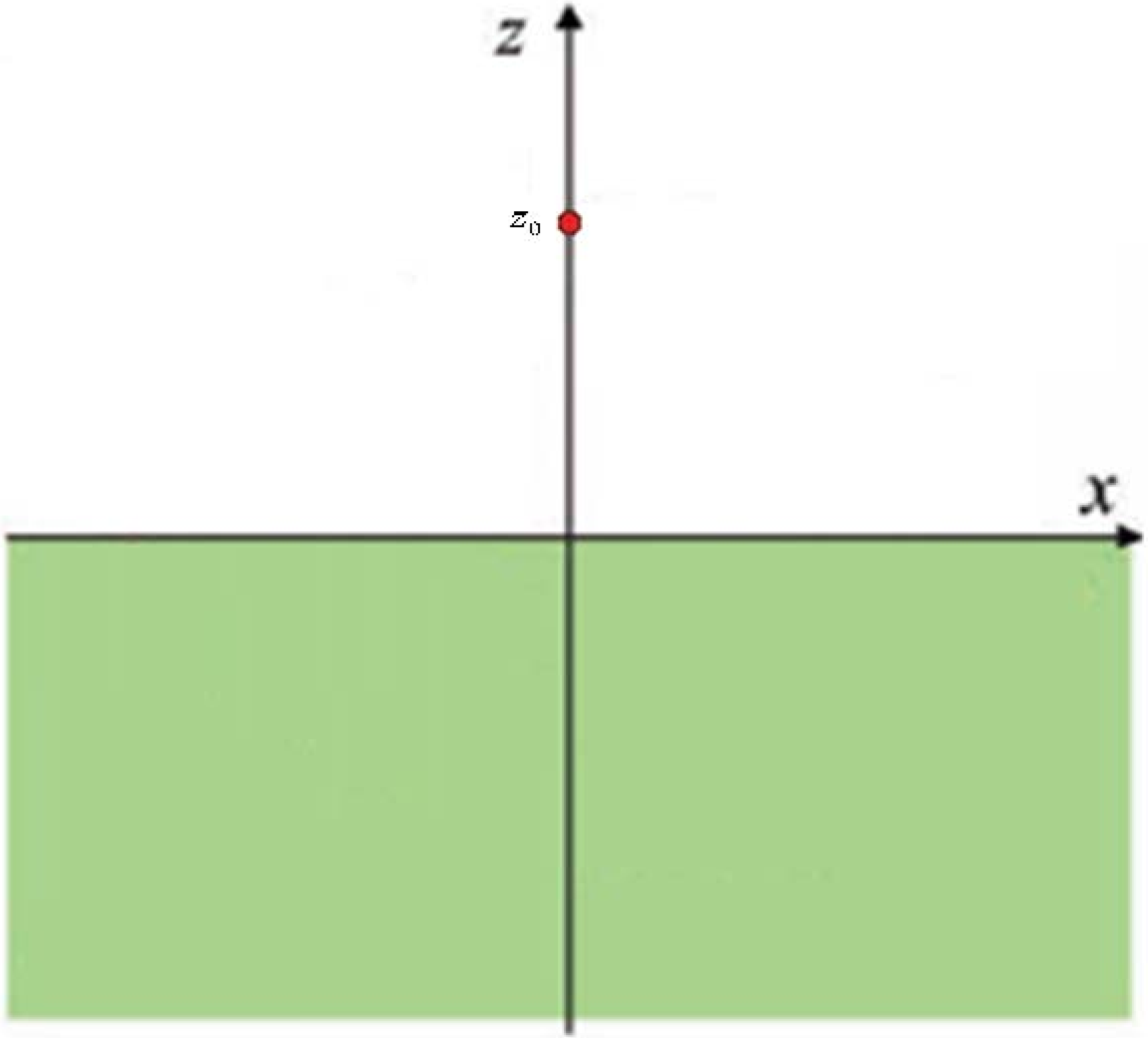
$$\oint_P \vec{E} \cdot d\vec{l} = 0$$

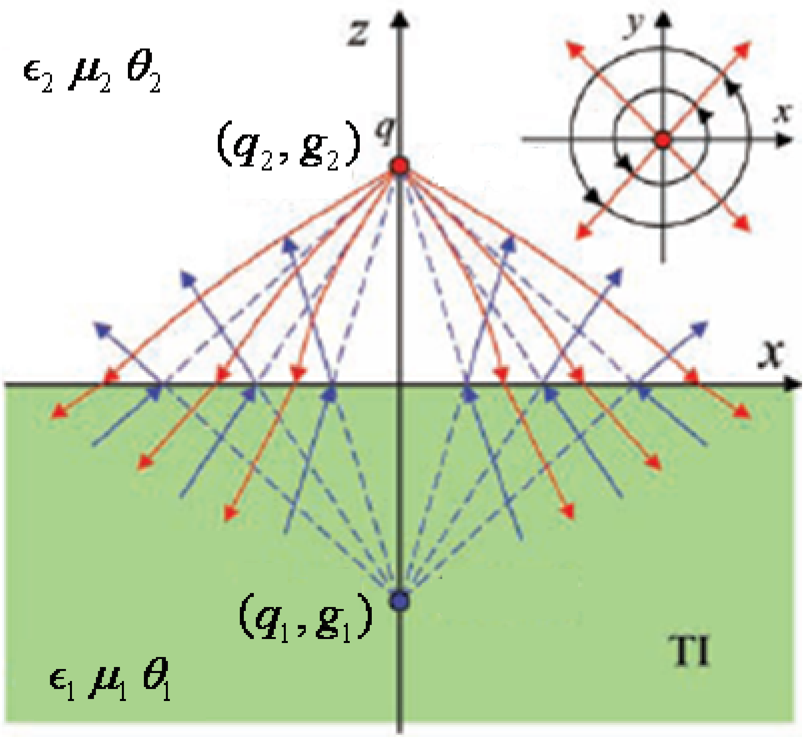
$$\vec{E}_{1,\parallel} = \vec{E}_{2,\parallel}$$

$$\oint_P \left(\frac{1}{\mu}\vec{B} - \frac{\theta\alpha}{\pi}\vec{E} \right) \cdot d\vec{l} = 0$$

$$\frac{1}{\mu_1}\vec{B}_{1,\parallel} - \frac{\theta_1\alpha}{\pi}\vec{E}_{1,\parallel} = \frac{1}{\mu_2}\vec{B}_{2,\parallel} - \frac{\theta_2\alpha}{\pi}\vec{E}_{2,\parallel}$$

