

(1+1) Dirac Hamiltonian

@ Klein Paradox

Group 4

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Introduction

DIRAC HAMILTONIAN

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$$H_D = -i\alpha_x \frac{\partial}{\partial x} + \beta m + V(x)$$

$$\alpha_x = \sigma_1 \quad \beta = \sigma_3$$

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0 (> 0), & 0 < x < a \\ 0, & x > a \end{cases}$$

CONFIRMATION

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$$\gamma^0 = \sigma_3$$

$$\gamma^1 = i\sigma_2$$

CONFIRMATION

$$\gamma^0 = \sigma_3 \qquad \gamma^1 = i\sigma_2$$

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$$

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$$\gamma^0 = \sigma_3 \quad \gamma^1 = i\sigma_2$$

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$$i\frac{\partial}{\partial t}\psi = H_D\psi$$

CONFIRMATION

$$\gamma^0 = \sigma_3 \qquad \gamma^1 = i\sigma_2$$

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$$

$$i\frac{\partial}{\partial t}\psi = H_D\psi \quad \longrightarrow \quad \partial_\mu(\bar{\psi}(x, t)\gamma^\mu\psi(x, t)) = 0$$

CONFIRMATION

$$\gamma^0 = \sigma_3 \qquad \gamma^1 = i\sigma_2$$

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$$i\frac{\partial}{\partial t}\psi = H_D\psi \quad \longrightarrow \quad \partial_\mu(\bar{\psi}(x, t)\gamma^\mu\psi(x, t)) = 0$$

$$i\frac{\partial}{\partial t}\psi = H\psi$$

CONFIRMATION

$$\gamma^0 = \sigma_3 \qquad \gamma^1 = i\sigma_2$$

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$$

$$i\frac{\partial}{\partial t}\psi = H_D\psi \quad \longrightarrow \quad \partial_\mu(\bar{\psi}(x, t)\gamma^\mu\psi(x, t)) = 0$$

$$i\frac{\partial}{\partial t}\psi = H\psi \quad \longrightarrow \quad \partial_t|\psi|^2 + \partial_x\left(\frac{1}{2m\hbar}(\psi^*(\partial_x\psi) - (\partial_x\psi^*)\psi)\right) = 0$$

CONFIRMATION

$$\partial_{\mu}(\bar{\psi}(x, t)\gamma^{\mu}\psi(x, t)) = 0$$

VS

$$\partial_t|\psi|^2 + \partial_x\left(\frac{1}{2m\imath}(\psi^*(\partial_x\psi) - (\partial_x\psi^*)\psi)\right) = 0$$

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$$\partial_{\mu}(\bar{\psi}(x, t)\gamma^{\mu}\psi(x, t)) = 0$$

VS

$$\partial_t|\psi|^2 + \partial_x\left(\frac{1}{2m\imath}(\psi^*(\partial_x\psi) - (\partial_x\psi^*)\psi)\right) = 0$$

Which term is continuous??

ANNUAL ANALYSIS I

GROUP 4

Introduction

Analysis

SQUARE BARRIER POTENTIAL PROBLEM

SQUARE BARRIER POTENTIAL PROBLEM

$$H_D = -i\sigma_1 \frac{\partial}{\partial x} + \sigma_3 m + V(x)$$

$$H\psi = E\psi \quad \psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

SQUARE BARRIER POTENTIAL PROBLEM

$$H_D = -i\sigma_1 \frac{\partial}{\partial x} + \sigma_3 m + V(x)$$

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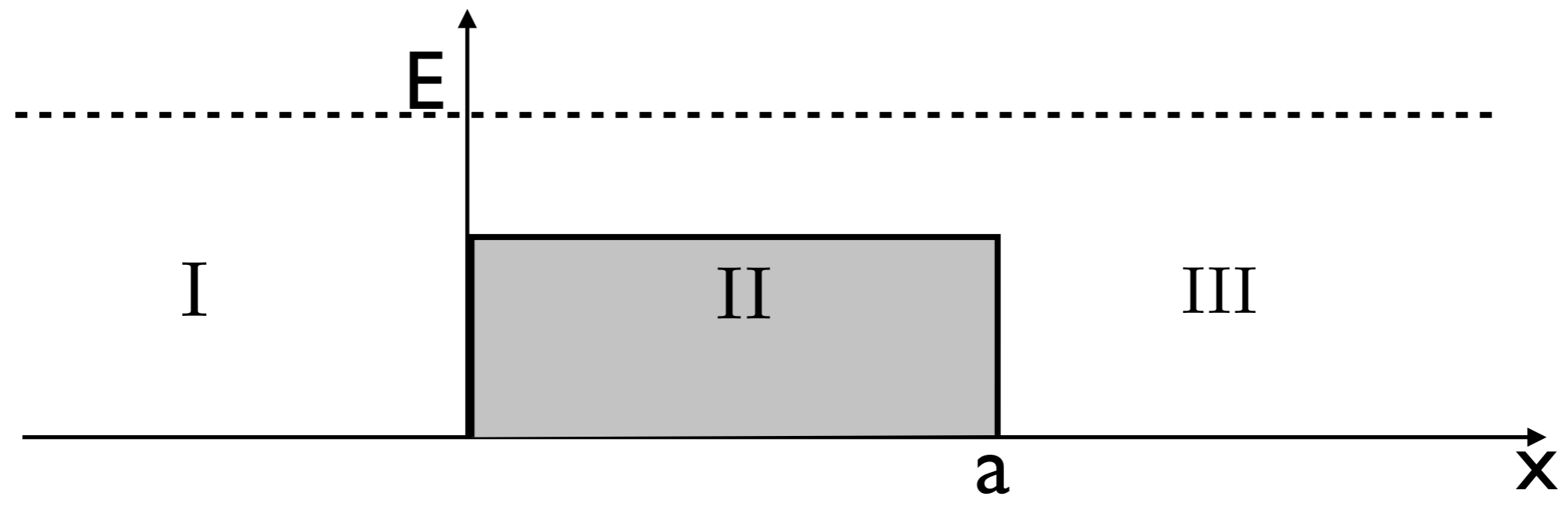
Plane wave solution(free)

$$\psi_p(x) = \begin{pmatrix} \sqrt{\frac{E+m}{p}} \\ \sqrt{\frac{p}{E+m}} \end{pmatrix} e^{ipx} \quad \psi_{-p}(x) = \begin{pmatrix} \sqrt{\frac{E+m}{p}} \\ -\sqrt{\frac{p}{E+m}} \end{pmatrix} e^{-ipx}$$

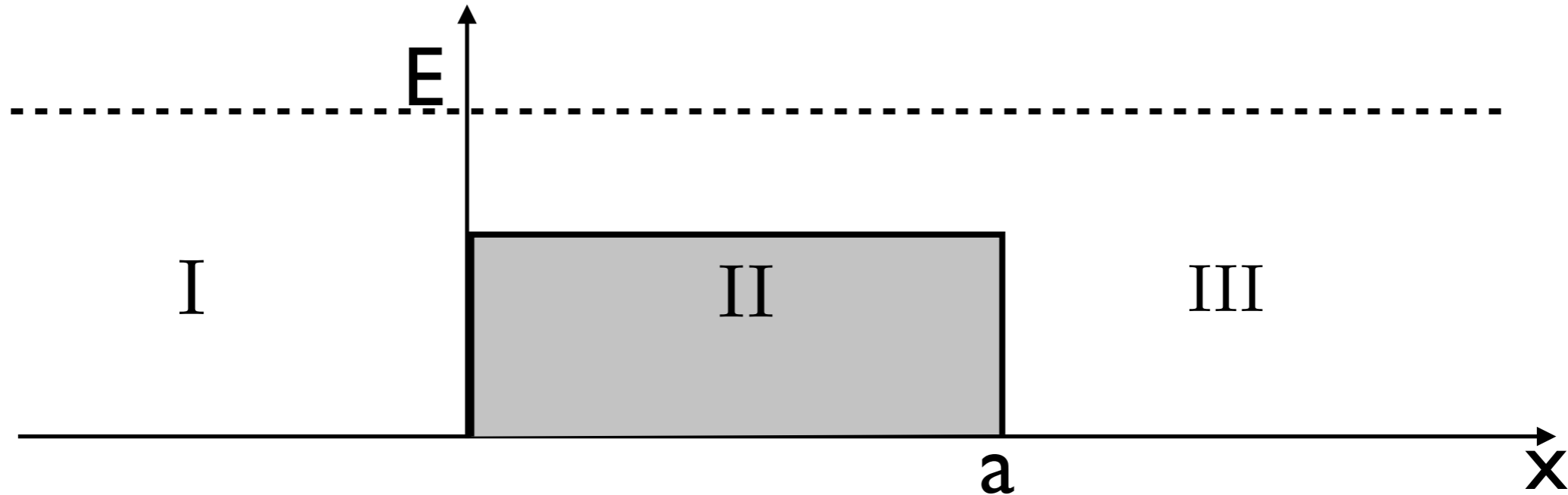
$$(p = \sqrt{E^2 - m^2})$$

WAVE FUNCTION & BOUNDARY CONDITION

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WAVE FUNCTION & BOUNDARY CONDITION

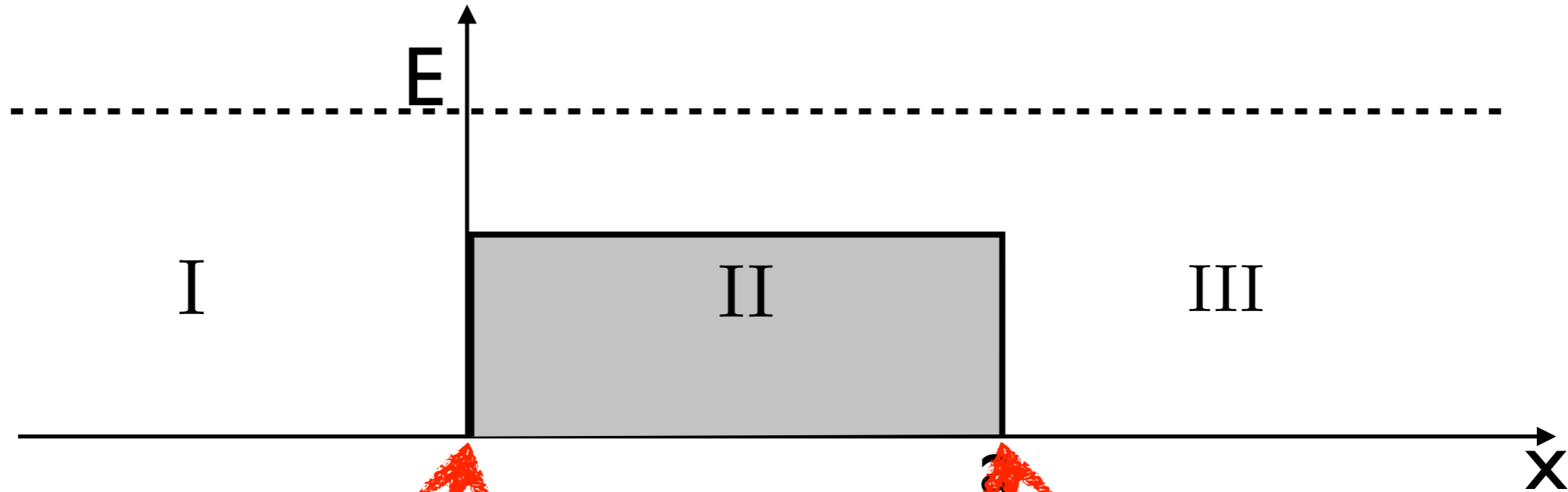


$$\text{I} \quad : \quad \psi_p(x) + r\psi_{-p}(x) \quad (p = \sqrt{E^2 - m^2})$$

$$\text{II} \quad : \quad A\psi_q(x) + B\psi_{-q}(x) \quad (q = \sqrt{(E - V_0)^2 - m^2})$$

$$\text{III} \quad : \quad t\psi_p(x)$$

WAVE FUNCTION & BOUNDARY CONDITION



I : $\psi_p(x) + r\psi_{-p}(x)$ ($p = \sqrt{E^2 - m^2}$)

II : $A\psi_q(x) + B\psi_{-q}(x)$ ($q = \sqrt{(E - V_0)^2 - m^2}$)

III : $t\psi_p(x)$

Continuity at $x=0, a$

TRANSMITTANCE & REFLECTANCE

TRANSMITTANCE & REFLECTANCE

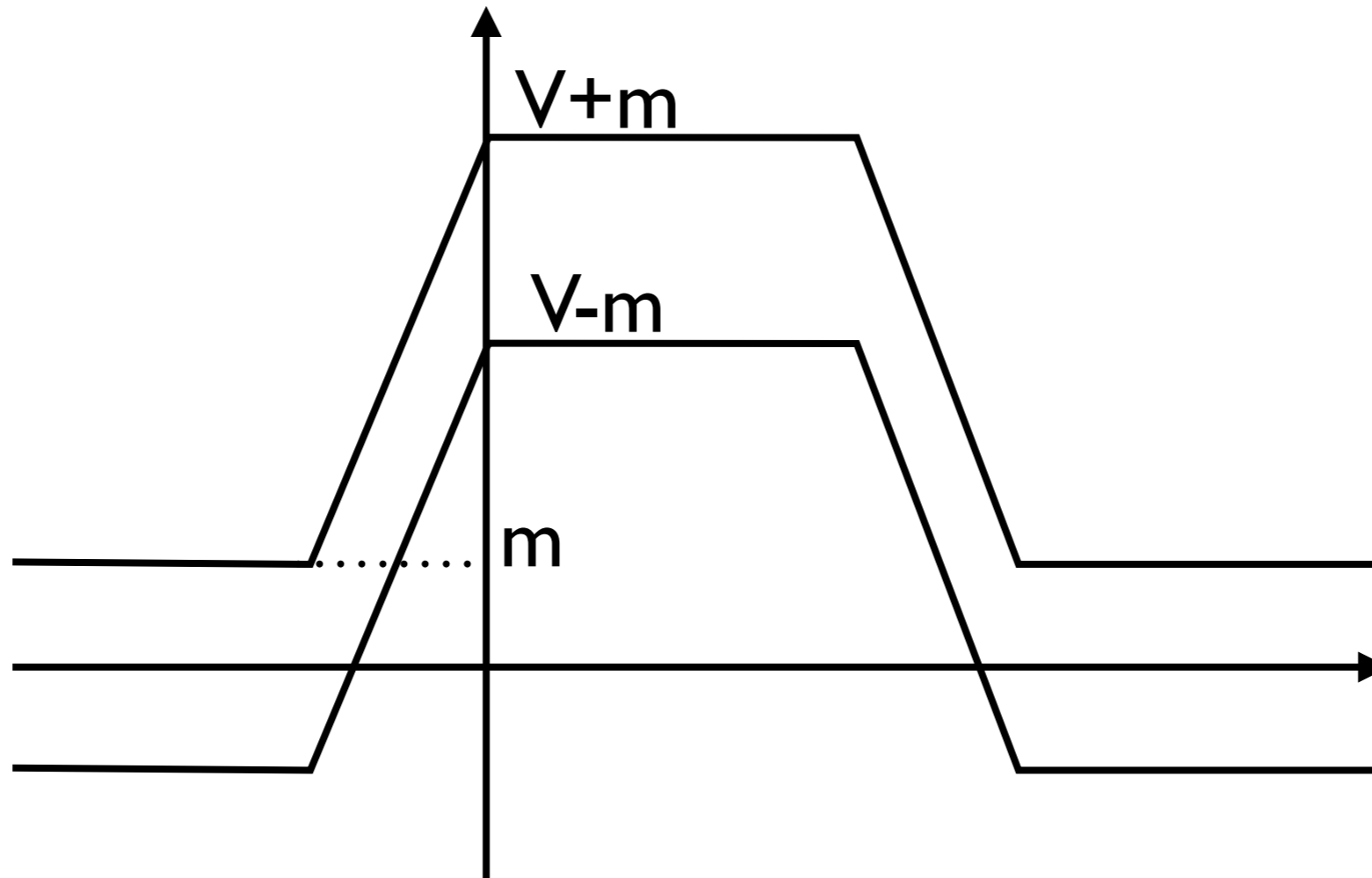
$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa}$$

$$R = |r|^2 = \frac{\frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa}$$

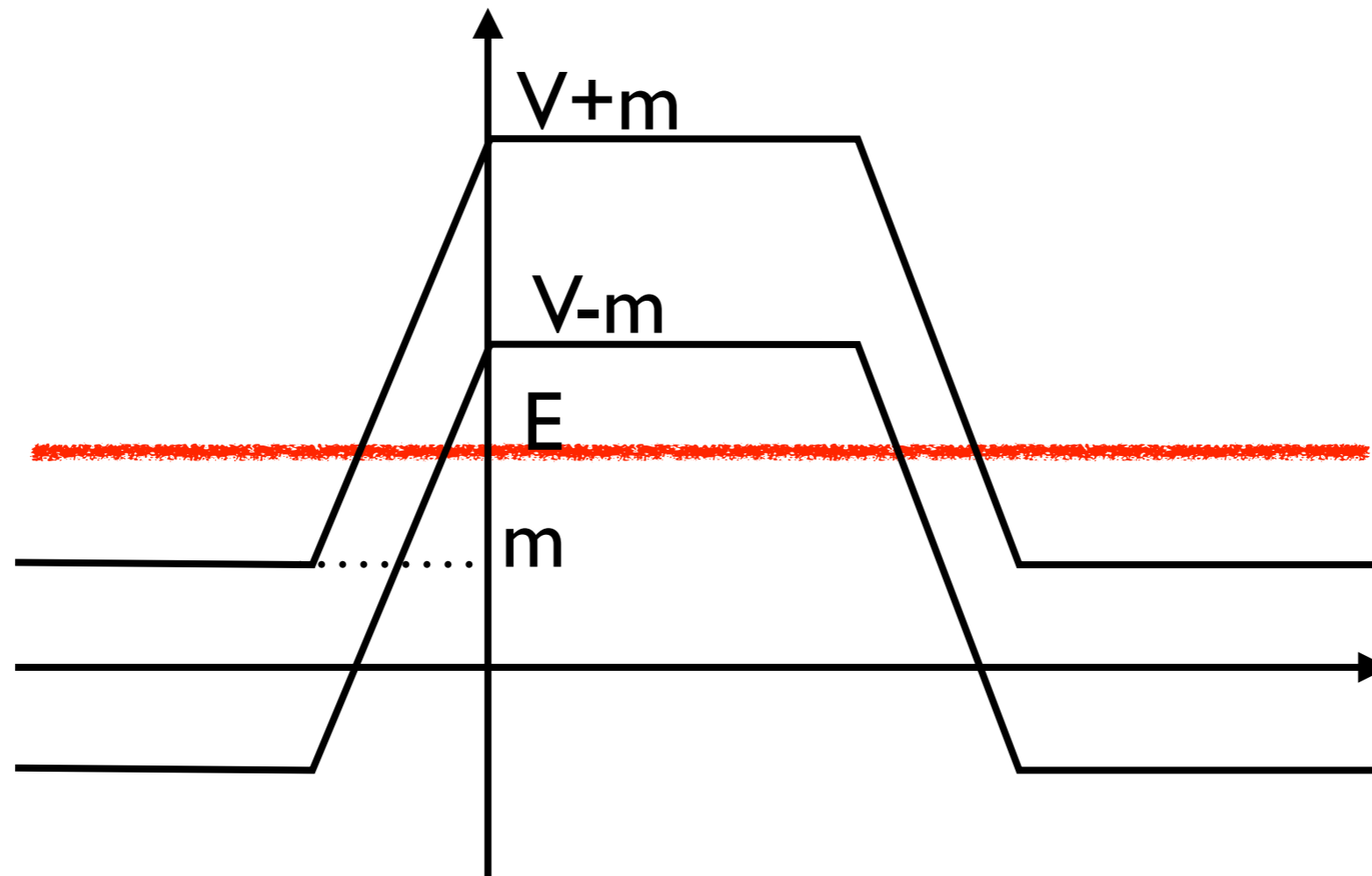
$$\left(\rho = \frac{q}{p} \frac{E + m}{E - V_0 + m}\right)$$

