2010년 2월 7일 일요일

(1+1) Dirac Hamiltonian

& Klein Paradox

Group 4

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Introduction

DIRAC HAMILTONIAN

DIRAC HAMILTONIAN

$$H_D = -i\alpha_x \frac{\partial}{\partial x} + \beta m + V(x)$$

$$\alpha_x = \sigma_1 \qquad \beta = \sigma_3$$

$$V(x) = \begin{cases} 0, & x<0 \\ V_0(>0), & 0<x<a \\ 0, & x>a \end{cases}$$

CONFIRMATION

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$$\gamma^0 = \sigma_3 \qquad \gamma^1 = \imath\sigma_2$$

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$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$$

CONFIRMATION

$$\gamma^0 = \sigma_3 \qquad\qquad \gamma^1 = \imath\sigma_2$$

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$$

$$\imath\frac{\partial}{\partial t}\psi = H_D\psi$$

CONFIRMATION

$$\gamma^0 = \sigma_3 \qquad \gamma^1 = \imath\sigma_2$$

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$$

$$\imath\frac{\partial}{\partial t}\psi = H_D\psi \longrightarrow \partial_\mu(\bar{\psi}(x,t)\gamma^\mu\psi(x,t)) = 0$$

CONFIRMATION

$$\gamma^0 = \sigma_3 \qquad \gamma^1 = \imath\sigma_2$$

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$$

$$\imath\frac{\partial}{\partial t}\psi = H_D\psi \quad \longrightarrow \quad \partial_\mu(\bar{\psi}(x,t)\gamma^\mu\psi(x,t)) = 0$$

$$\imath\frac{\partial}{\partial t}\psi = H\psi$$

CONFIRMATION

$$\gamma^0 = \sigma_3 \qquad\qquad \gamma^1 = \imath\sigma_2$$

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$$

$$\imath\frac{\partial}{\partial t}\psi = H_D\psi \longrightarrow \partial_\mu(\bar{\psi}(x,t)\gamma^\mu\psi(x,t)) = 0$$

$$\imath\frac{\partial}{\partial t}\psi = H\psi \longrightarrow \partial_t|\psi|^2 + \partial_x(\frac{1}{2m\imath}(\psi^*(\partial_x\psi) - (\partial_x\psi^*)\psi) = 0$$

CONFIRMATION

$$\partial_\mu(\bar{\psi}(x,t)\gamma^\mu\psi(x,t)) = 0$$

vs

$$\partial_t|\psi|^2 + \partial_x(\frac{1}{2m\imath}(\psi^*(\partial_x\psi) - (\partial_x\psi^*)\psi) = 0$$

CONFIRMATION

$$\partial_\mu(\bar{\psi}(x,t)\gamma^\mu\psi(x,t)) = 0$$

vs

$$\partial_t|\psi|^2 + \partial_x(\frac{1}{2m\imath}(\psi^*(\partial_x\psi) - (\partial_x\psi^*)\psi) = 0$$

Which term is continuous??

NALYSI

GROUP 4

Introduction

Analysis

SQUARE BARRIER POTENTIAL PROBLEM

SQUARE BARRIER POTENTIAL PROBLEM

$$H_D = -i\sigma_1 \frac{\partial}{\partial x} + \sigma_3 m + V(x)$$

$$H\psi = E\psi \qquad \psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

SQUARE BARRIER POTENTIAL PROBLEM

$$H_D = -i\sigma_1 \frac{\partial}{\partial x} + \sigma_3 m + V(x)$$

$$H\psi = E\psi \qquad \psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Plane wave solution(free)

$$\psi_p(x) = \begin{pmatrix} \sqrt{\frac{E+m}{p}} \\ \sqrt{\frac{p}{E+m}} \end{pmatrix} e^{\imath px} \qquad \psi_{-p}(x) = \begin{pmatrix} \sqrt{\frac{E+m}{p}} \\ -\sqrt{\frac{p}{E+m}} \end{pmatrix} e^{-\imath px}$$

$$(p = \sqrt{E^2 - m^2})$$

WAVE FUNCTION & BOUNDARY CONDITION

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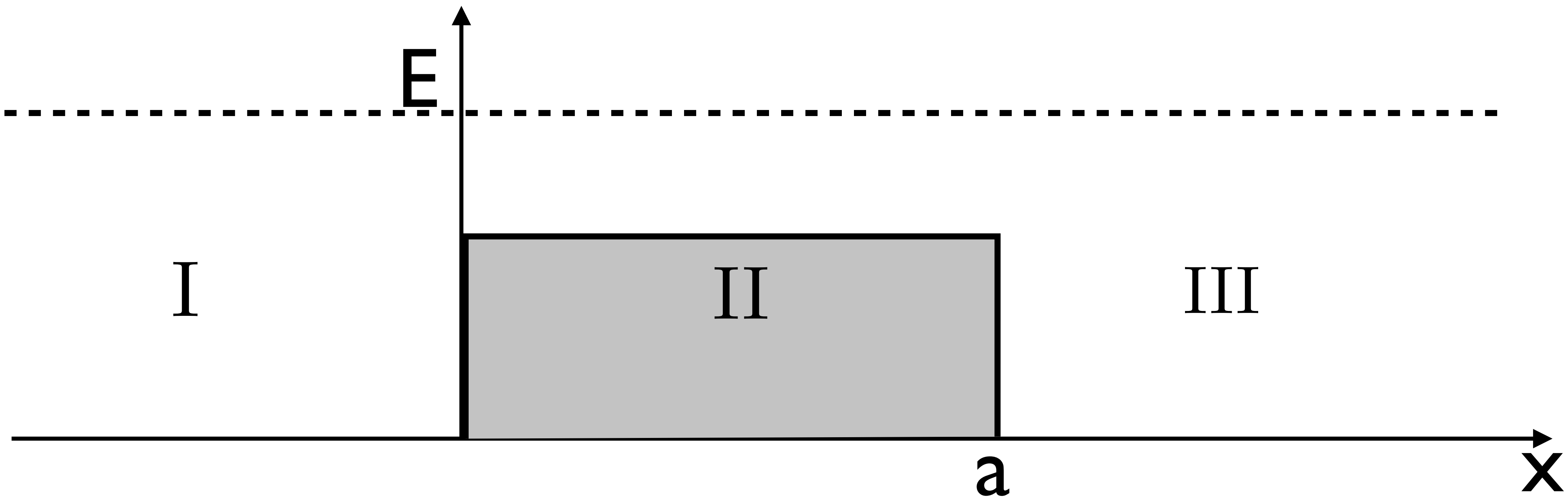
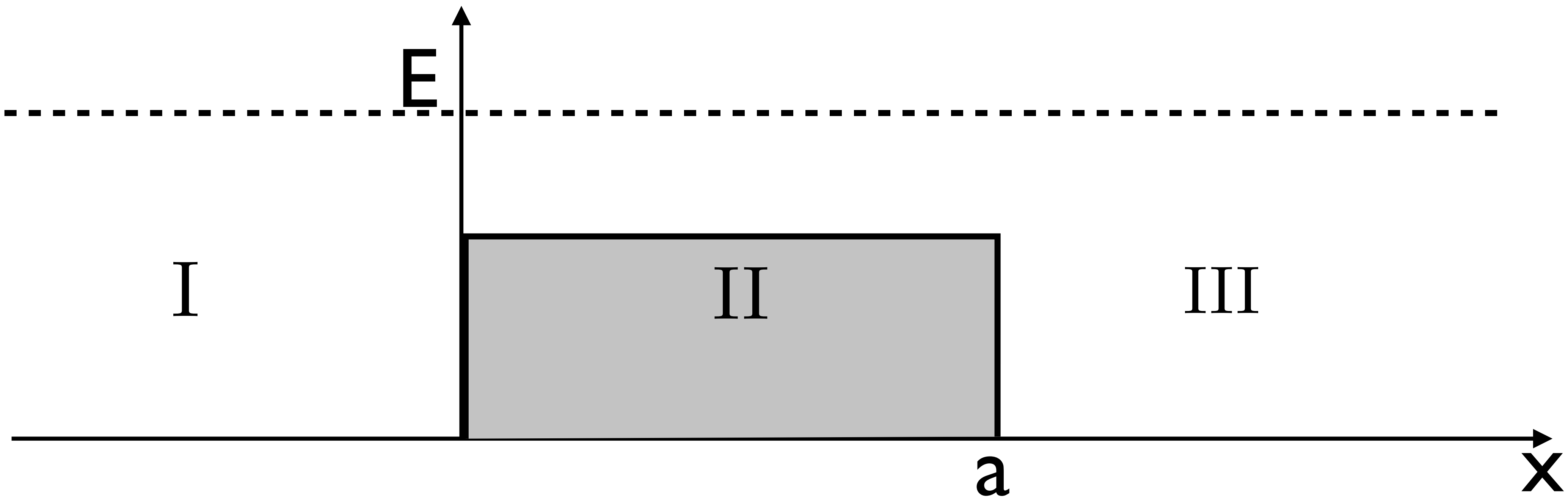


WAVE FUNCTION & BOUNDARY CONDITION

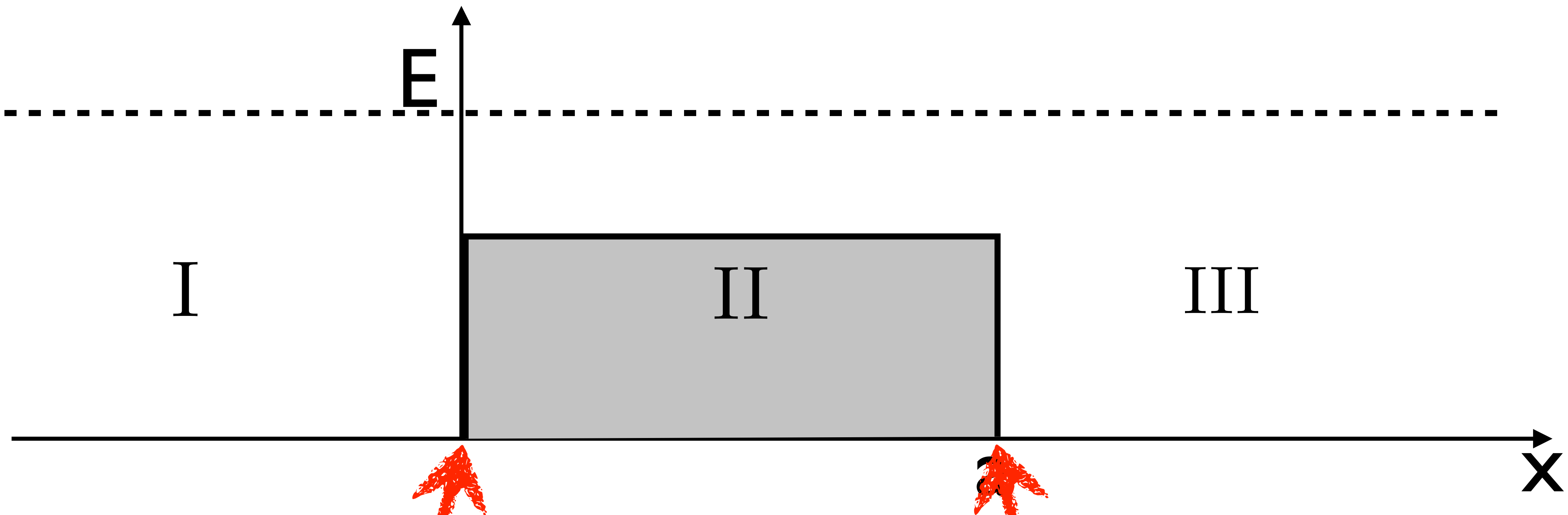


I : $\psi_p(x) + r\psi_{-p}(x)$ $(p = \sqrt{E^2 - m^2})$

II : $A\psi_q(x) + B\psi_{-q}(x)$ $(q = \sqrt{(E - V_0)^2 - m^2})$

III : $t\psi_p(x)$

WAVE FUNCTION & BOUNDARY CONDITION



I	:	$\psi_p(x) + r\psi_{-p}(x)$	$(p = \sqrt{E^2 - m^2})$
II	:	$A\psi_q(x) + B\psi_{-q}(x)$	$(q = \sqrt{(E - V_0)^2 - m^2})$
III	:	$t\psi_p(x)$	

Continuity at x=0, a

TRANSMITTANCE & REFLECTANCE

TRANSMITTANCE & REFLECTANCE

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa}$$

$$R = |r|^2 = \frac{\frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa}$$

$$(\rho = \frac{q}{p}\frac{E+m}{E - V_0 + m})$$

ENERGY RANGE OF INTEREST

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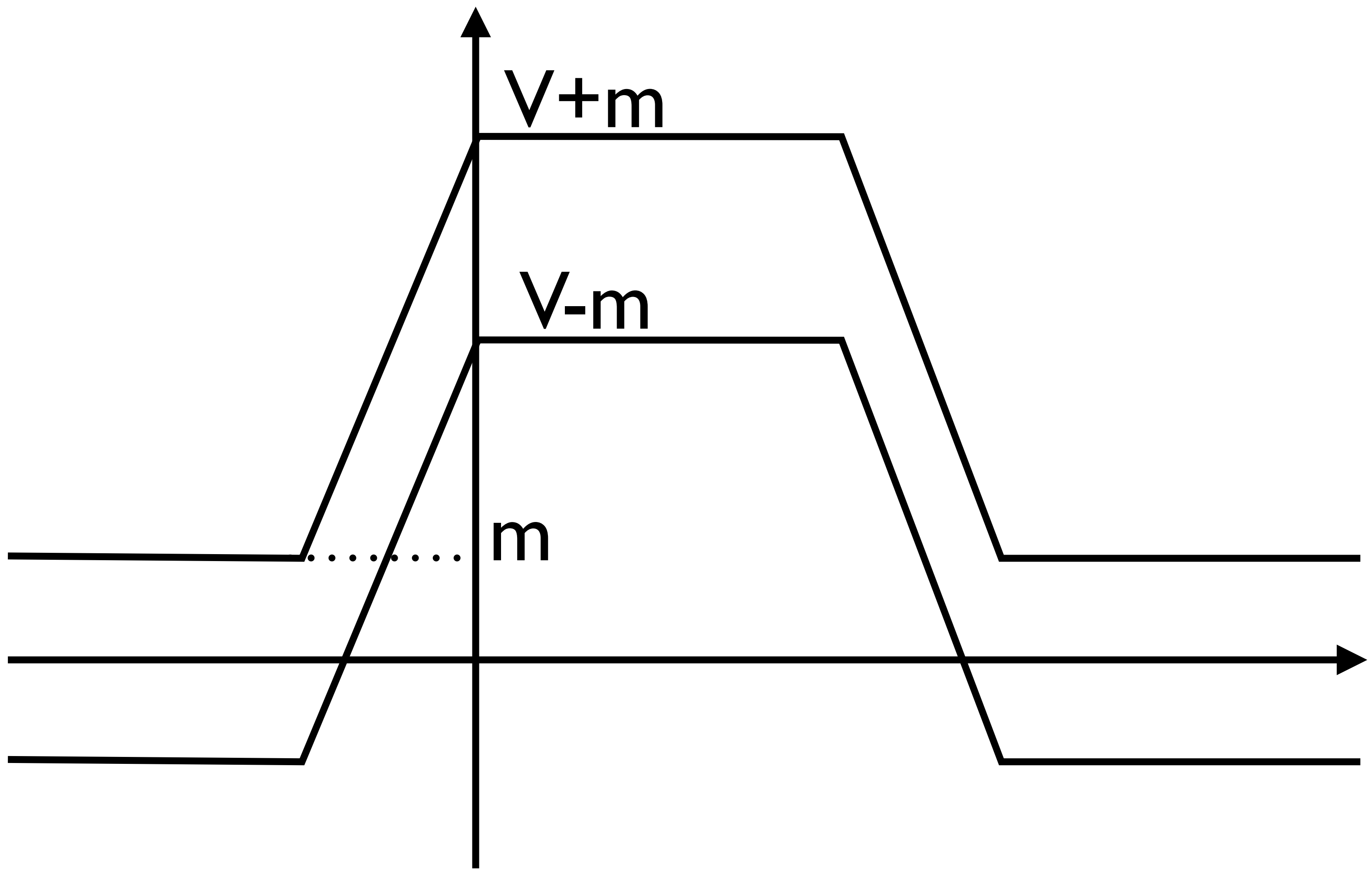

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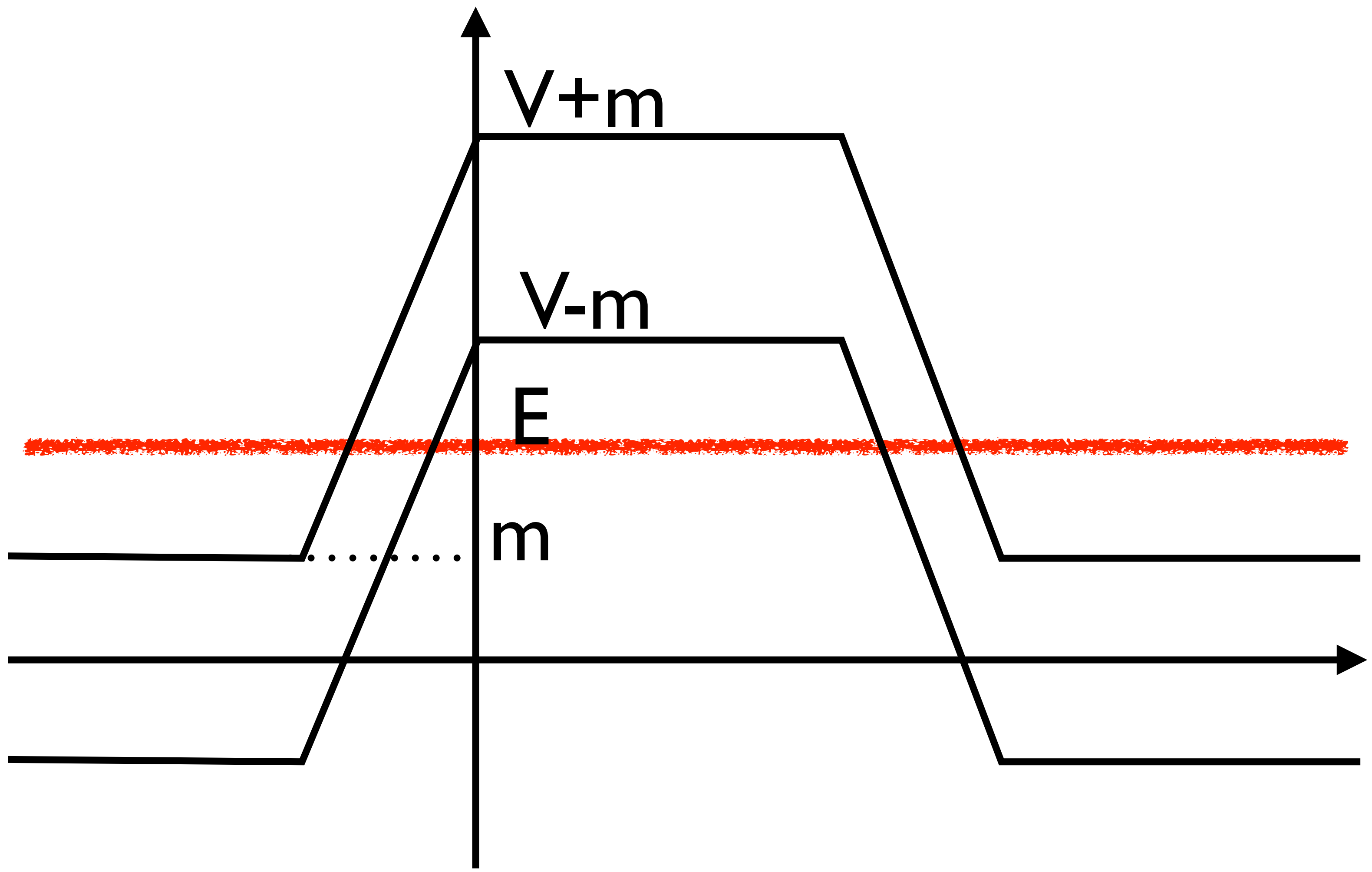

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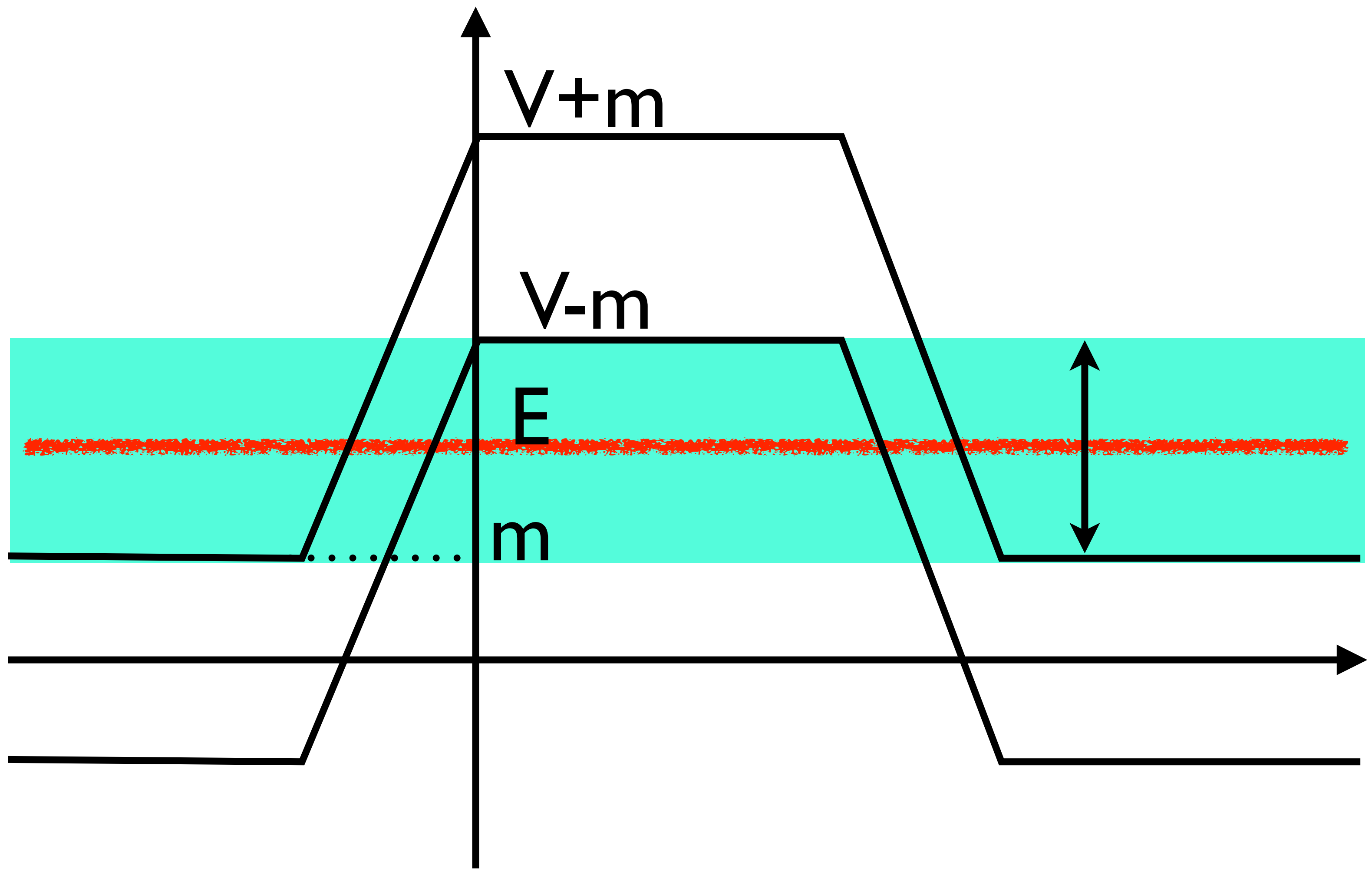
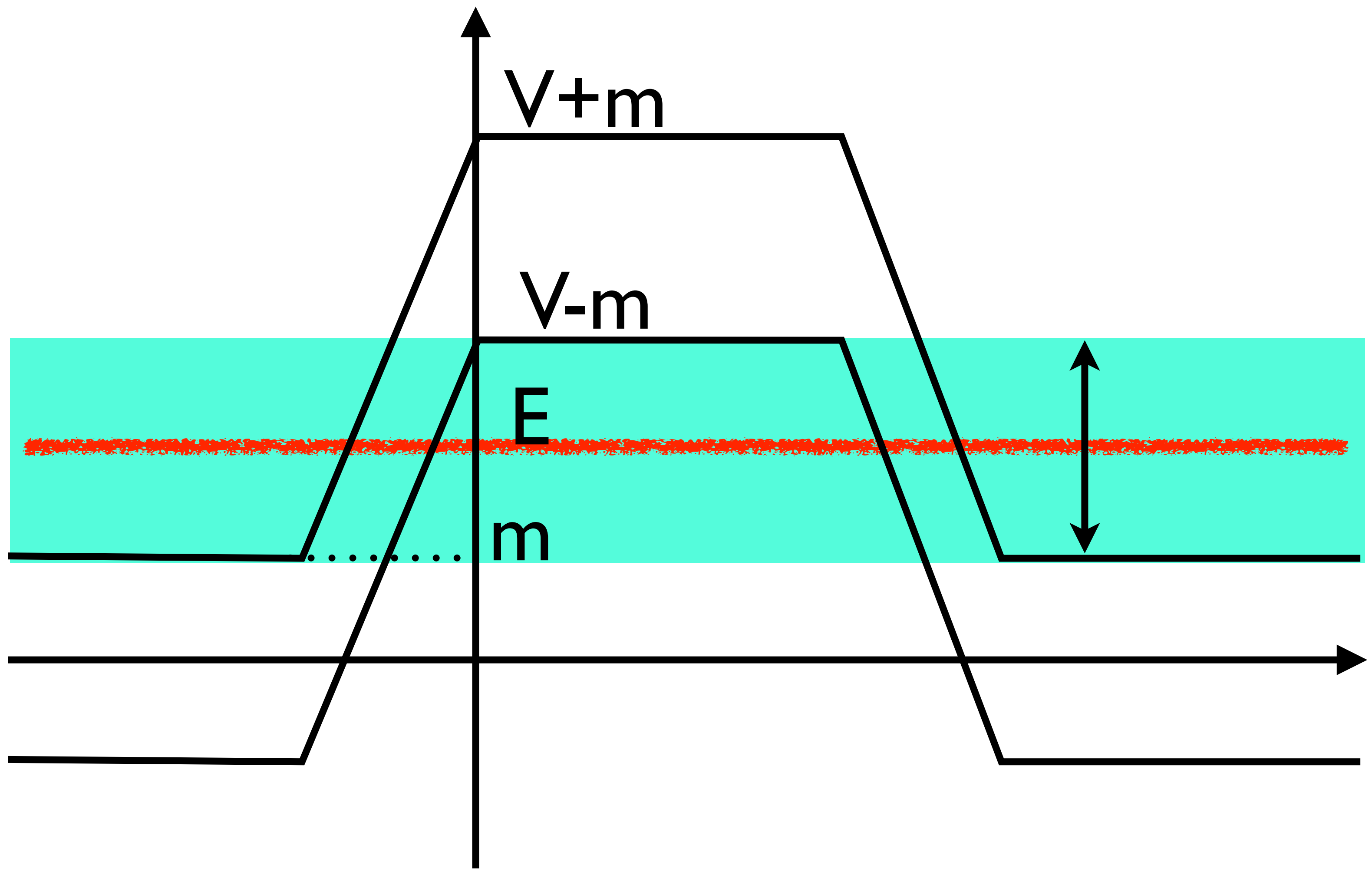