Electric charge





아래(z<0)에서 볼 때

$(0, 0, z_0)$ 에

electric charge

$q/\epsilon_2, q_1$

Magnetic monopole g_2

위(z>0)에서 볼 때

$(0, 0, z_0)$ 에 Electric charge q/ϵ_2

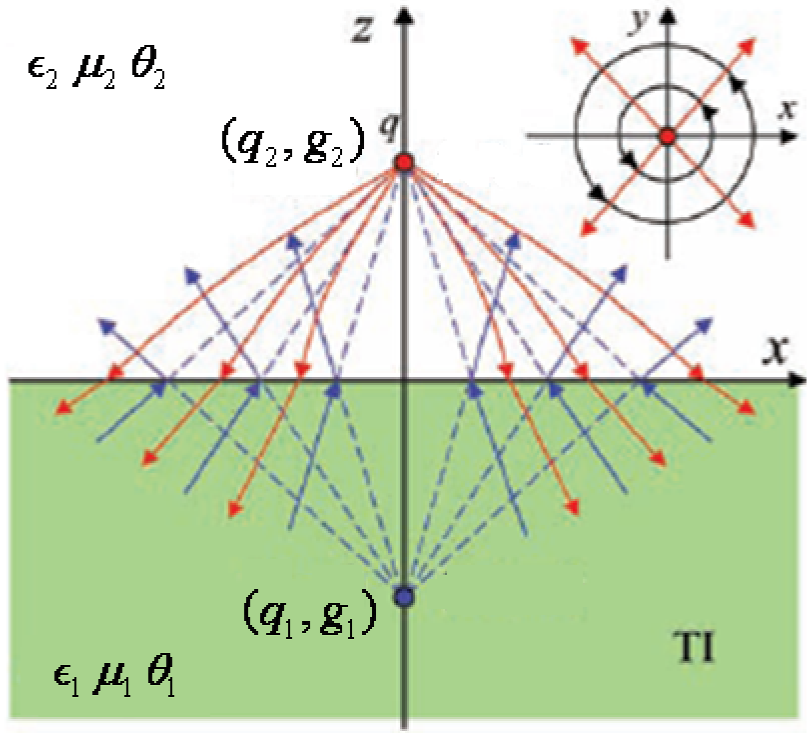
$(0, 0, -z_0)$ 에 Electric charge q_1

Magnetic monopole g_1

아래(z<0)

$$\vec{E} = \frac{q_2 + q/\epsilon_2}{\left(x^2 + y^2 + (z - z_0)^2\right)^{3/2}}(x, y, z - z_0)$$

$$\vec{B} = \frac{g_2}{\left(x^2 + y^2 + (z - z_0)^2\right)^{3/2}}(x, y, z - z_0)$$