ENERGY RANGE OF INTEREST

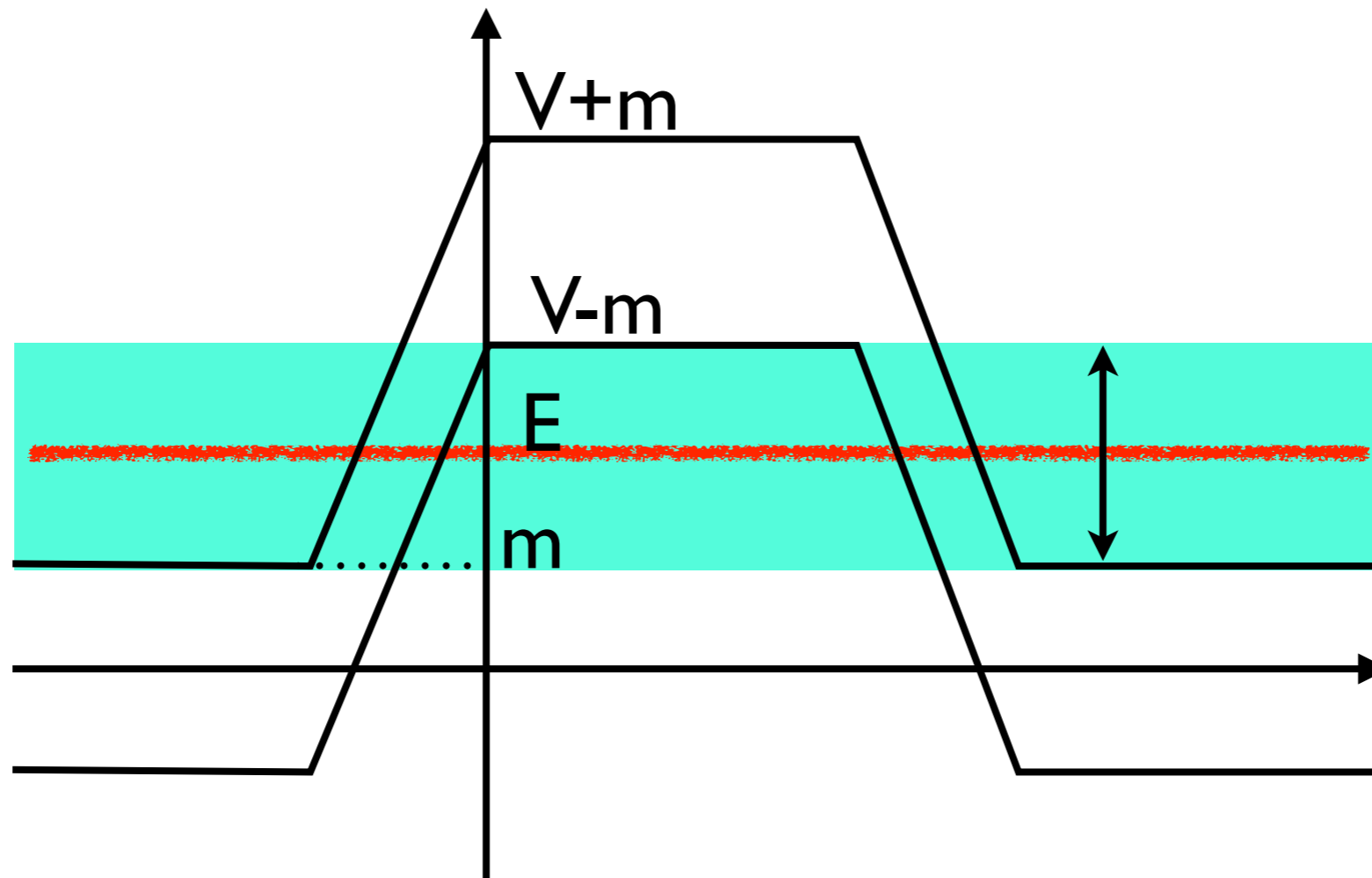
ENERGY RANGE OF INTEREST



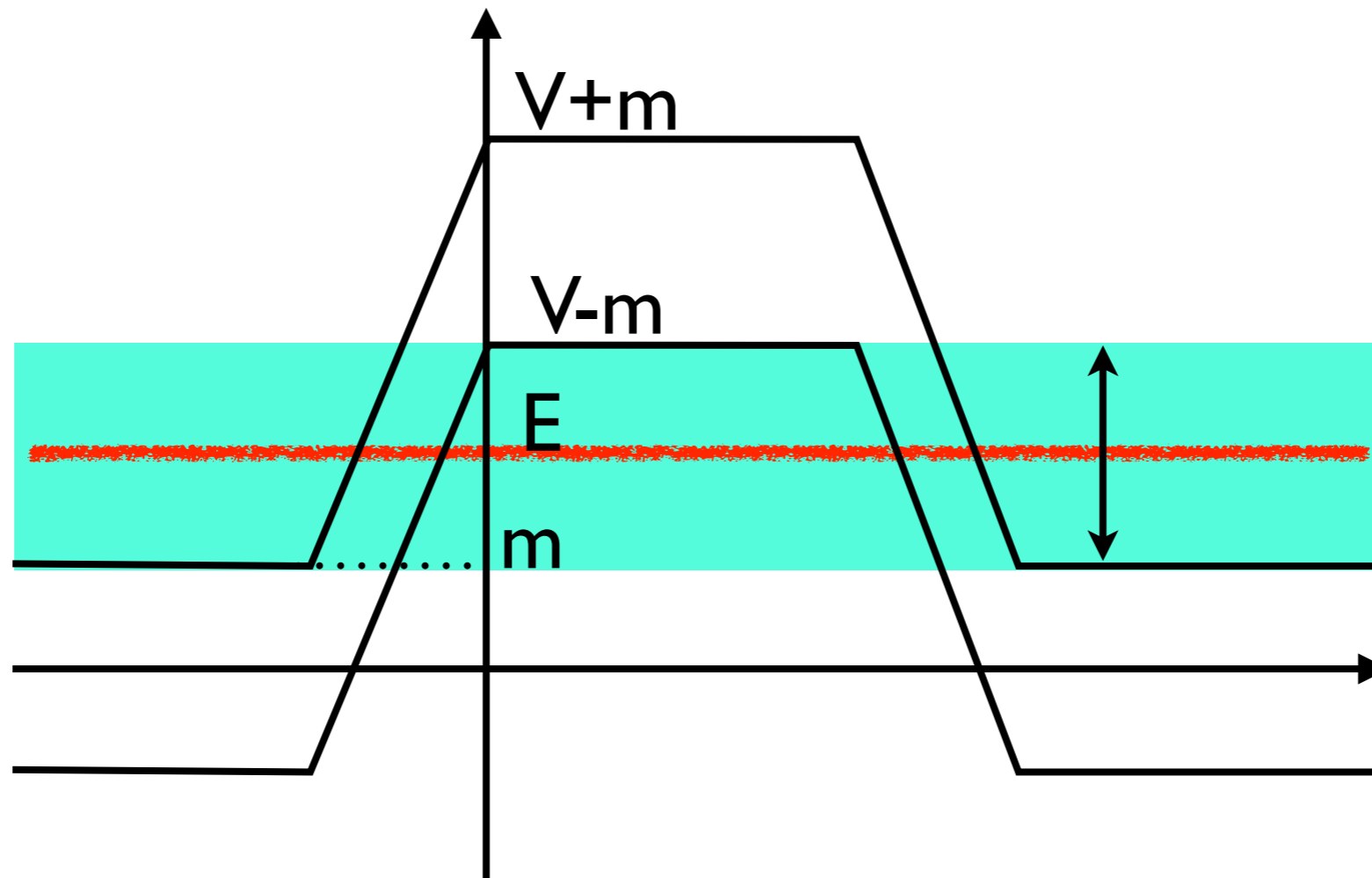
ENERGY RANGE OF INTEREST



ENERGY RANGE OF INTEREST



ENERGY RANGE OF INTEREST



$$q = \sqrt{(E - V_0)^2 - m^2} \quad : \text{Real} \quad m < E < V - m$$

NON-RELATIVISTIC LIMIT

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$$V_0 \ll m$$

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$$V_0 \ll m \quad E = m + K \quad K \ll m$$

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NON-RELATIVISTIC LIMIT

$$V_0 \ll m \quad E = m + K \quad K \ll m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa}$$
$$\approx \frac{1}{1 + \frac{V_0^2}{4K(K-V_0)} \sin^2(a\sqrt{2m(K-V_0)})}$$

NON-RELATIVISTIC LIMIT

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$$R = 1 - T$$

NON-RELATIVISTIC LIMIT

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$$\approx \frac{1}{1 + \frac{V_0^2}{4K(K-V_0)} \sin^2(a\sqrt{2m(K-V_0)})}$$

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Exactly Same!!

INFINITE POTENTIAL LIMIT

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$$m \ll V$$

$$m < E < V - m$$

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$$m \ll V$$

$$m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \approx \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

INFINITE POTENTIAL LIMIT

$$m \ll V$$

$$m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K$ $K \ll m$

INFINITE POTENTIAL LIMIT

$$m \ll V \qquad m < E < V - m$$

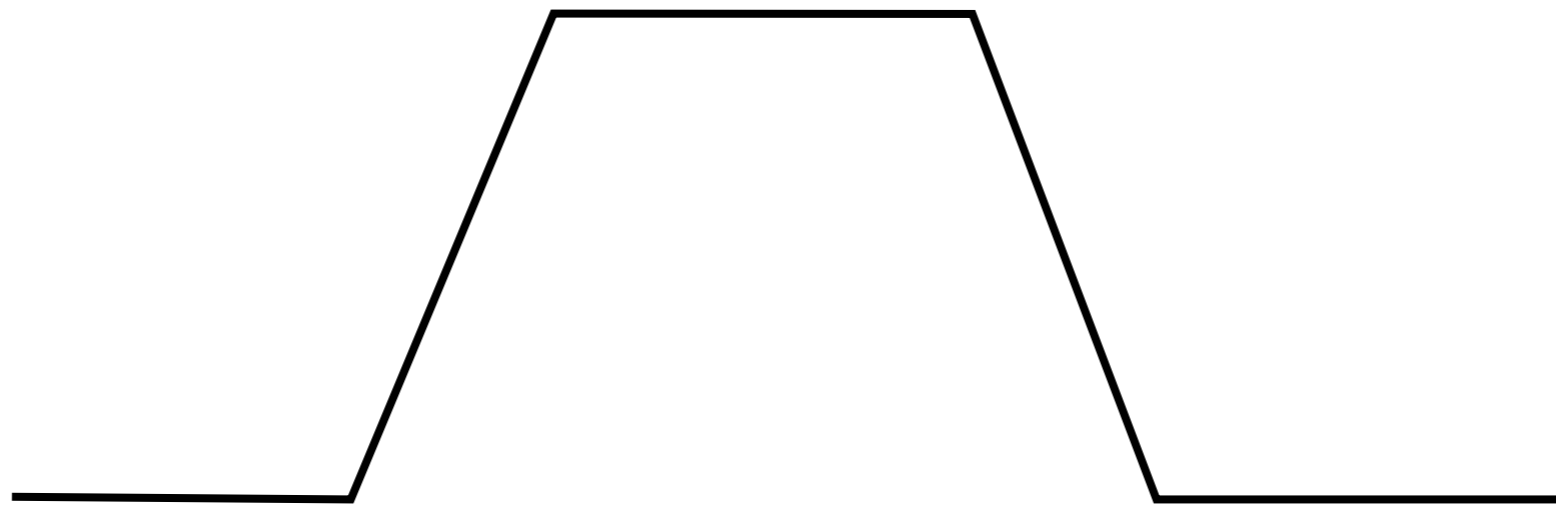
$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

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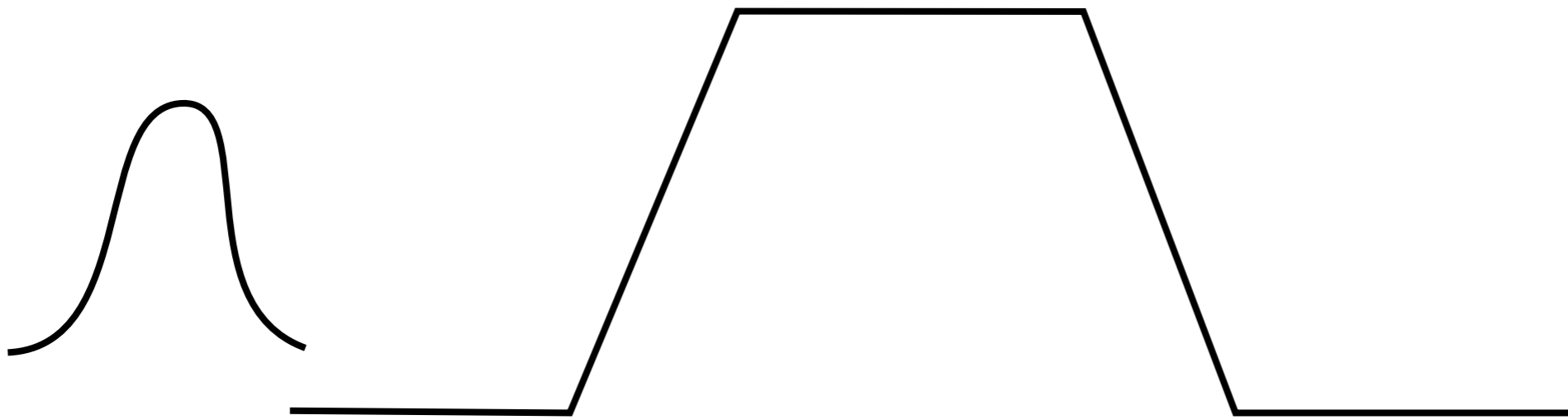
Non-relativistic limit $E = m + K \quad K \ll m$

$$T \simeq 0$$

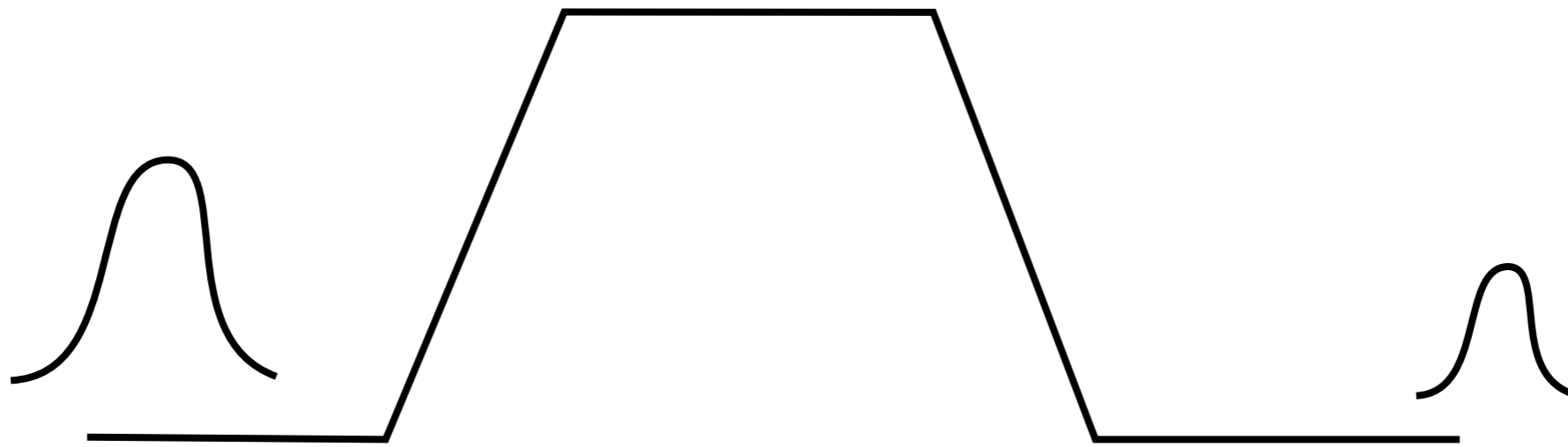
Analysis



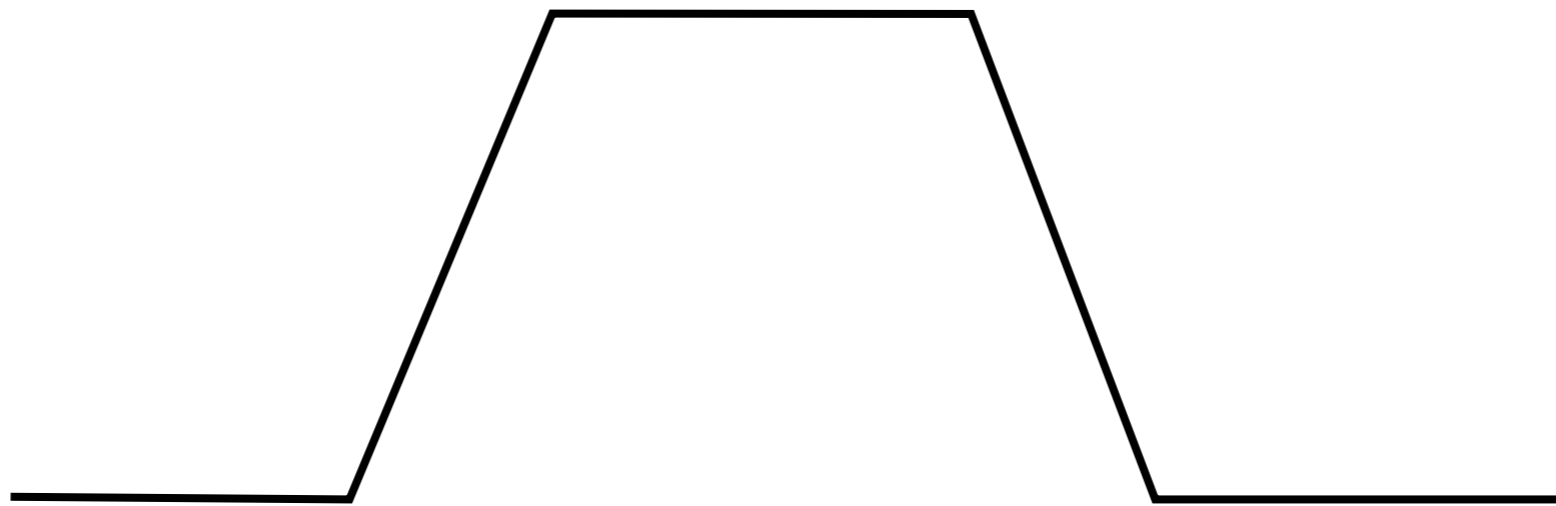
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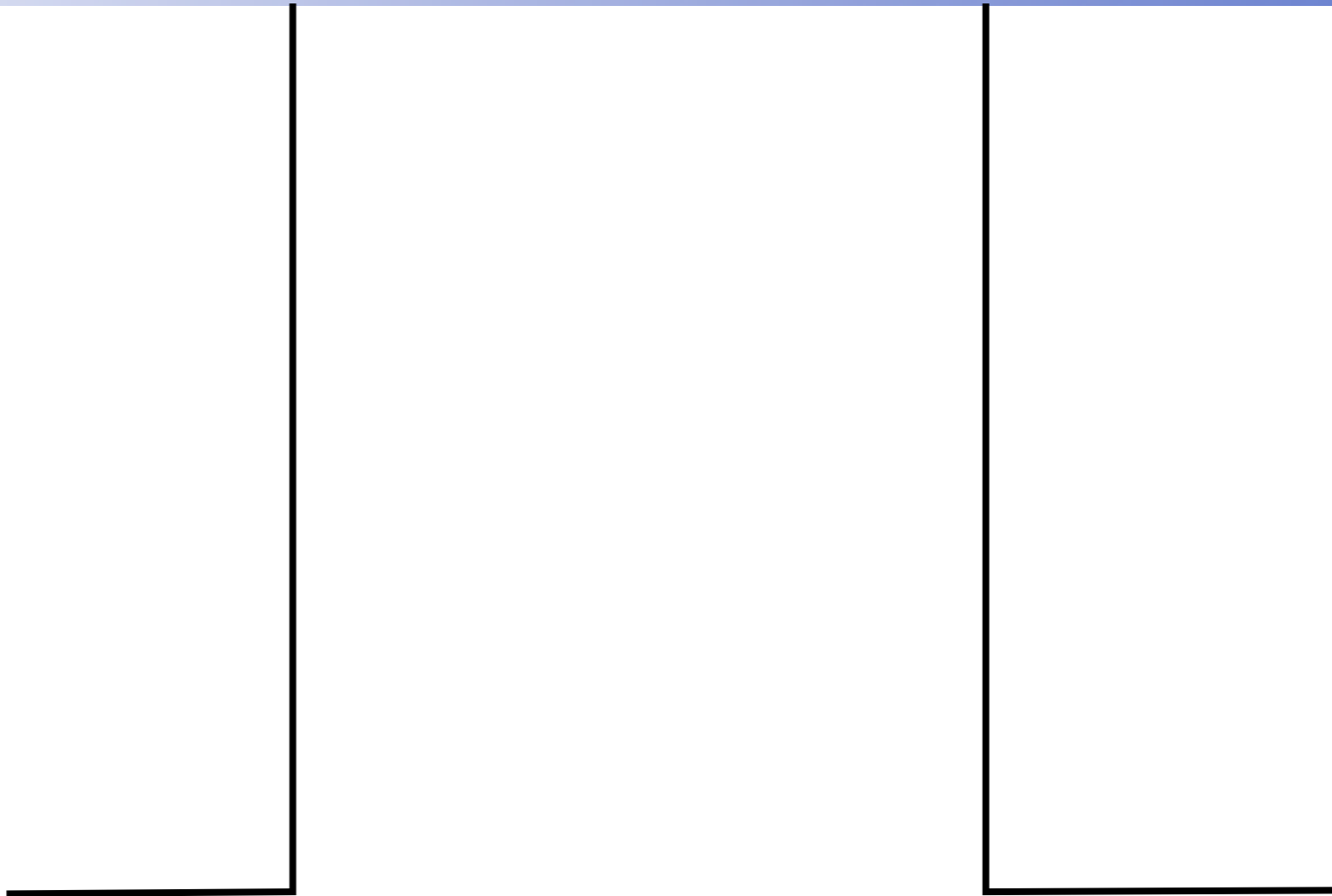
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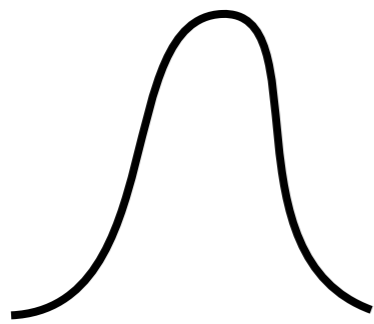
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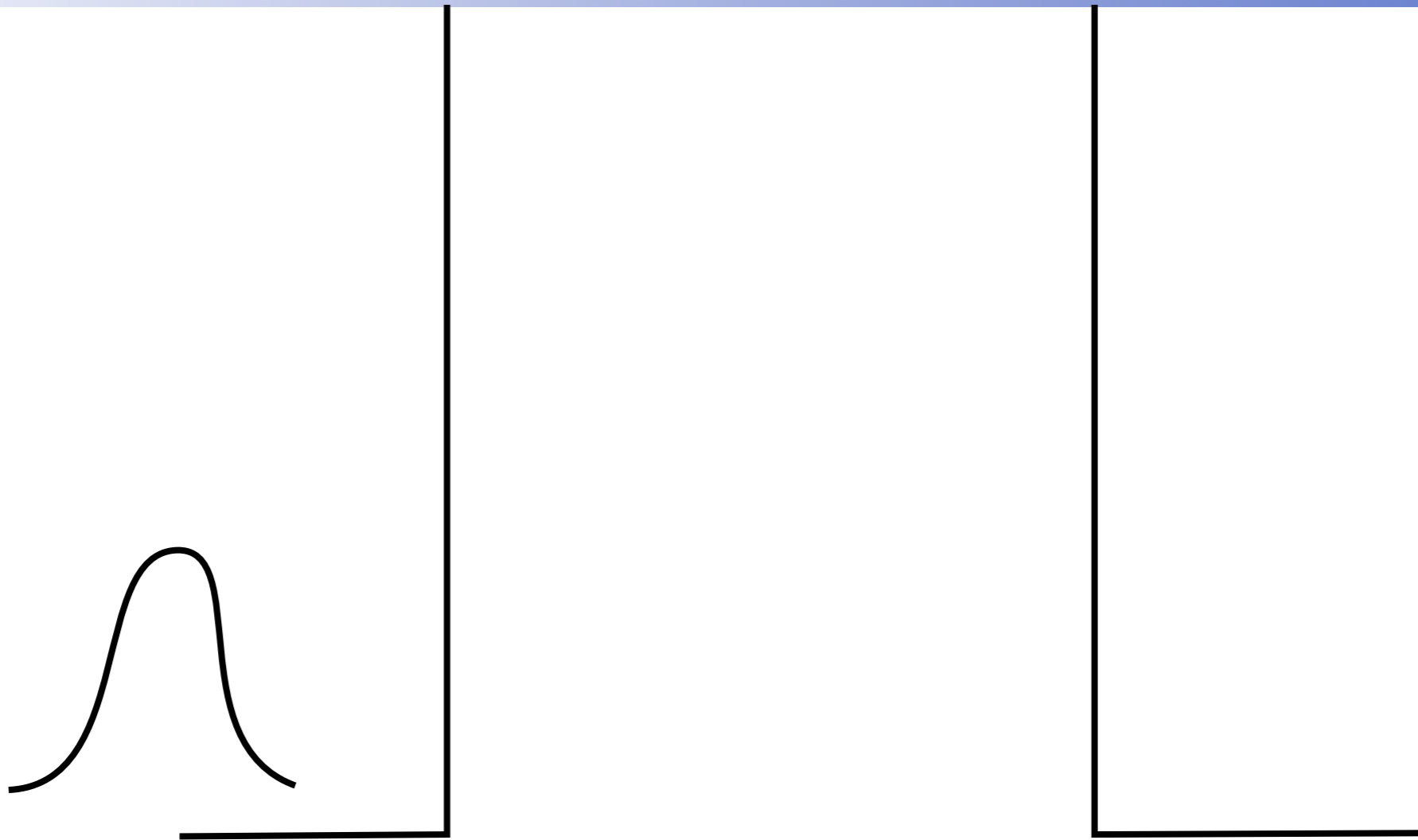
Analysis



Analysis



Analysis



INFINITE POTENTIAL LIMIT

$$m \ll V$$

$$m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K$ $K \ll m$

$$T \simeq 0$$

INFINITE POTENTIAL LIMIT

$$m \ll V$$

$$m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K$ $K \ll m$

$T \simeq 0$ **CONSISTENT!!**

INFINITE POTENTIAL LIMIT

$$m \ll V$$

$$m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K$ $K \ll m$

$T \simeq 0$ **CONSISTENT!!**

Ultra-relativistic limit $m \ll E$ $m \ll E \ll V$

INFINITE POTENTIAL LIMIT

$$m \ll V \qquad m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K \quad K \ll m$

$$T \simeq 0 \quad \text{CONSISTENT!!}$$

Ultra-relativistic limit $m \ll E \quad m \ll E \ll V$

$$T \simeq 1$$

INFINITE POTENTIAL LIMIT

$$m \ll V \qquad m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K \quad K \ll m$