ENERGY RANGE OF INTEREST



$q = \sqrt{(E - V_0)^2 - m^2}$: Real $m < E < V - m$

NON-RELATIVISTIC LIMIT

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$V_0 << m$

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$V_0 << m \qquad E = m + K \qquad K \ll m$

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$$\simeq \frac{1}{1 + \frac{V_0^2}{4K(K - V_0)} \sin^2(a\sqrt{2m(K - V_0)})}$$

NON-RELATIVISTIC LIMIT

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$$R = 1 - T$$

NON-RELATIVISTIC LIMIT

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$$\simeq \frac{1}{1 + \frac{V_0^2}{4K(K-V_0)} \sin^2(a\sqrt{2m(K - V_0)})}$$

$$R = 1 - T$$

Exactly Same!!

INFINITE POTENTIAL LIMIT

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$m \ll V \qquad m < E < V - m$

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$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

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$$R = 1 - T$$

Non-relativistic limit $E = m + K \quad K \ll m$

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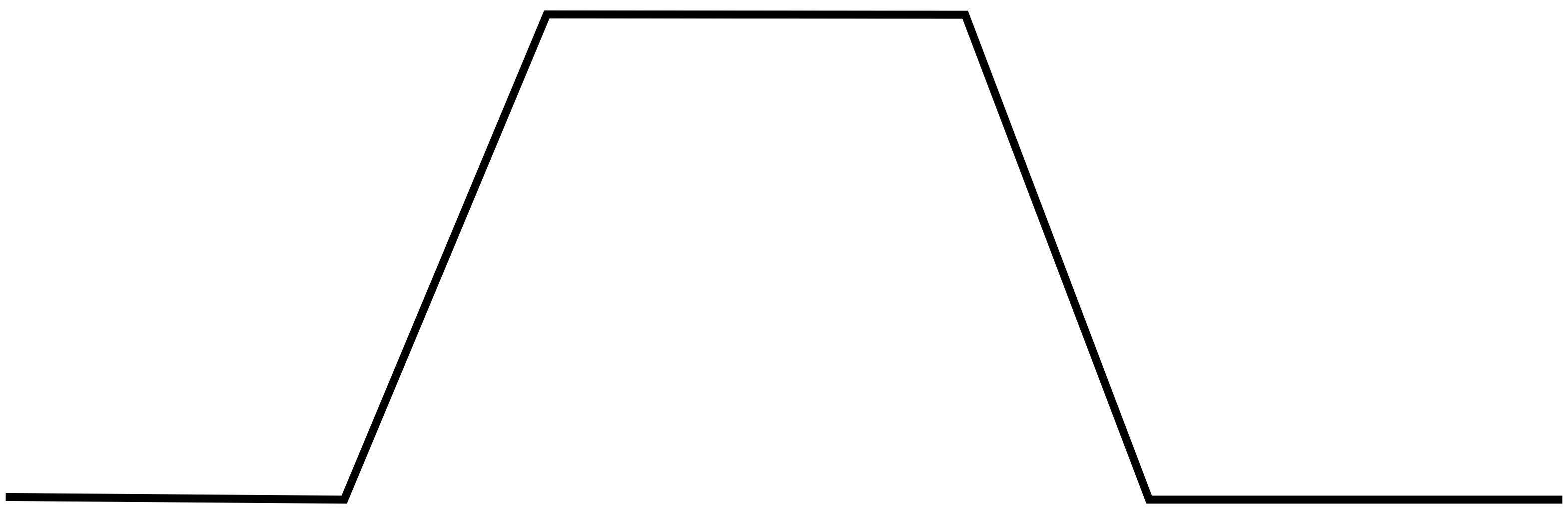
$$R = 1 - T$$

Non-relativistic limit $E = m + K \quad K \ll m$

$$T \simeq 0$$

Analysis



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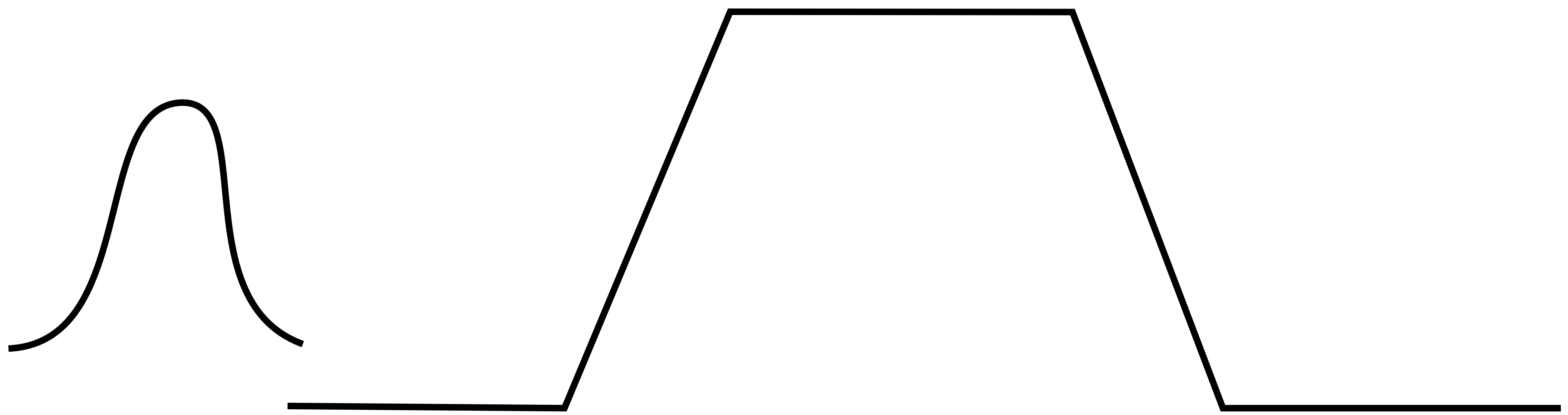

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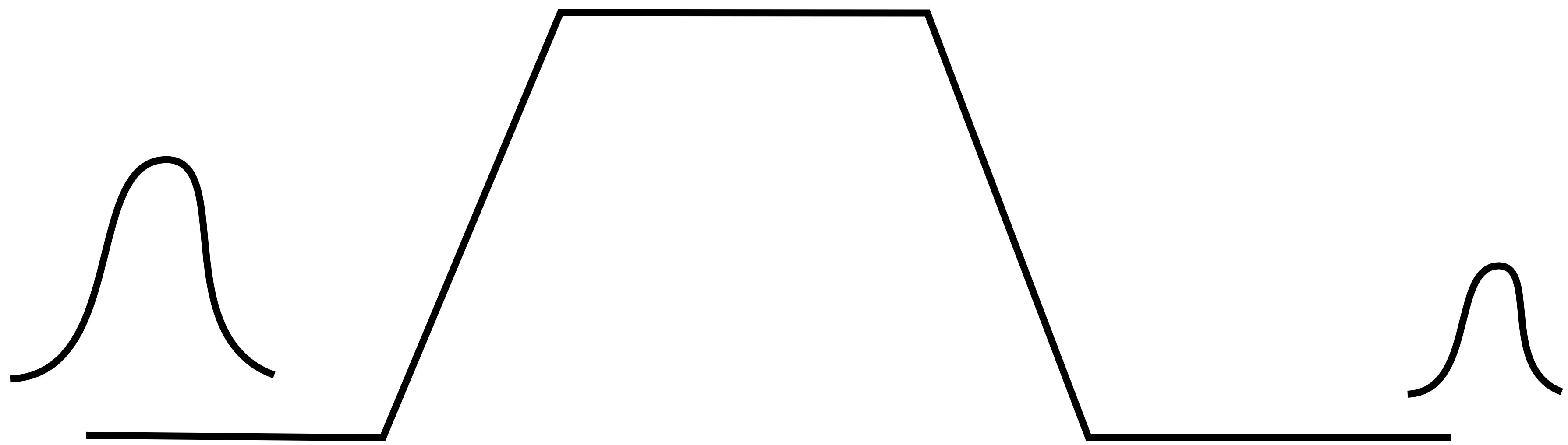

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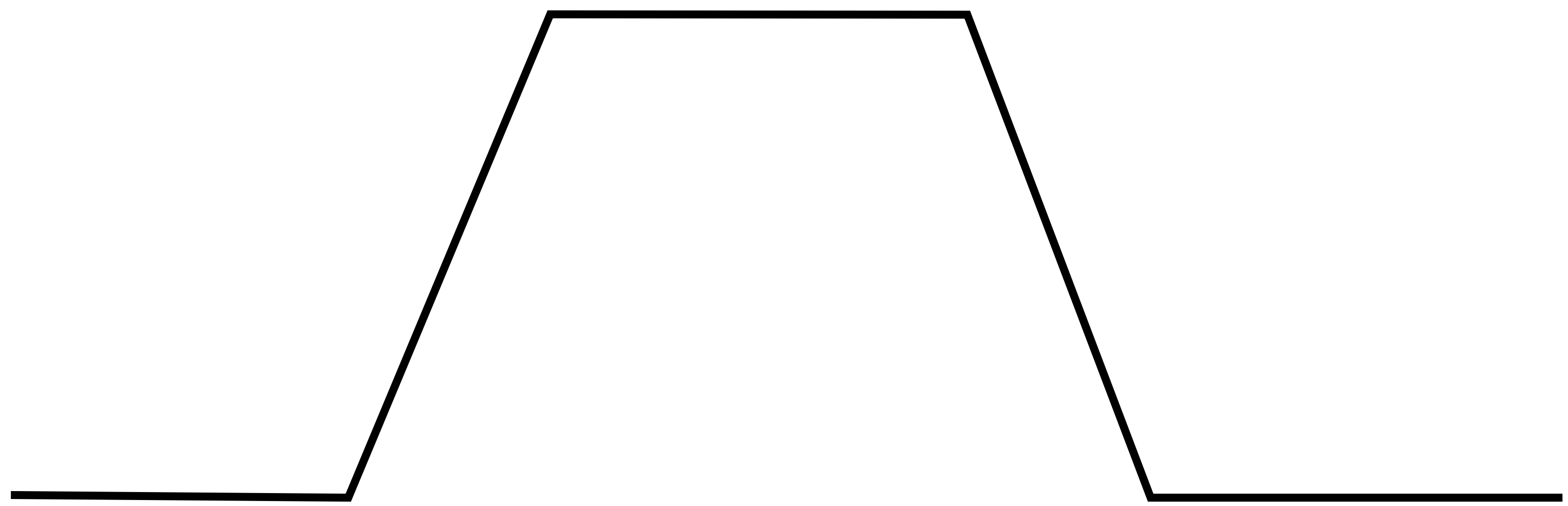

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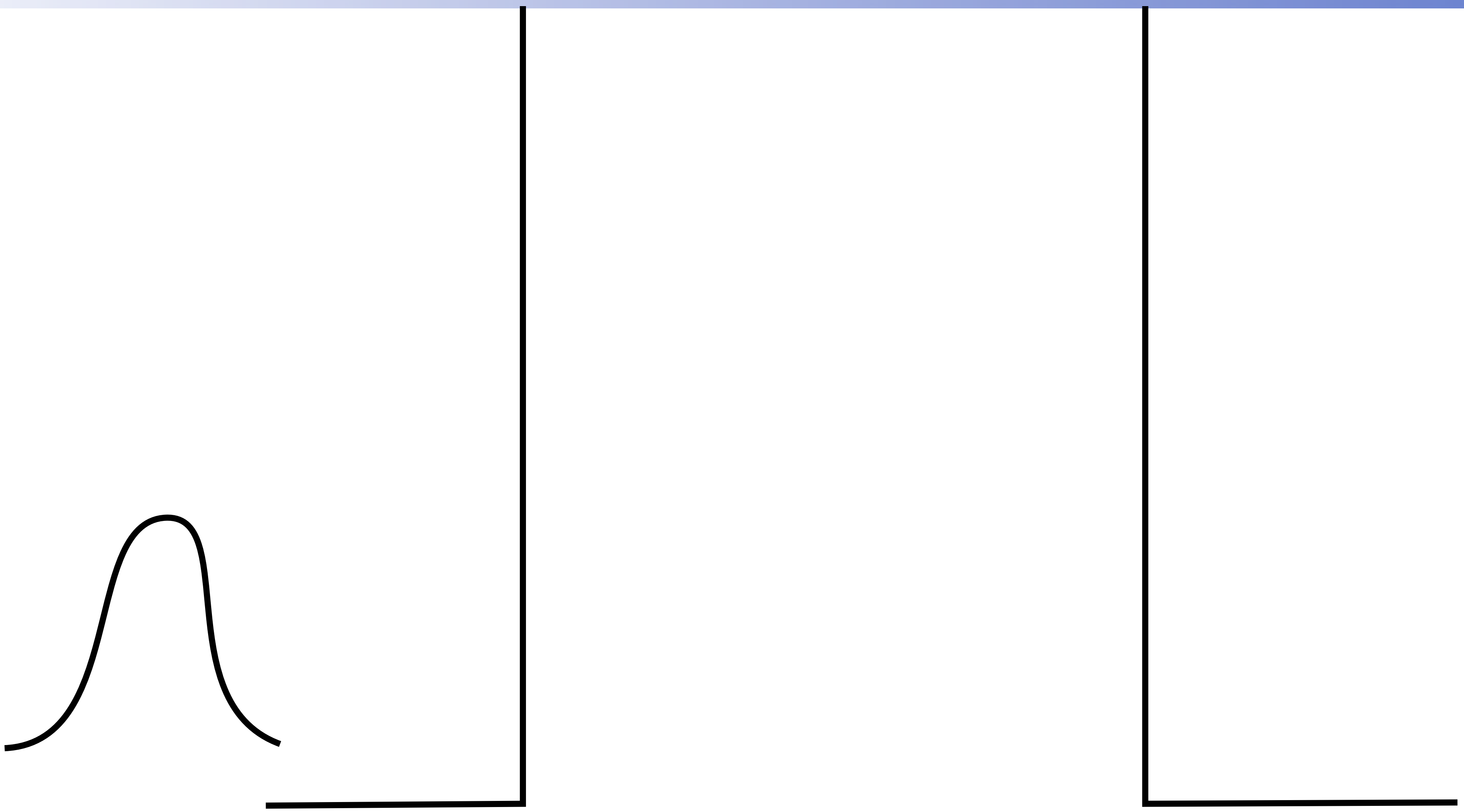
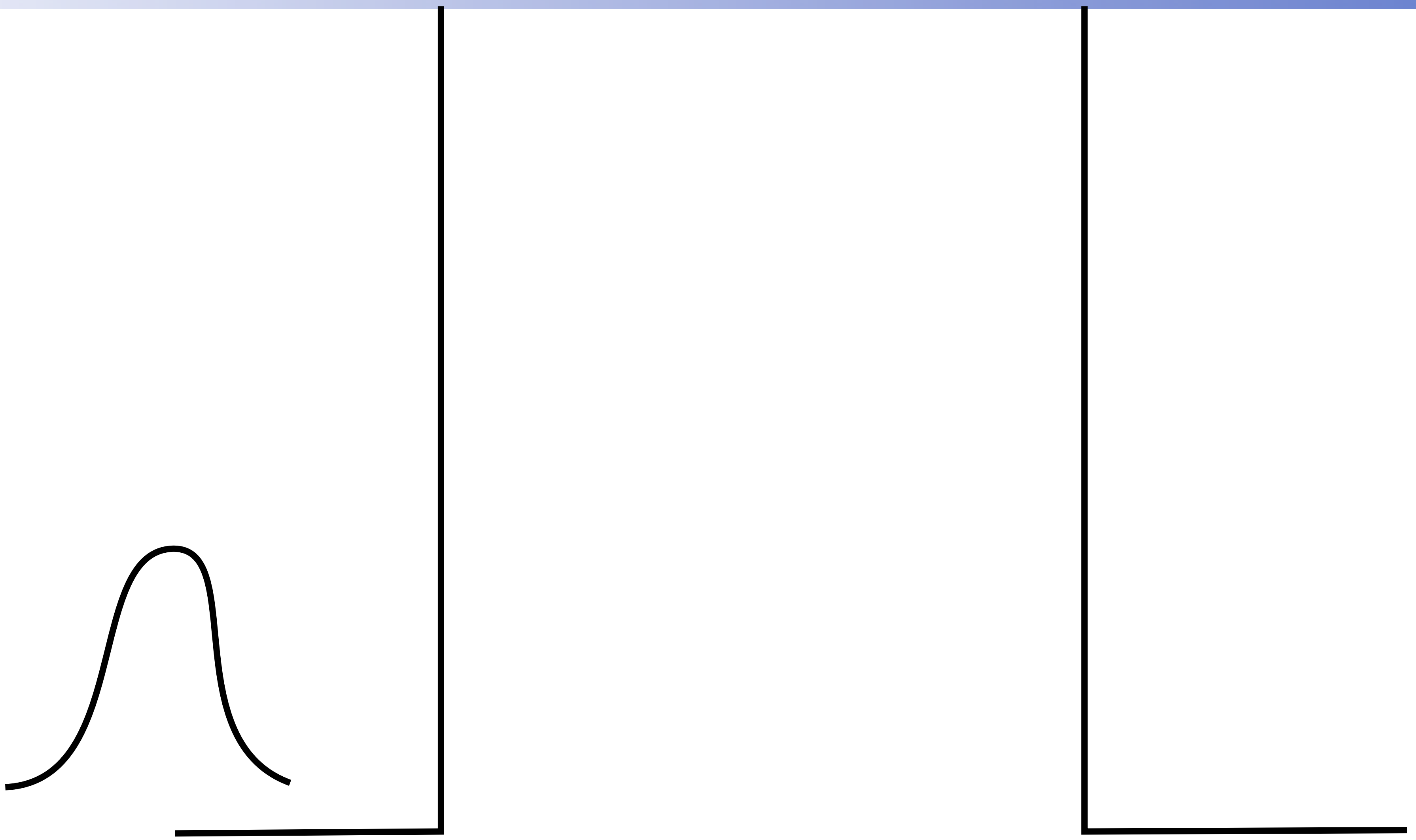




INFINITE POTENTIAL LIMIT

$$m \ll V \qquad\qquad m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K \quad K \ll m$

$$T \simeq 0$$

INFINITE POTENTIAL LIMIT

$$m \ll V \qquad m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K \quad K \ll m$

$T \simeq 0$ CONSISTENT!!

INFINITE POTENTIAL LIMIT

$$m \ll V \qquad m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K$ $K \ll m$

$T \simeq 0$ CONSISTENT!!

Ultra-relativistic limit $m \ll E$ $m \ll E \ll V$

INFINITE POTENTIAL LIMIT

$$m \ll V \qquad m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K \quad K \ll m$

$$T \simeq 0$$ CONSISTENT!!

Ultra-relativistic limit $m \ll E \quad m \ll E \ll V$

$$T \simeq 1$$

INFINITE POTENTIAL LIMIT

$$m \ll V \qquad m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K \quad K \ll m$

$$T \simeq 0$$

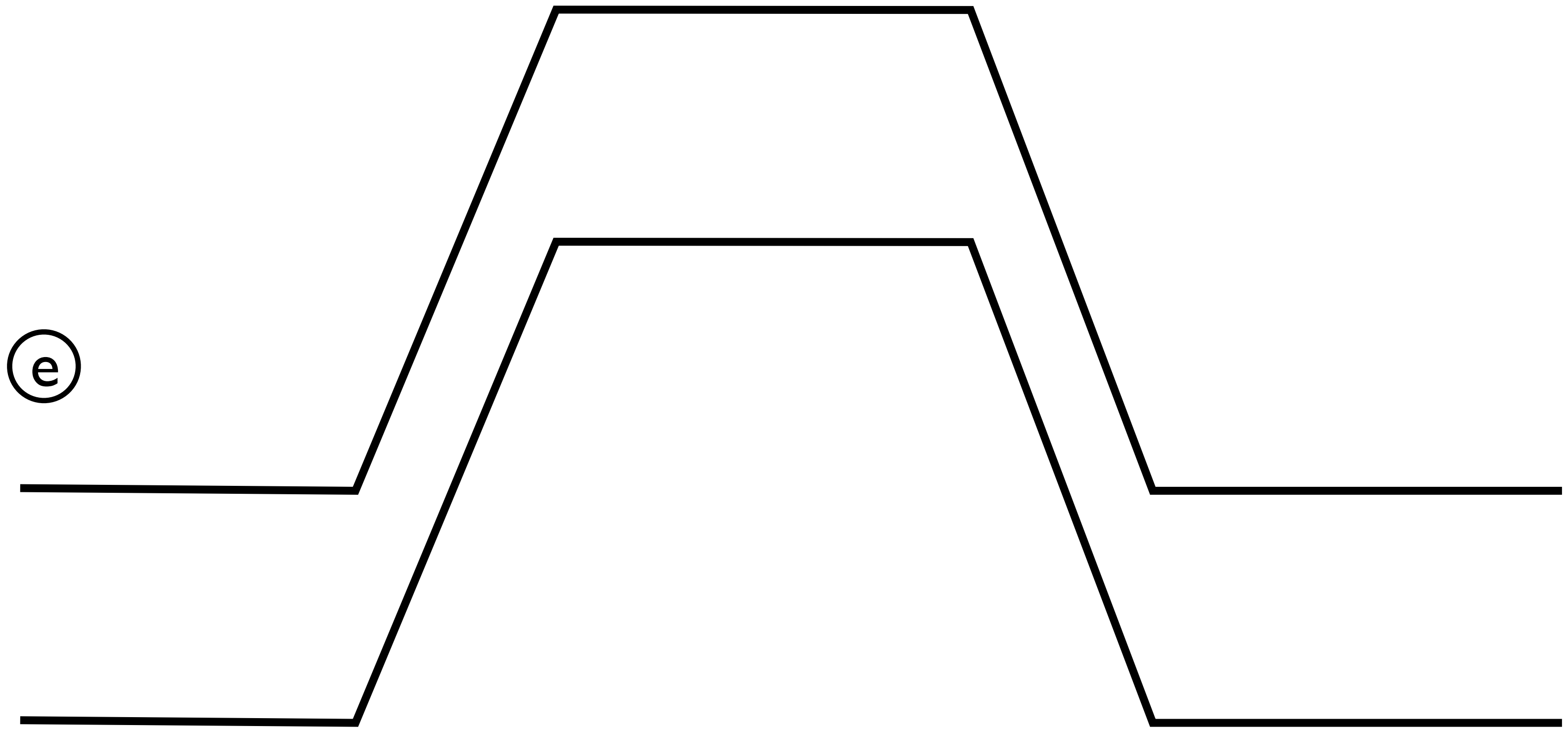
CONSISTENT!!