위(z>0)

$$\vec{E} = \frac{q/\epsilon_2}{\left(x^2 + y^2 + (z - z_0)^2\right)^{3/2}}(x, y, z - z_0) + \frac{q_1}{\left(x^2 + y^2 + (z + z_0)^2\right)^{3/2}}(x, y, z + z_0)$$

$$\vec{B} = \frac{g_1}{\left(x^2 + y^2 + (z + z_0)^2\right)^{3/2}}(x, y, z + z_0)$$

$$\vec{B}_{1,\perp} = \vec{B}_{1,\perp} \qquad \vec{E}_{1,\parallel} = \vec{E}_{2,\parallel}$$

$$\Rightarrow \quad g_1 = -g_2 \qquad \Rightarrow \quad q_1 = q_2$$

$$\epsilon_1 \vec{E}_{1,\perp} + \frac{\theta_1 \alpha}{\pi} \vec{B}_{1,\perp} = \epsilon_2 \vec{E}_{2,\perp} + \frac{\theta_2 \alpha}{\pi} \vec{B}_{2,\perp}$$

$$\Rightarrow \quad (\epsilon_1 + \epsilon_2) q_1 = q\left(1 - \frac{\epsilon_1}{\epsilon_2}\right) + \frac{\alpha}{\pi}(\theta_2 - \theta_1) g_1$$

$$\frac{1}{\mu_1} \vec{B}_{1,\parallel} - \frac{\theta_1 \alpha}{\pi} \vec{E}_{1,\parallel} = \frac{1}{\mu} \vec{B}_{2,\parallel} - \frac{\theta_2 \alpha}{\pi} \vec{E}_{2,\parallel}$$

$$\Rightarrow \quad g_1\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right) = \frac{\alpha}{\pi}\left(q_1 + \frac{q}{\epsilon_2}\right)(\theta_1 - \theta_2)$$

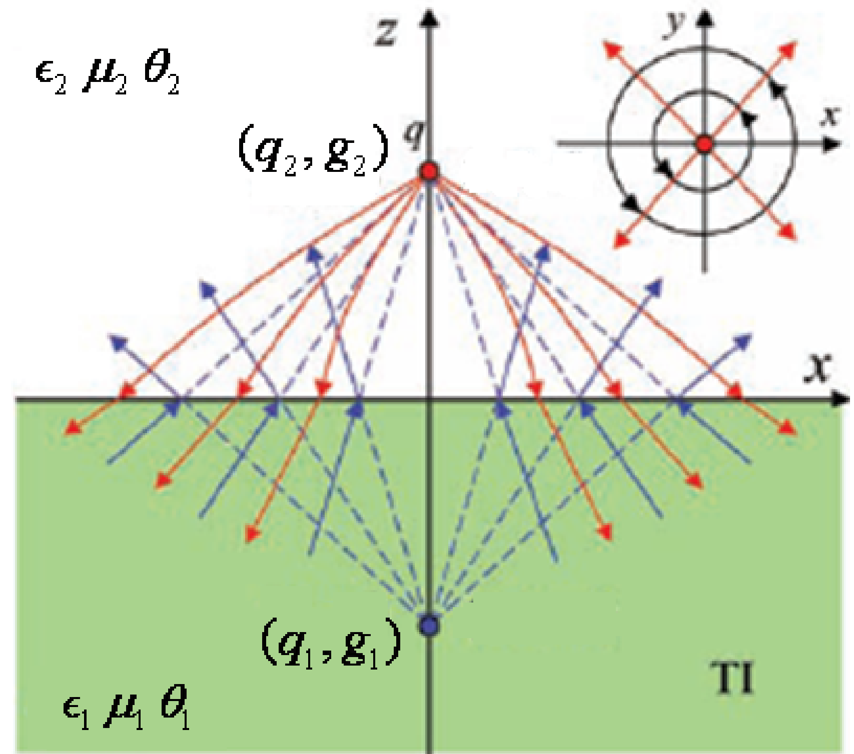
$$q_1=\frac{1}{\epsilon_2}\frac{\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)(\epsilon_2-\epsilon_1)-\left(\frac{\alpha}{\pi}\right)^2(\theta_1-\theta_2)^2}{\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)(\epsilon_1+\epsilon_2)+\left(\frac{\alpha}{\pi}\right)^2(\theta_1-\theta_2)^2}q \qquad g_1=\frac{\alpha}{\pi}\frac{2(\theta_1-\theta_2)}{\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)(\epsilon_1+\epsilon_2)+\left(\frac{\alpha}{\pi}\right)^2(\theta_1-\theta_2)^2}q$$