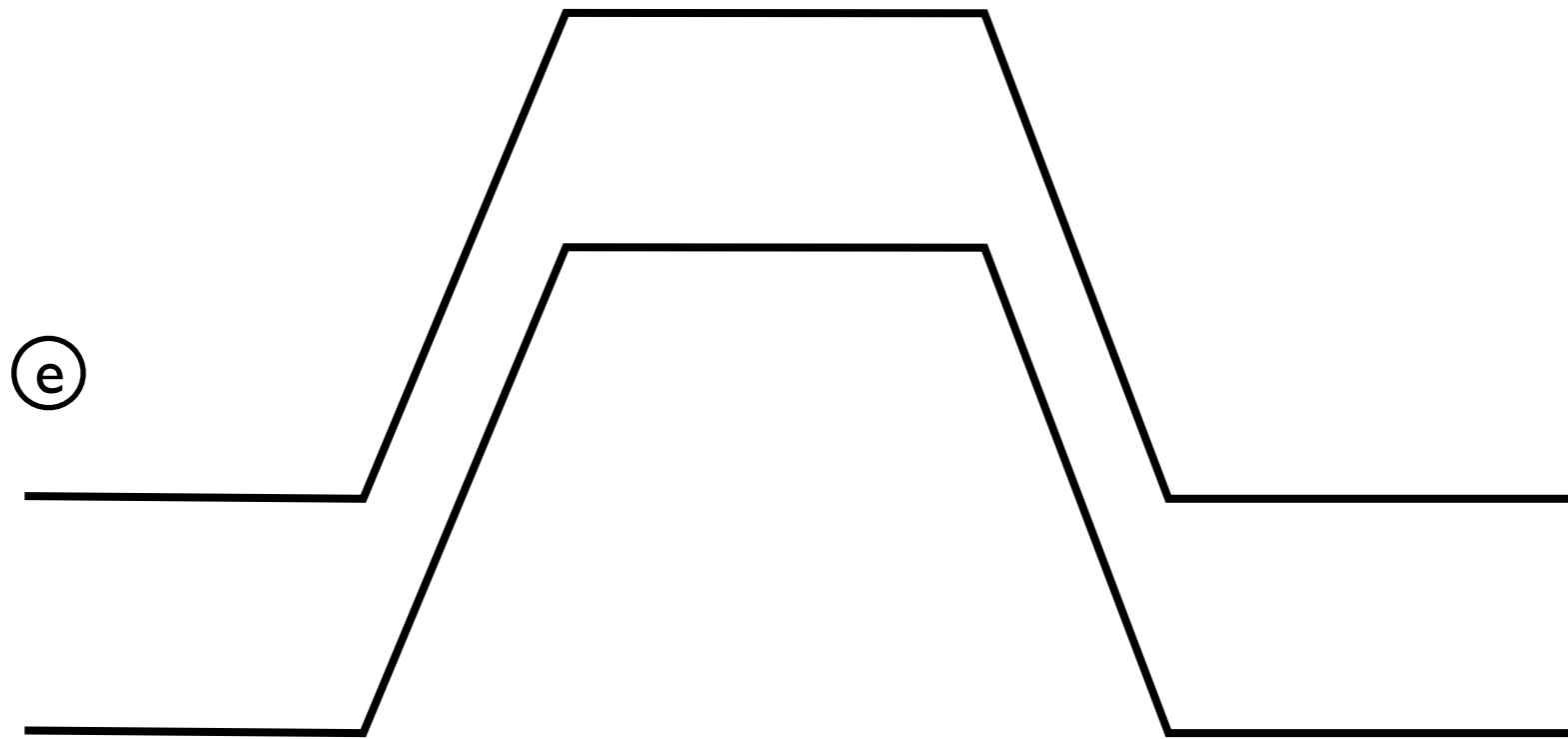
$$T \simeq 0 \quad \text{CONSISTENT!!}$$

Ultra-relativistic limit $m \ll E \quad m \ll E \ll V$

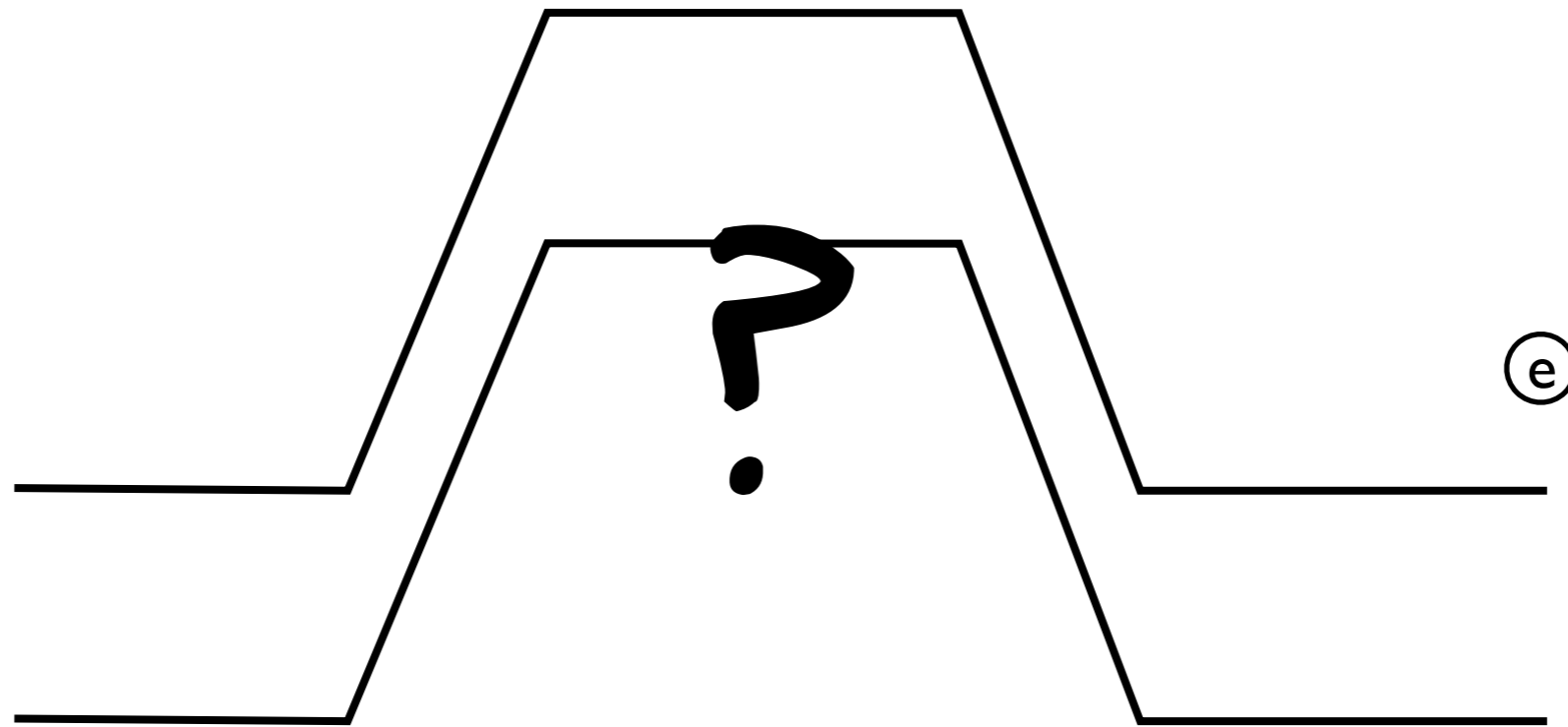
$$T \simeq 1$$

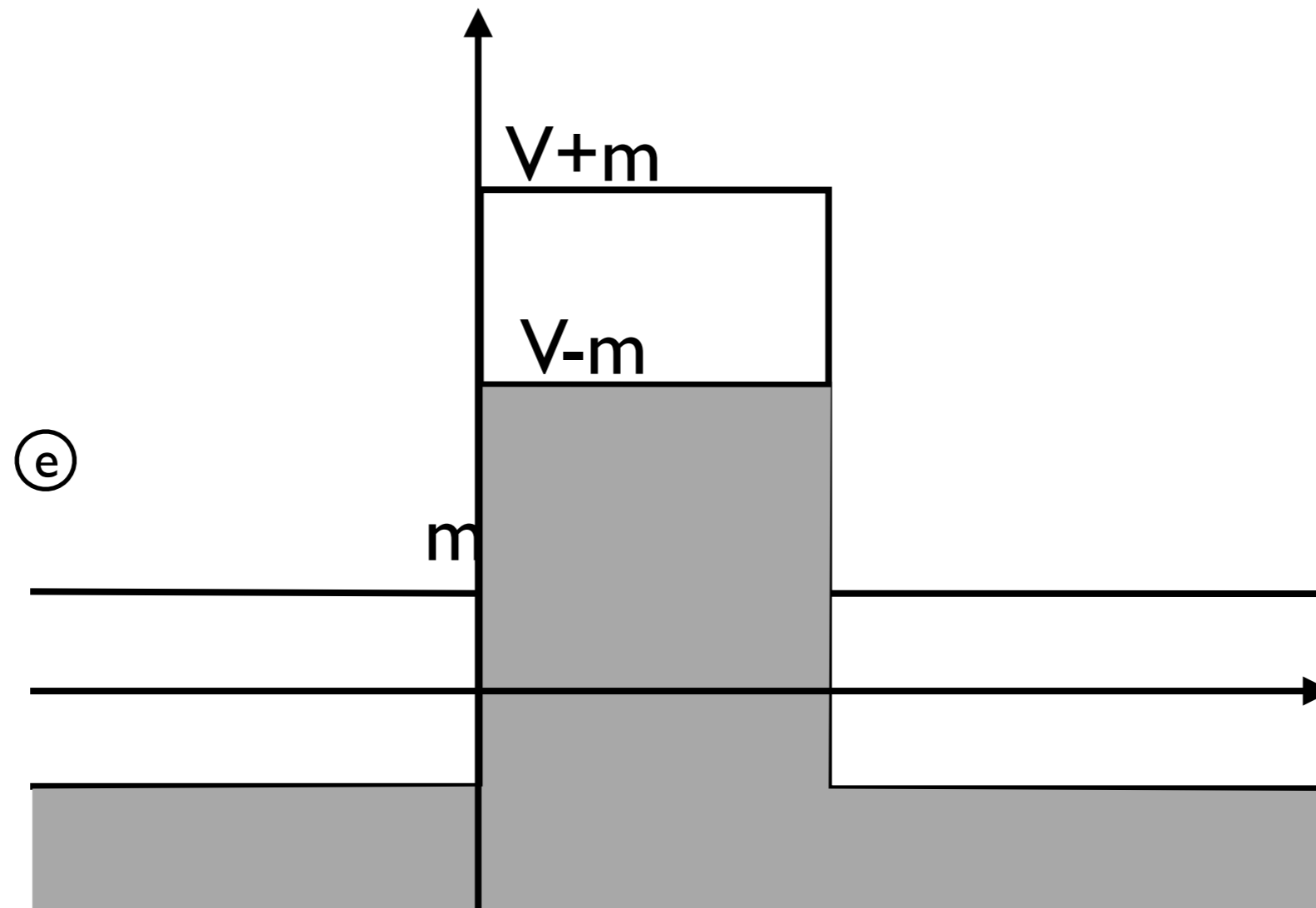
REASONABLE??