Ultra-relativistic limit $m \ll E \quad m \ll E \ll V$

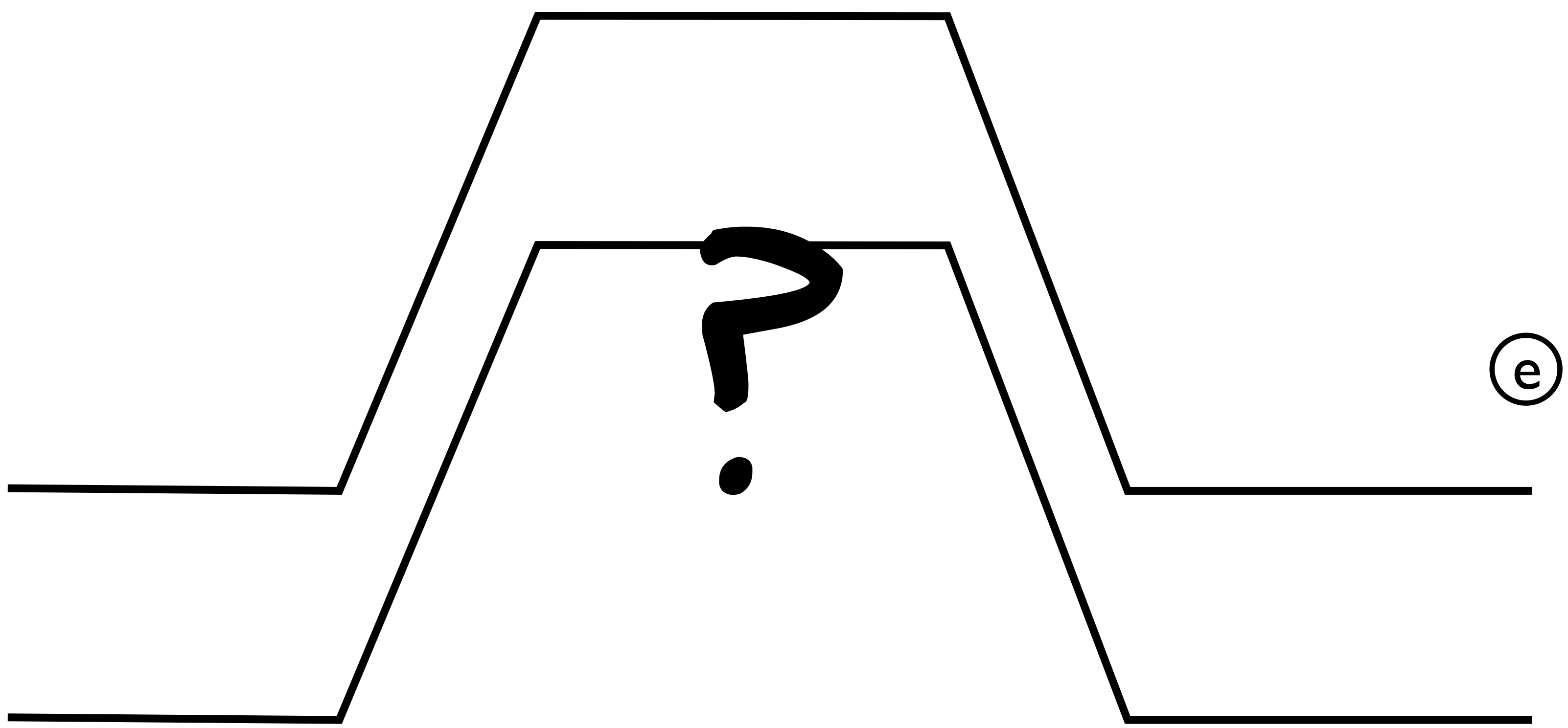
$$T \simeq 1$$

REASONABLE??

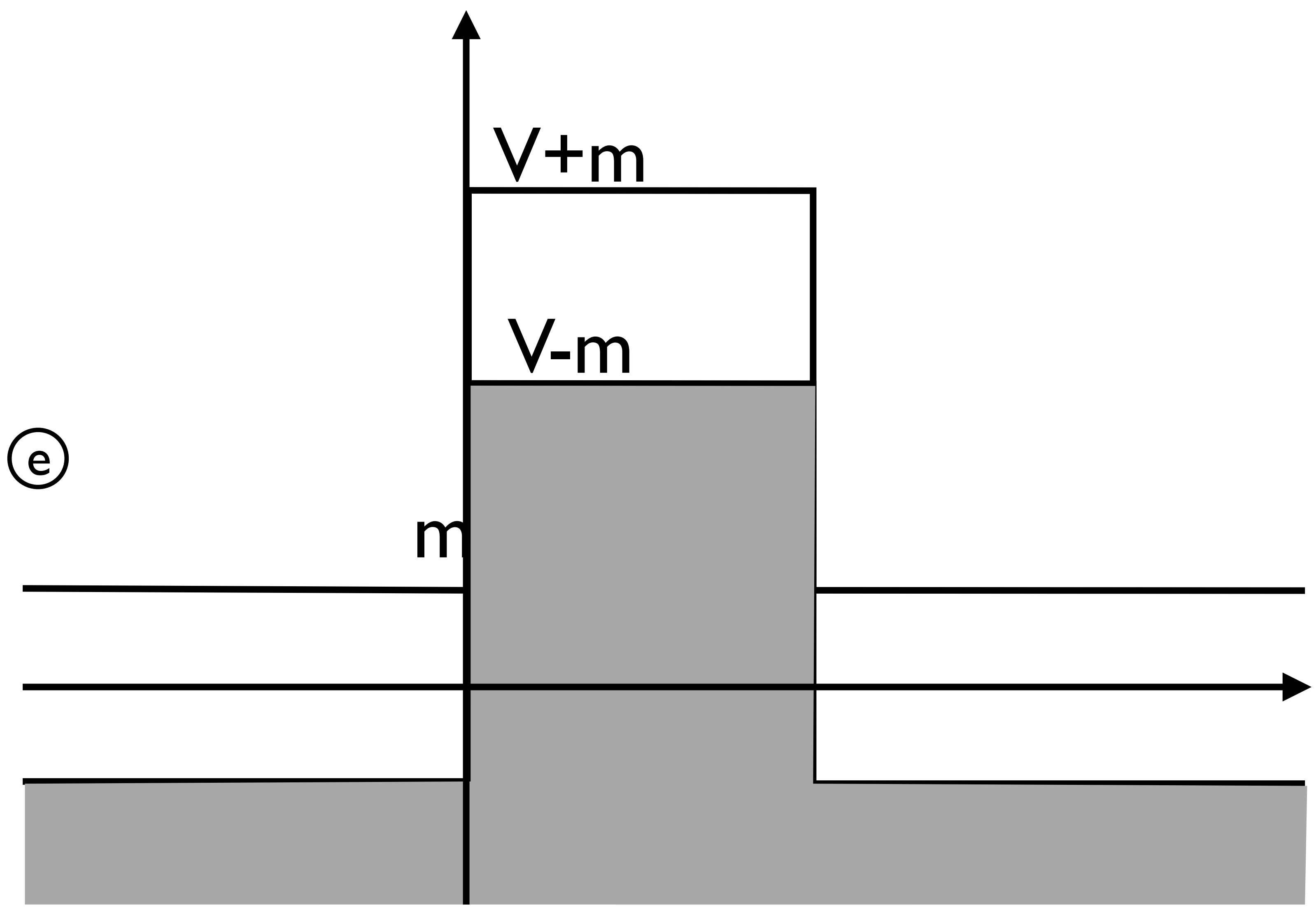
PROCESS



PROCESS



Analysis



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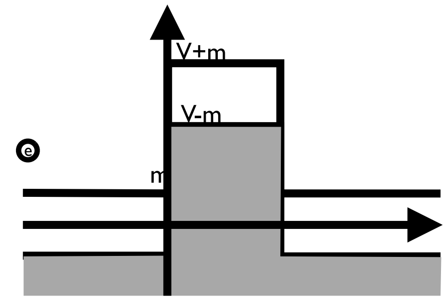

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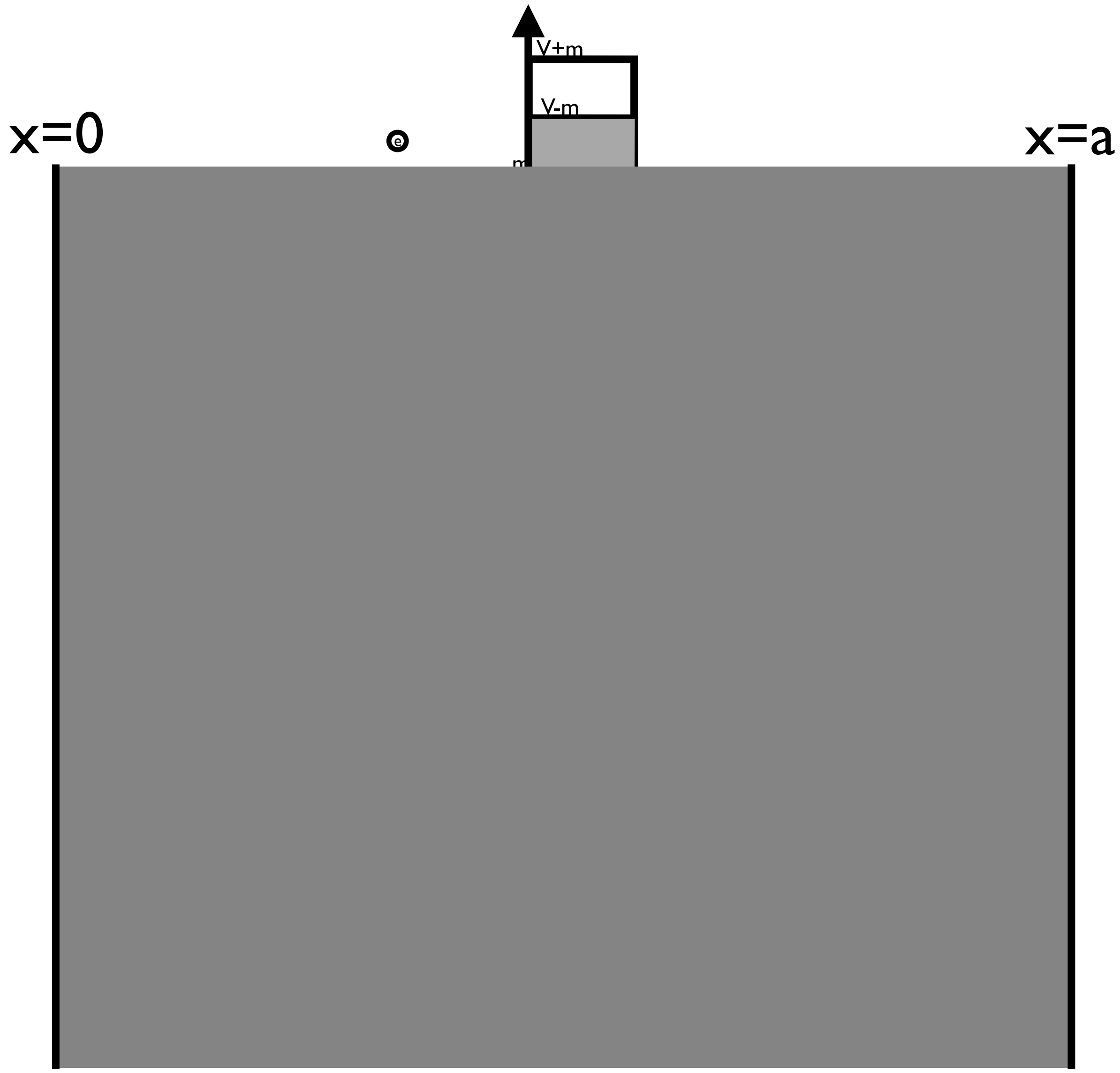
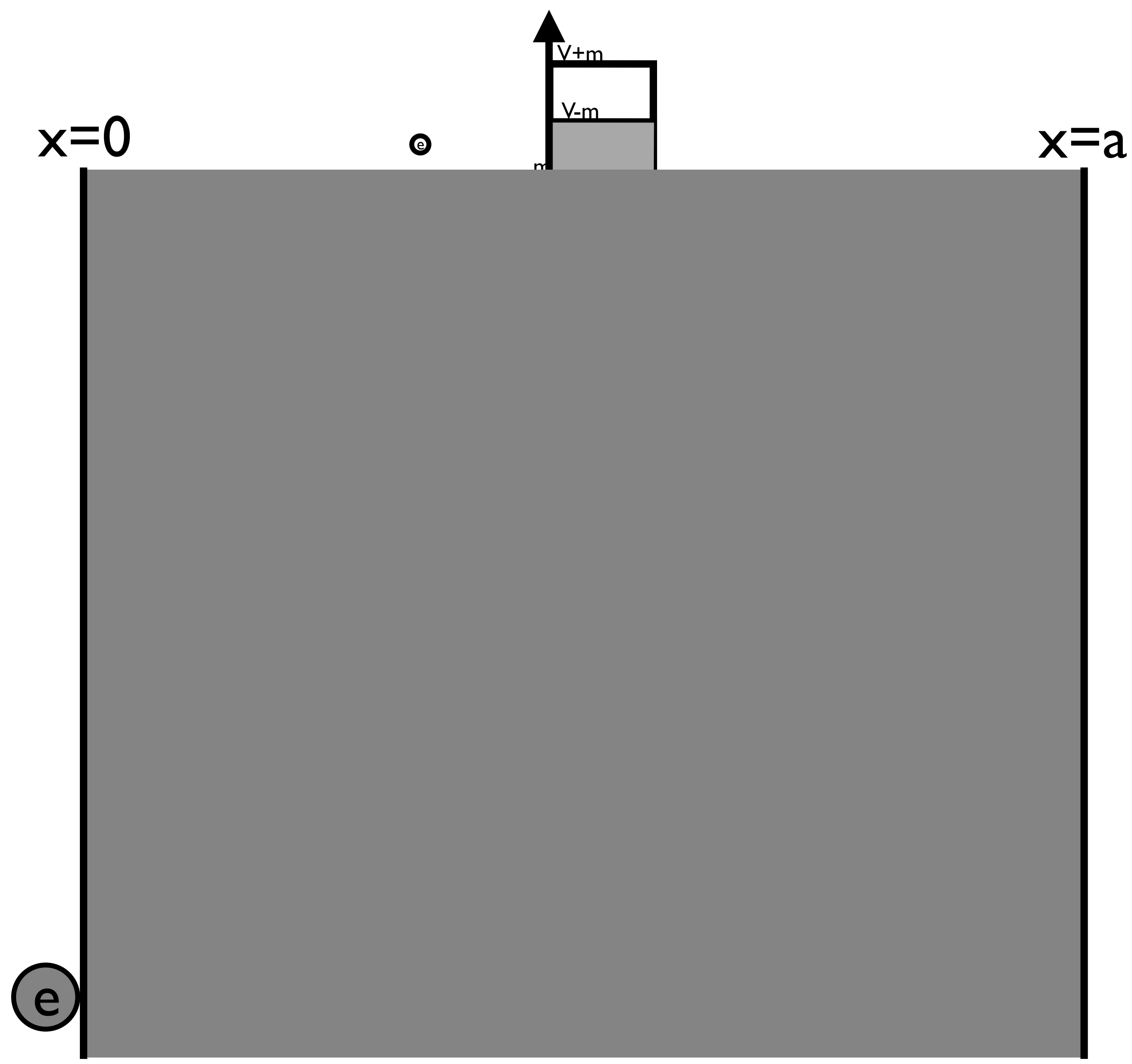
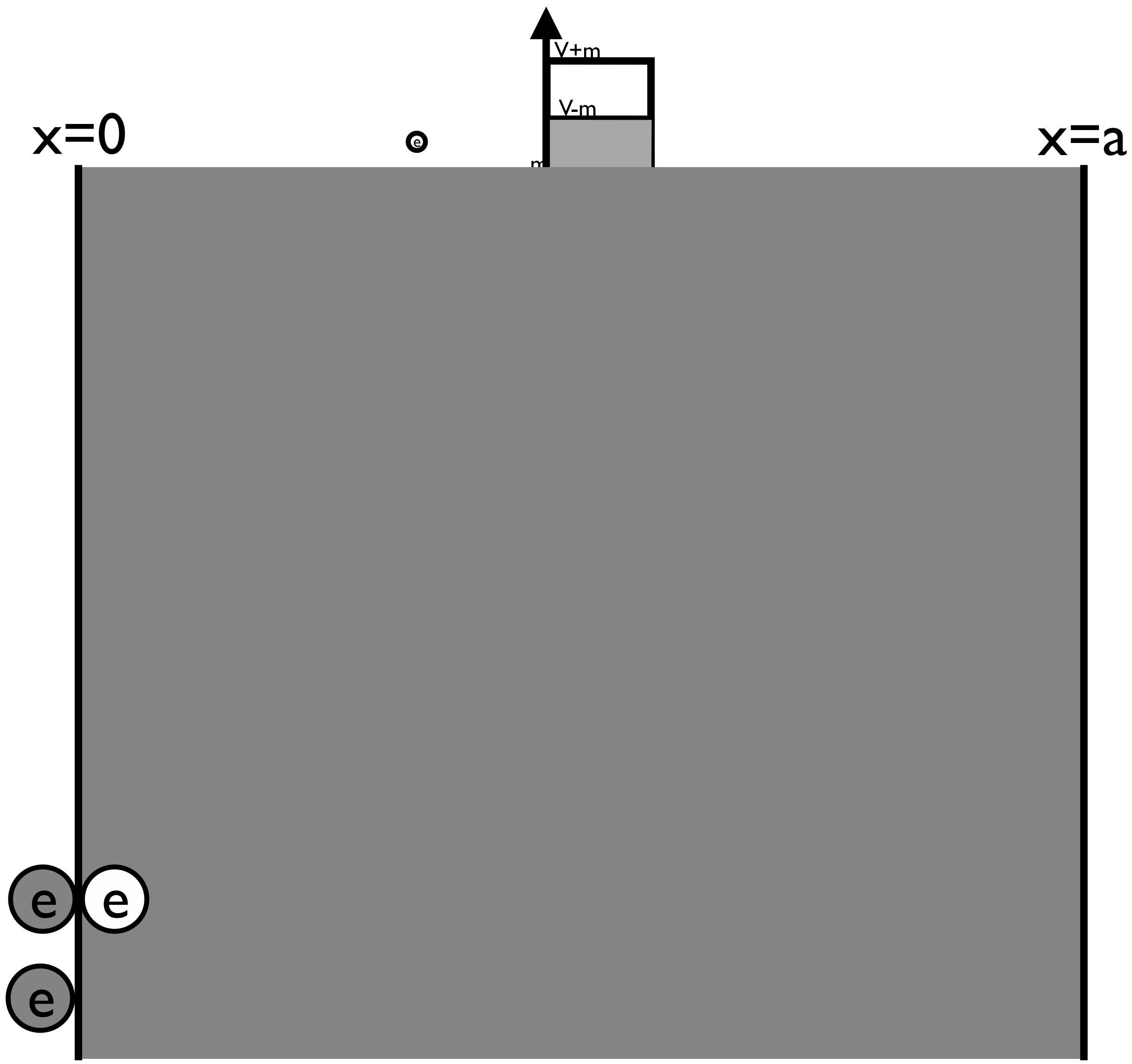



V+m
V-m
x=0
x=a
e

Analysis



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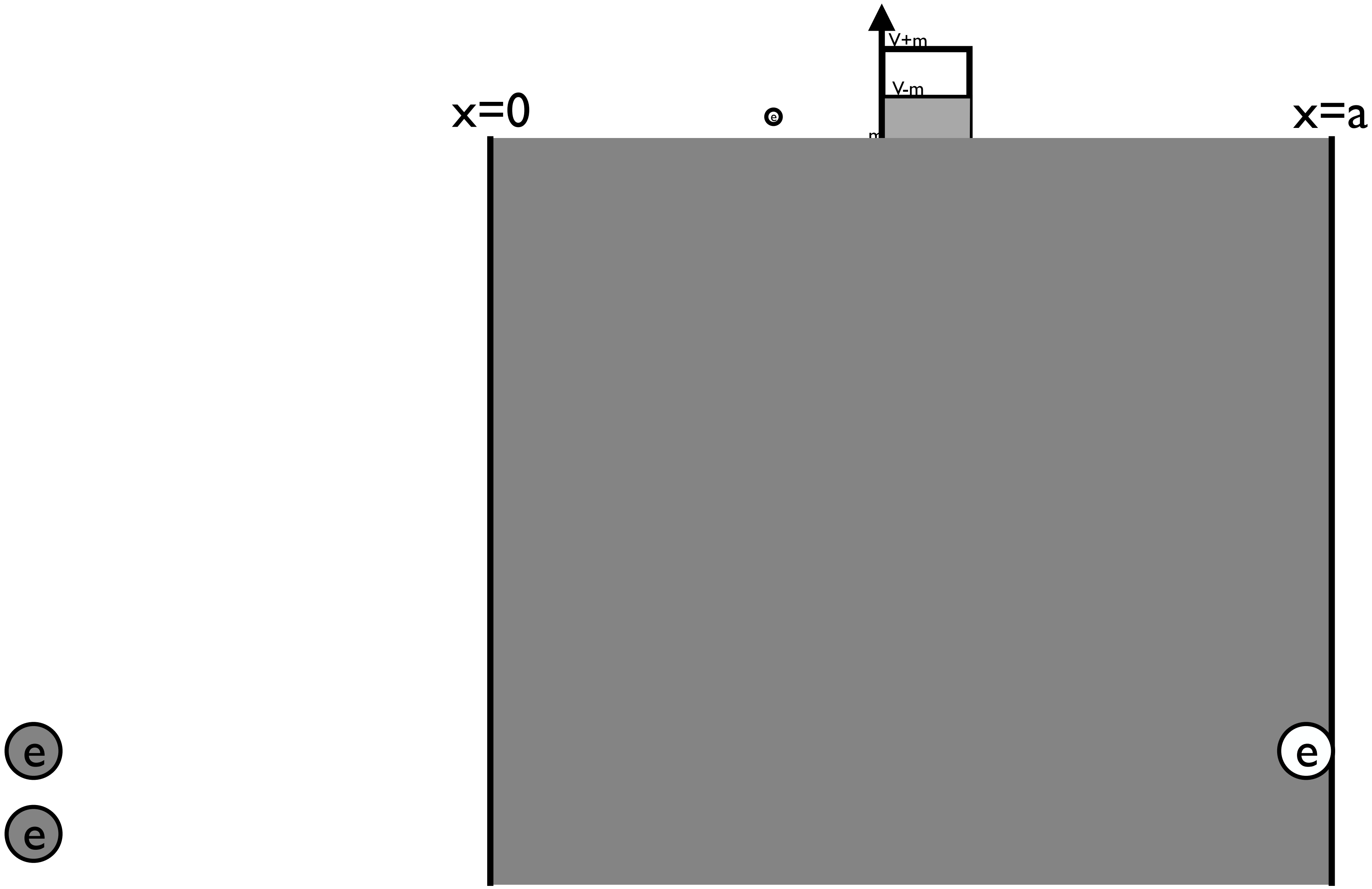