$$\overrightarrow{E_1}=\frac{q/\epsilon_2}{\left(x^2+y^2+(z-z_0)^2\right)^{3/2}}(x,y,z-z_0)+\frac{q_1}{\left(x^2+y^2+(z+z_0)^2\right)^{3/2}}(x,y,z+z_0)$$

$$\overrightarrow{B_1}=\frac{g_1}{\left(x^2+y^2+(z+z_0)^2\right)^{3/2}}(x,y,z+z_0)$$

$$\overrightarrow{B_2}=\frac{-g_1}{\left(x^2+y^2+(z-z_0)^2\right)^{3/2}}(x,y,z-z_0)$$

$$\overrightarrow{E_2}=\frac{q_1+q/\epsilon_2}{\left(x^2+y^2+(z-z_0)^2\right)^{3/2}}(x,y,z-z_0)$$

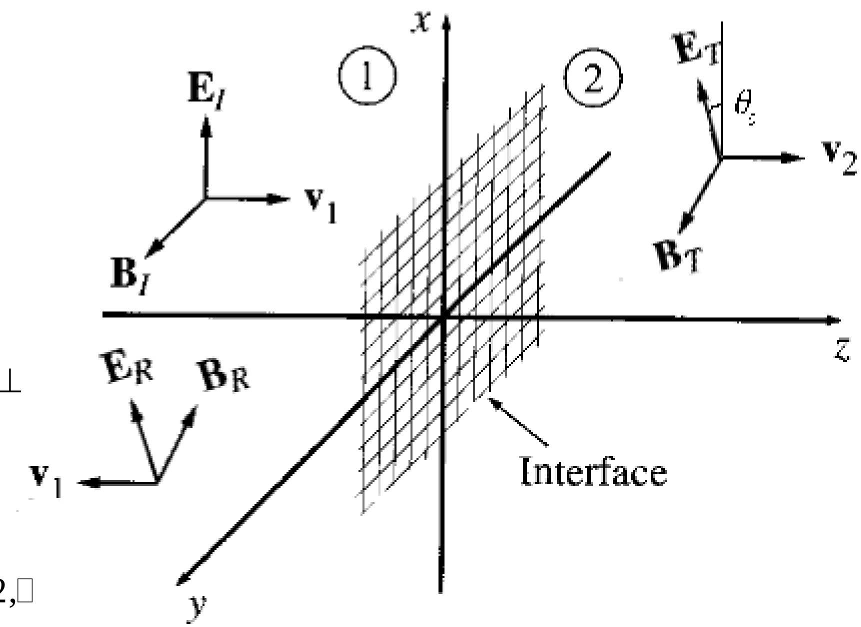


Electromagnetic Wave

$$\vec{B}_{1,\perp} = \vec{B}_{1,\perp} \qquad \vec{E}_{1,\parallel} = \vec{E}_{2,\parallel}$$

$$\varepsilon_1 \vec{E}_{1,\perp} + \frac{\theta_1 \alpha}{\pi} \vec{B}_{1,\perp} = {}_2 \vec{E}_{2,\perp} + \frac{\theta_2 \alpha}{\pi} \vec{B}_{2,\perp}$$

$$\frac{1}{\mu_1} \vec{B}_{1,\parallel} - \frac{\theta_1 \alpha}{\pi} \vec{E}_{1,\parallel} = \frac{1}{\mu} \vec{B}_{2,\parallel} - \frac{\theta_2 \alpha}{\pi} \vec{E}_{2,\parallel}$$



$$\tan \theta_c = \frac{\frac{(\theta_1 - \theta_2)\alpha}{\pi}}{\sqrt{\varepsilon_1 / \mu_1} + \sqrt{\varepsilon_2 / \mu_2}}$$

Conclusion

- A topological insulator is a band insulator which is characterized by a topological number and which has gapless excitations at its boundaries.
- A topological insulator has many interesting properties.

감사합니다