PROCESS

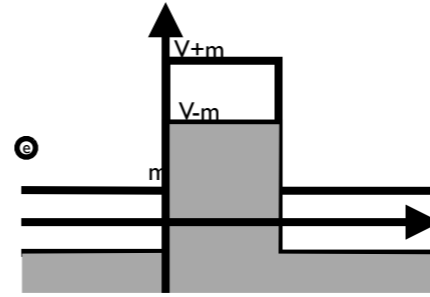


PROCESS

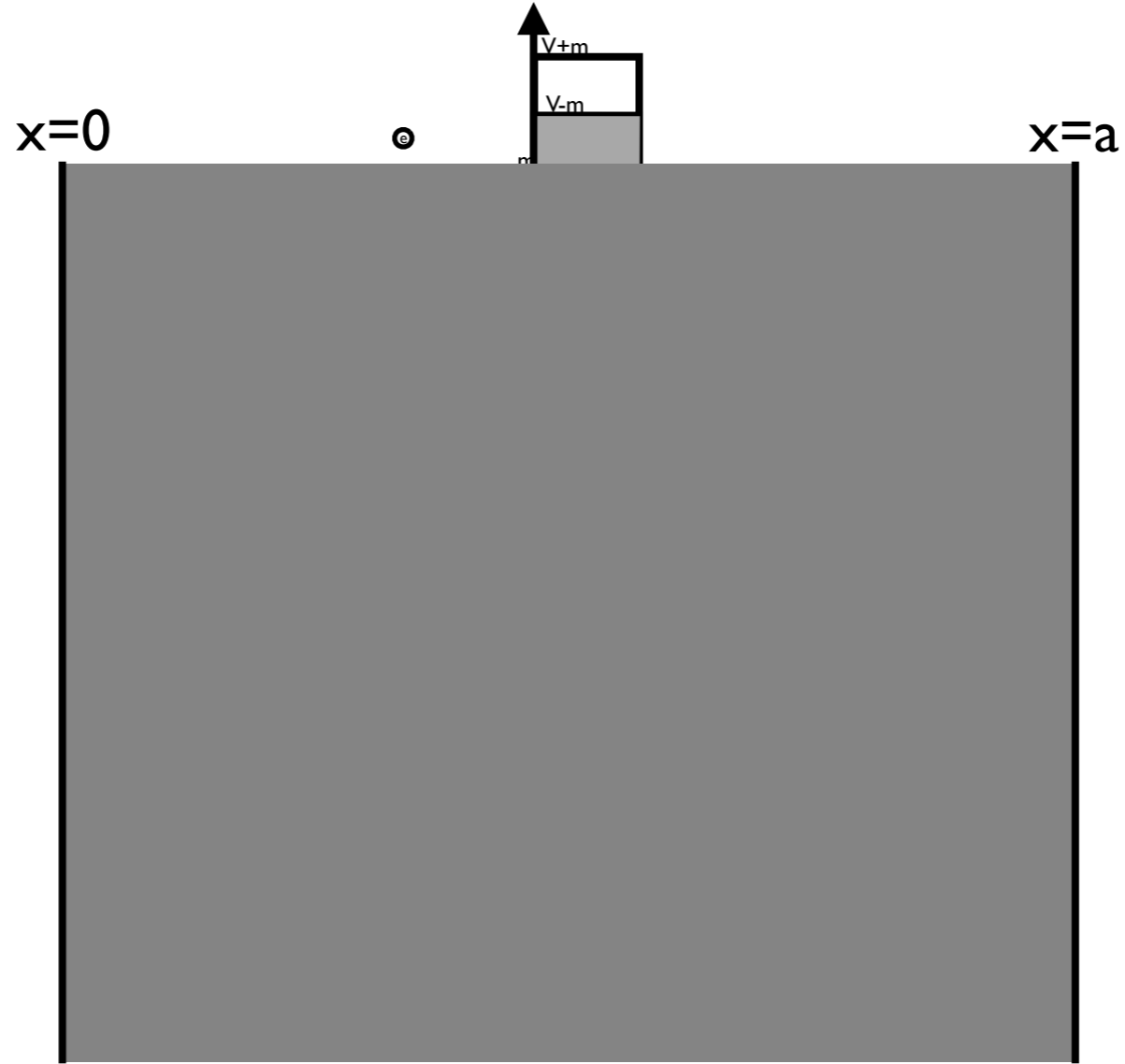




Analysis

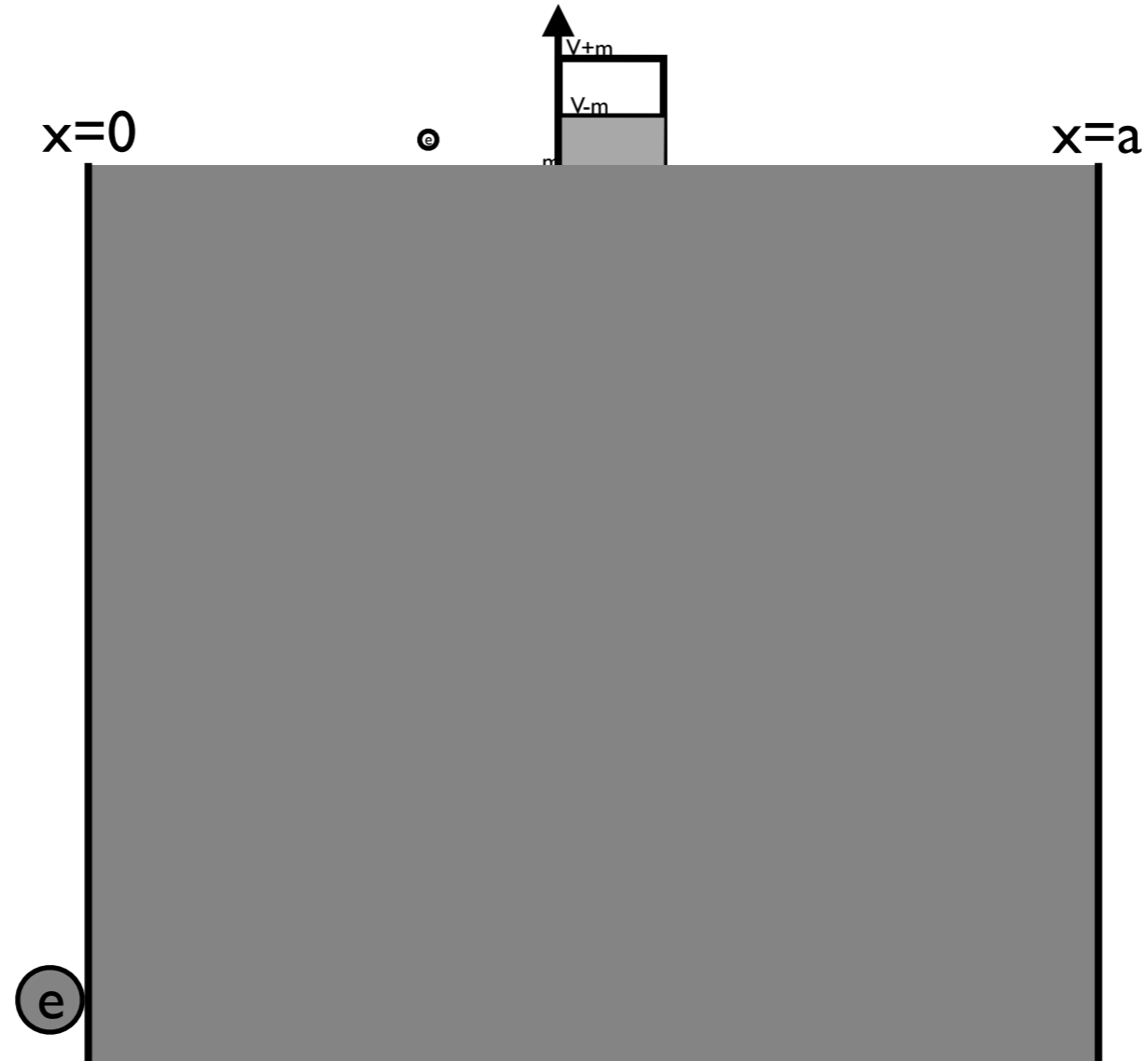


Analysis

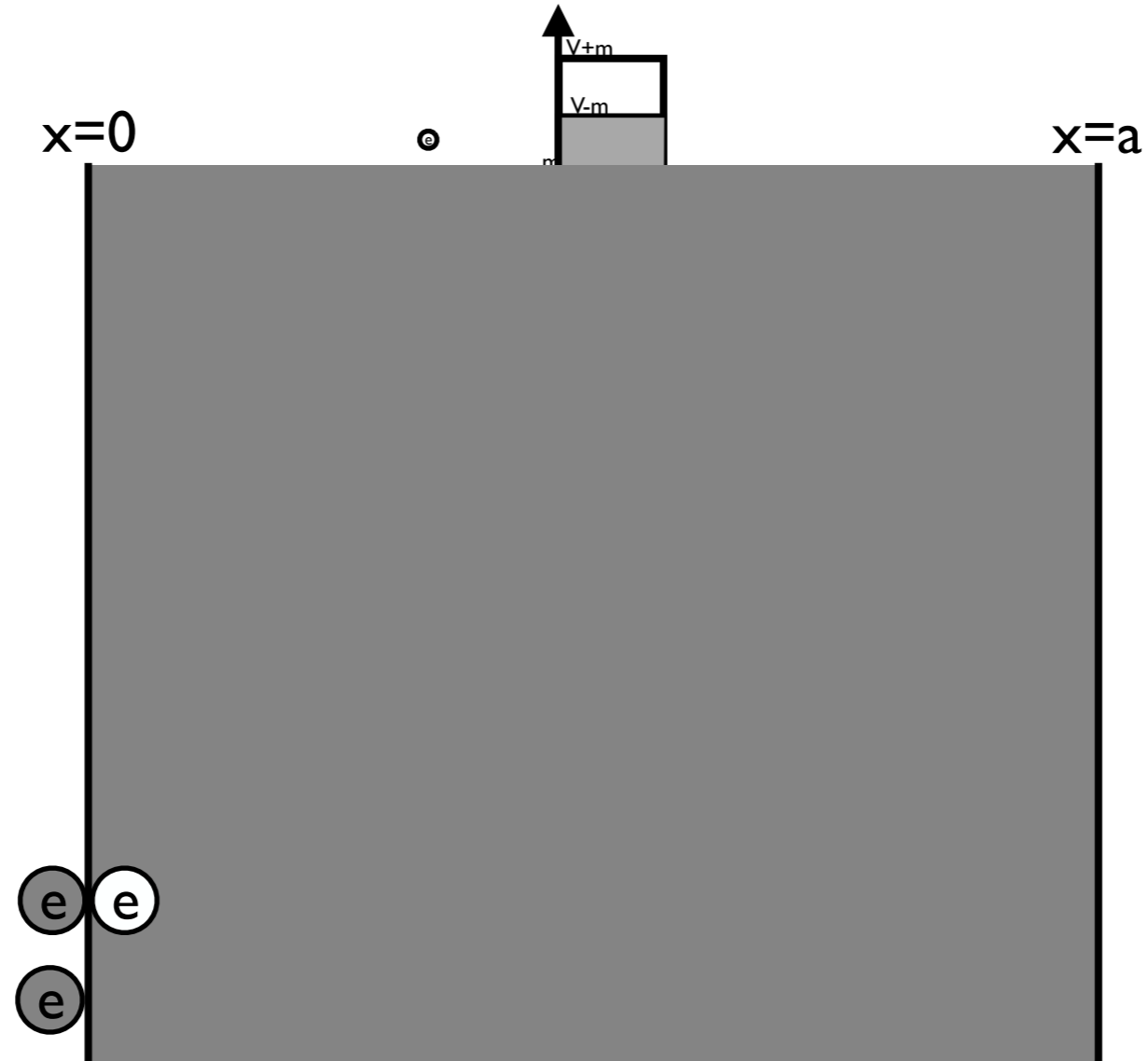


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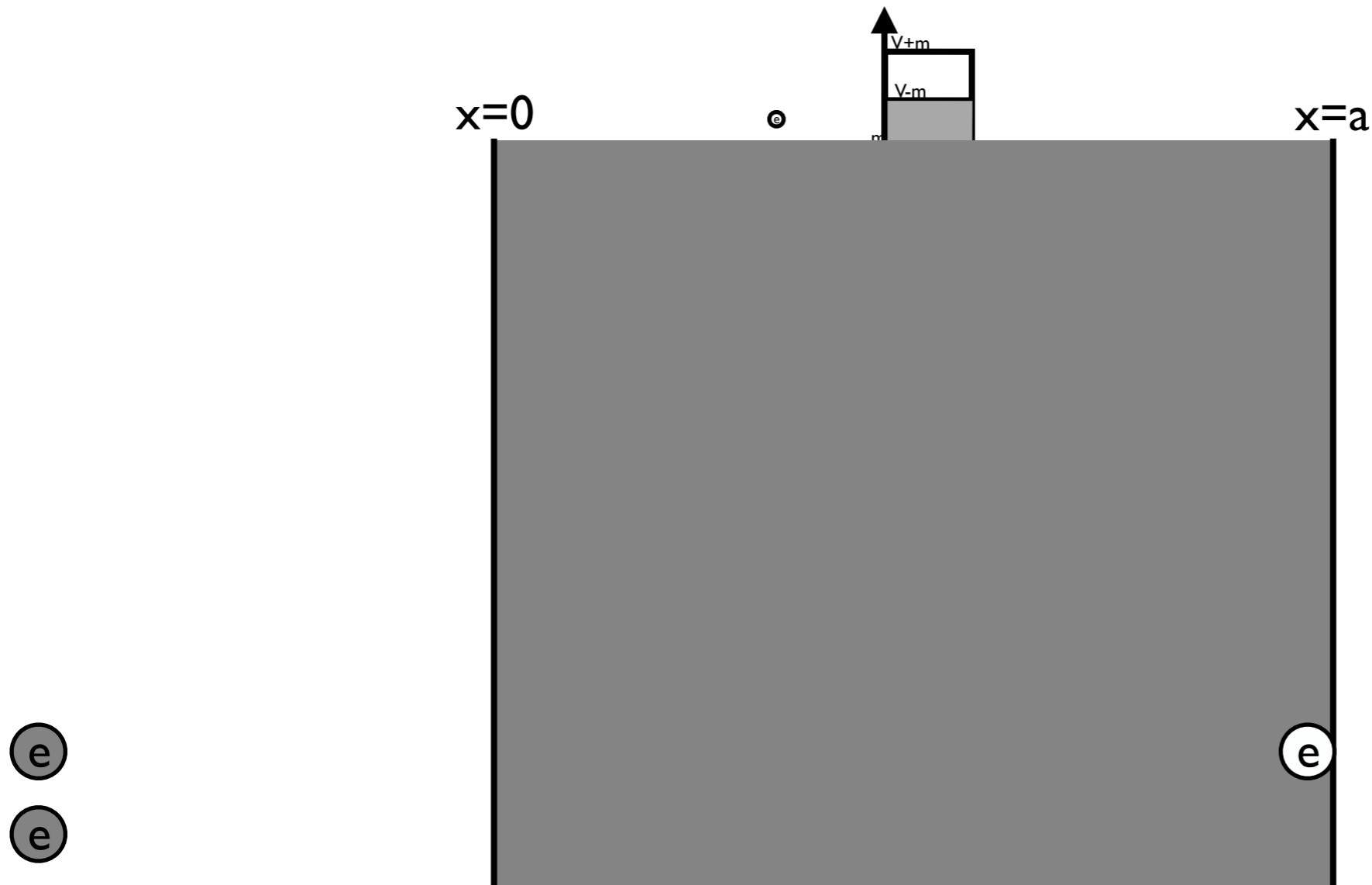
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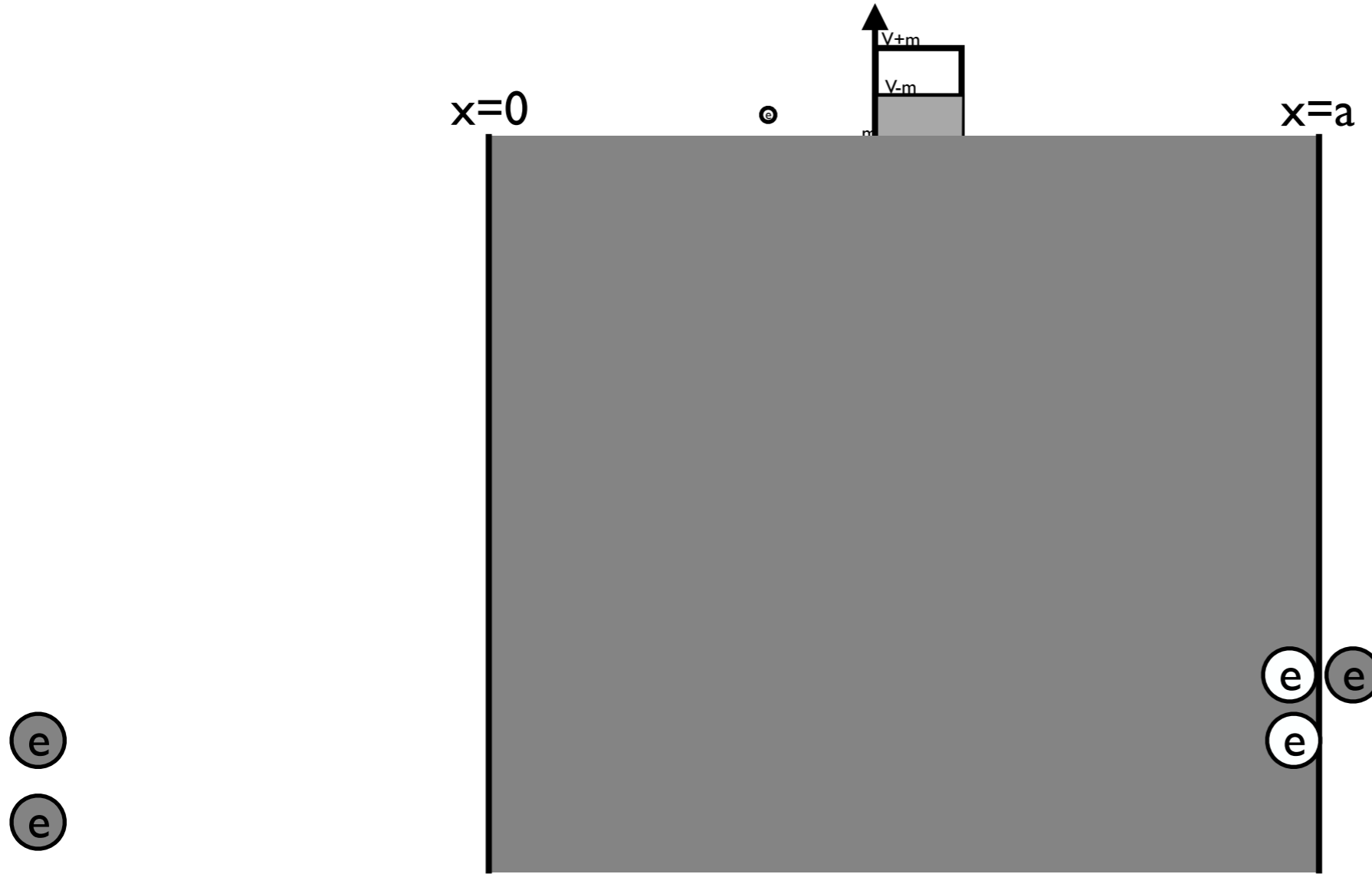
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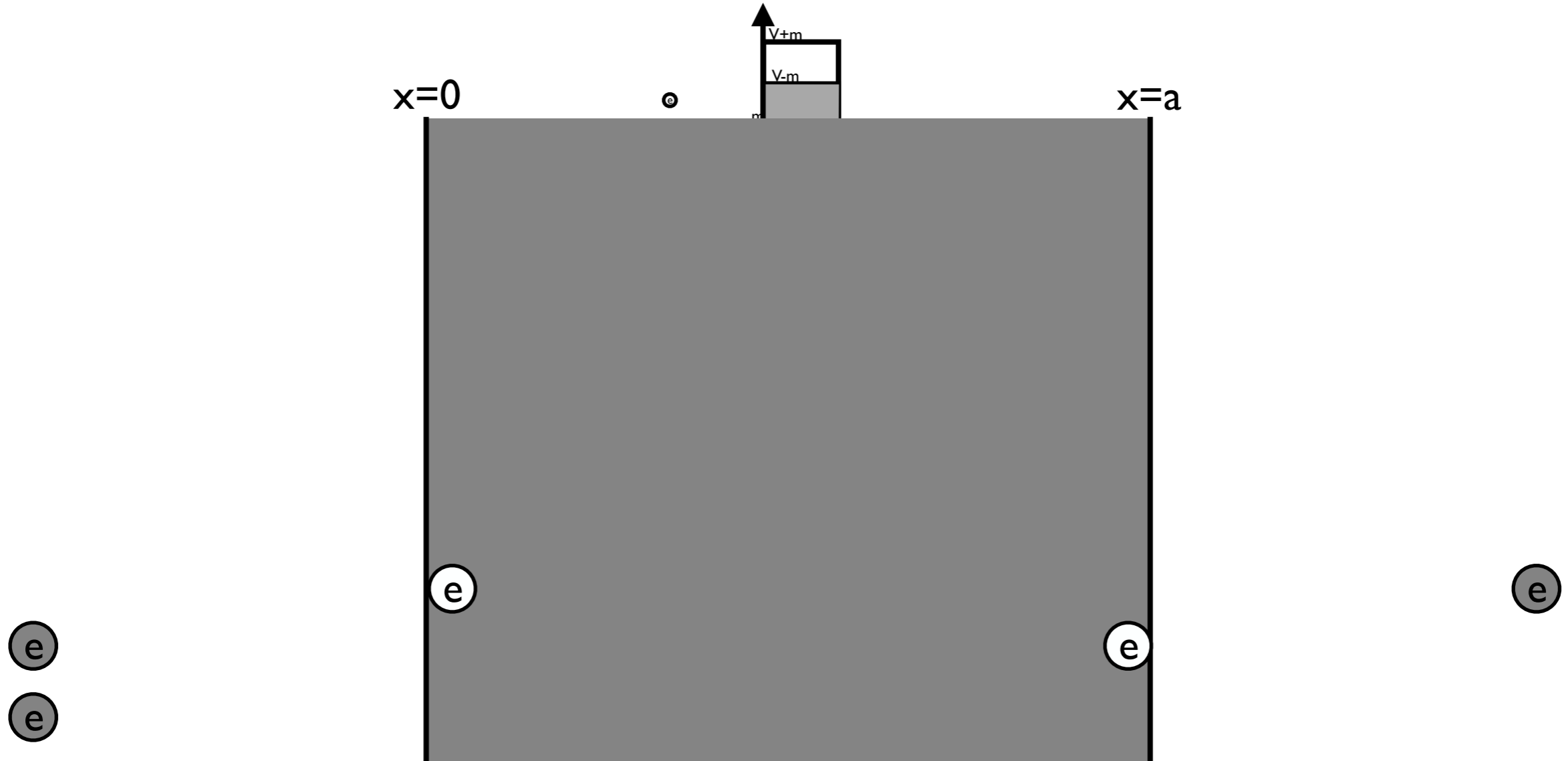
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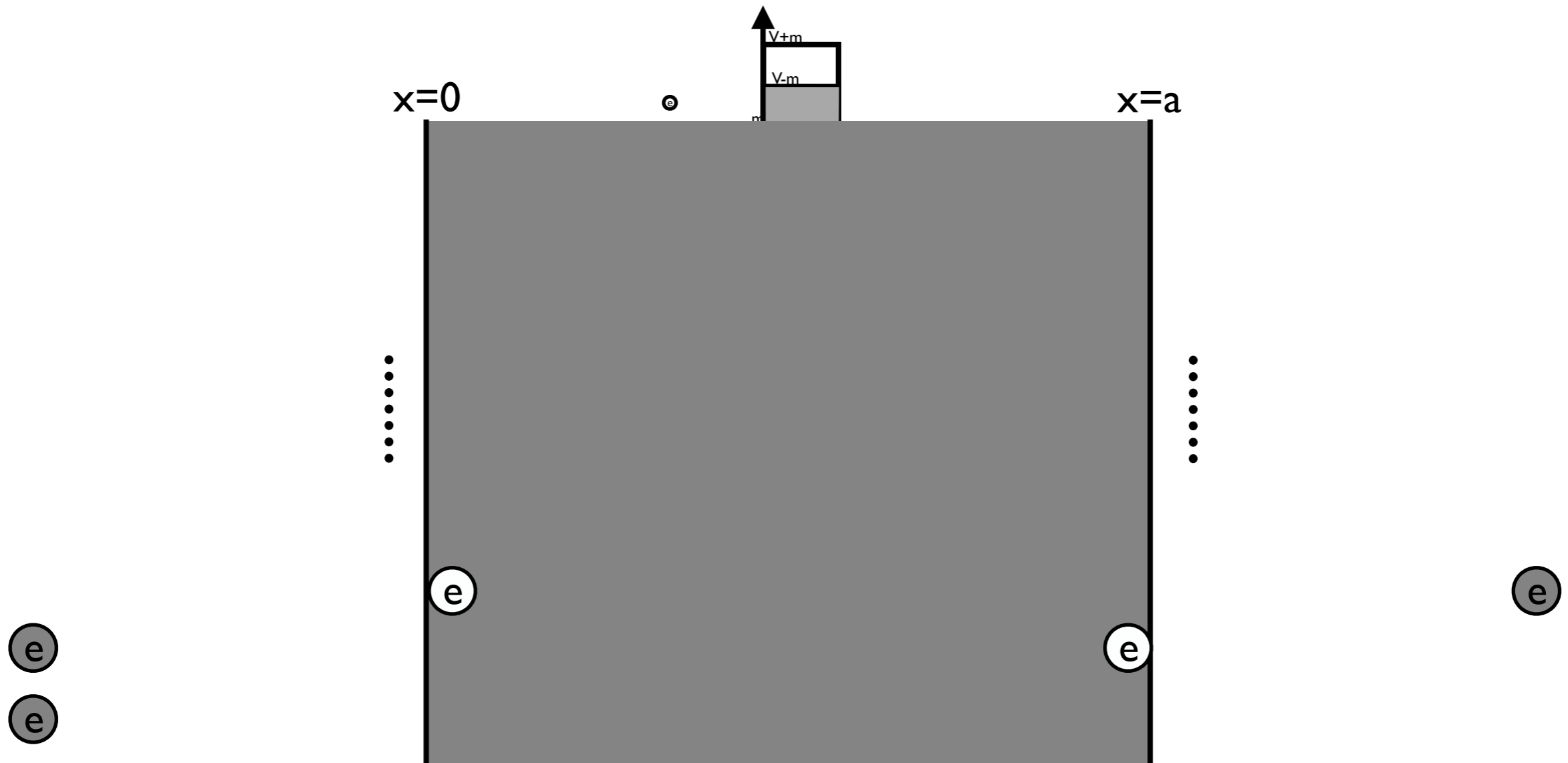
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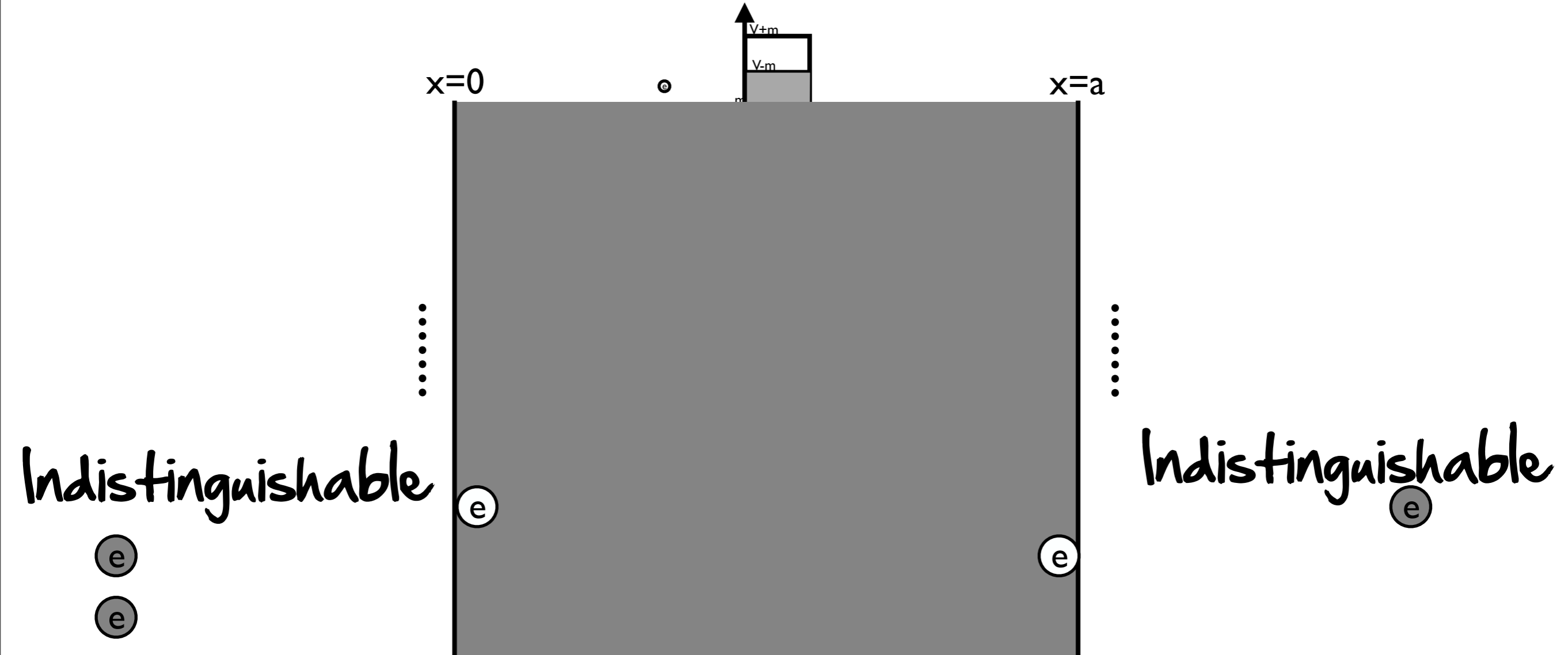
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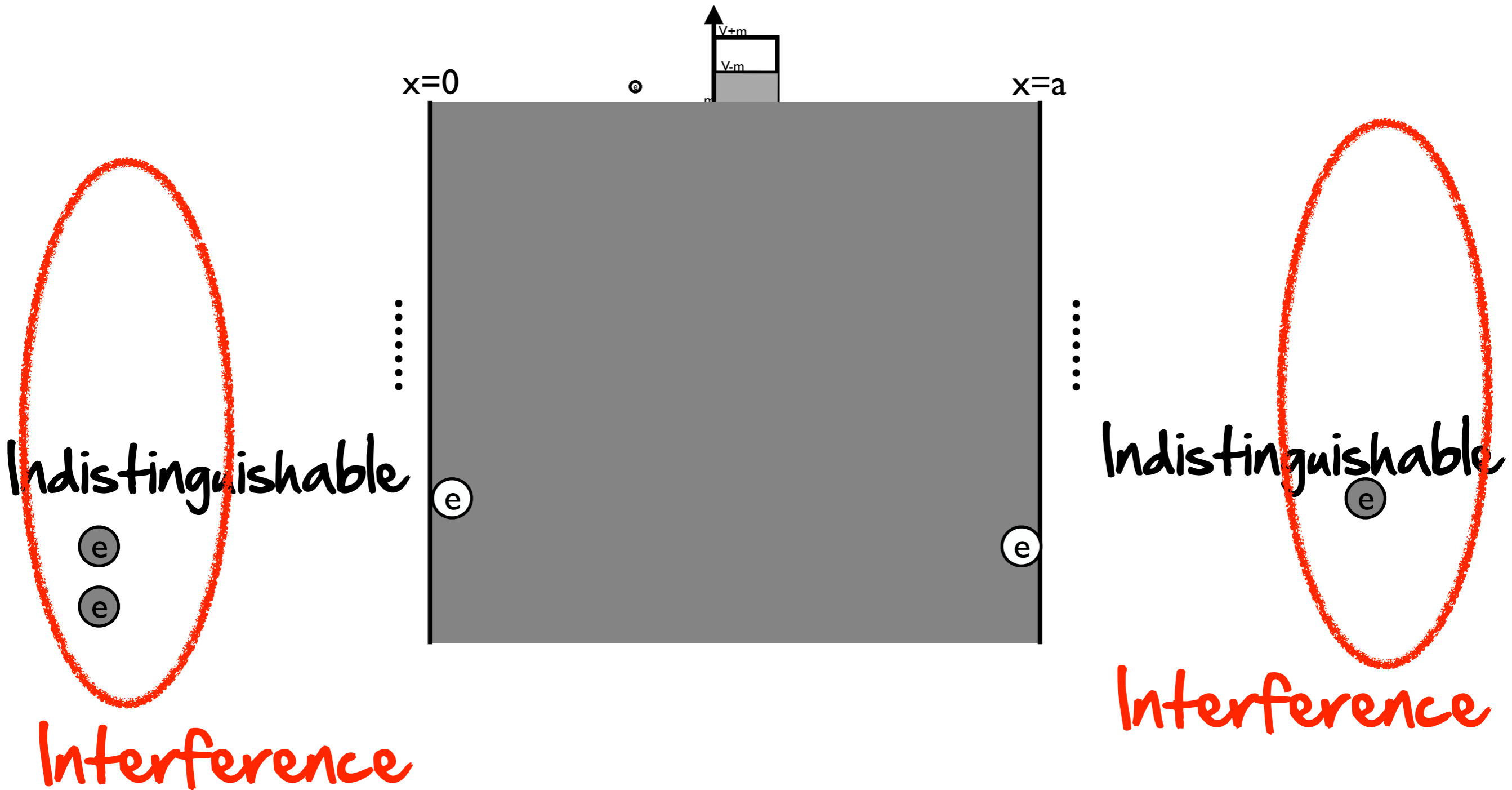
Analysis