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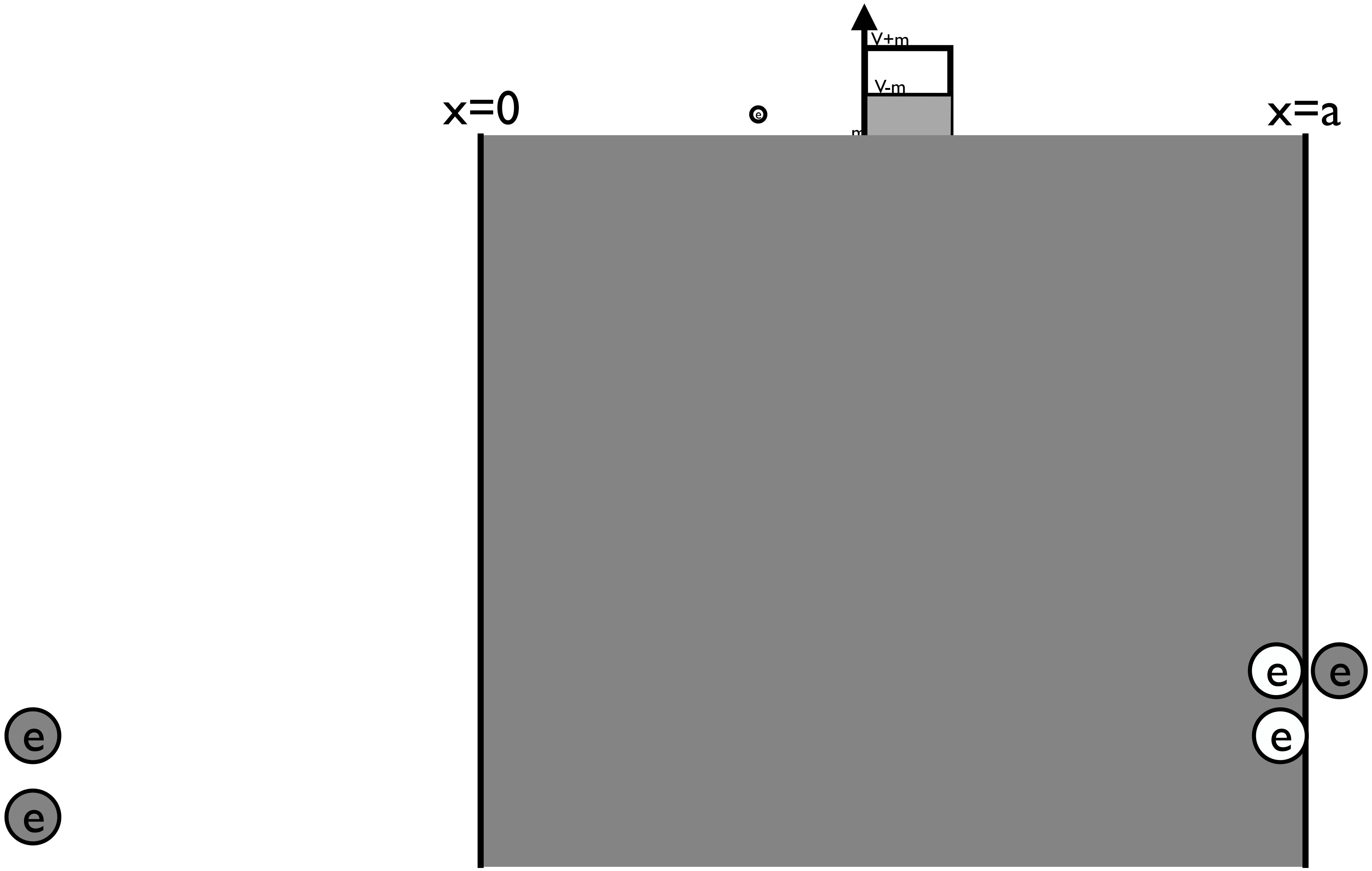

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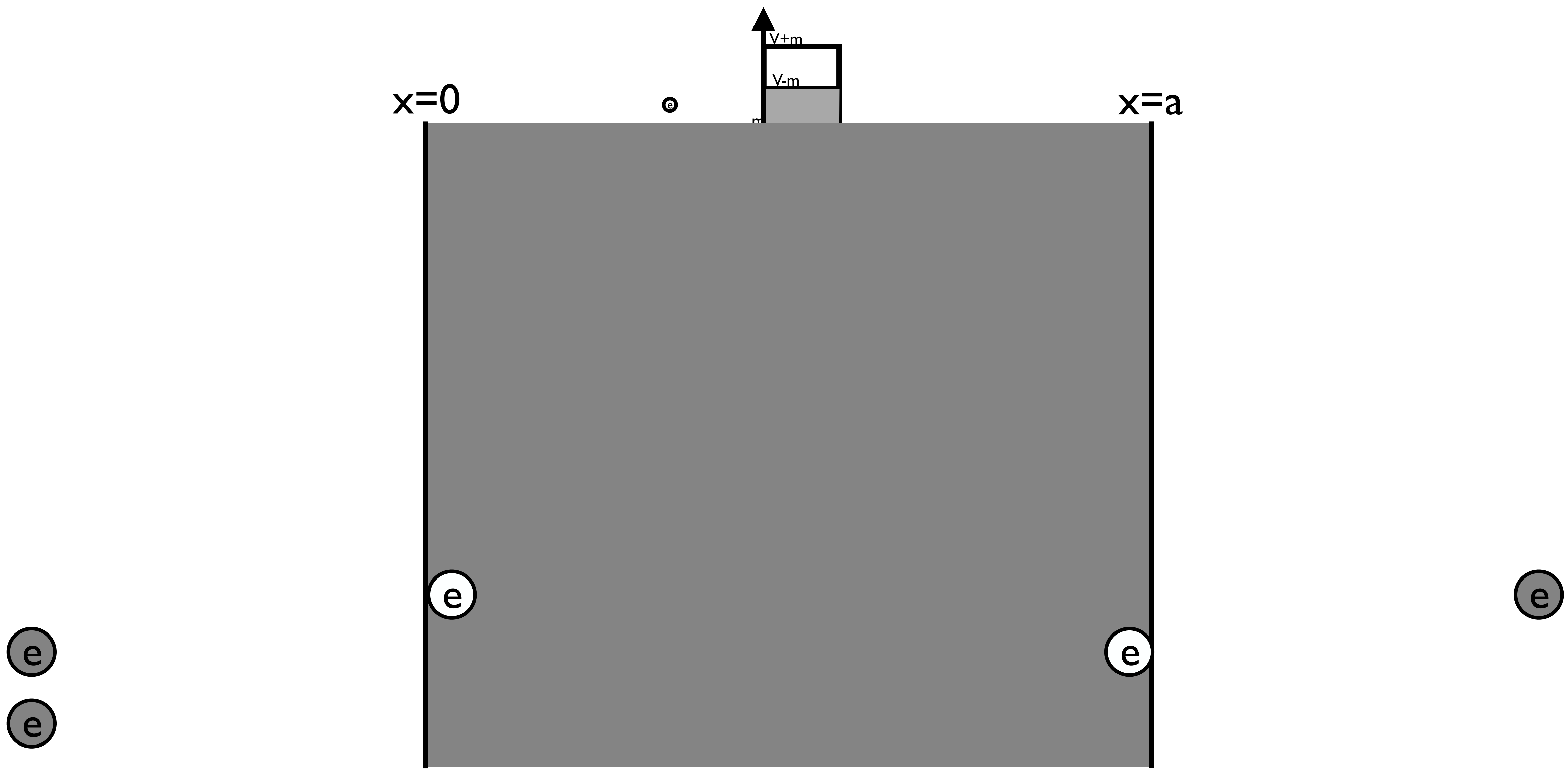

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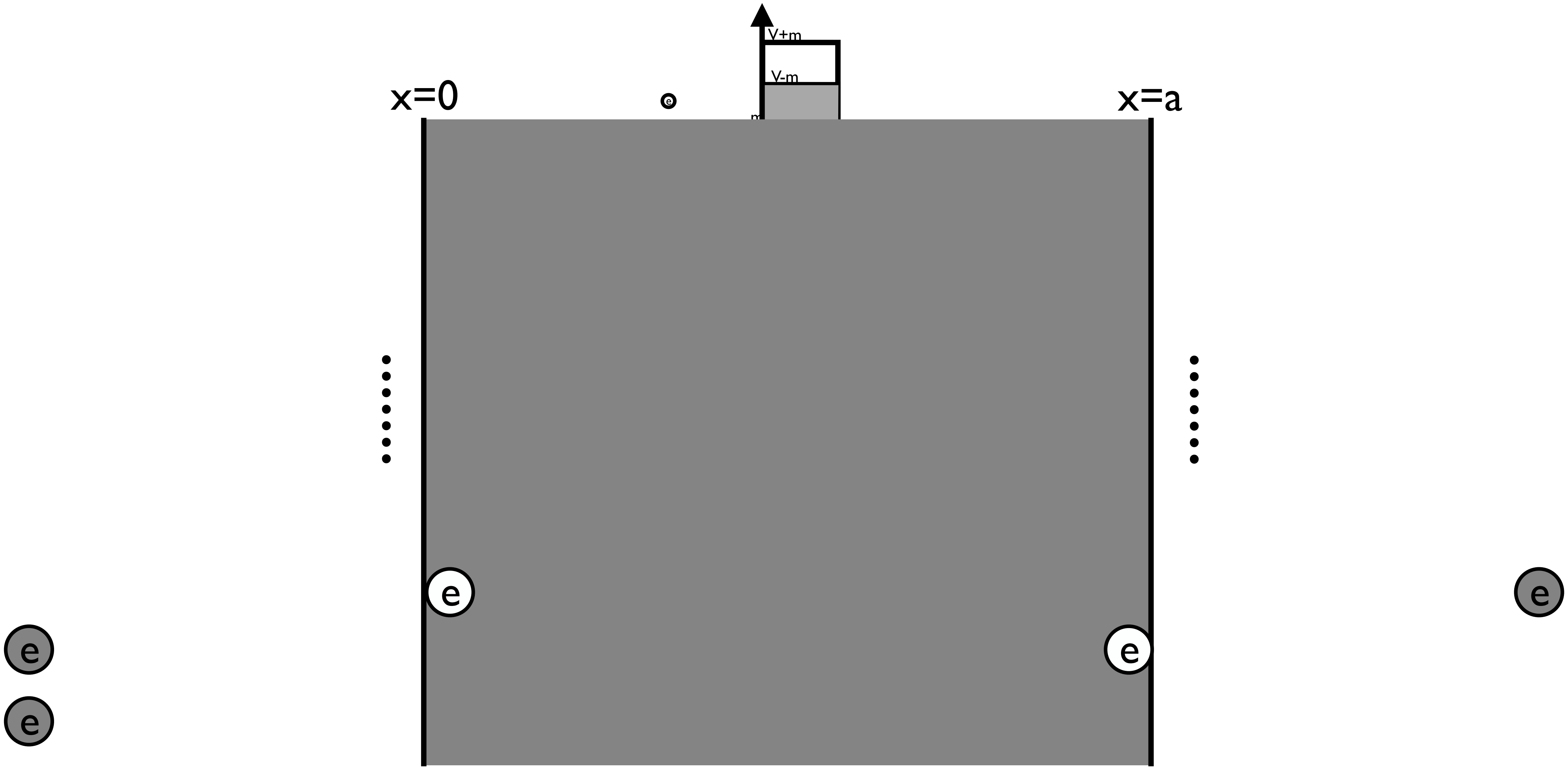

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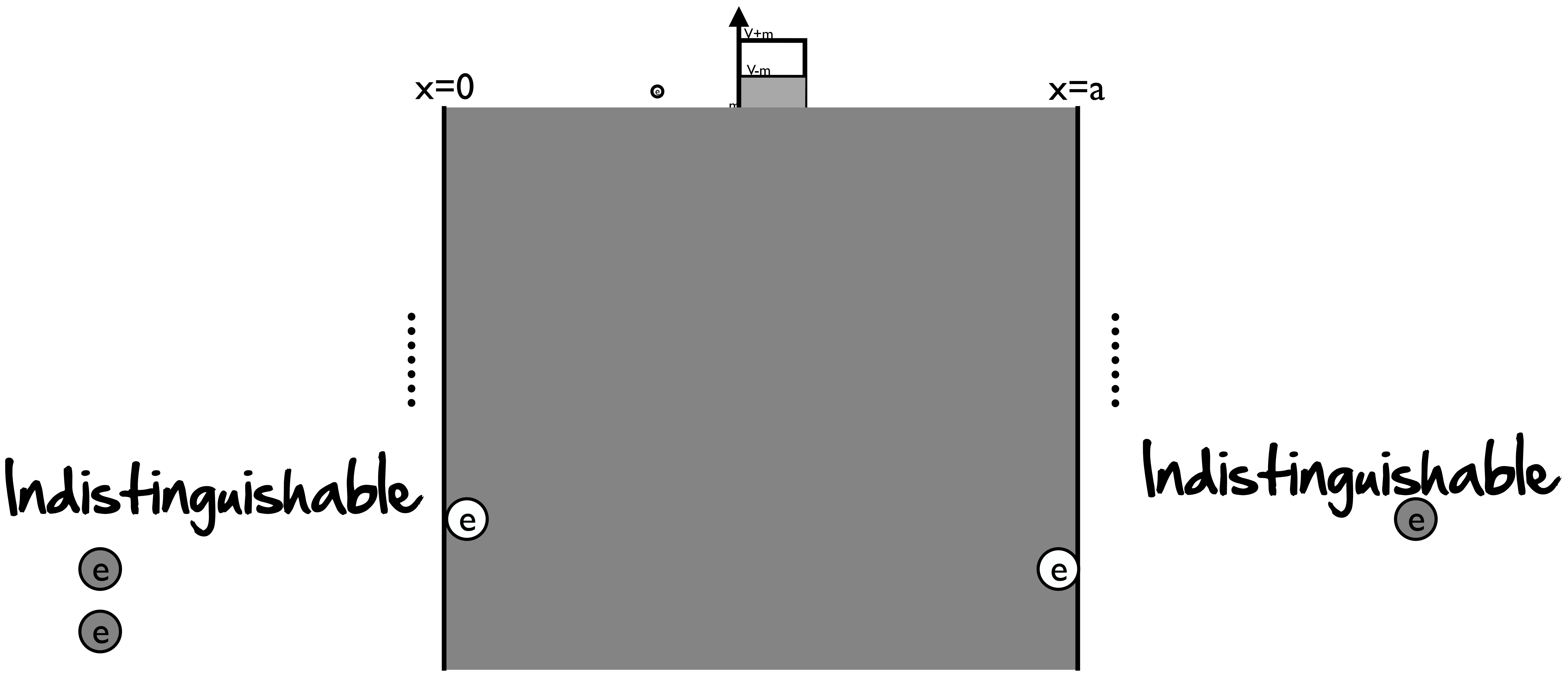

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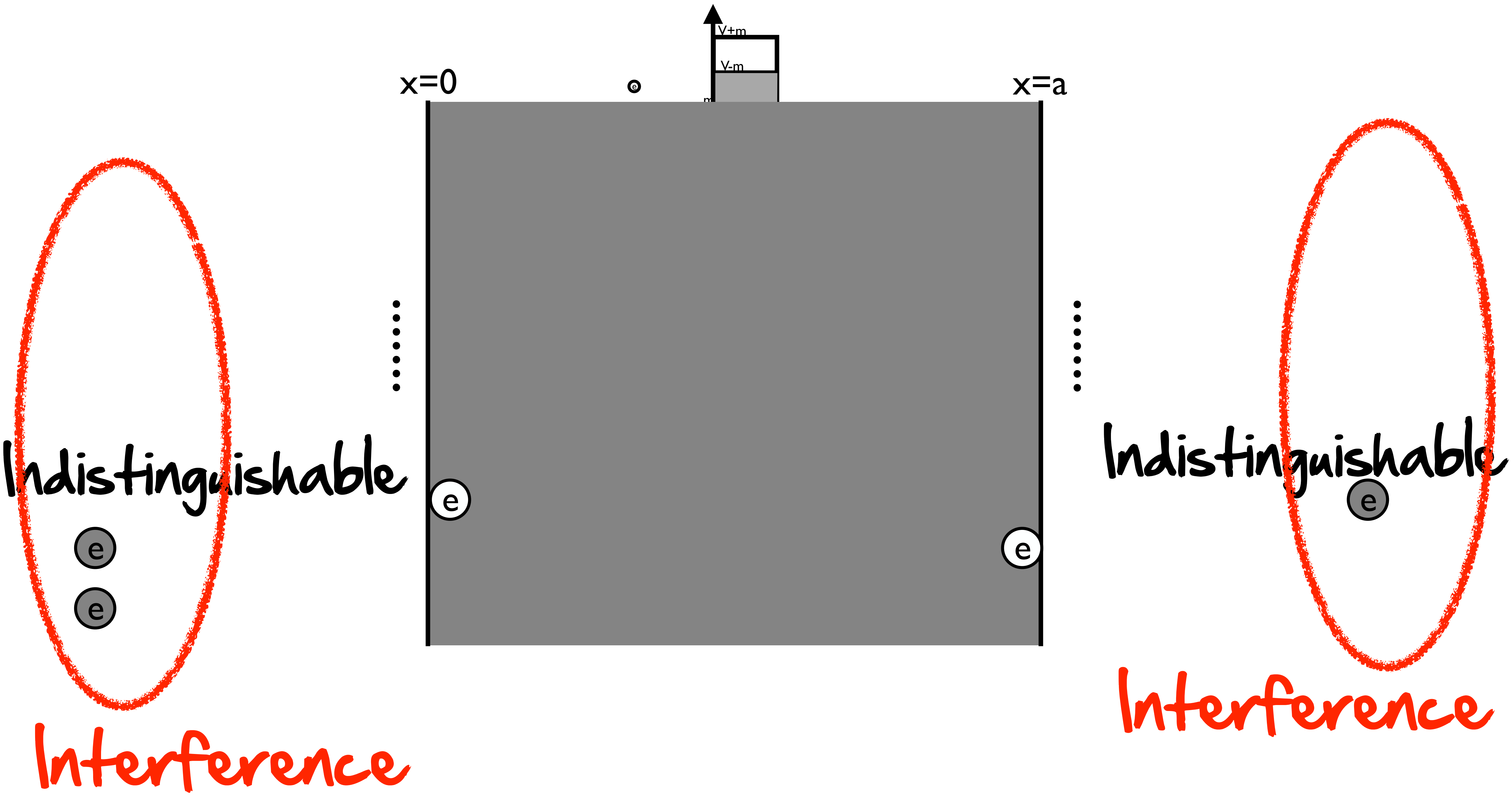


V+m
V-m
x=0
x=a
e
e
e
e
e
Indistinguishable
Indistinguishable
Interference
Interference

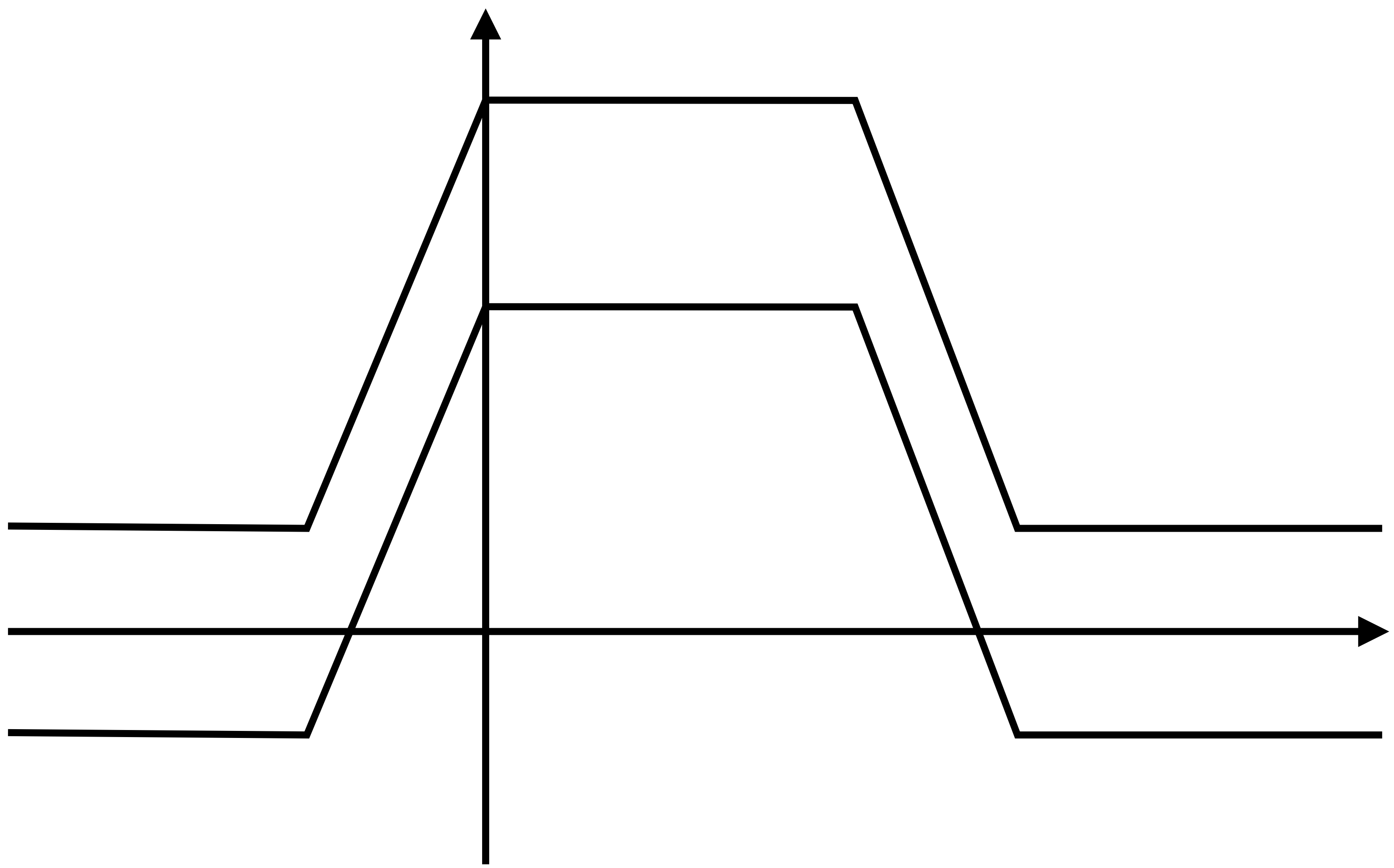
SCUSSIO

GROUP 4

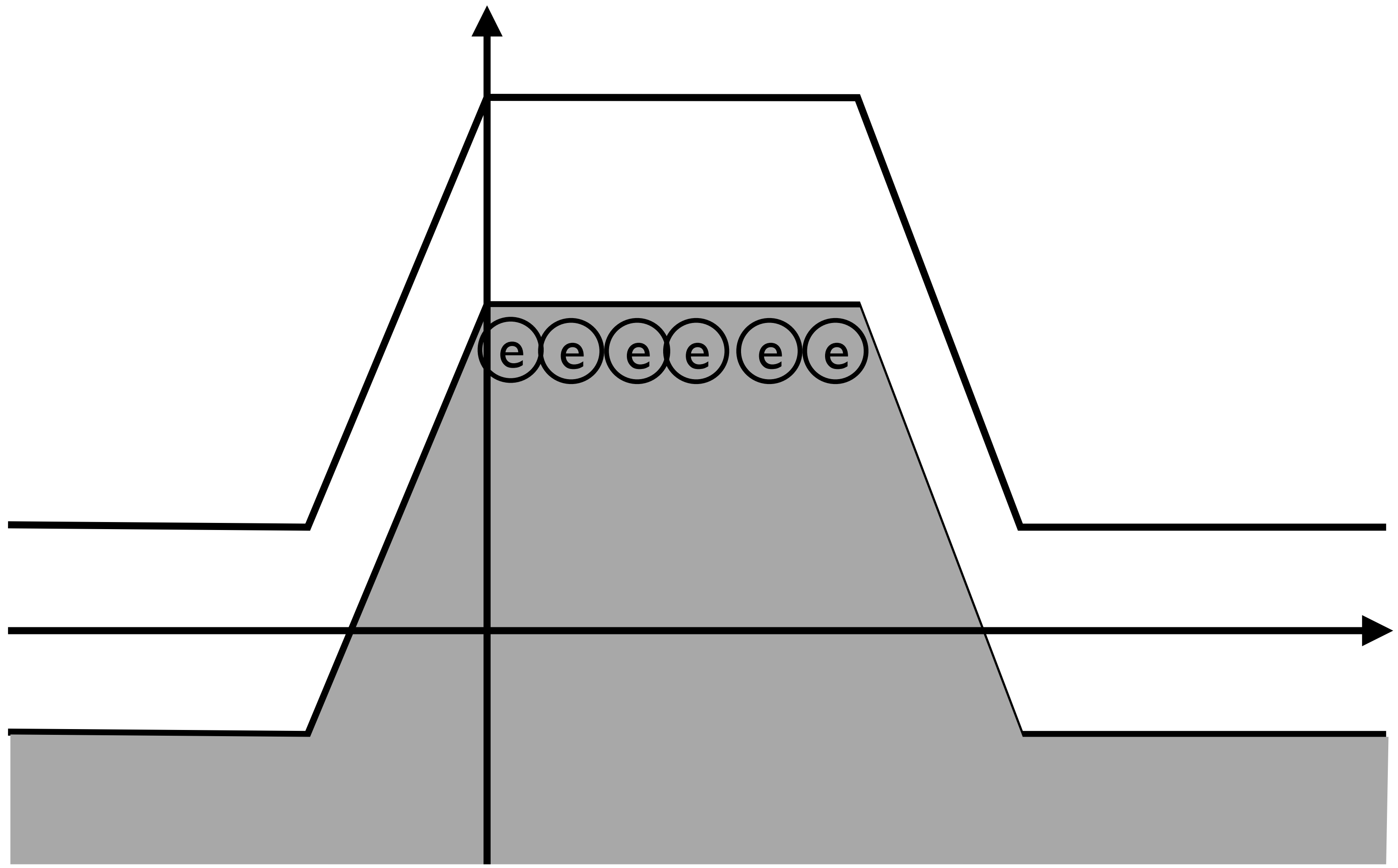
Analysis

Discussion

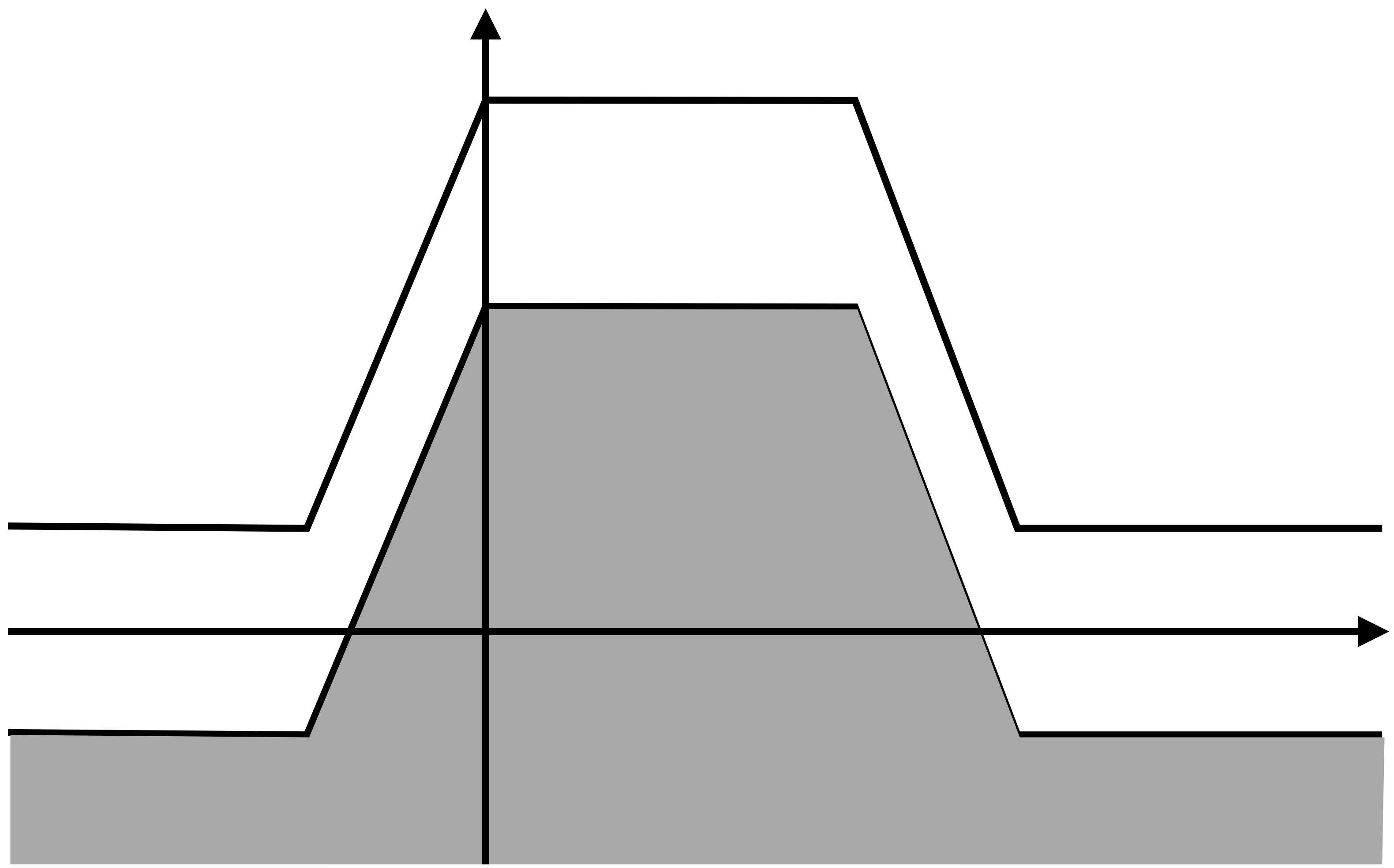
STATIONARY??



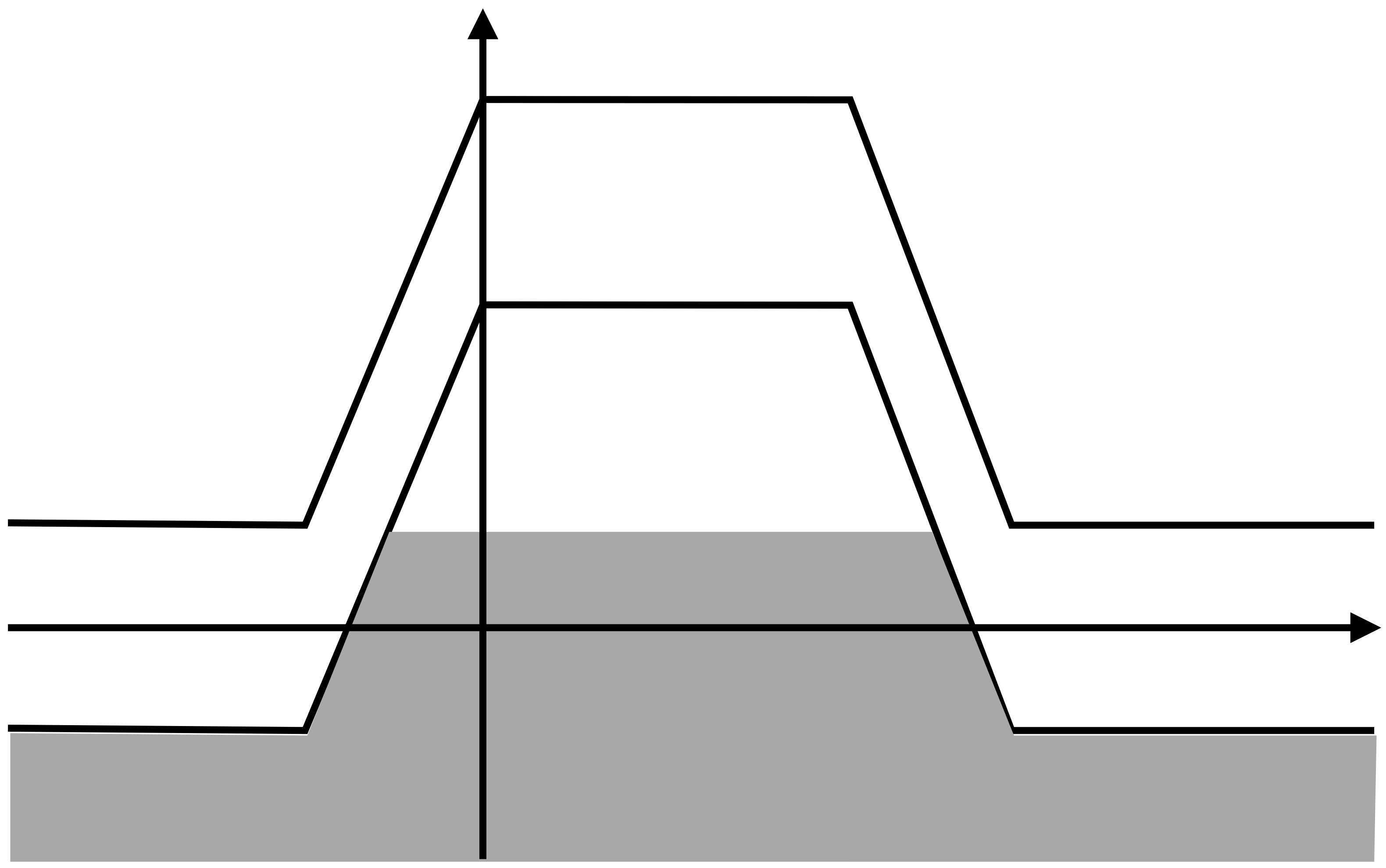
STATIONARY??



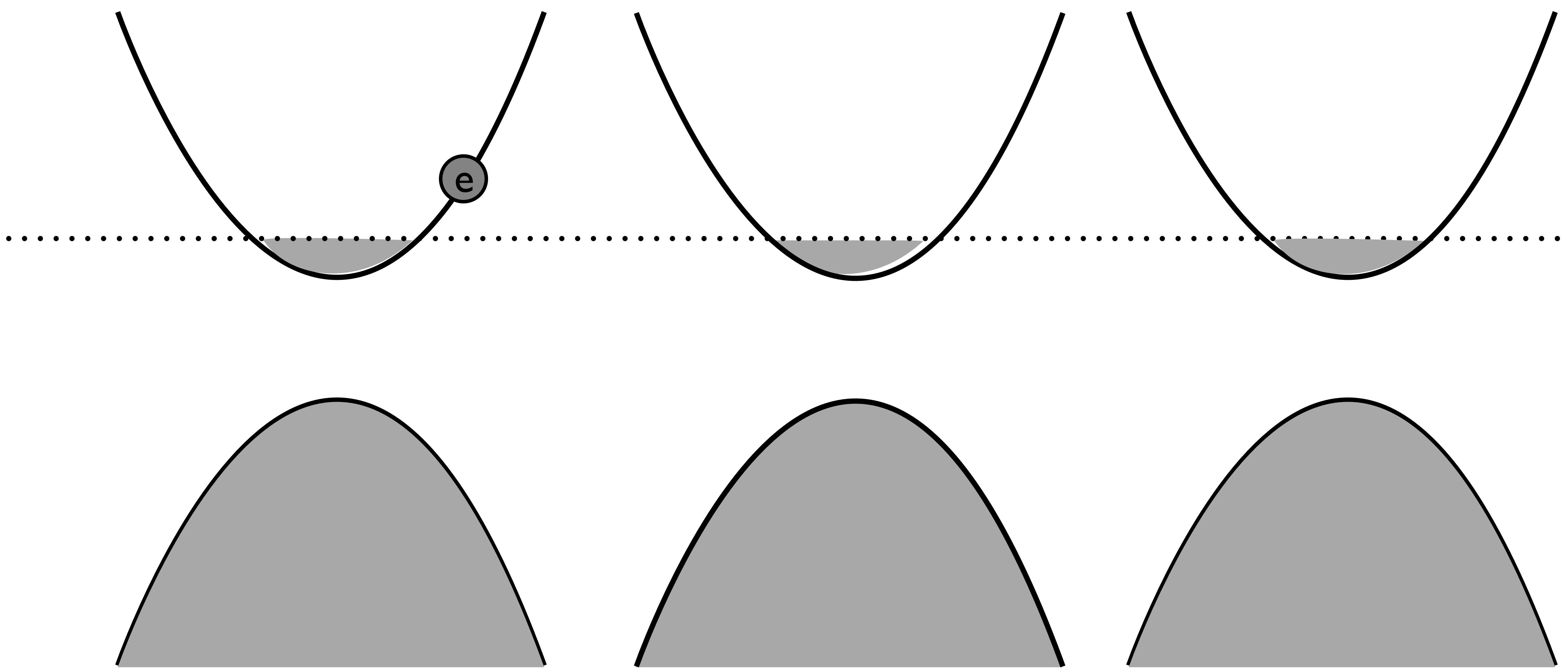
STATIONARY??



STATIONARY??



ELECTRON IN GRAPHENE



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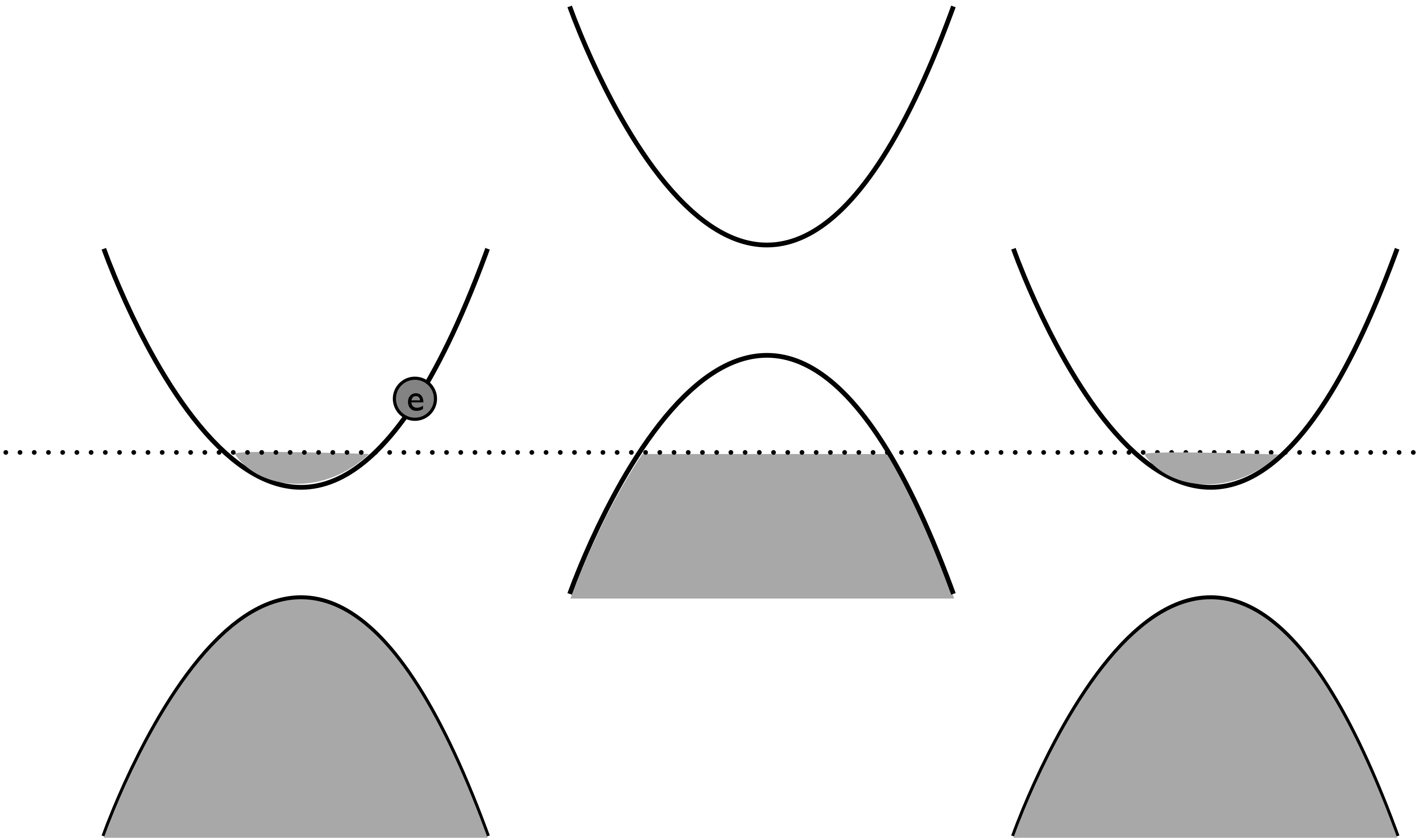

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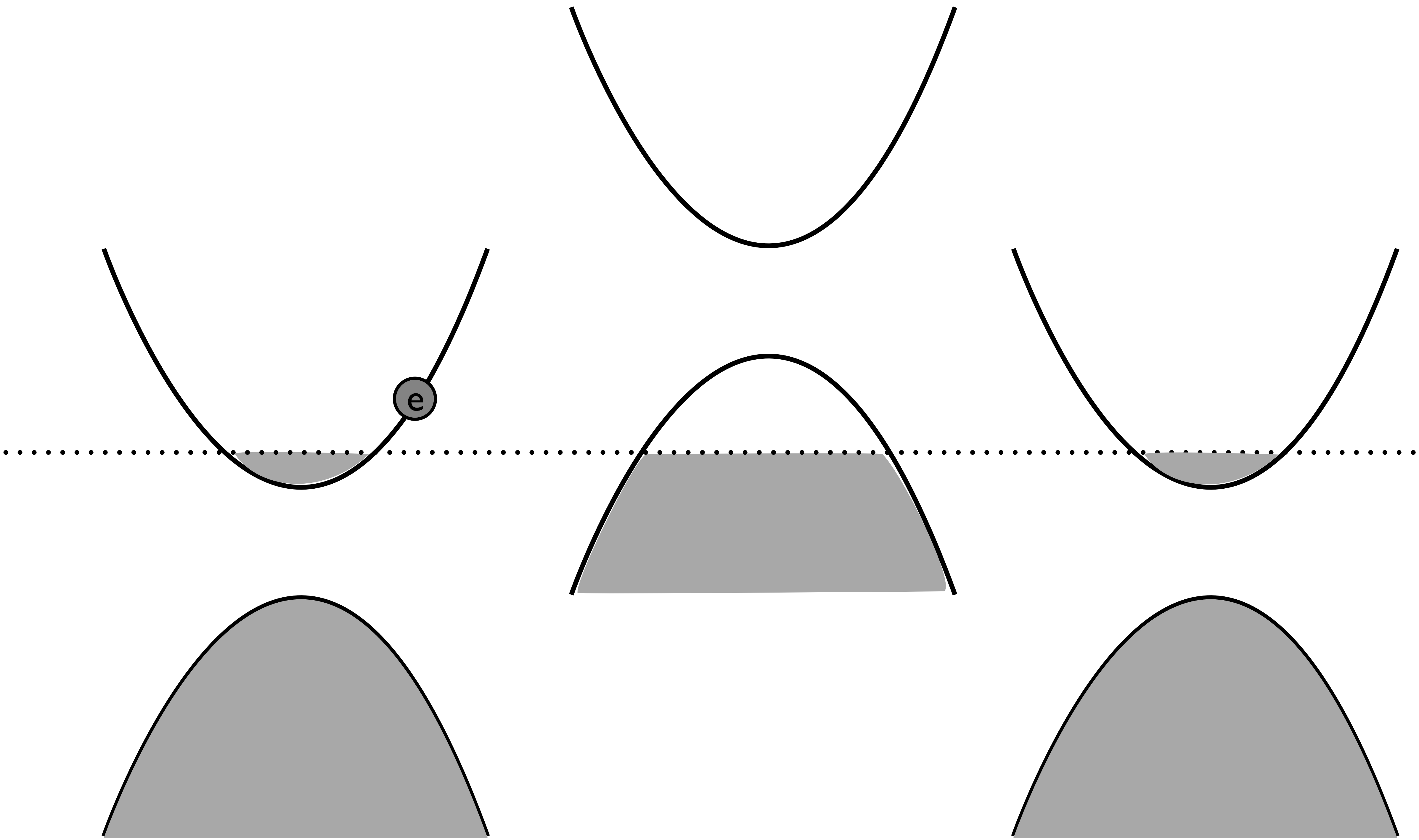

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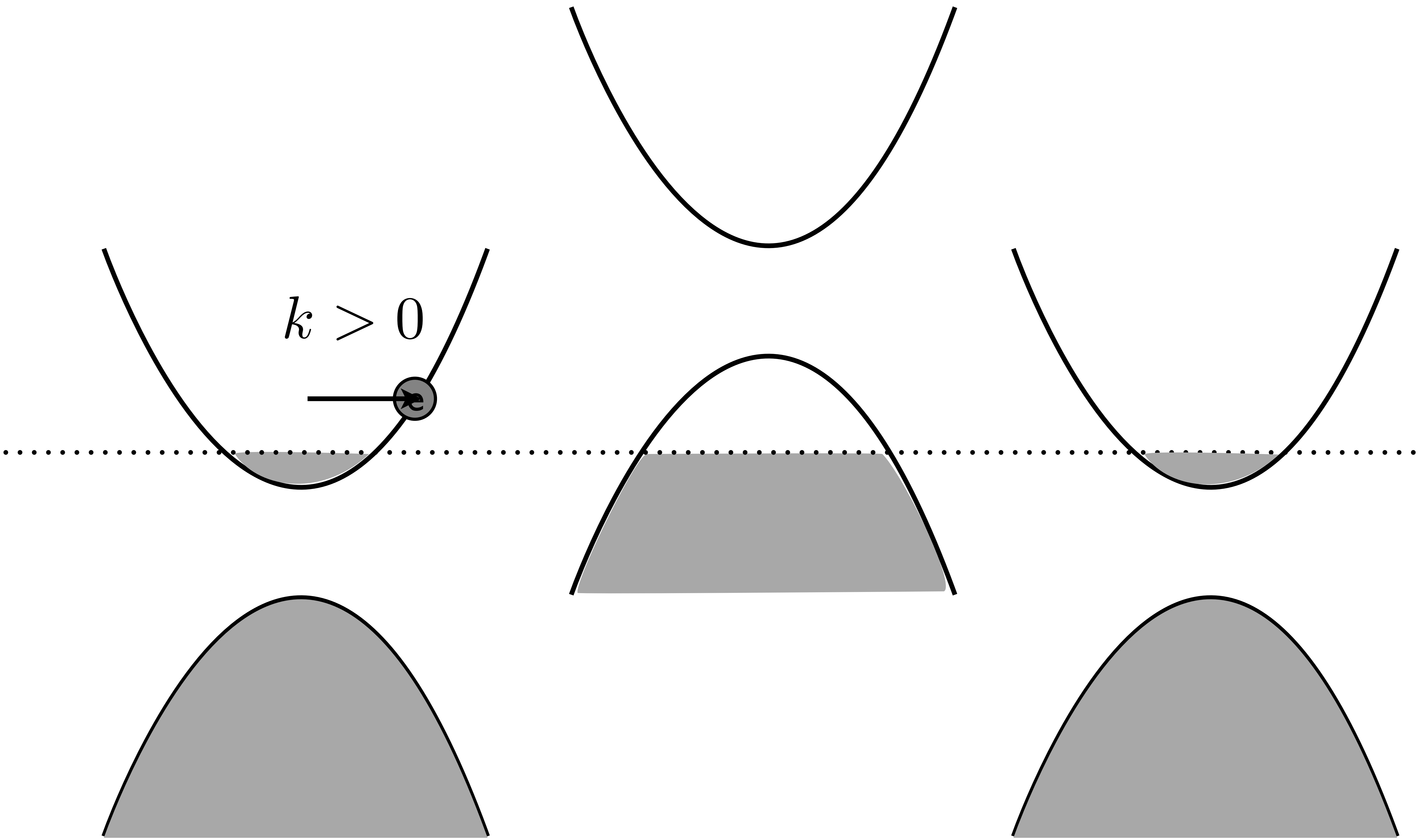