Analysis



Analysis



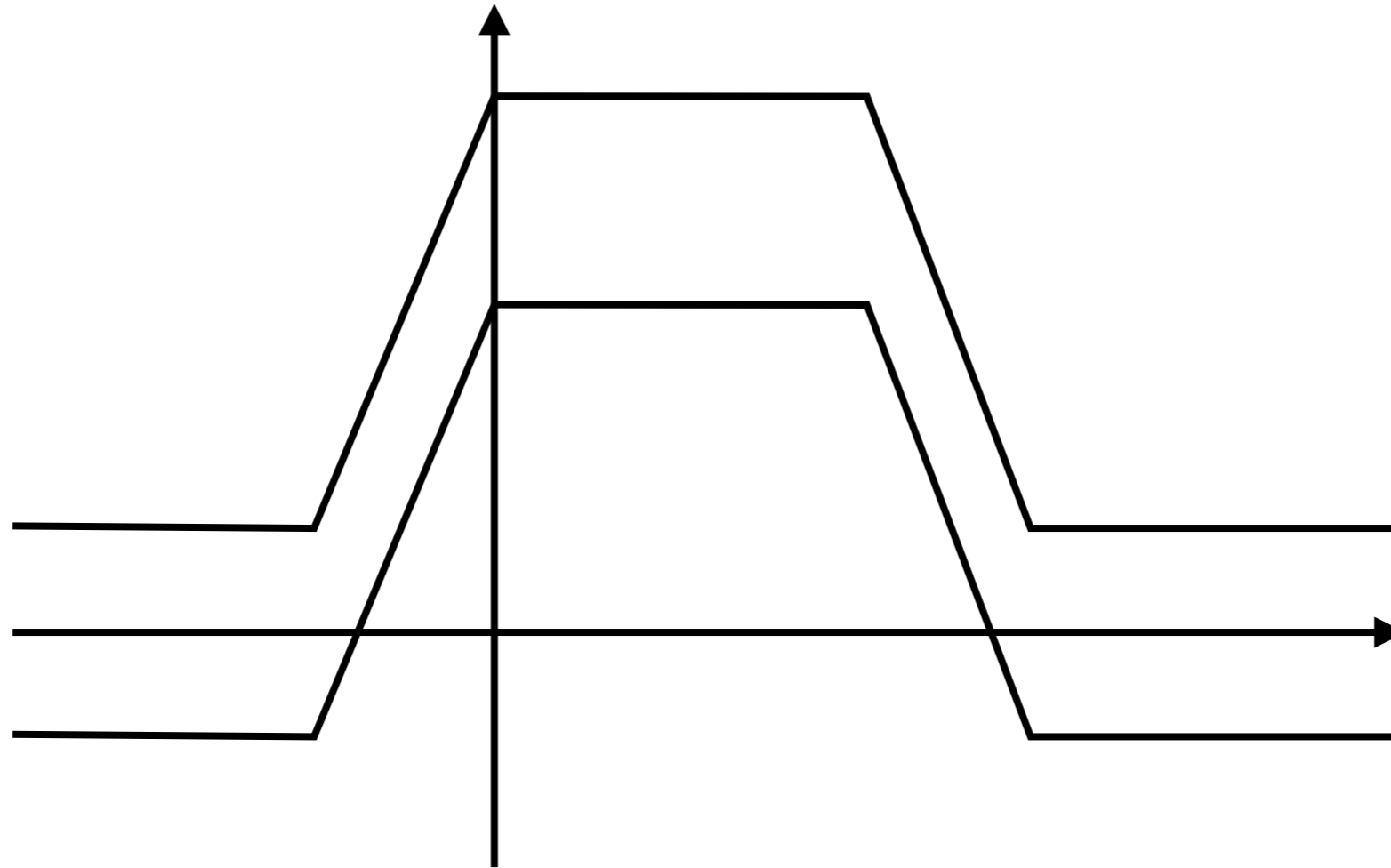
SCUSSIC

GROUP 4

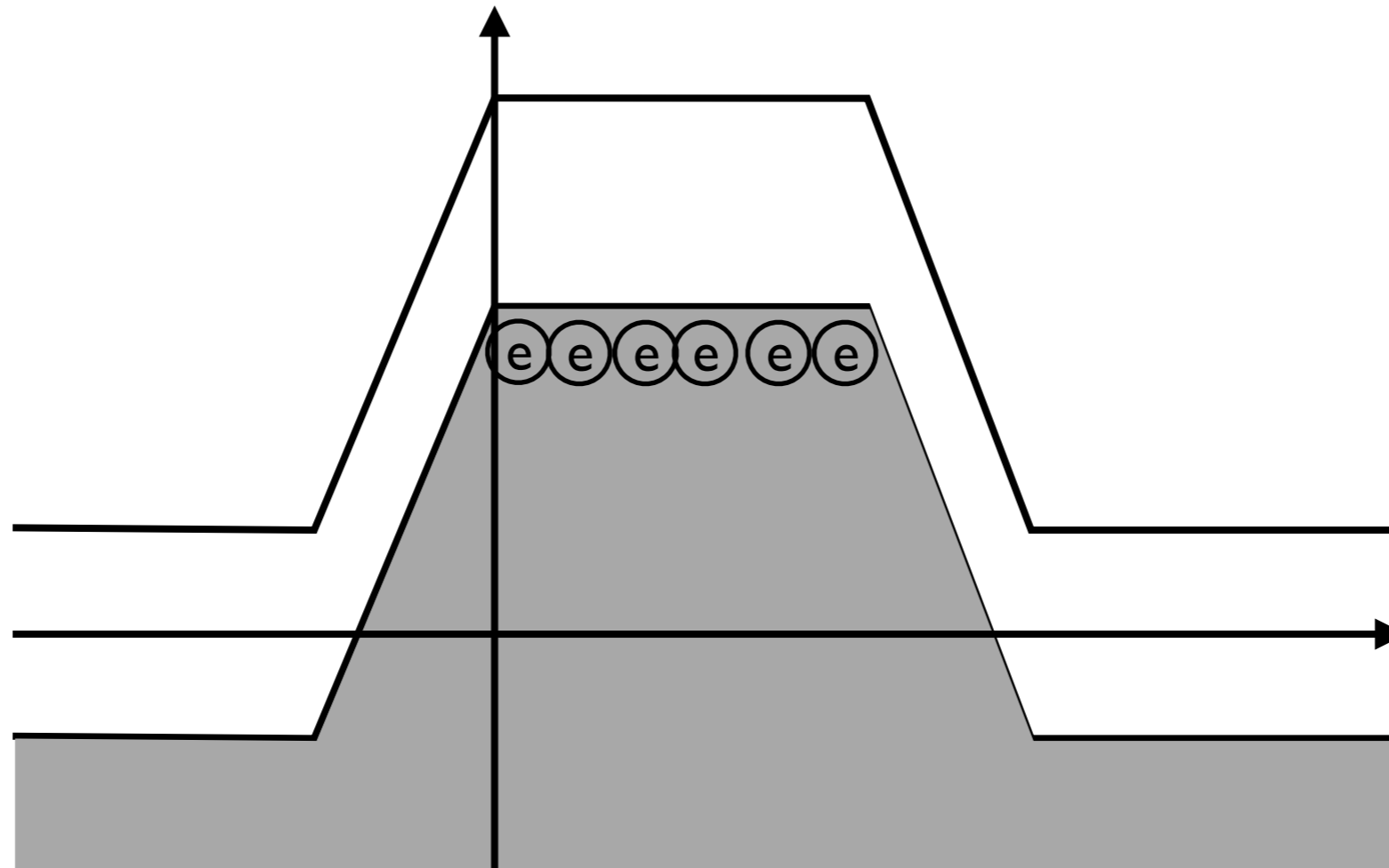
Analysis

Discussion

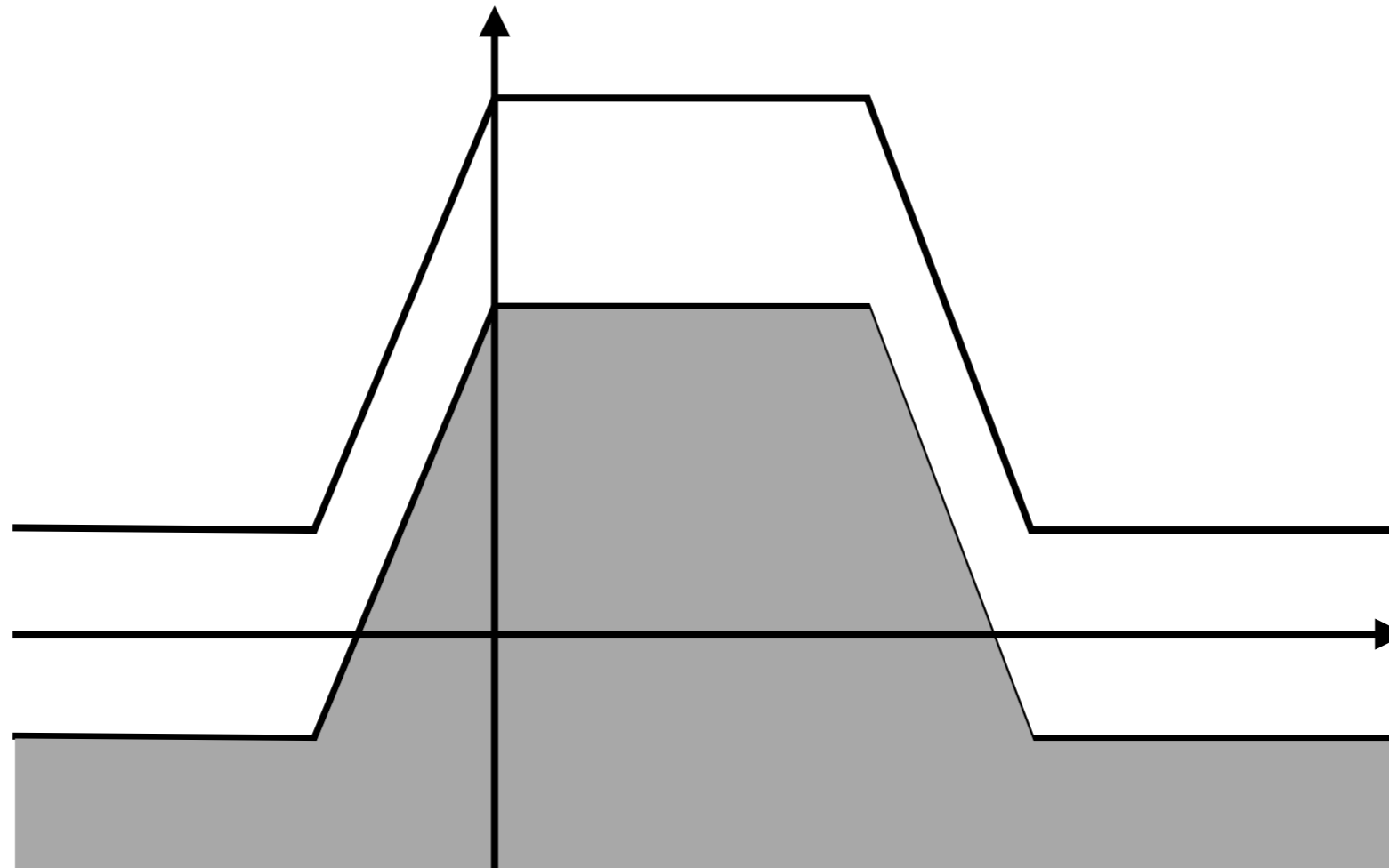
STATIONARY??



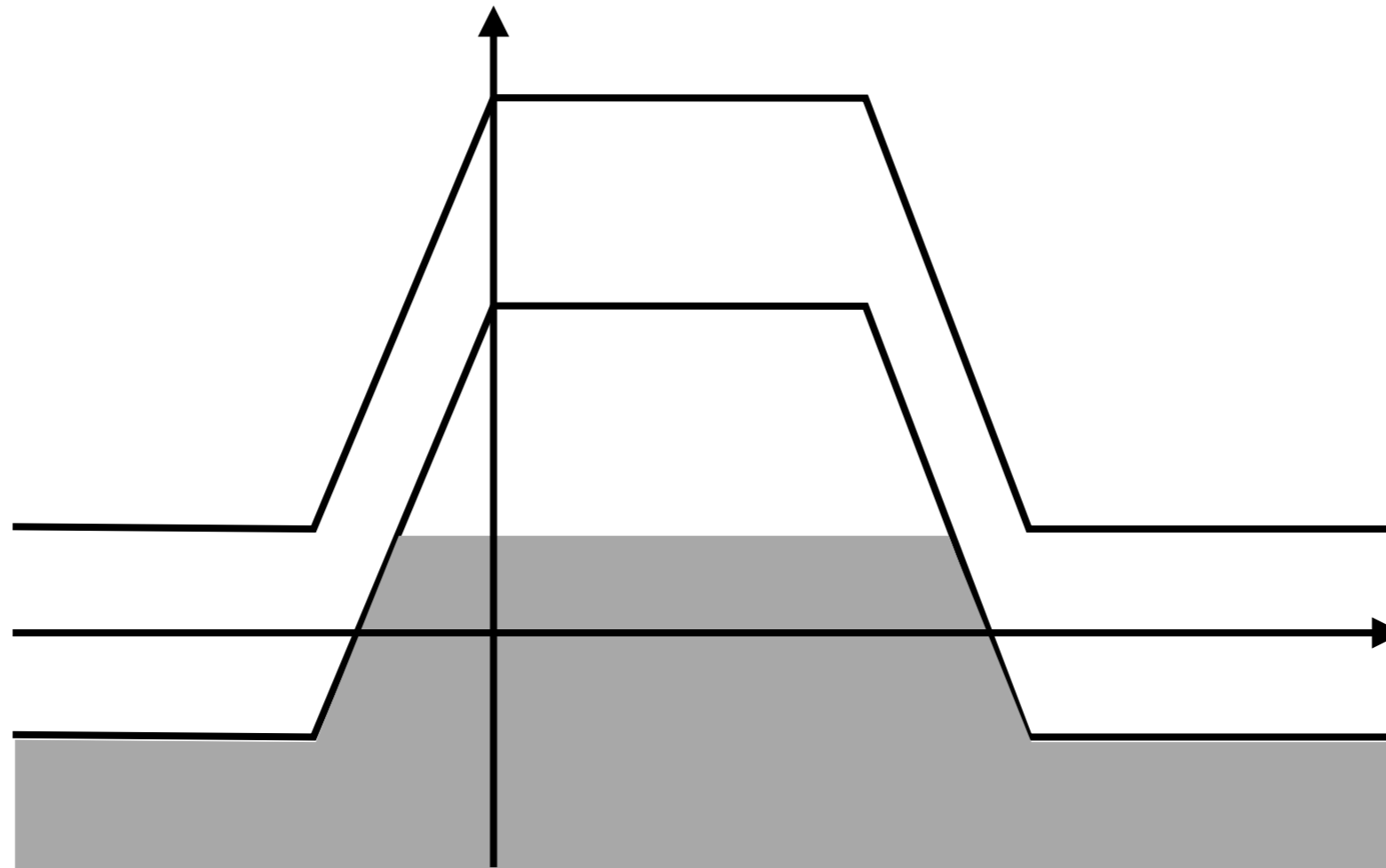
STATIONARY??



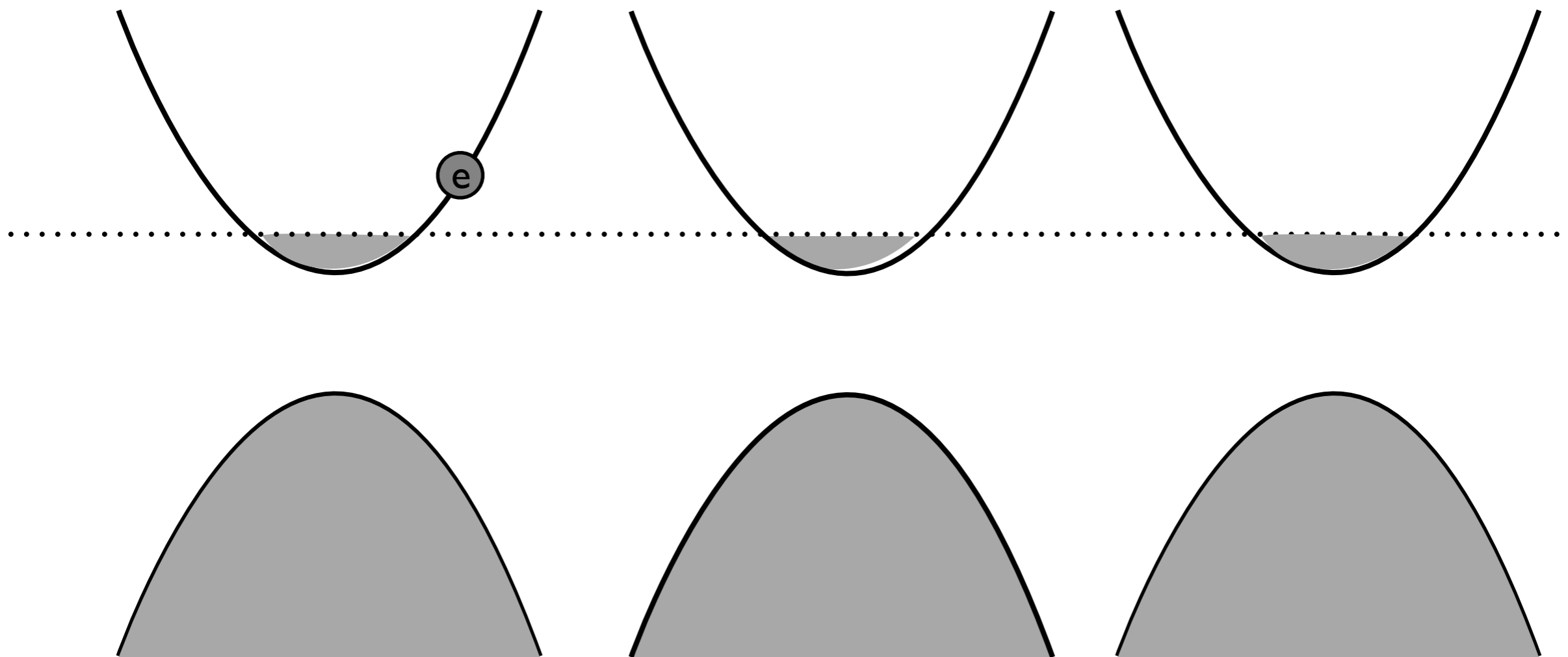
STATIONARY??



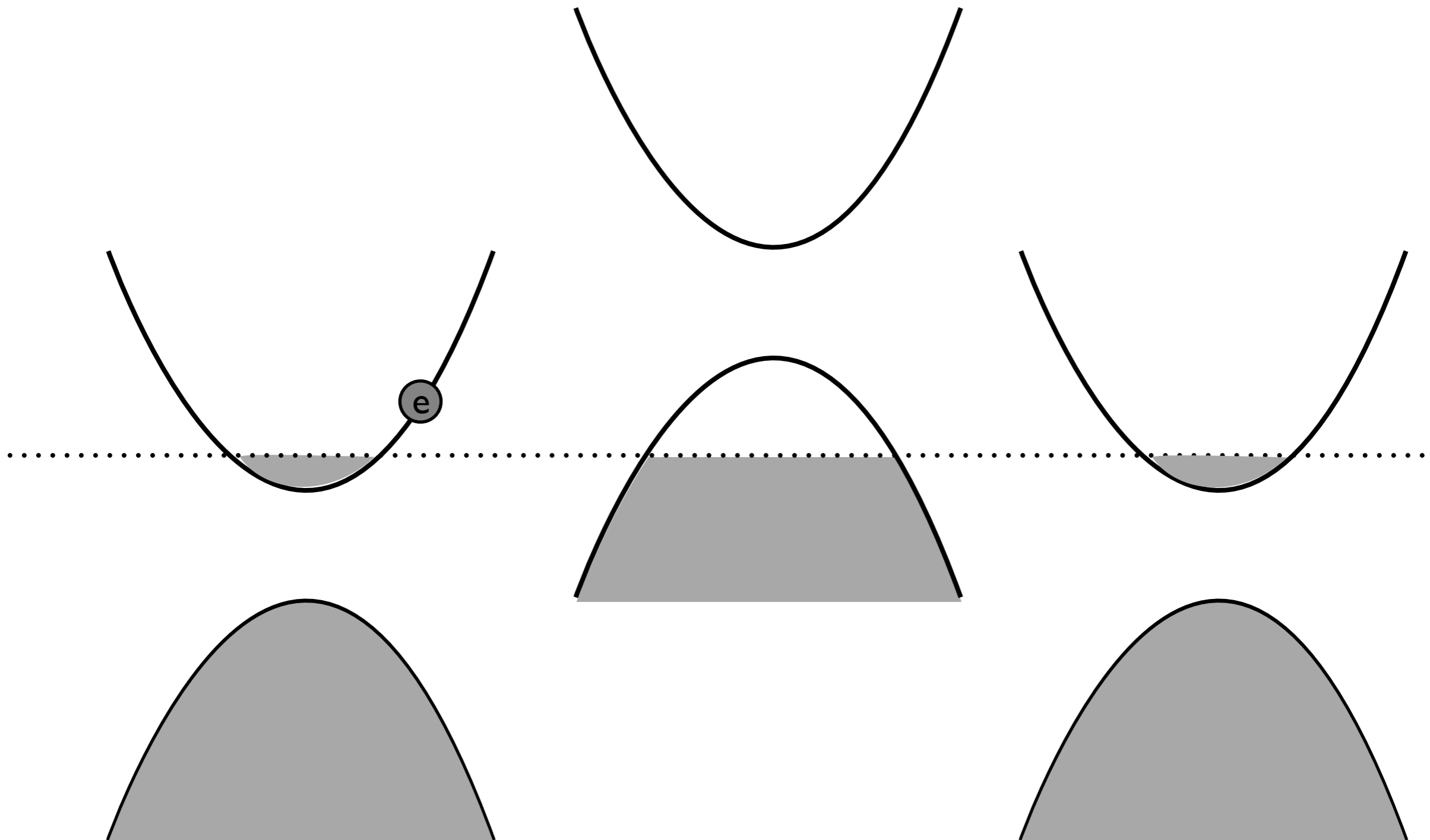
STATIONARY??