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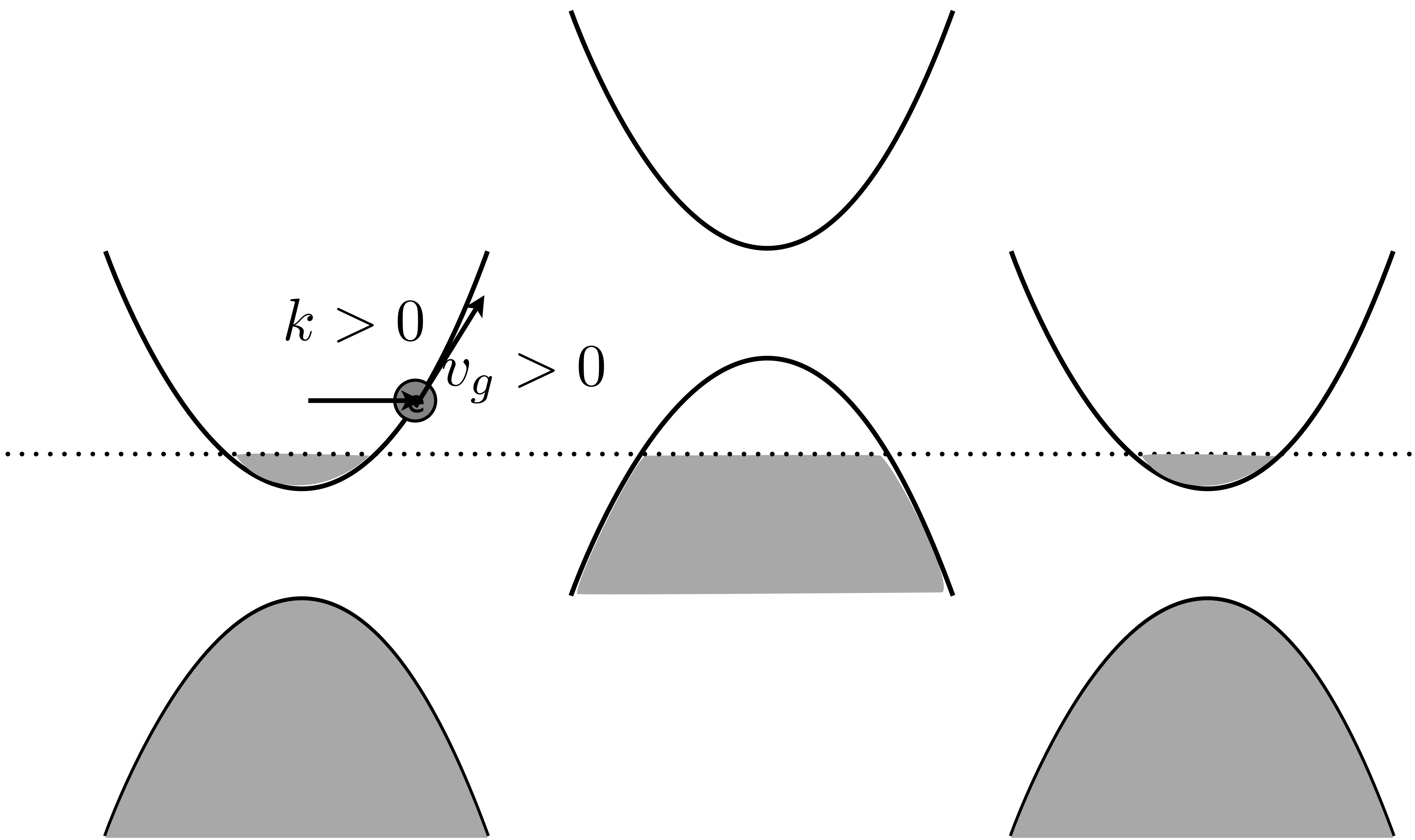

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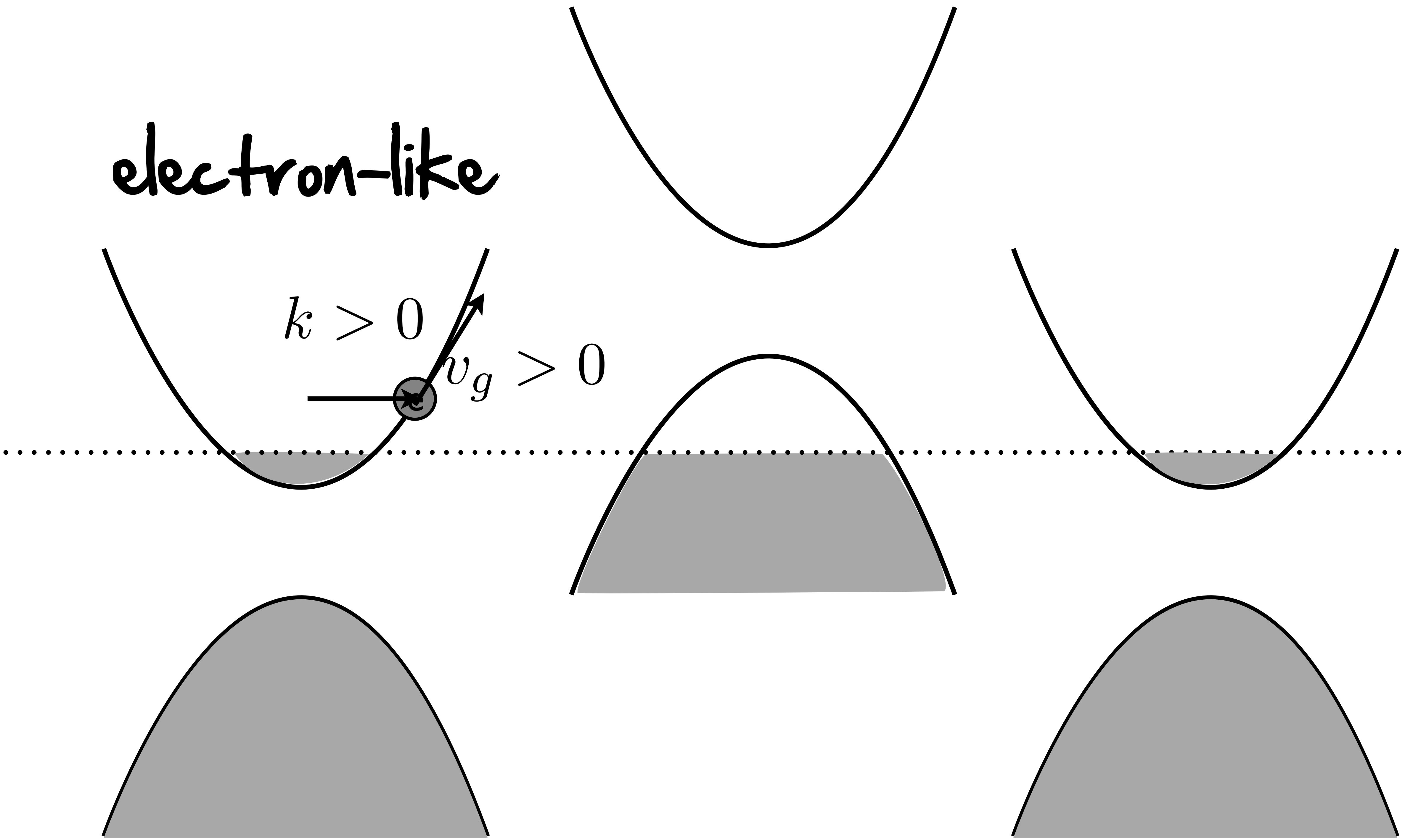

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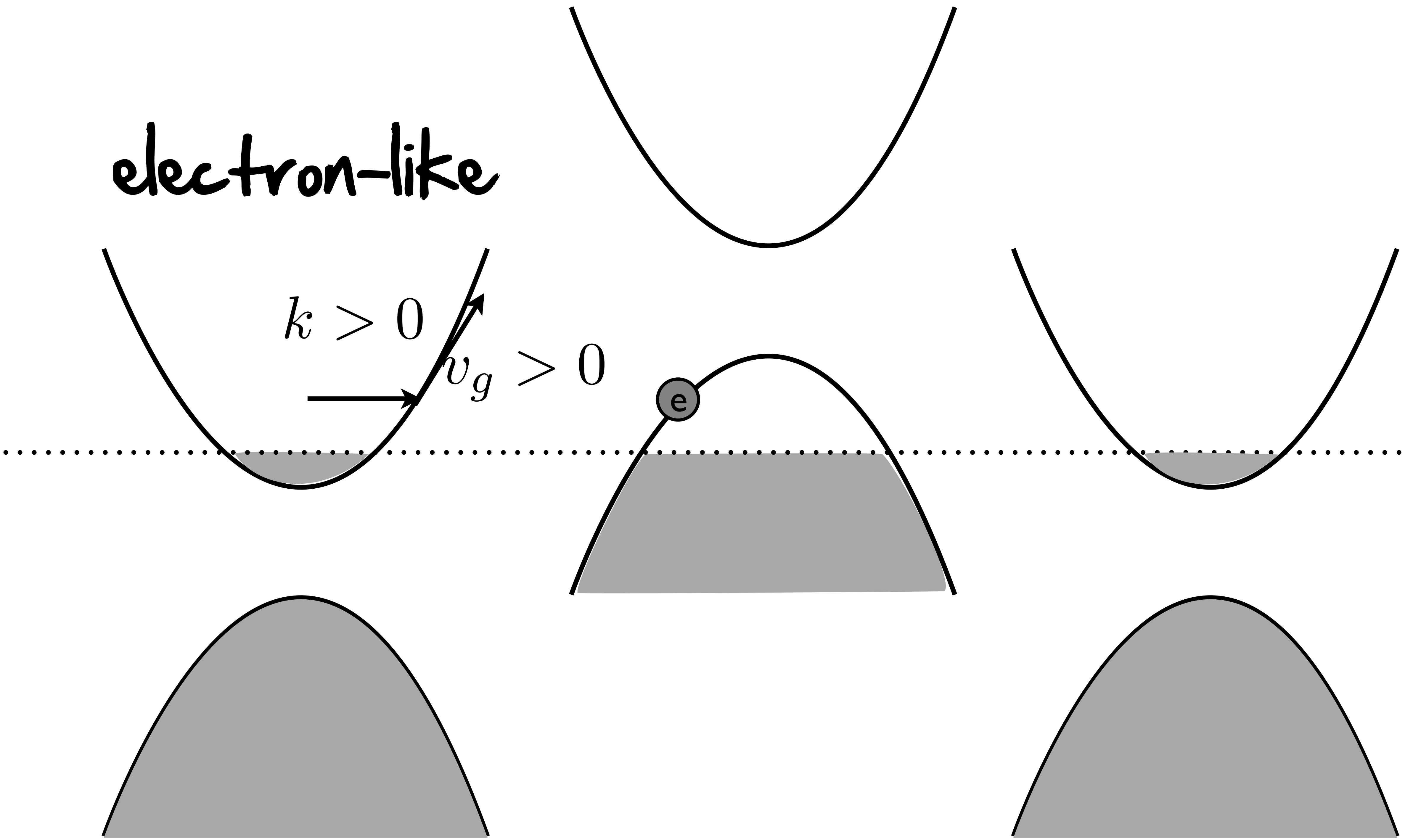

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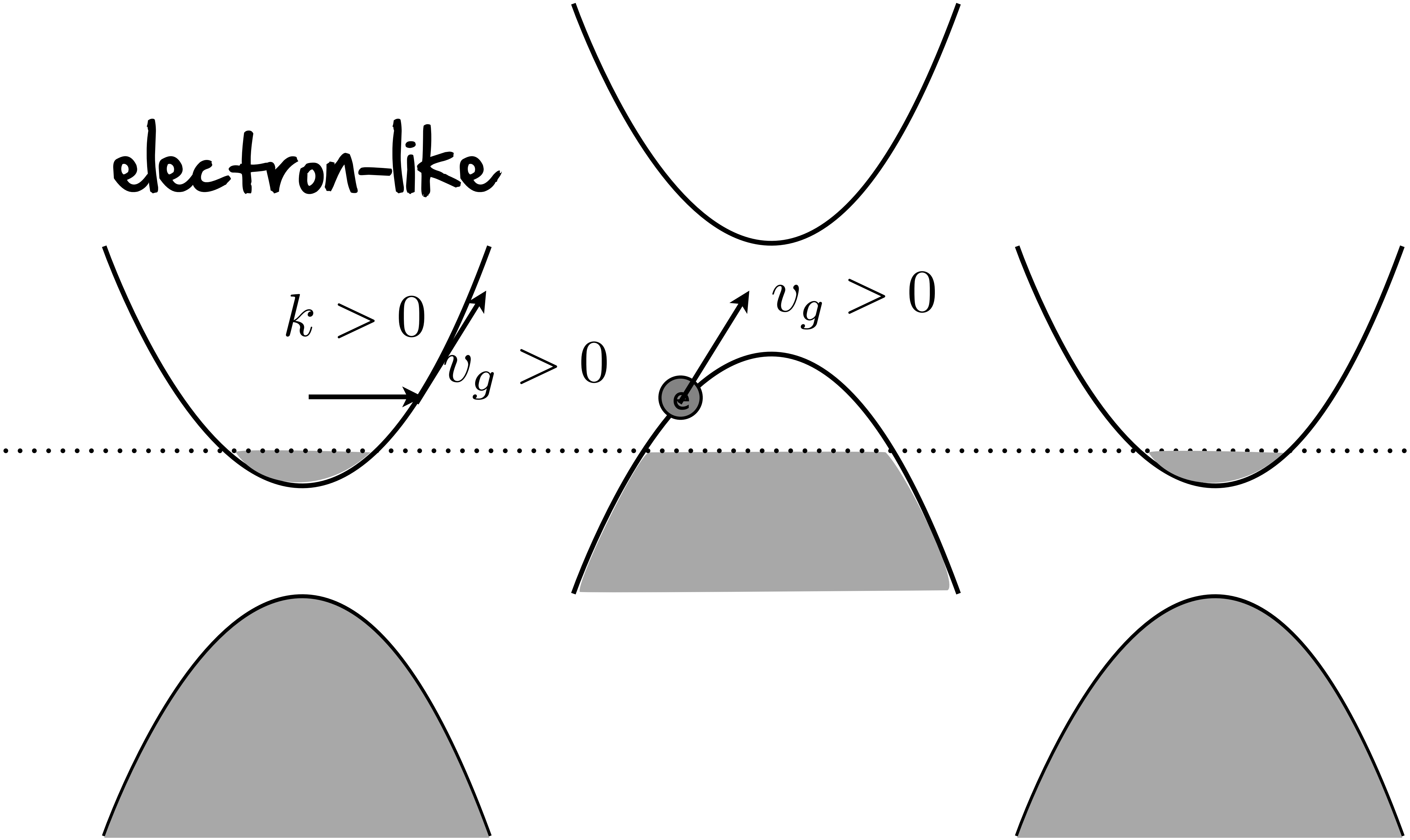

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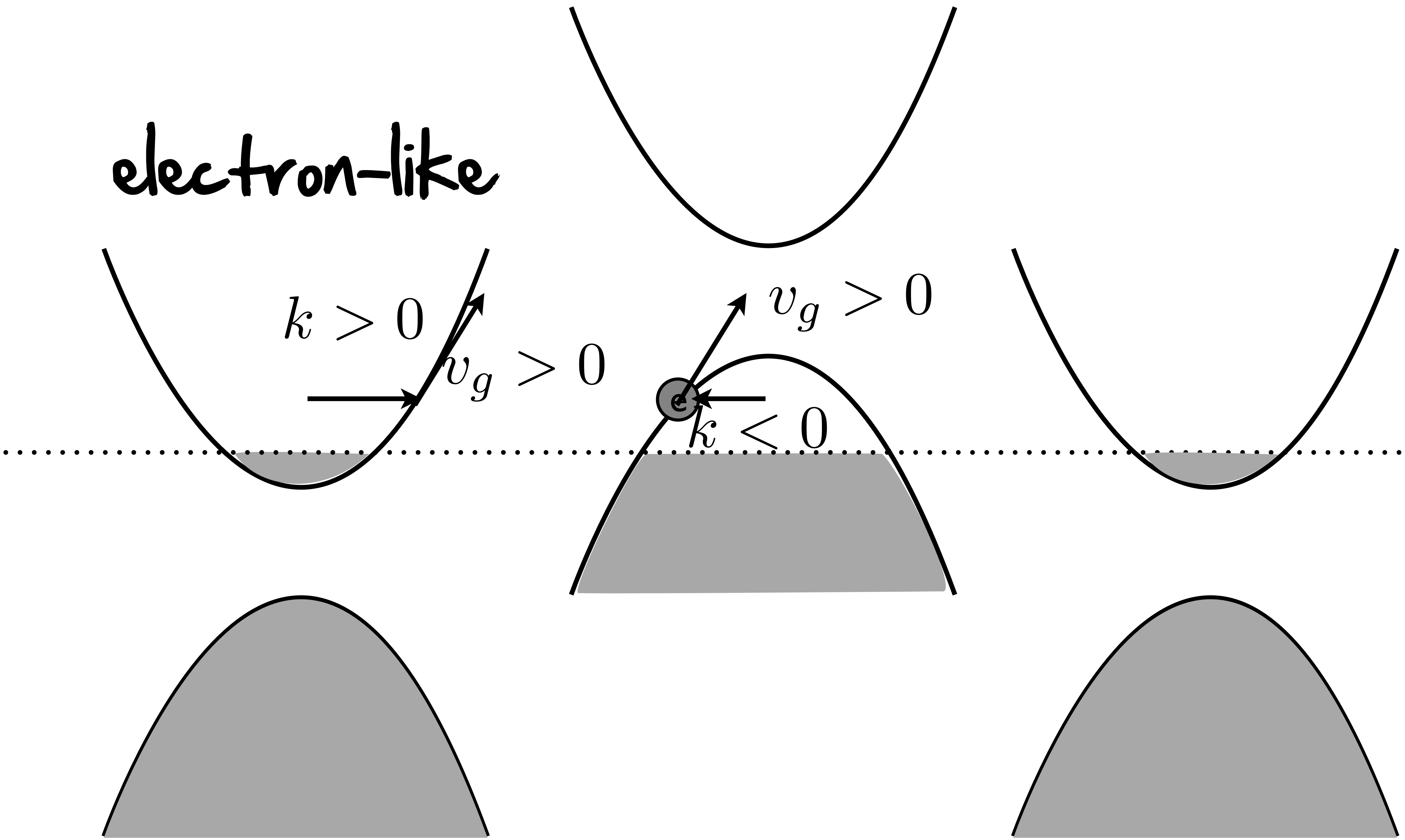
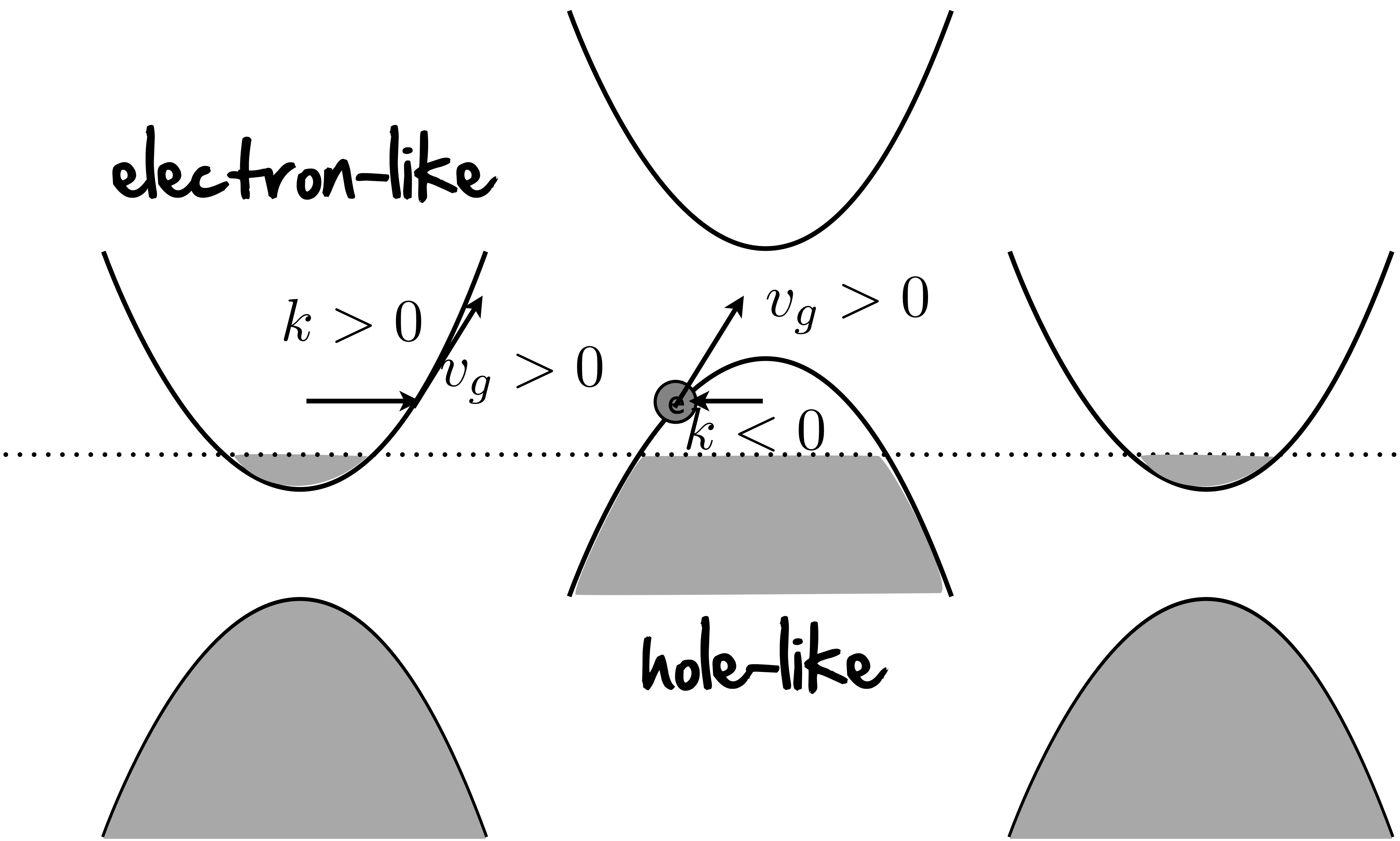
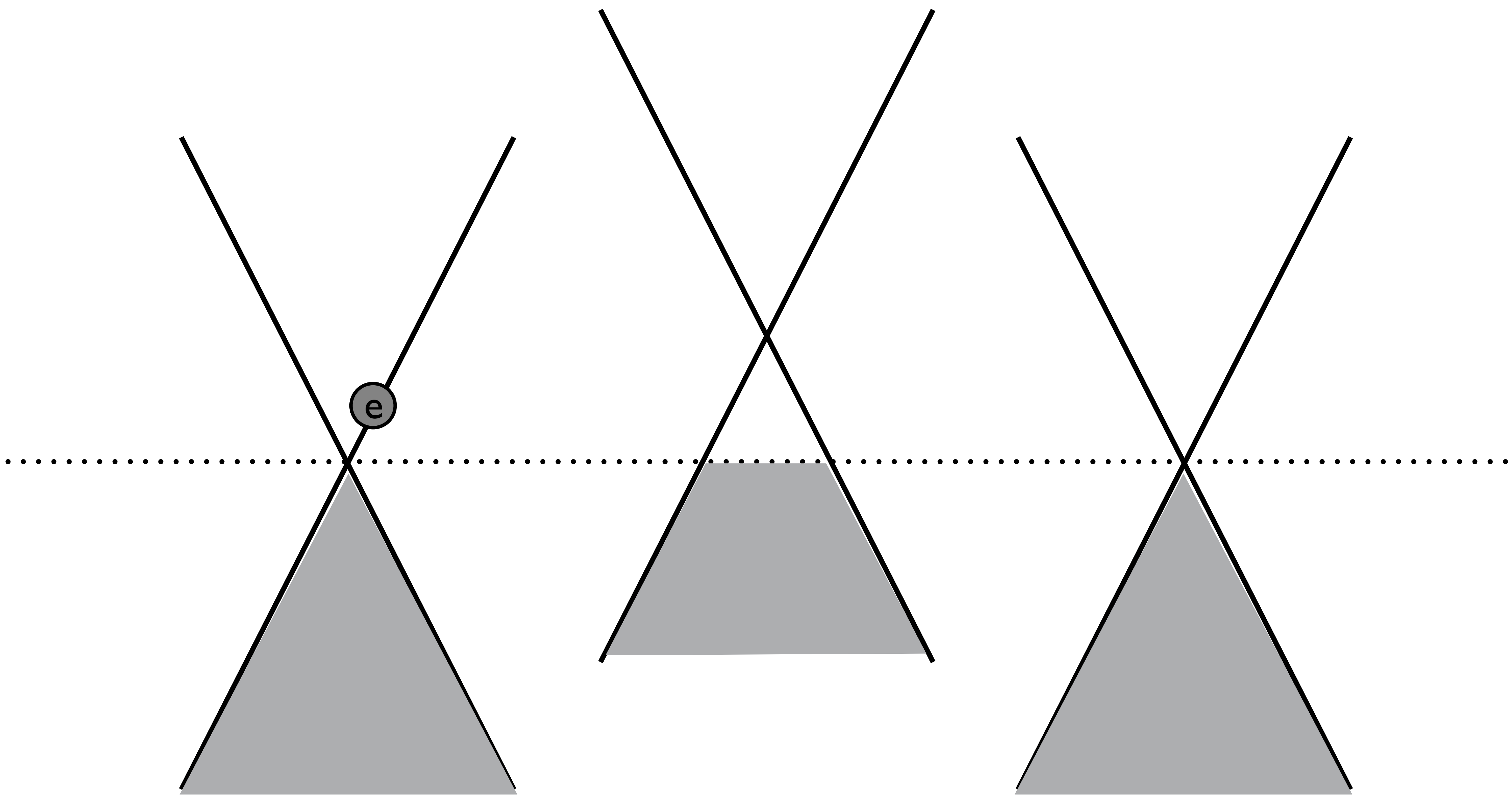