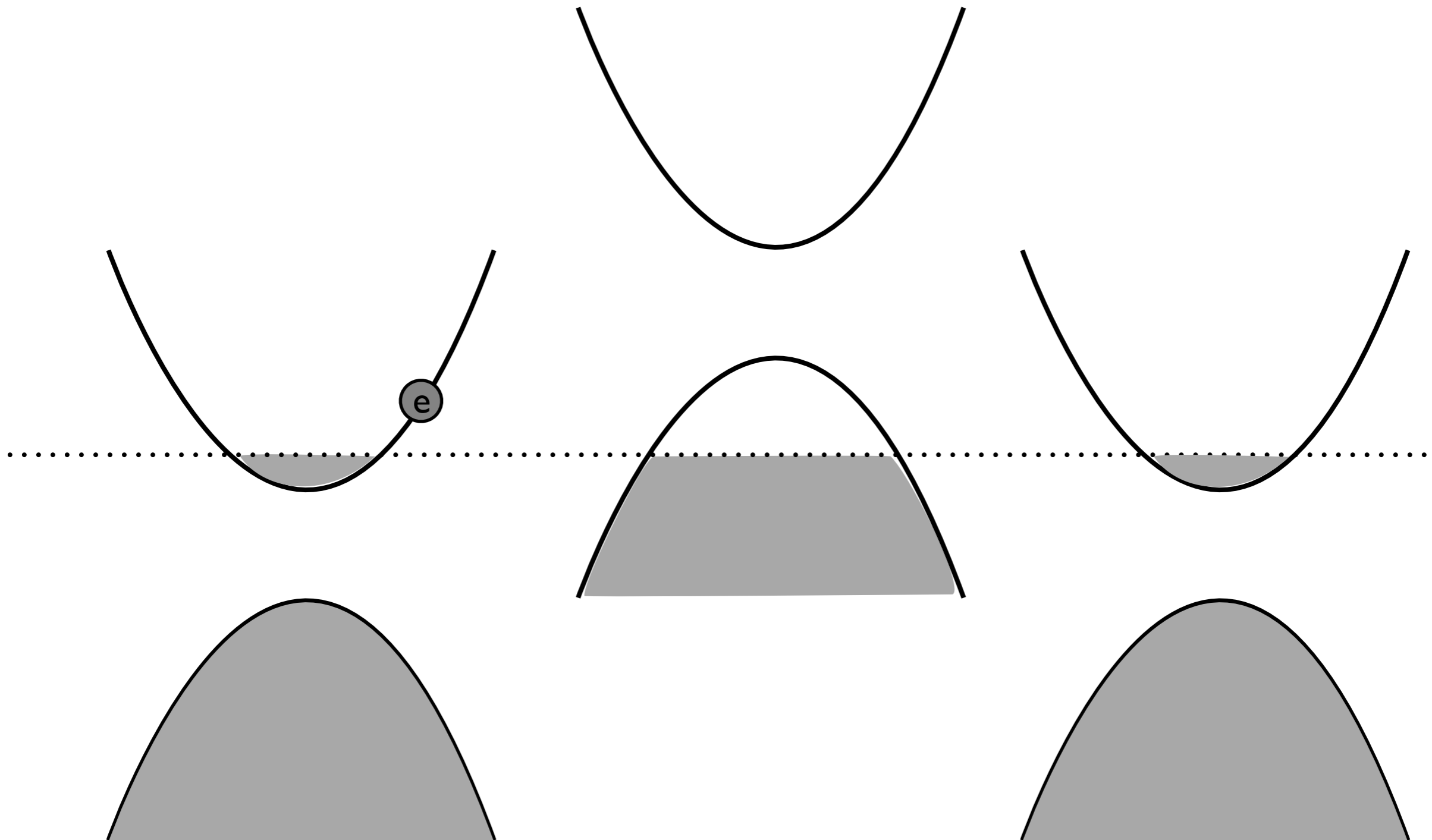
ELECTRON IN GRAPHENE



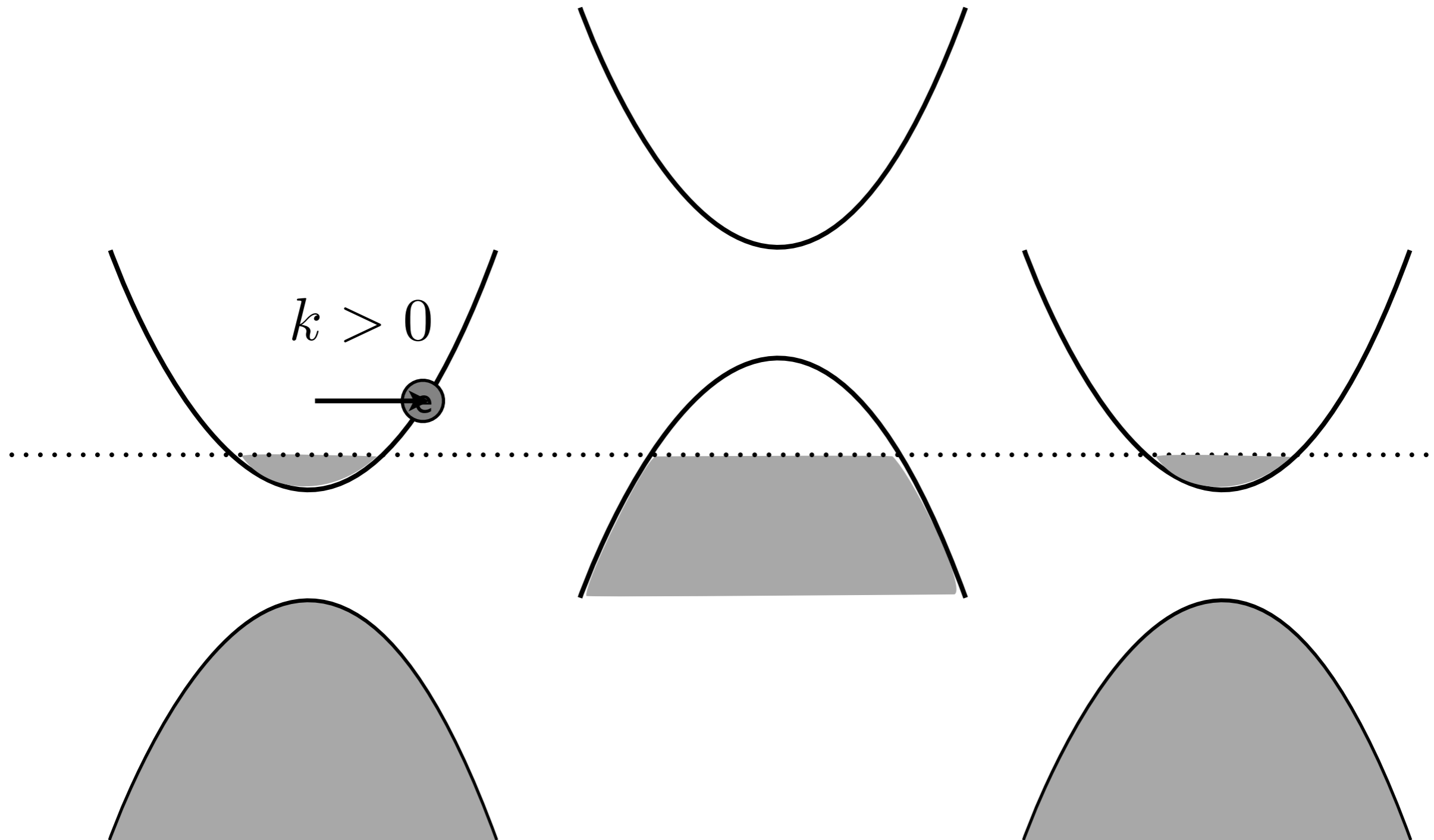
ELECTRON IN GRAPHENE



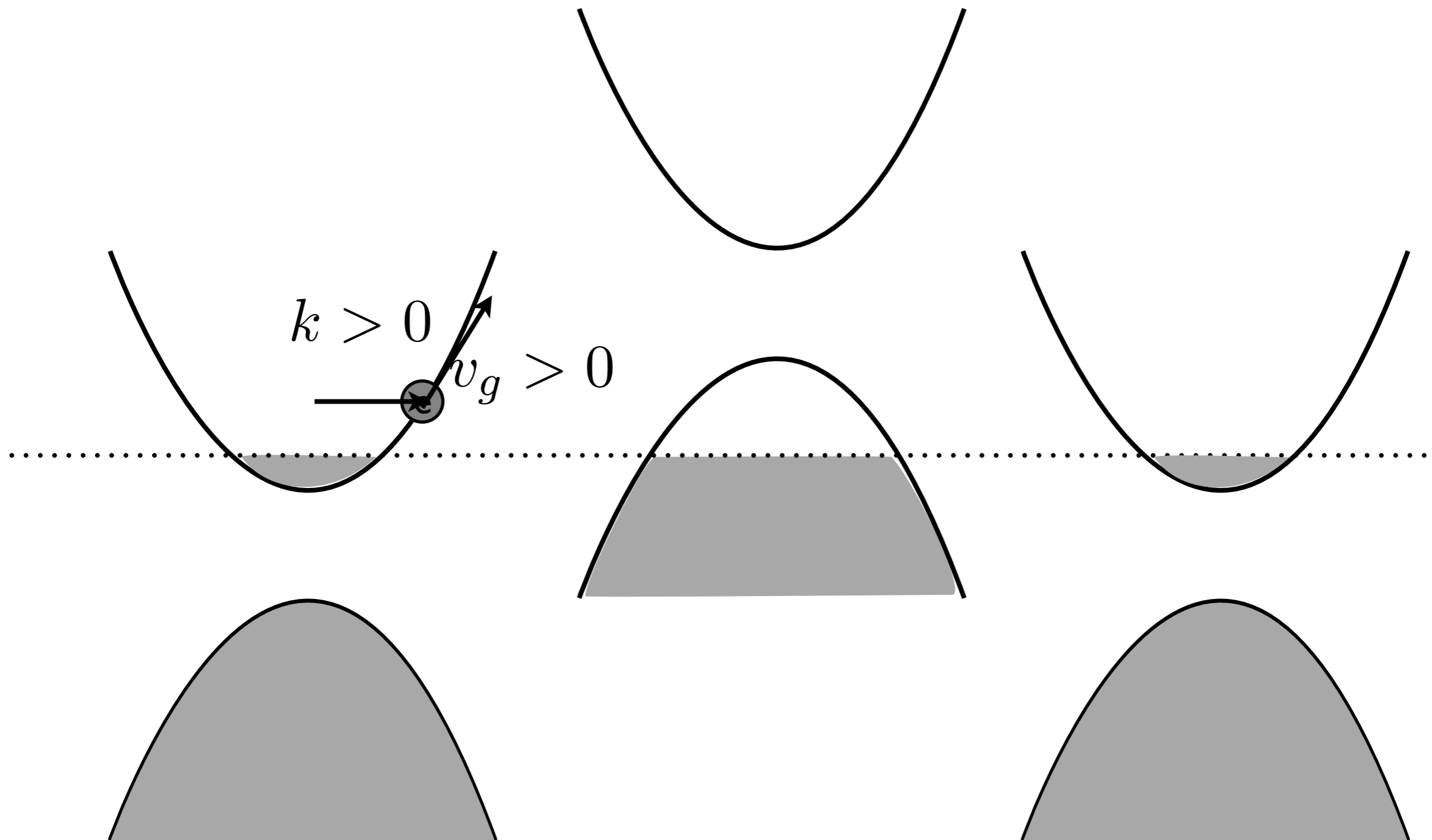
ELECTRON IN GRAPHENE



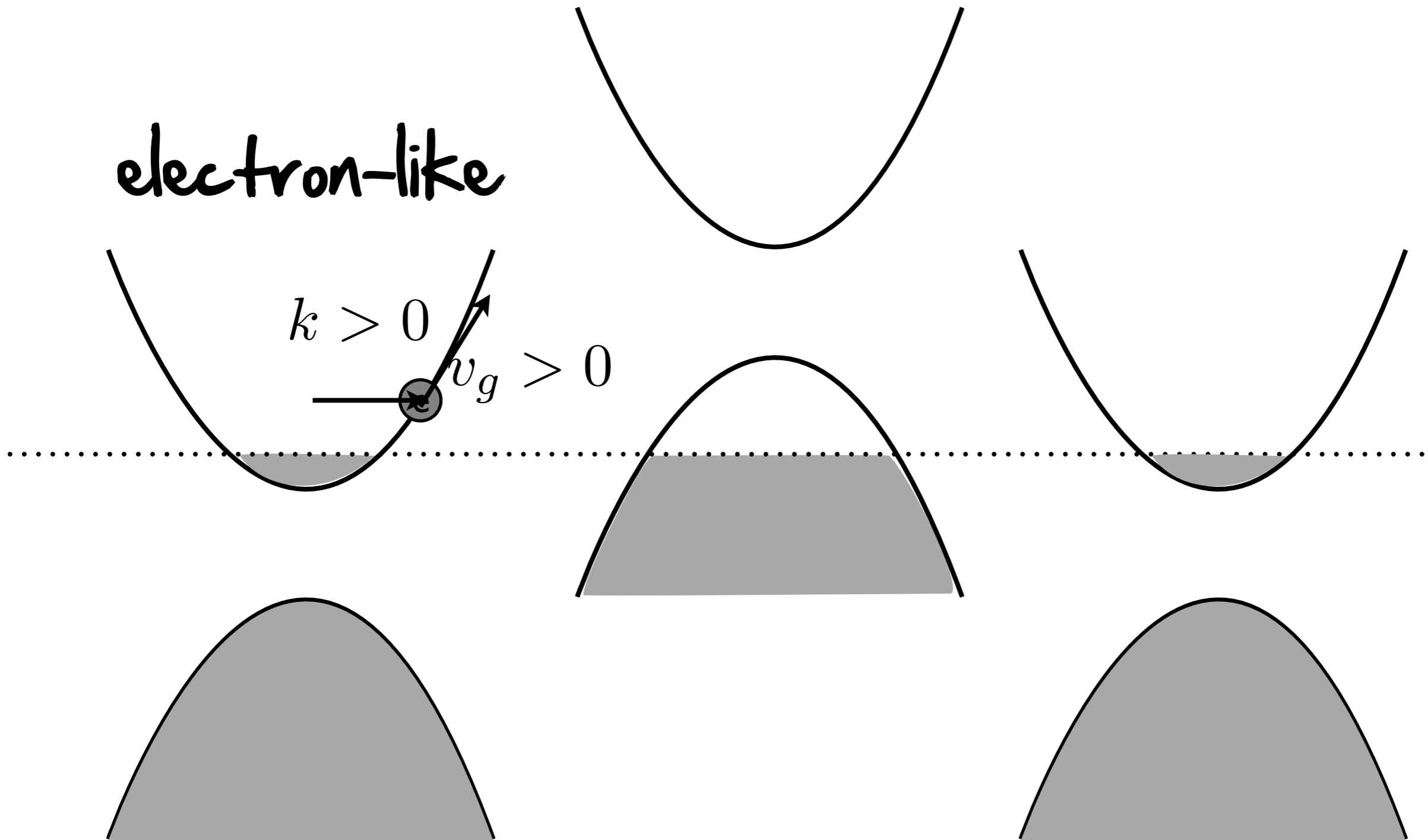
ELECTRON IN GRAPHENE



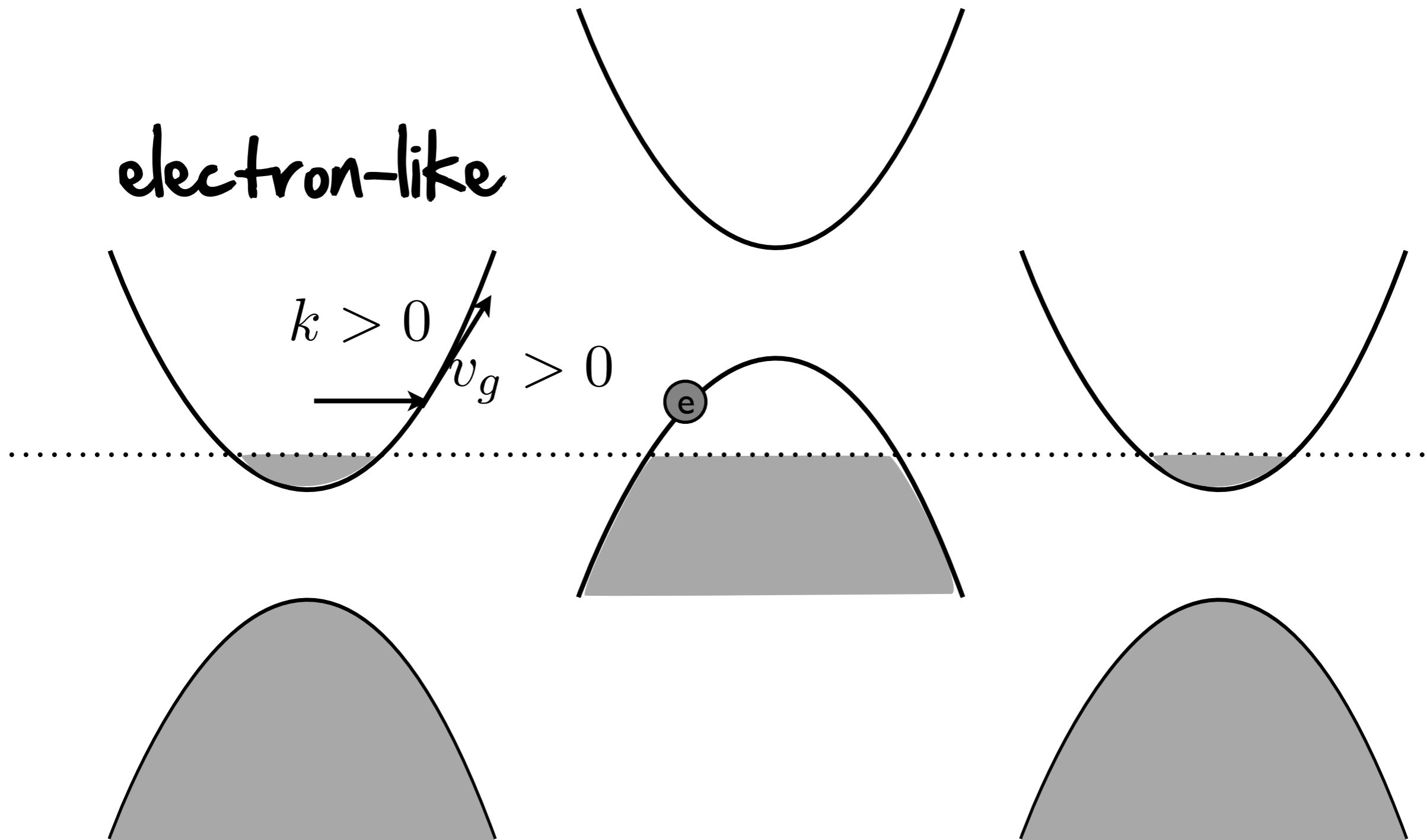
ELECTRON IN GRAPHENE



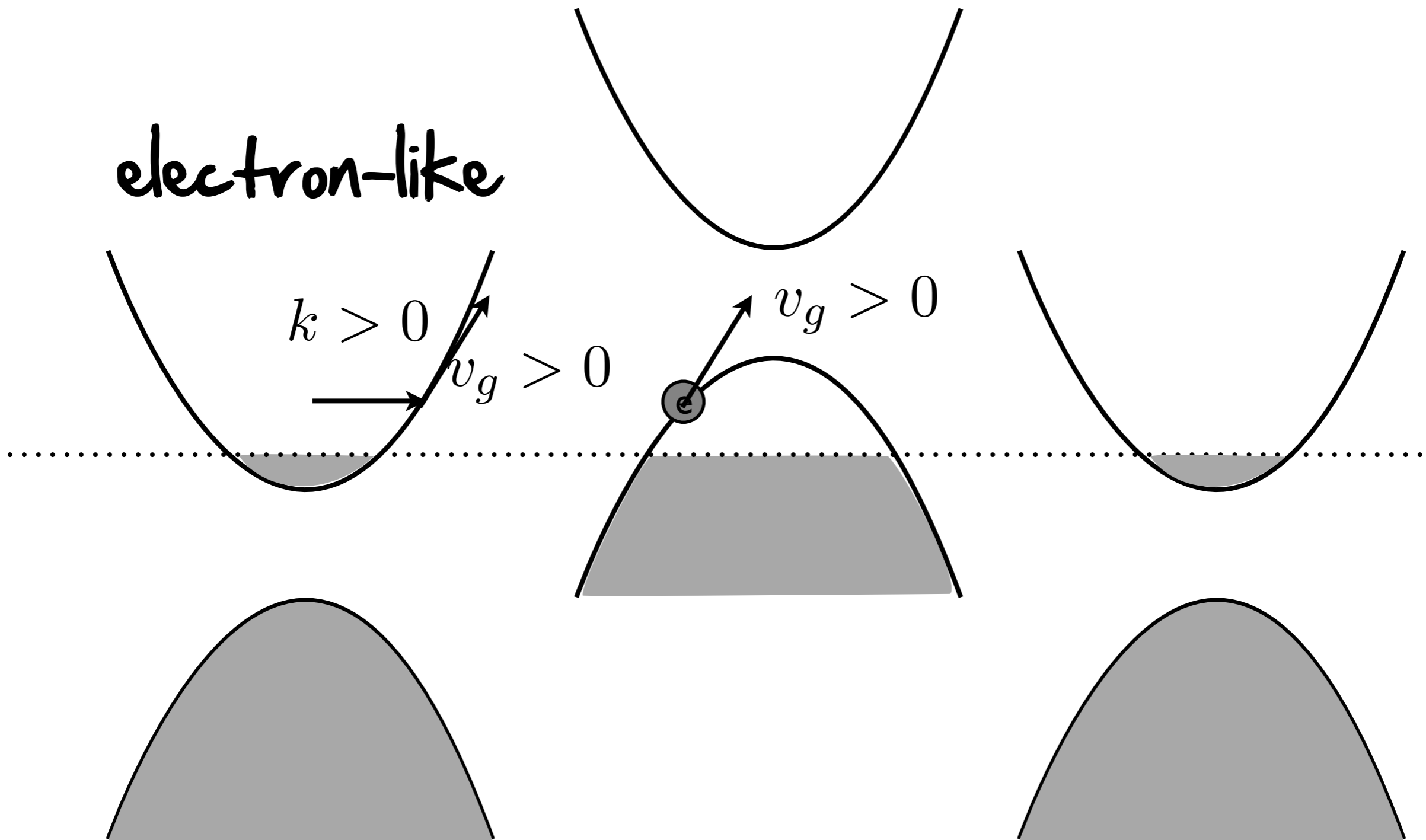
ELECTRON IN GRAPHENE



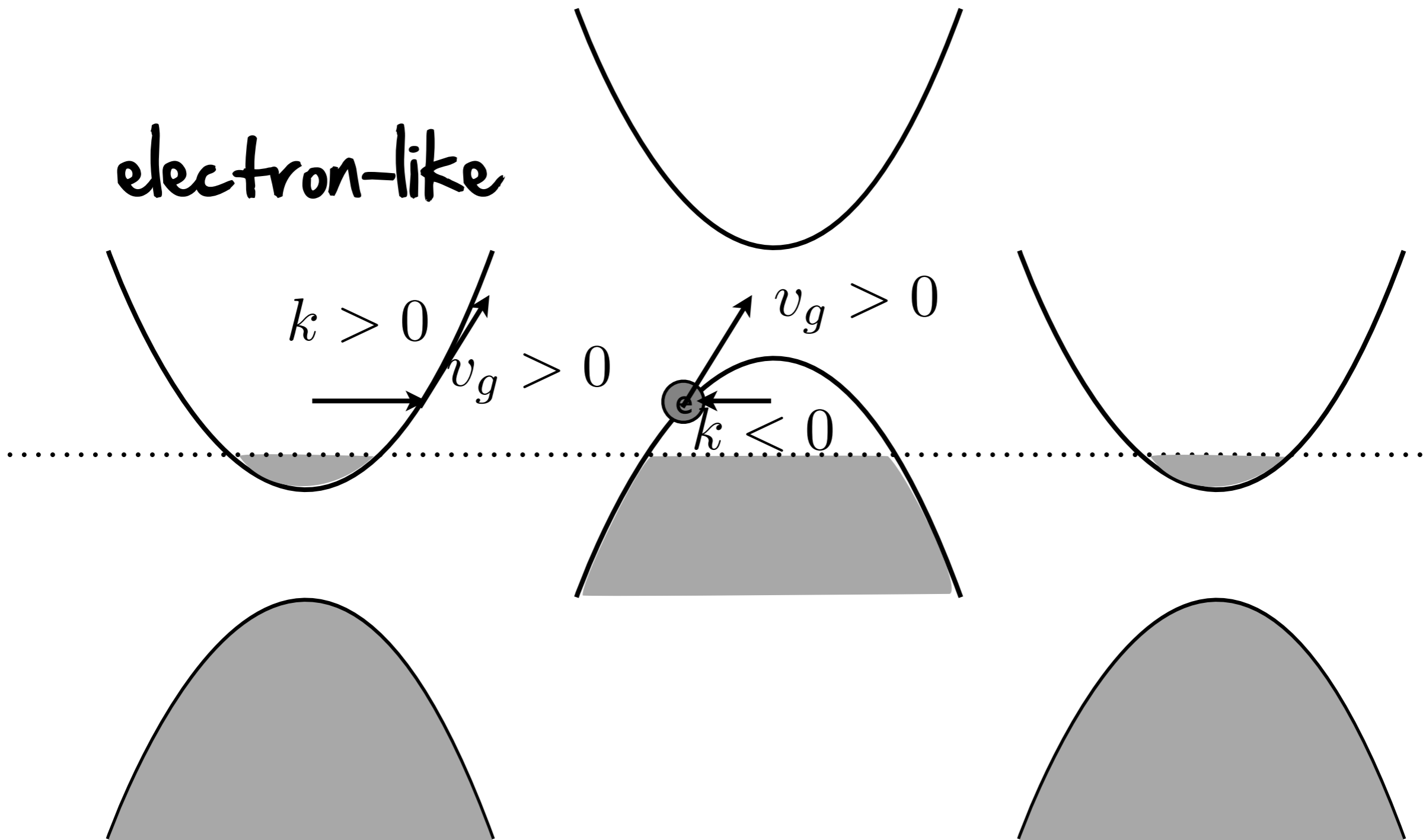
ELECTRON IN GRAPHENE



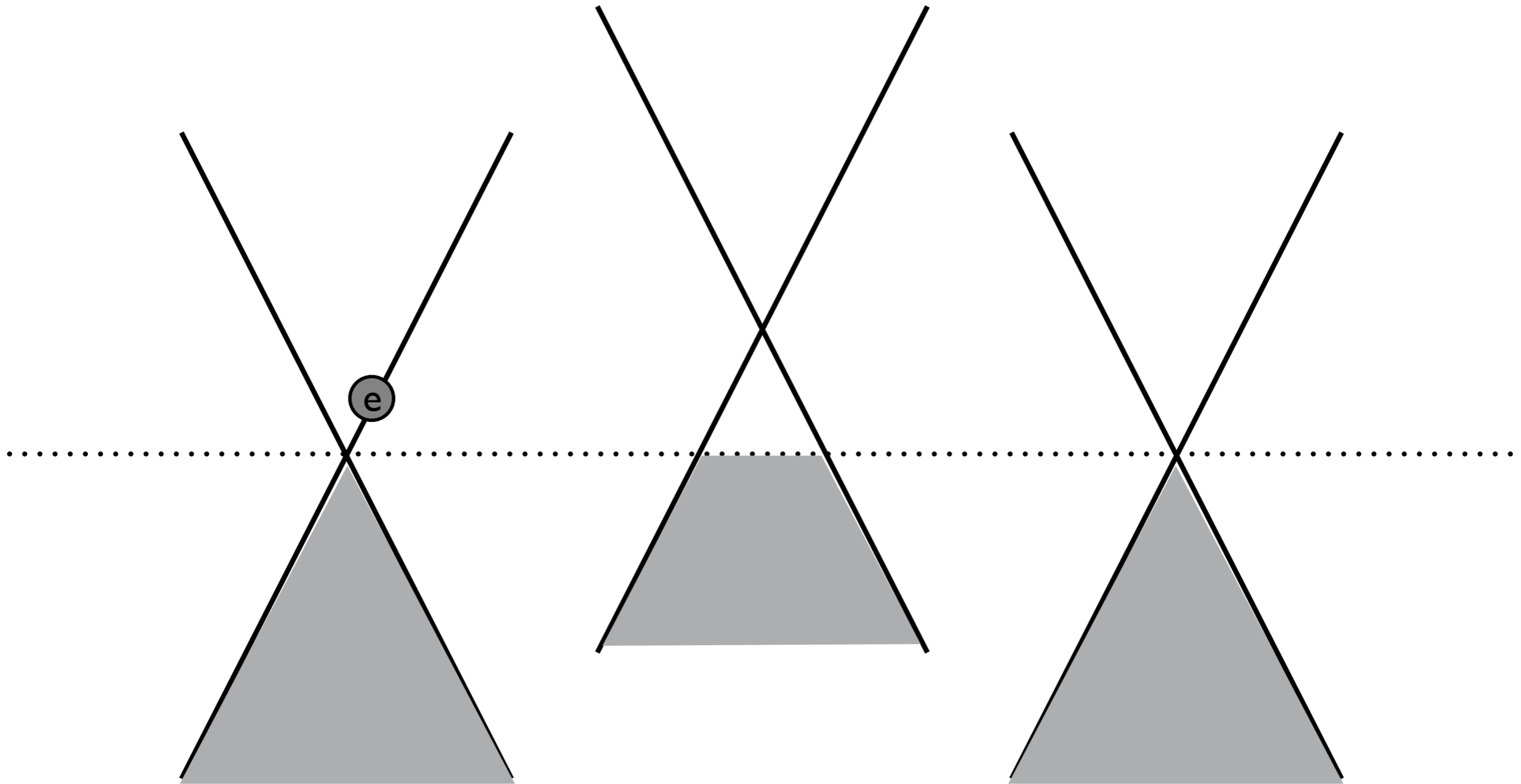
ELECTRON IN GRAPHENE



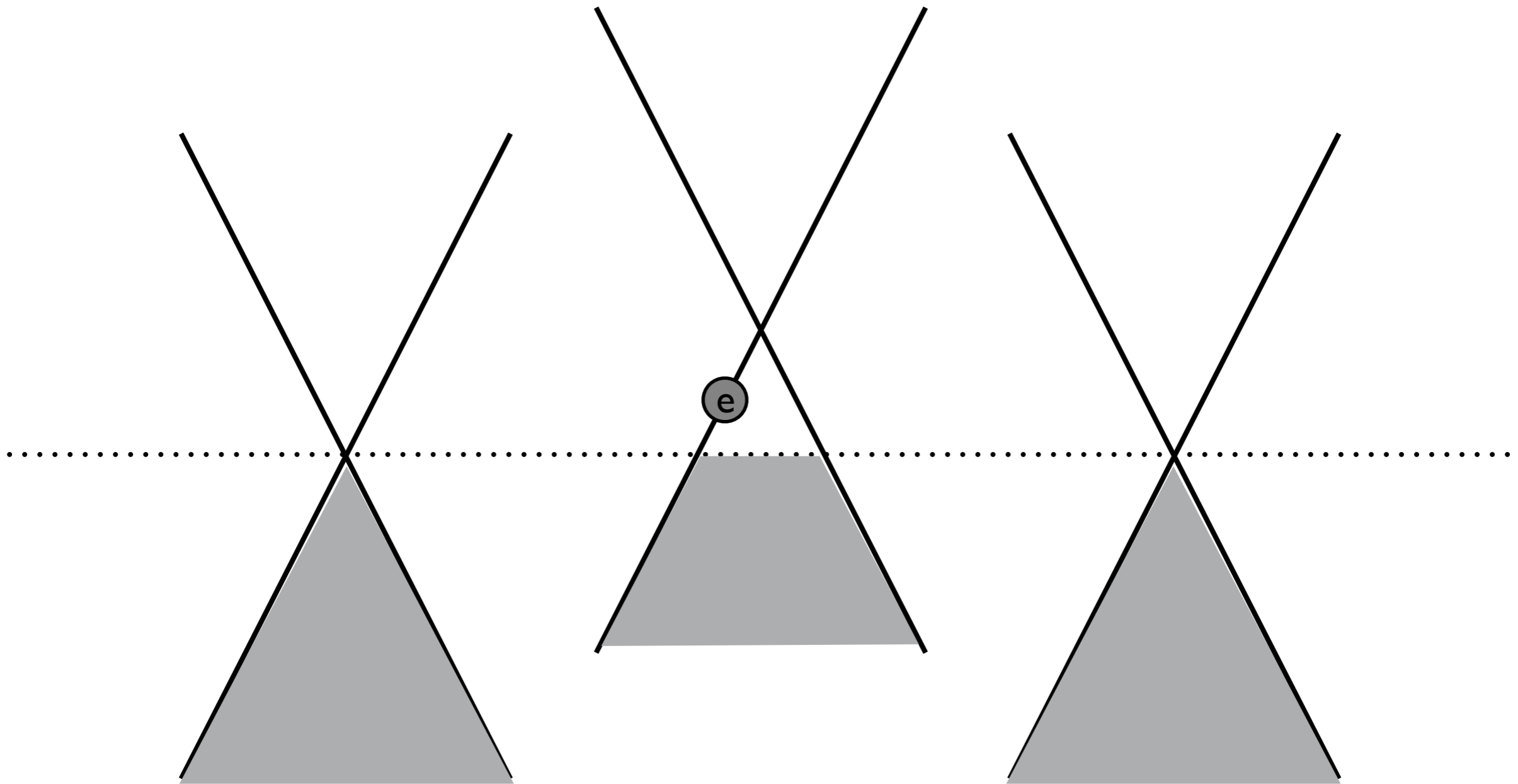
ELECTRON IN GRAPHENE



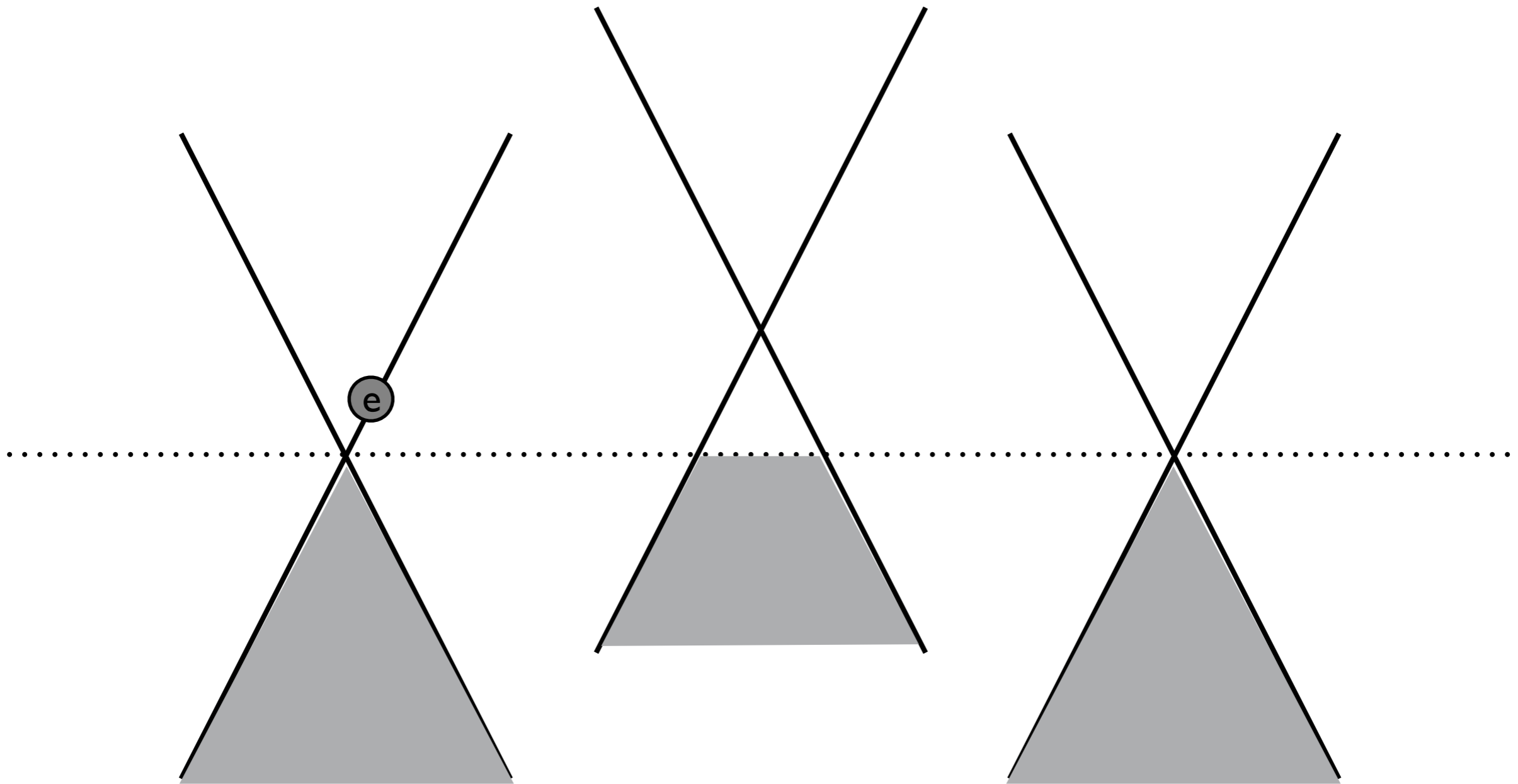
MASSLESS LIMIT



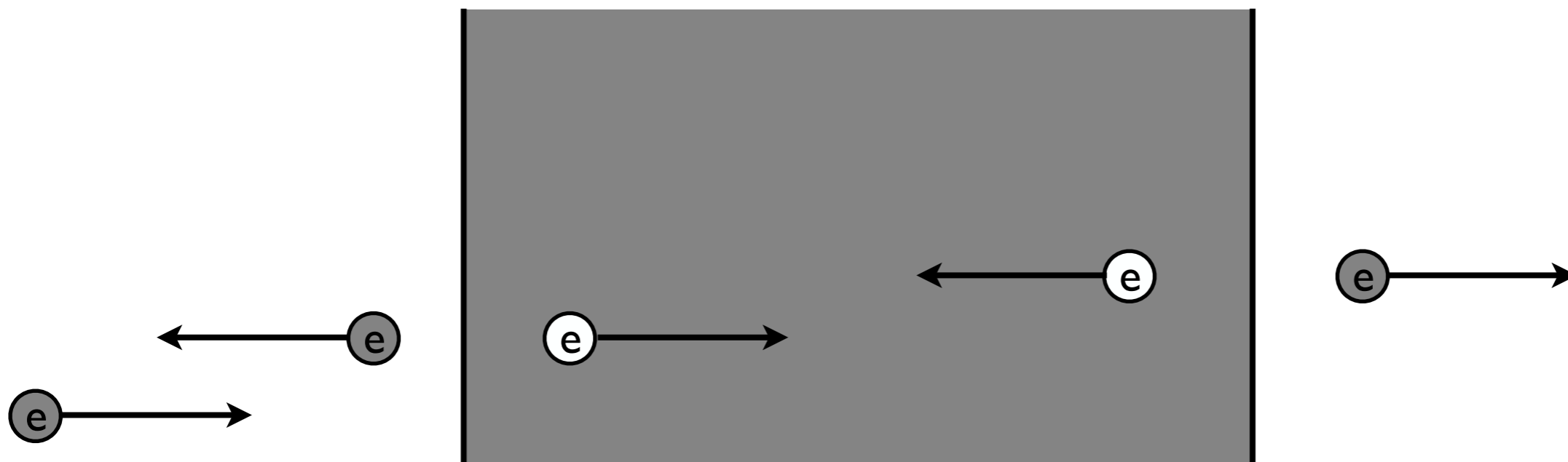
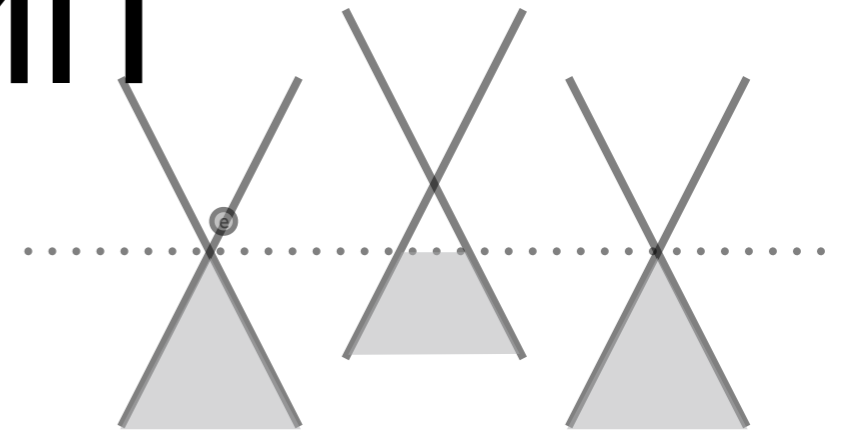