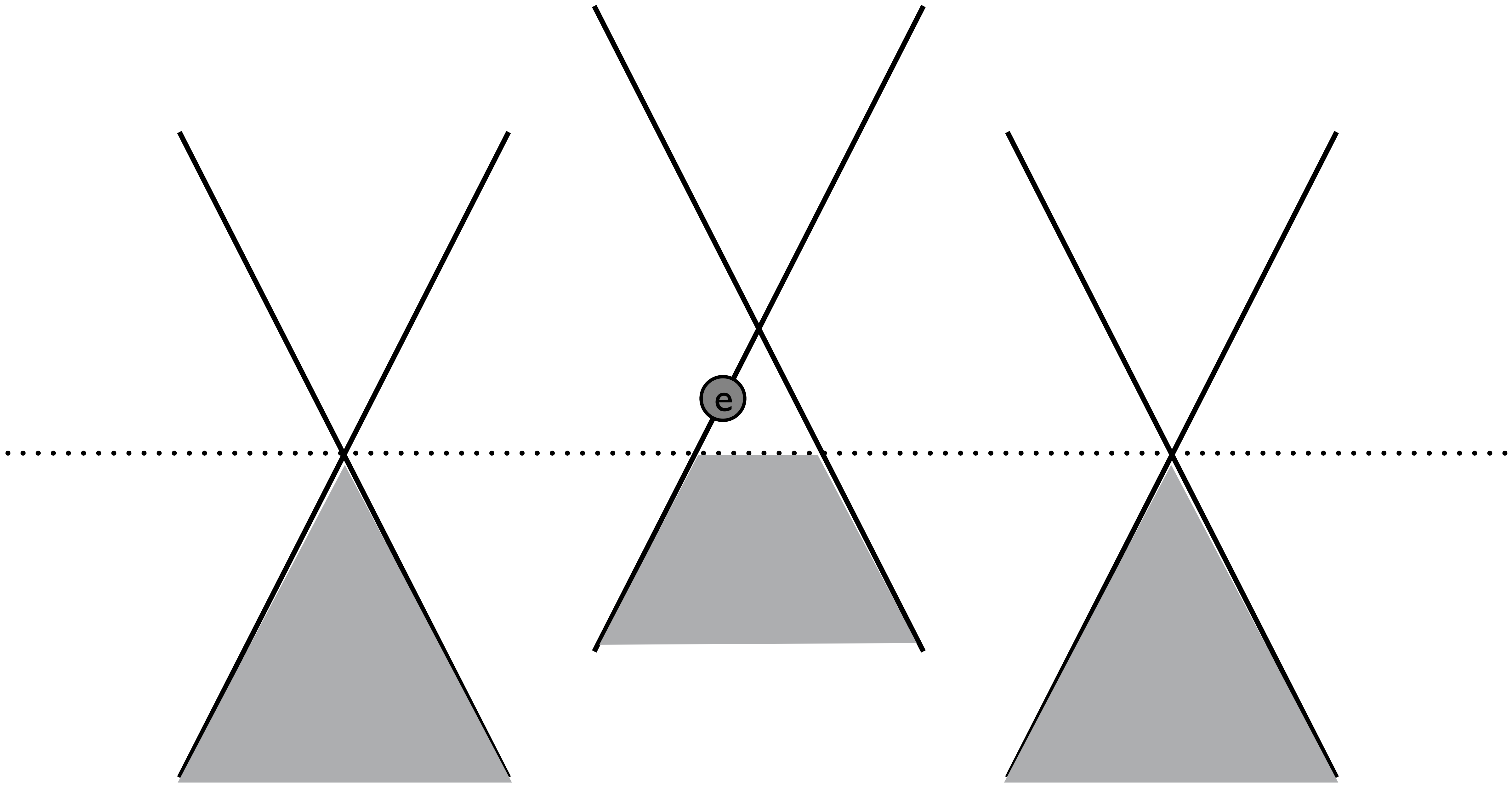
ELECTRON IN GRAPHENE



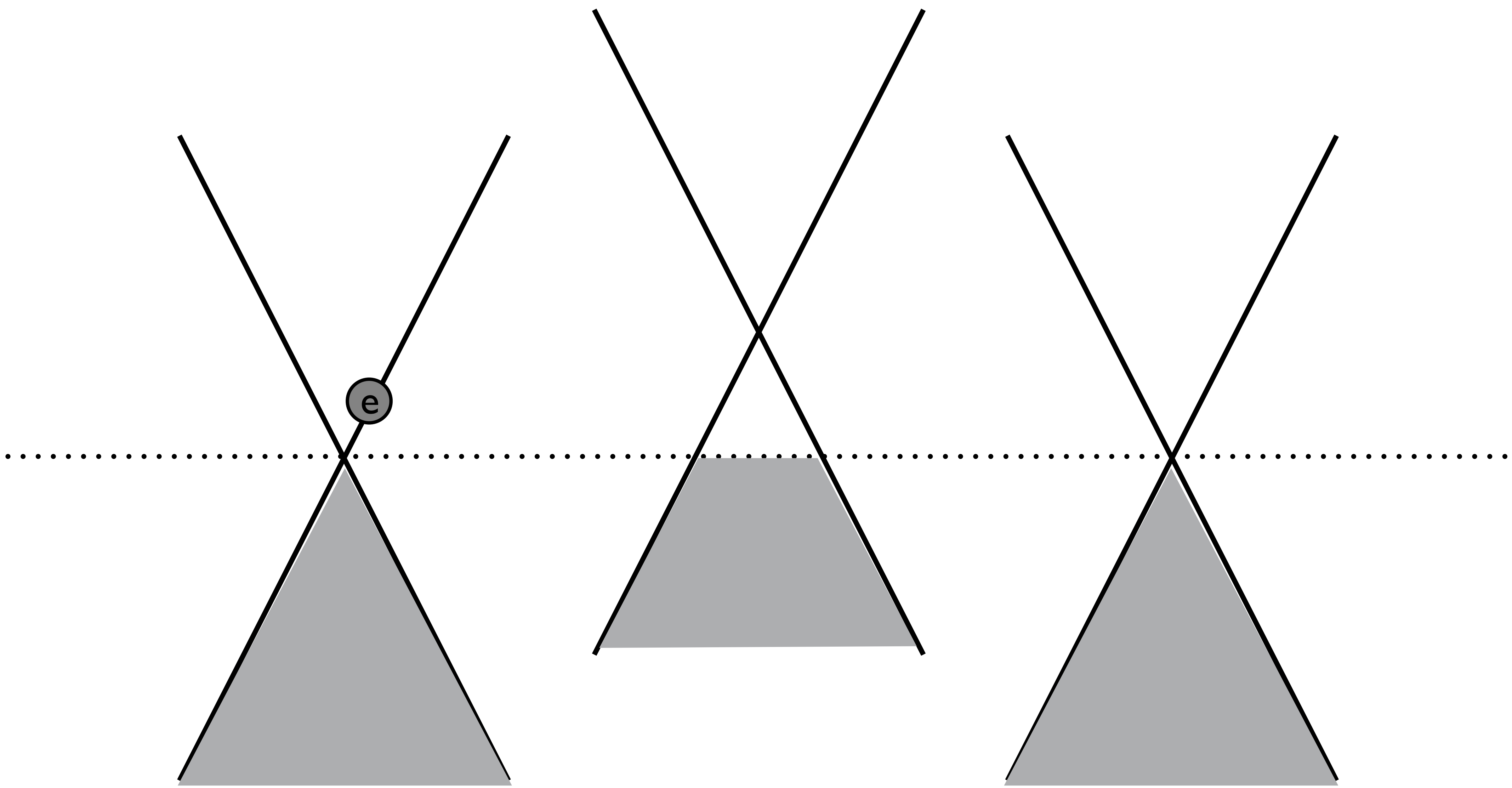
MASSLESS LIMIT



MASSLESS LIMIT



MASSLESS LIMIT



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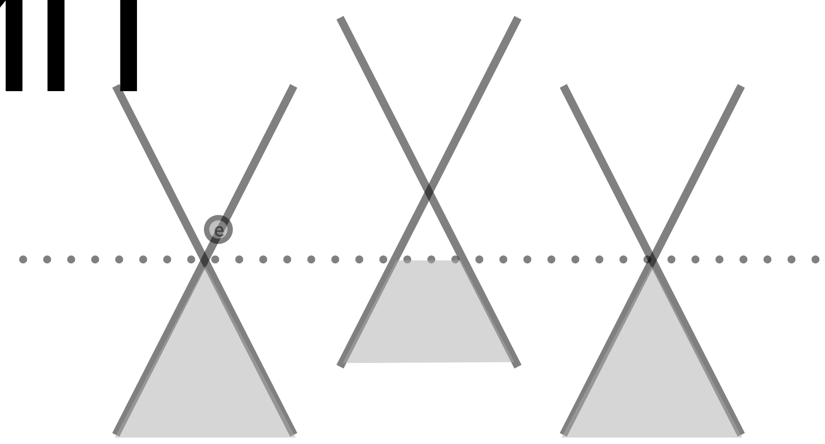

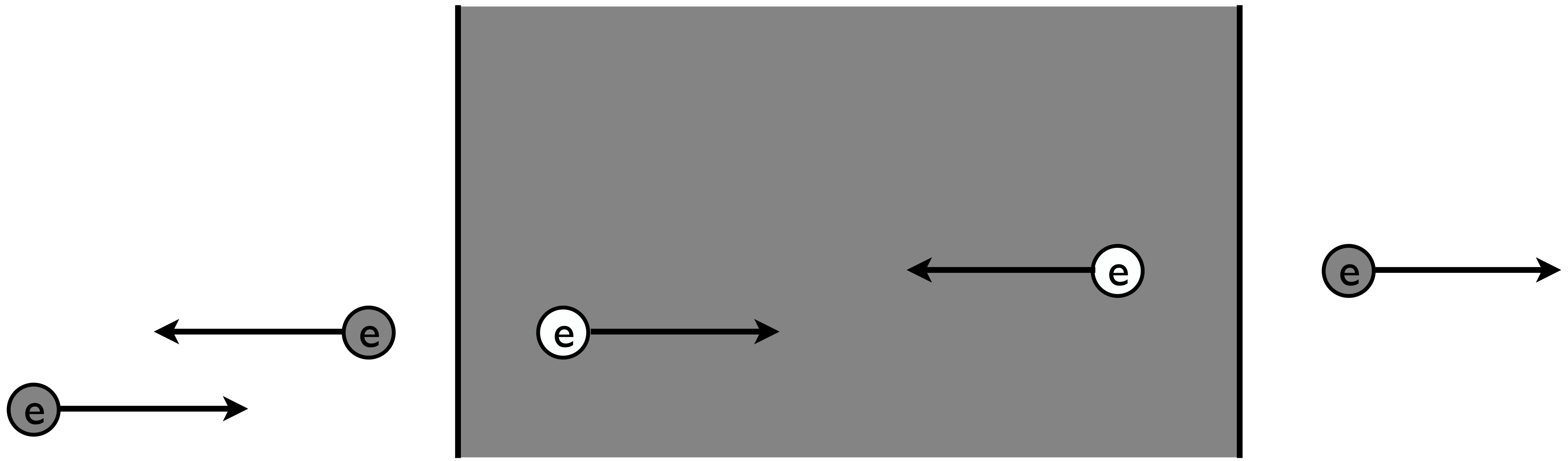

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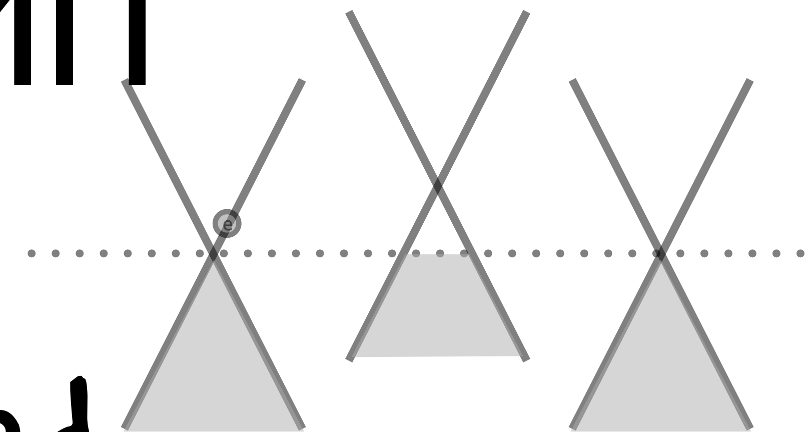
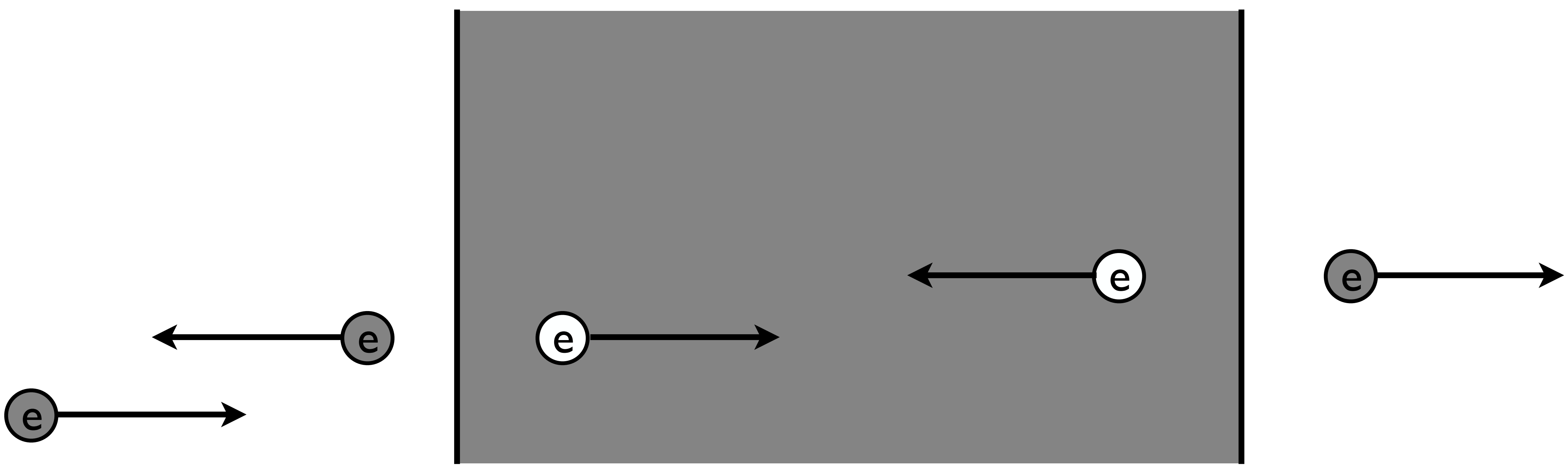
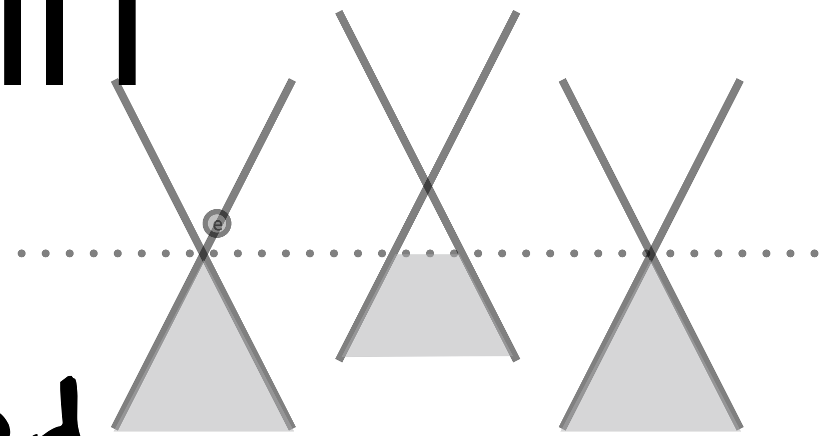


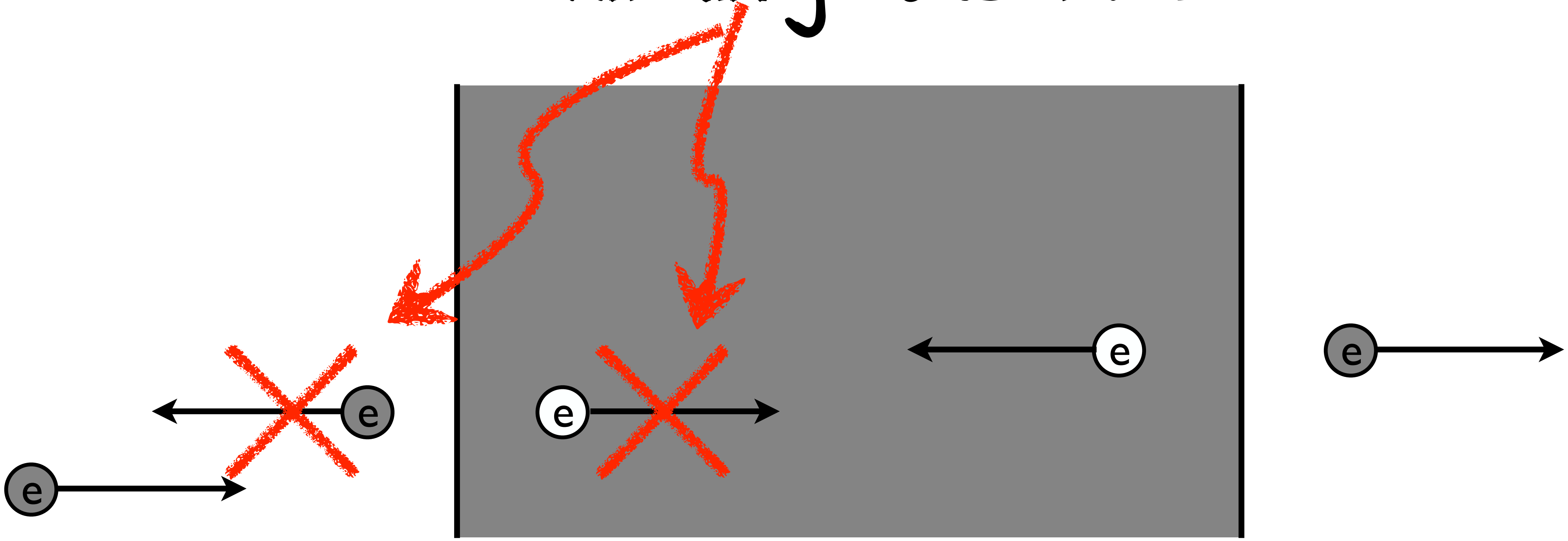
Chirality conserved



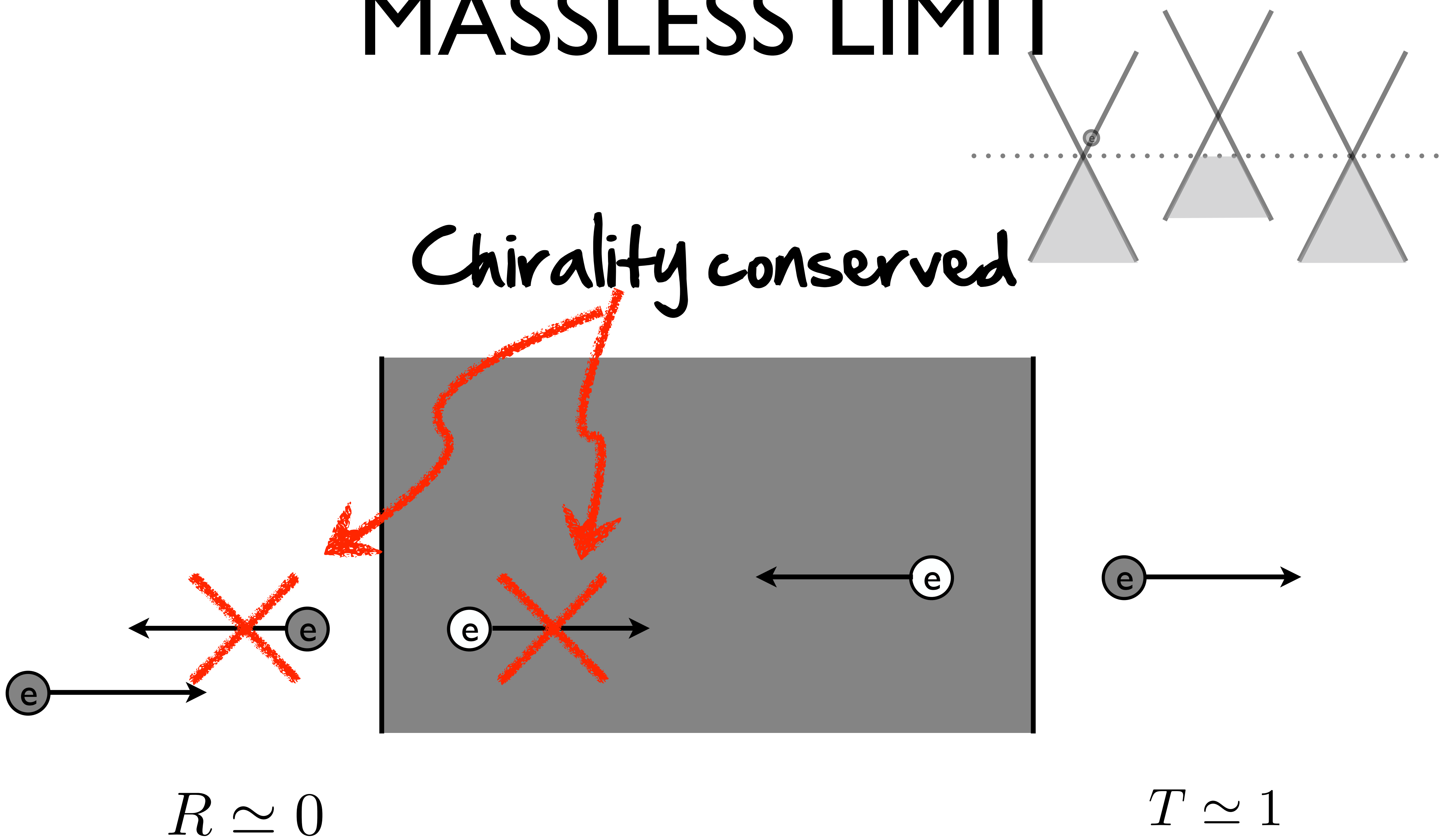
MASSLESS LIMIT



Chirality conserved



MASSLESS LIMIT



INFINITE POTENTIAL LIMIT

$$m \ll V \qquad m < E < V - m$$

$$T = |t|^2 = \frac{1}{1 + \frac{1}{4}(\rho - \frac{1}{\rho})^2 \sin^2 qa} \simeq \frac{E^2 - m^2}{E^2 - \frac{1}{2}m^2}$$

$$R = 1 - T$$

Non-relativistic limit $E = m + K$ $K \ll m$

$$T \simeq 0$$

CONSISTENT!!

Ultra-relativistic limit $m \ll E$ $m \ll E \ll V$

$$T \simeq 1$$

REASONABLE??