MASSLESS LIMIT



MASSLESS LIMIT

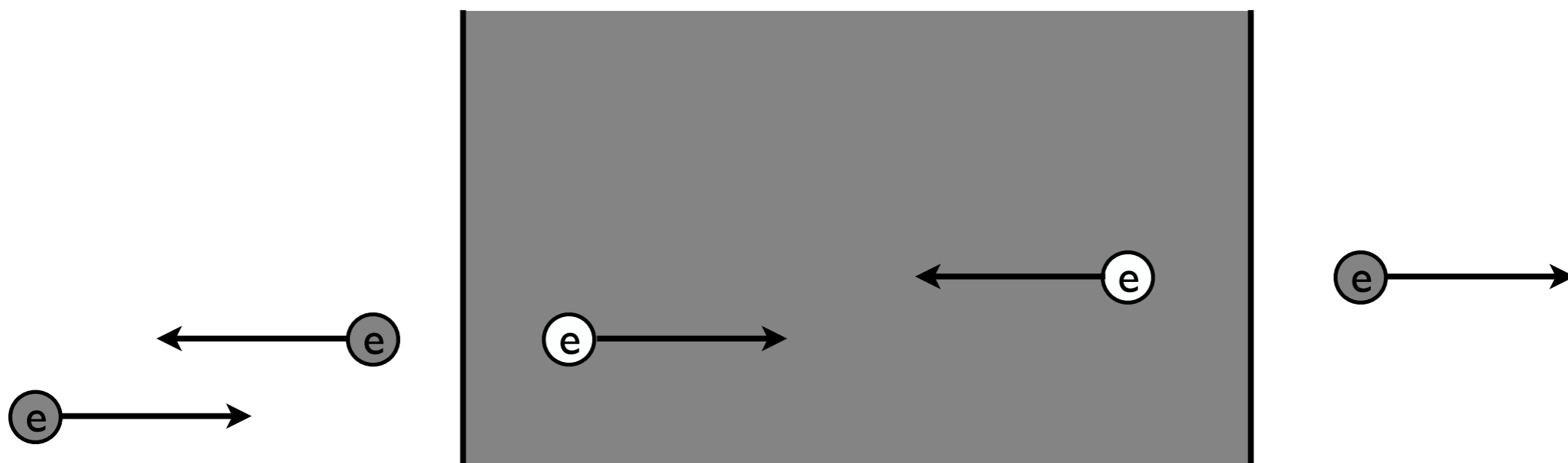
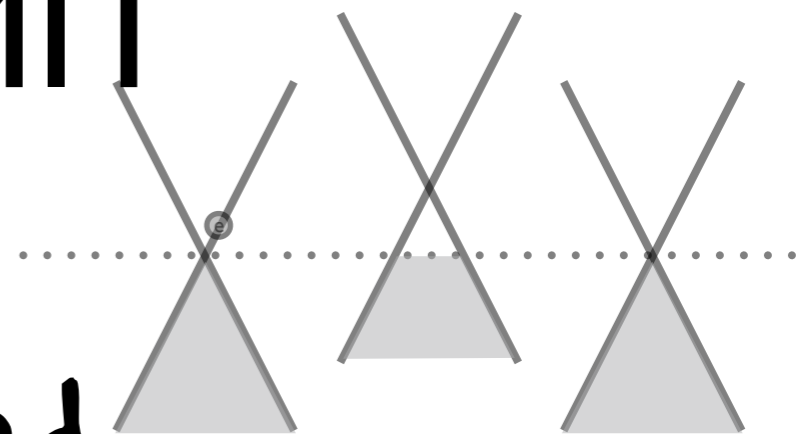


MASSLESS LIMIT



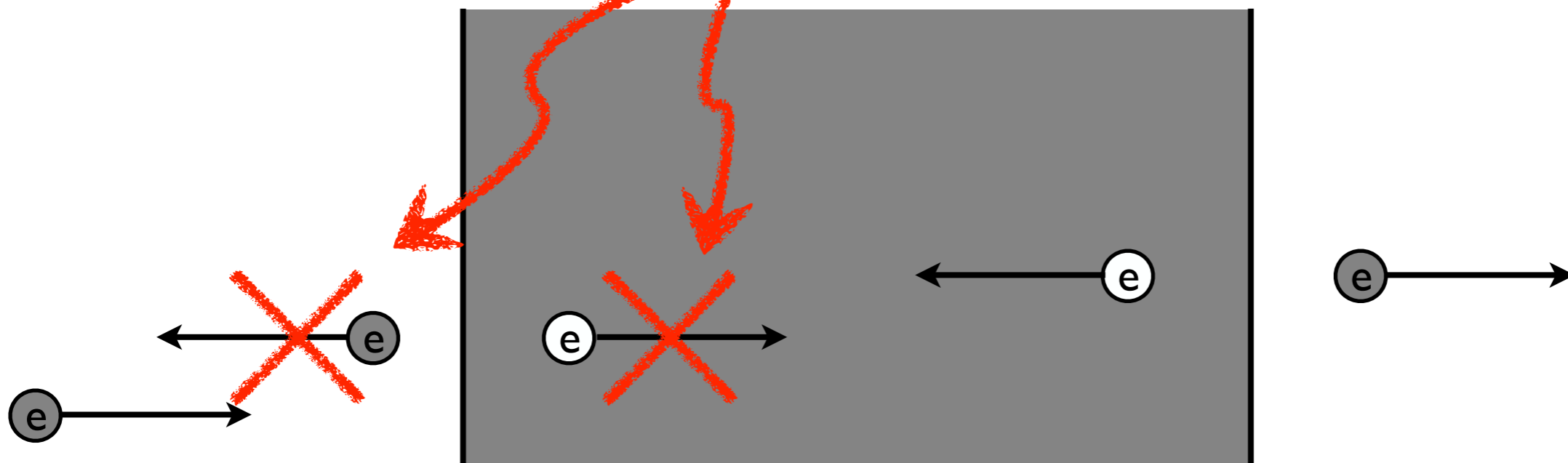
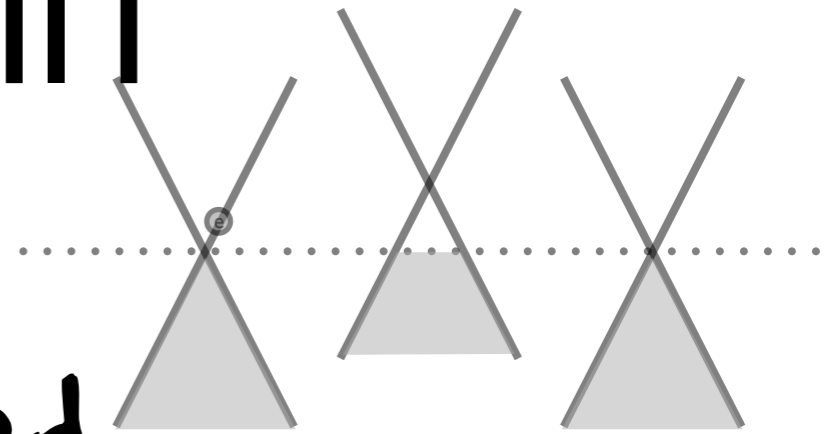
MASSLESS LIMIT

Chirality conserved



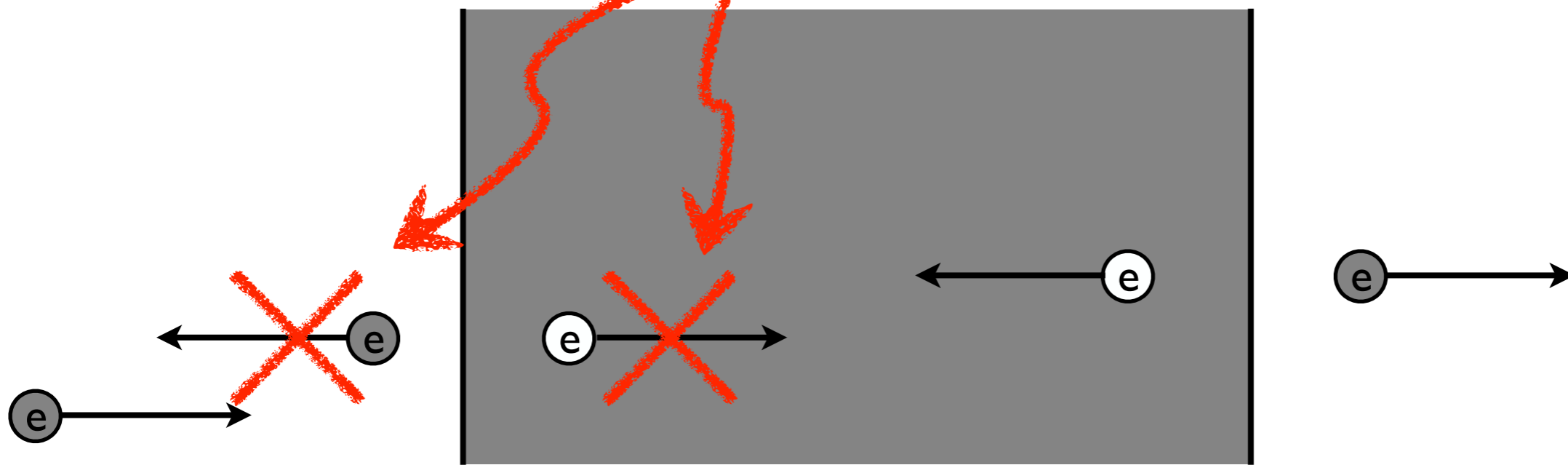
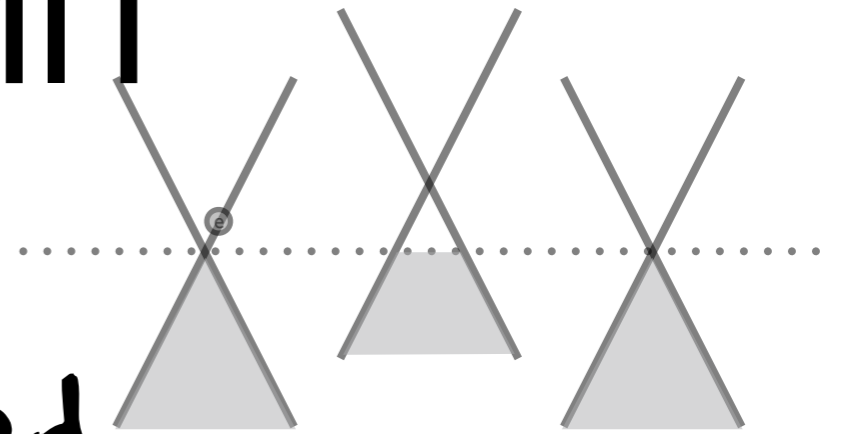
MASSLESS LIMIT

Chirality conserved



MASSLESS LIMIT

Chirality conserved



$$R \simeq 0$$

$$T \simeq 1$$

INFINITE POTENTIAL LIMIT

$$m \ll V \qquad m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \approx \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K \quad K \ll m$

$T \simeq 0$ **CONSISTENT!!**

Ultra-relativistic limit $m \ll E \quad m \ll E \ll V$

$T \simeq 1$

REASONABLE??

INFINITE POTENTIAL LIMIT

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$$T \simeq 0 \quad \text{CONSISTENT!!}$$

MASSLESS LIMIT

Ultra-relativistic limit $m \ll E \quad m \ll E \ll V$

$$T \simeq 1$$

REASONABLE??

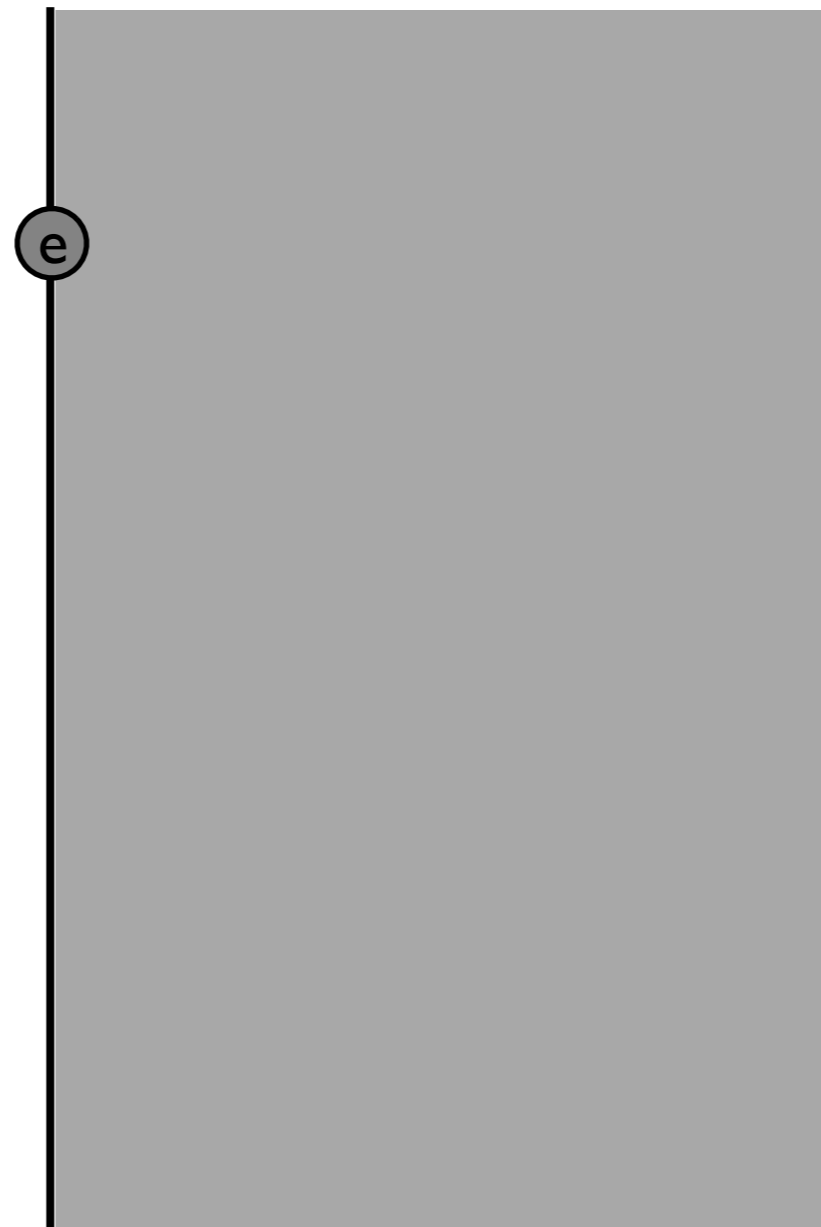
YES

THE FOCUSSED OF ELECTRON FLOW

e

A diagram illustrating the focusing of electron flow. On the left, a small grey circle with the letter 'e' inside represents an electron. To its right is a vertical grey bar. The electron is positioned to the left of the bar, and the bar's left edge is a solid vertical line. The bar itself is a solid grey rectangle.

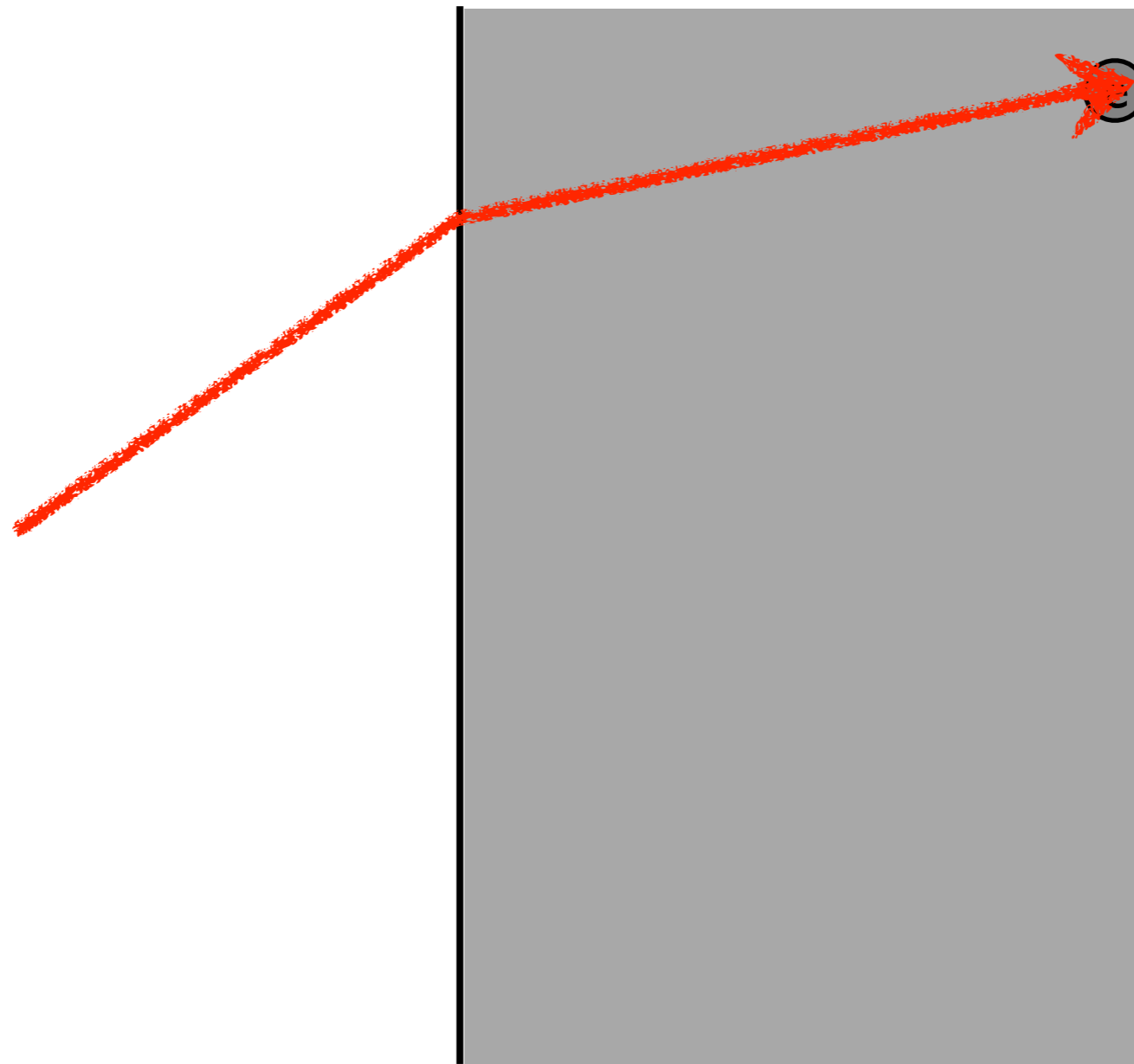
THE FOCUSSED OF ELECTRON FLOW



THE FOCUSSED OF ELECTRON FLOW



THE FOCUSSED OF ELECTRON FLOW



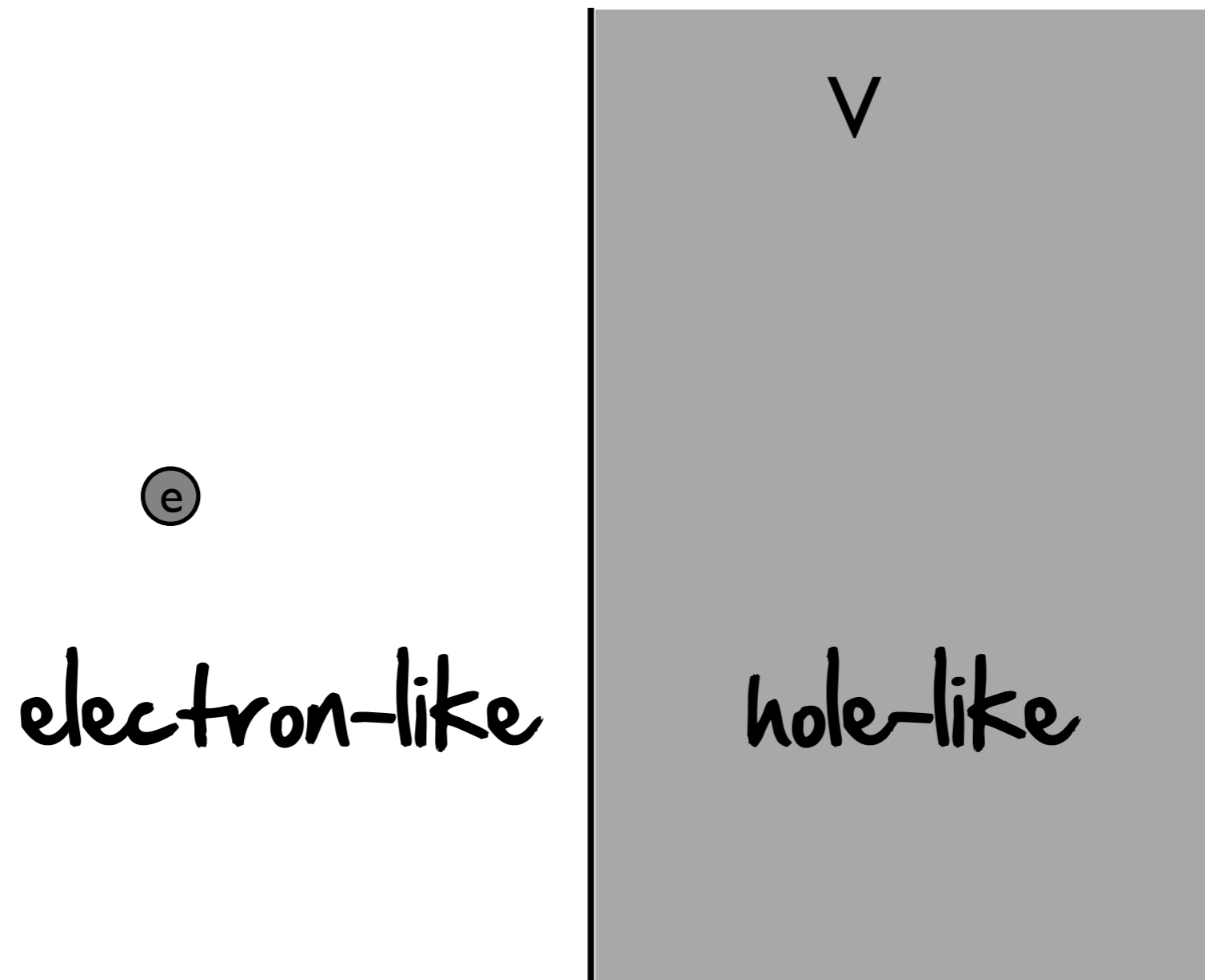
THE FOCUSSED OF ELECTRON FLOW

e

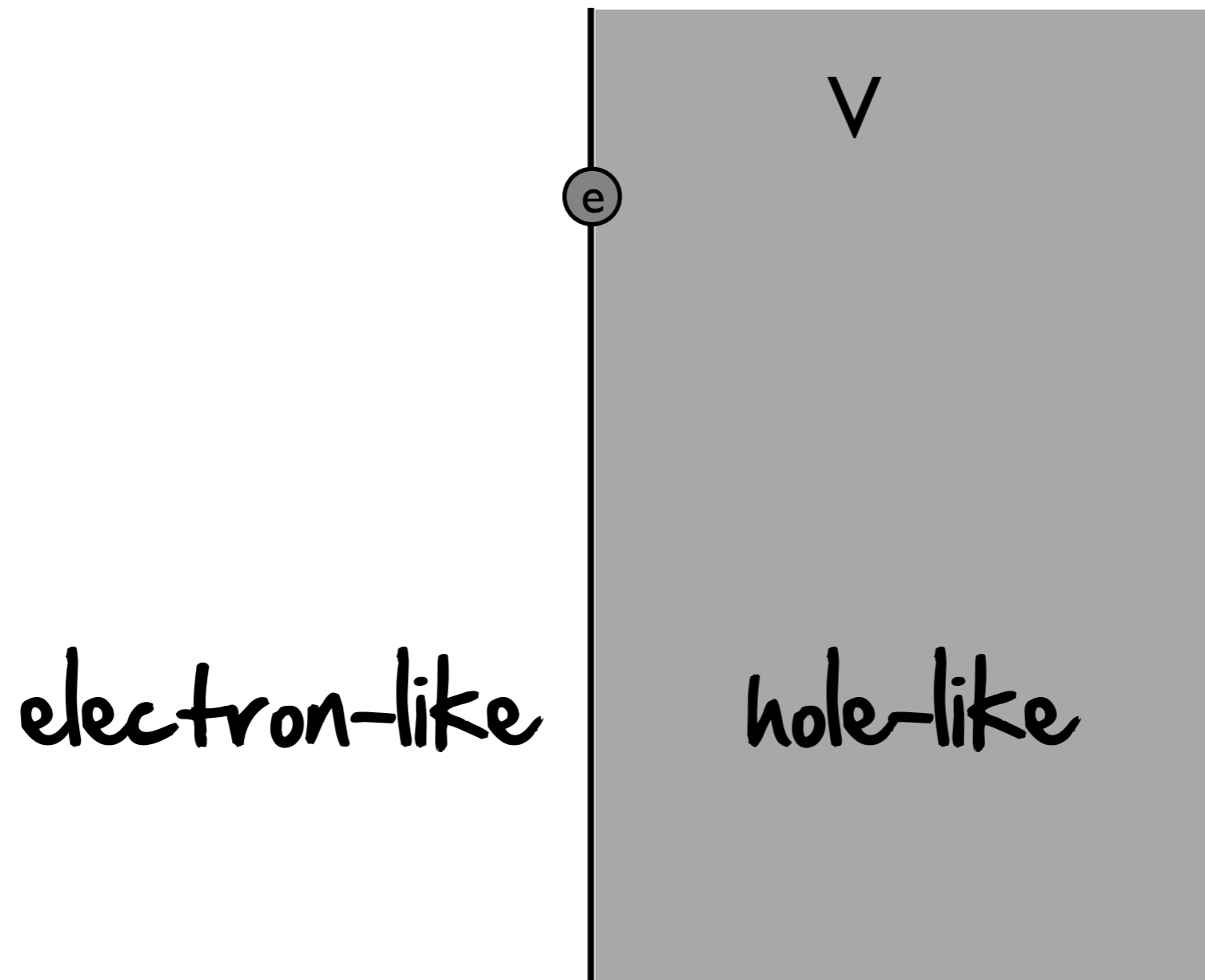
v

A diagram illustrating the focusing of electron flow. On the left, a small grey circle contains the letter 'e', representing an electron. To its right is a vertical grey rectangular bar. Inside the top portion of this bar is the letter 'v'. The bar is positioned such that it appears to be focusing or directing the electron's path.

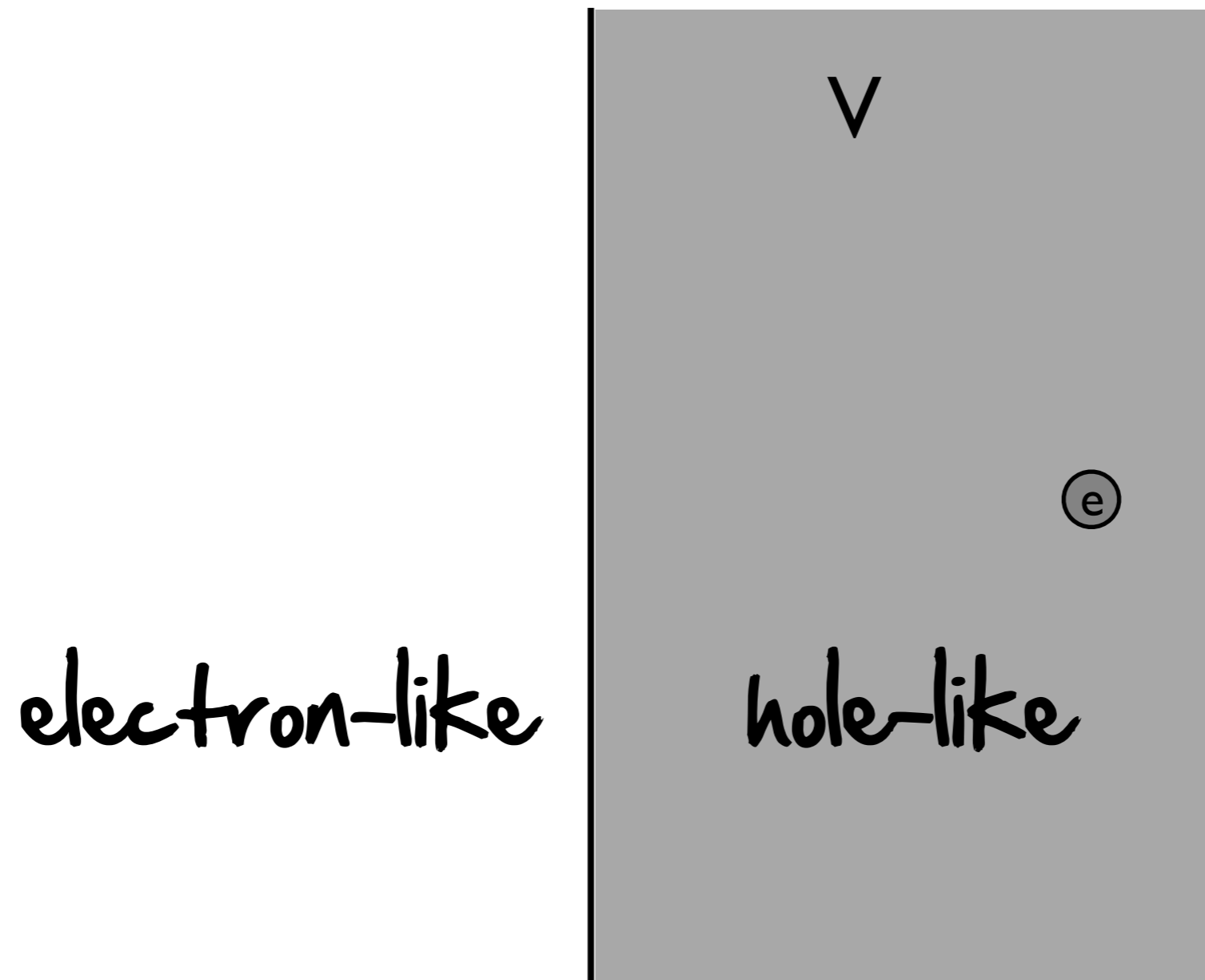
THE FOCUSSED OF ELECTRON FLOW



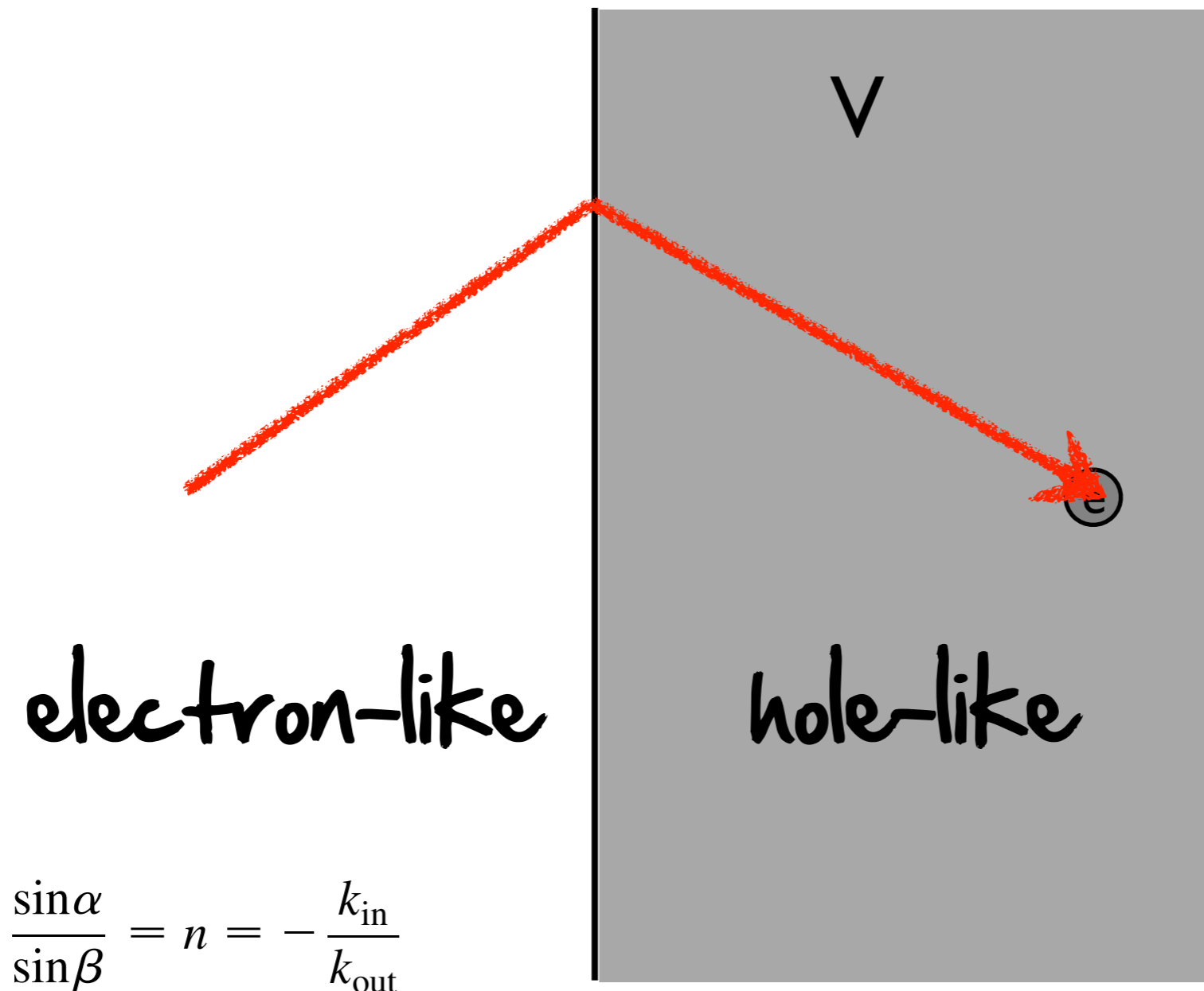
THE FOCUSSED OF ELECTRON FLOW



THE FOCUSSED OF ELECTRON FLOW



THE FOCUSSED OF ELECTRON FLOW



$$\frac{\sin\alpha}{\sin\beta} = n = -\frac{k_{\text{in}}}{k_{\text{out}}}$$

Discussion

The Focusing of Electron Flow and a Veselago Lens in Graphene p-n Junctions Vadim V. Cheianov, et al. Science 315, 1252 (2007);