INFINITE POTENTIAL LIMIT

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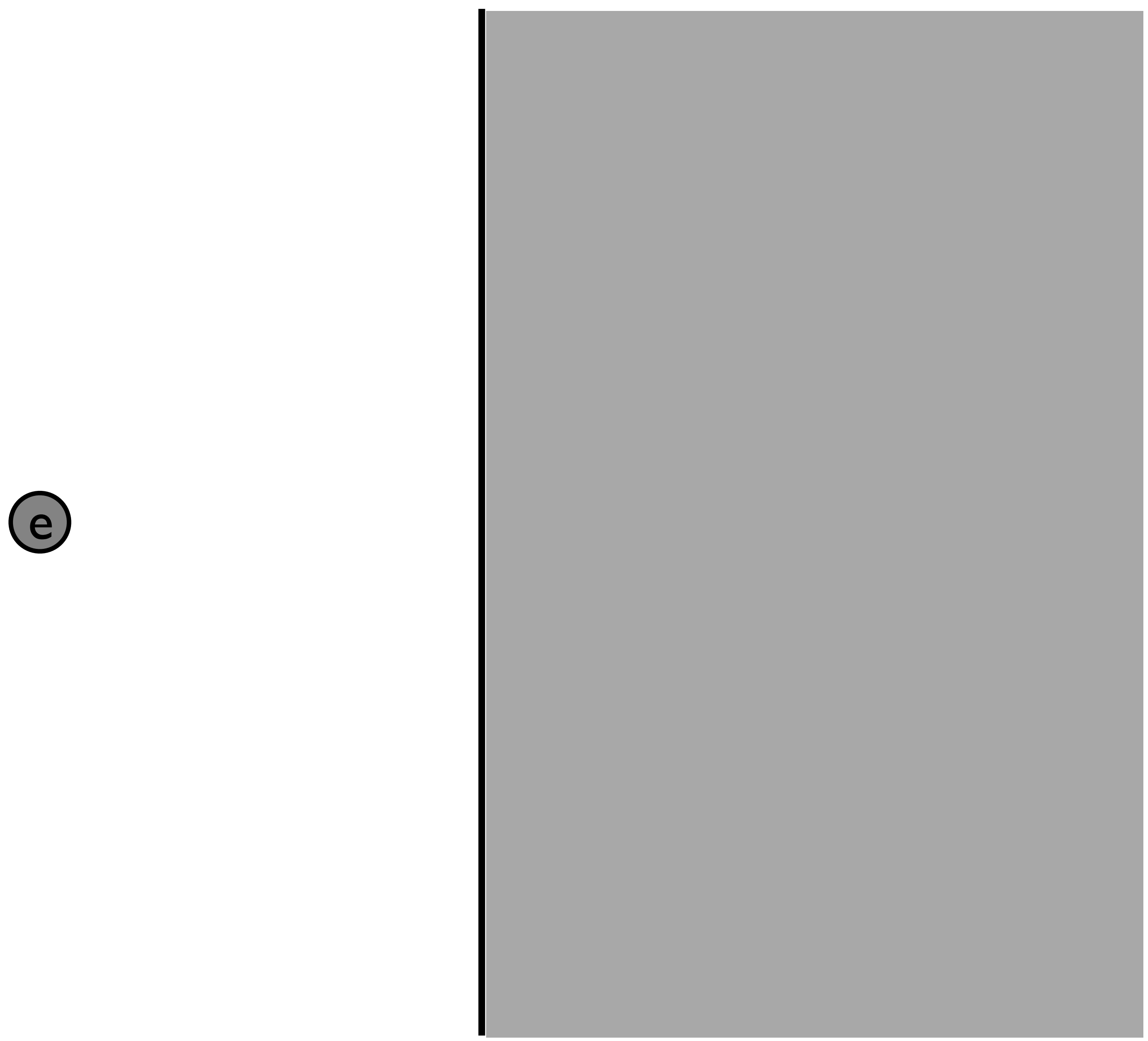
MASSLESS LIMIT

Ultra-relativistic limit $m \ll E \quad m \ll E \ll V$

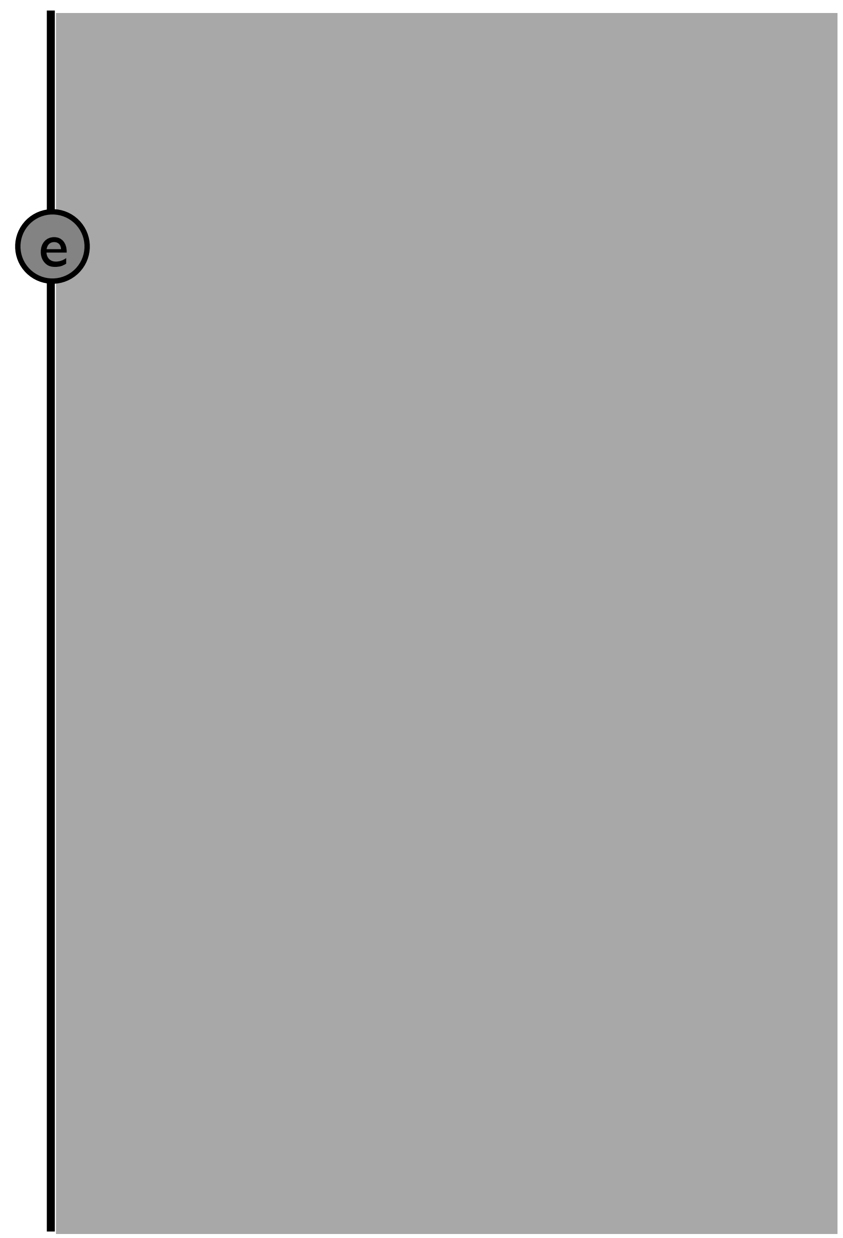
$$T \simeq 1$$

REASONABLE?? YES

THE FOCUSSING OF ELECTRON FLOW



THE FOCUSSING OF ELECTRON FLOW



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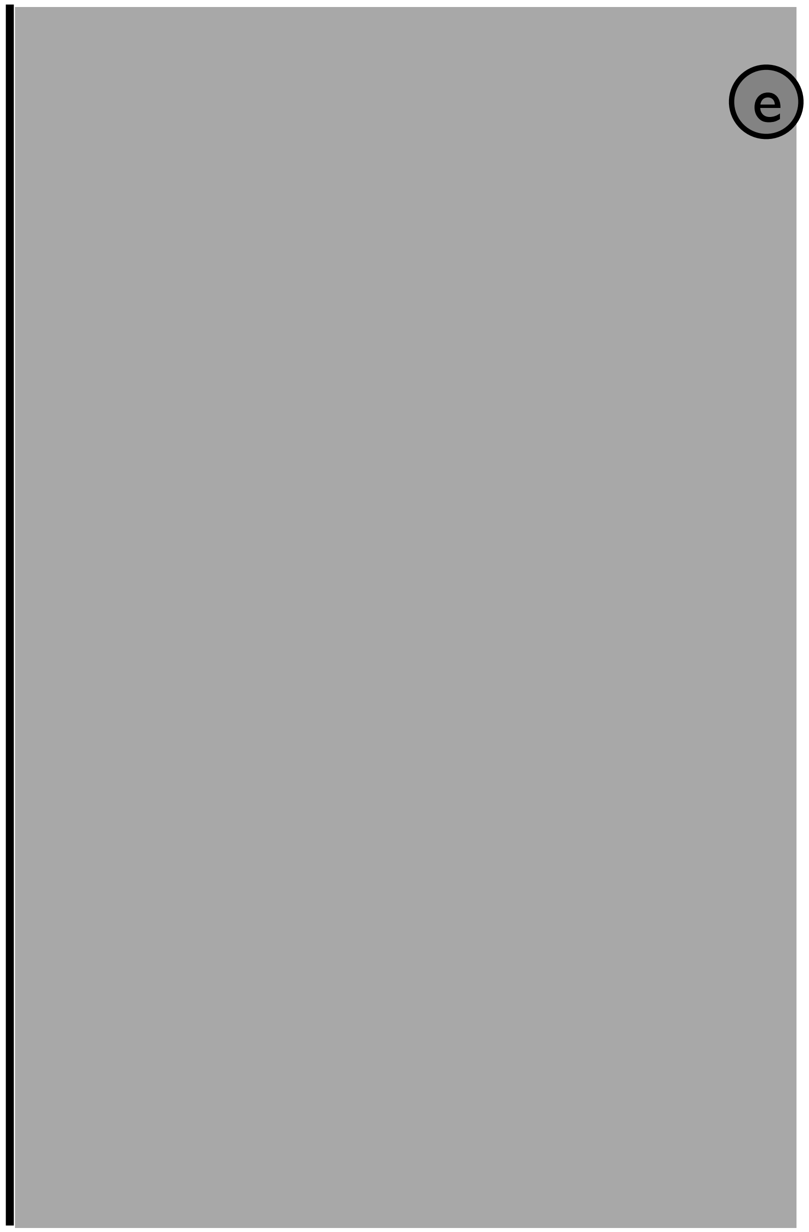

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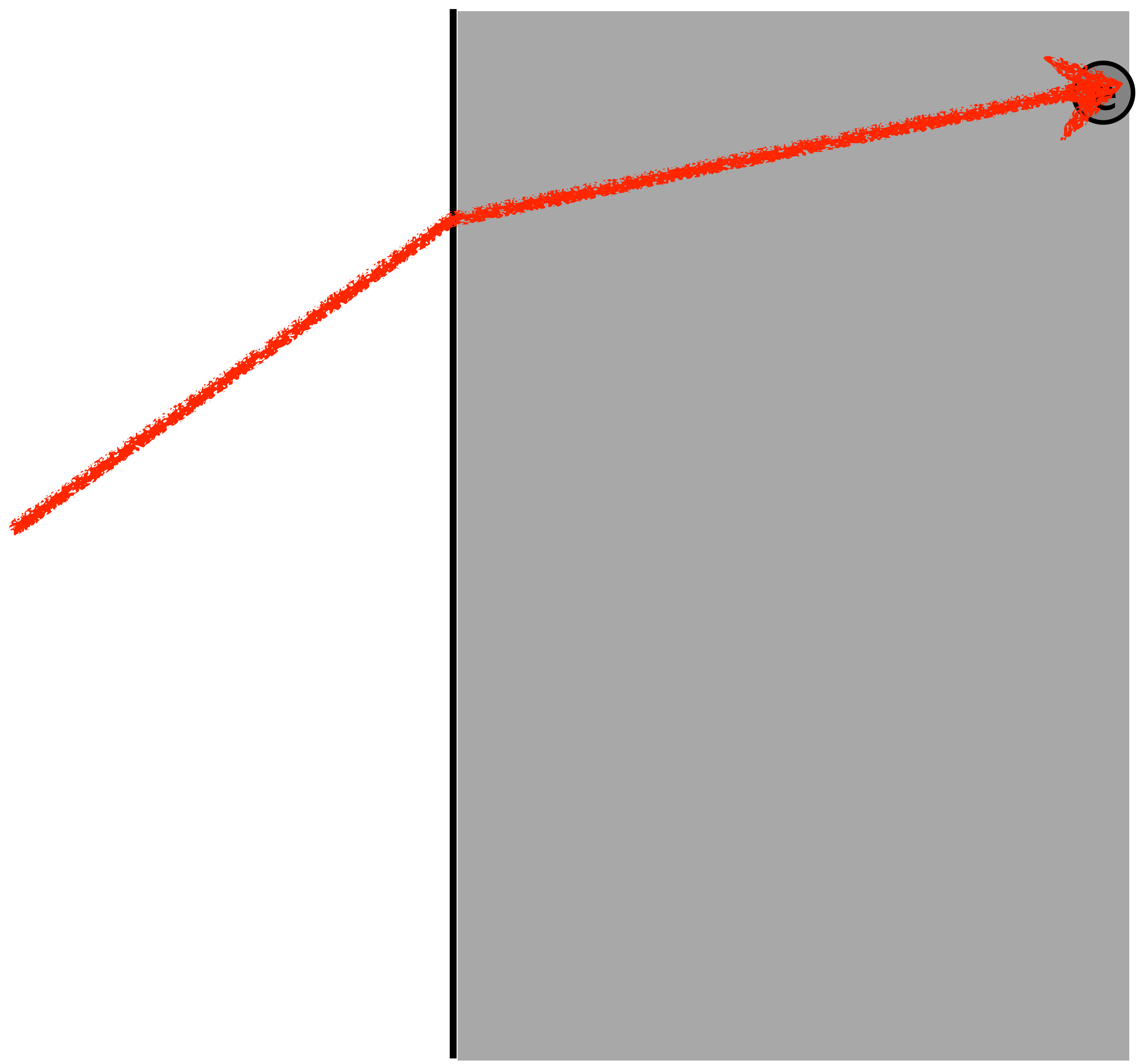

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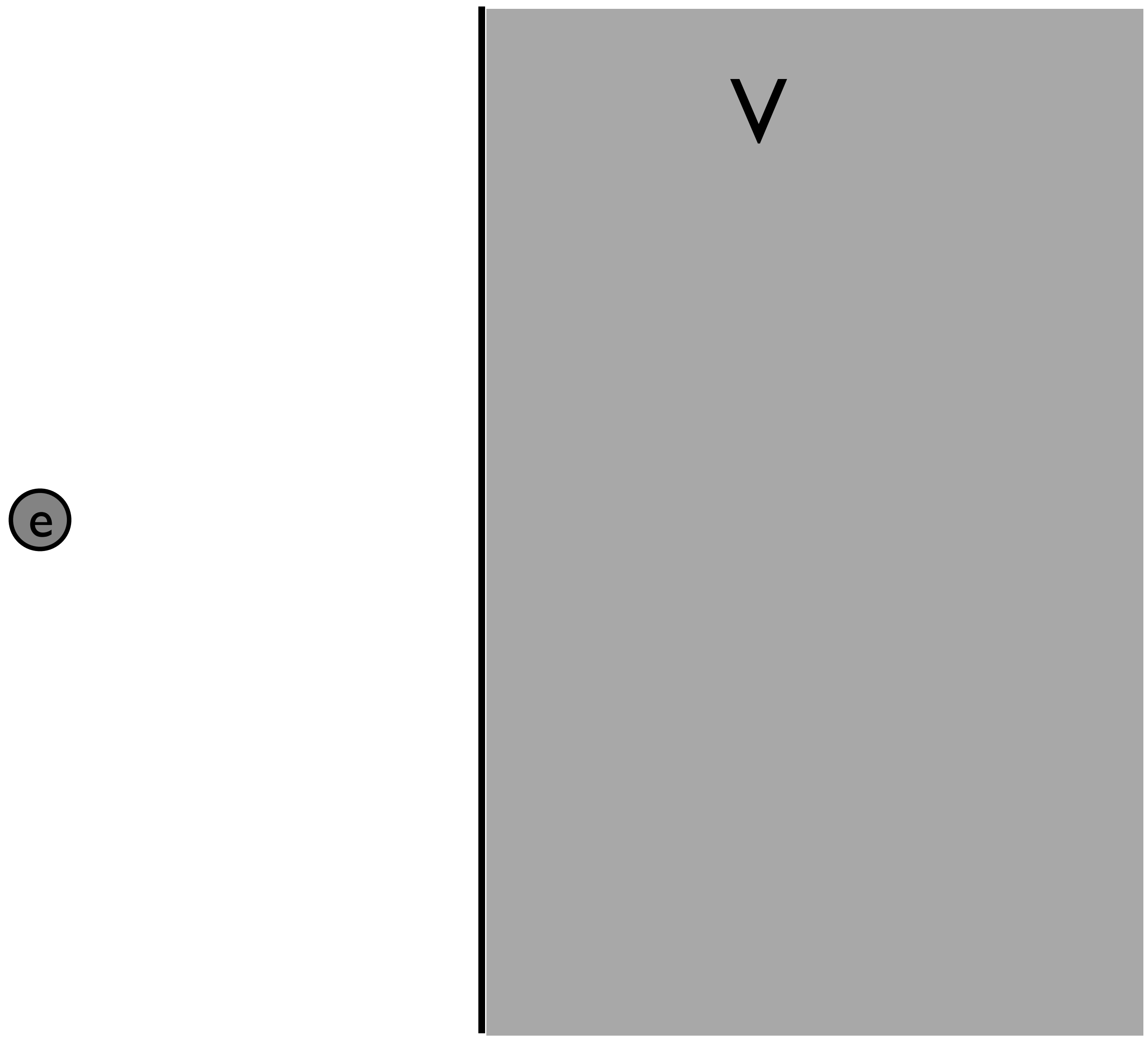

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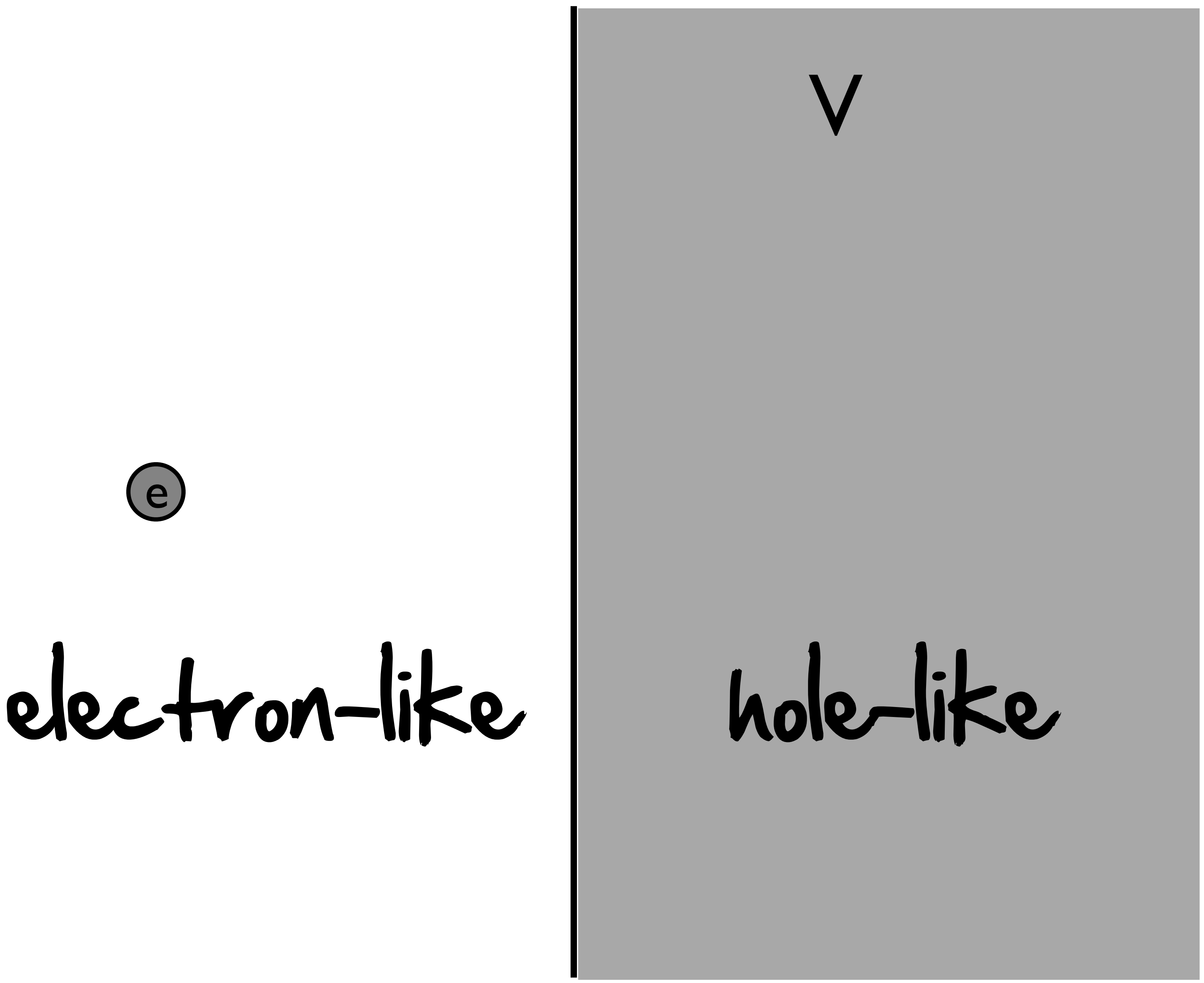

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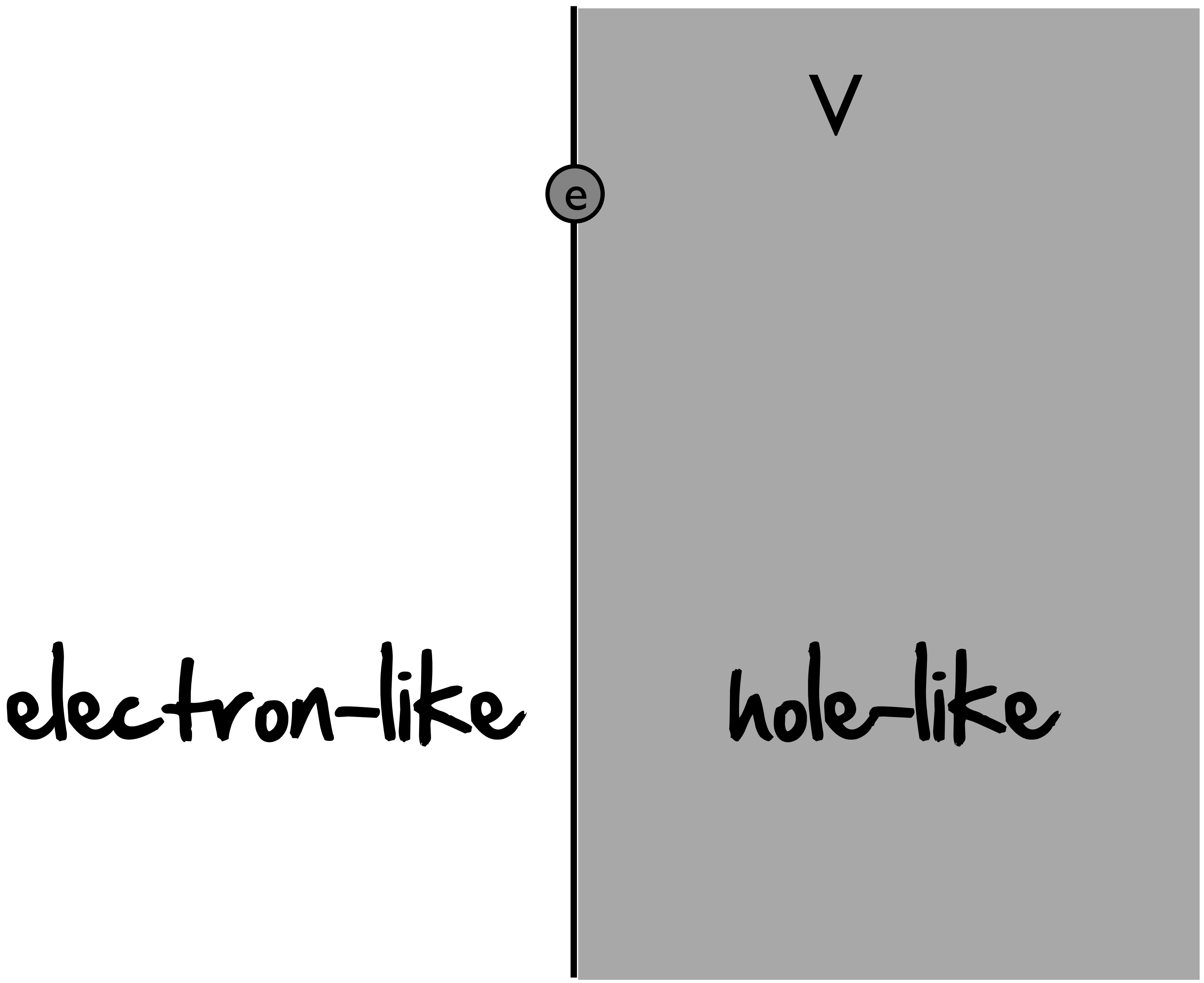

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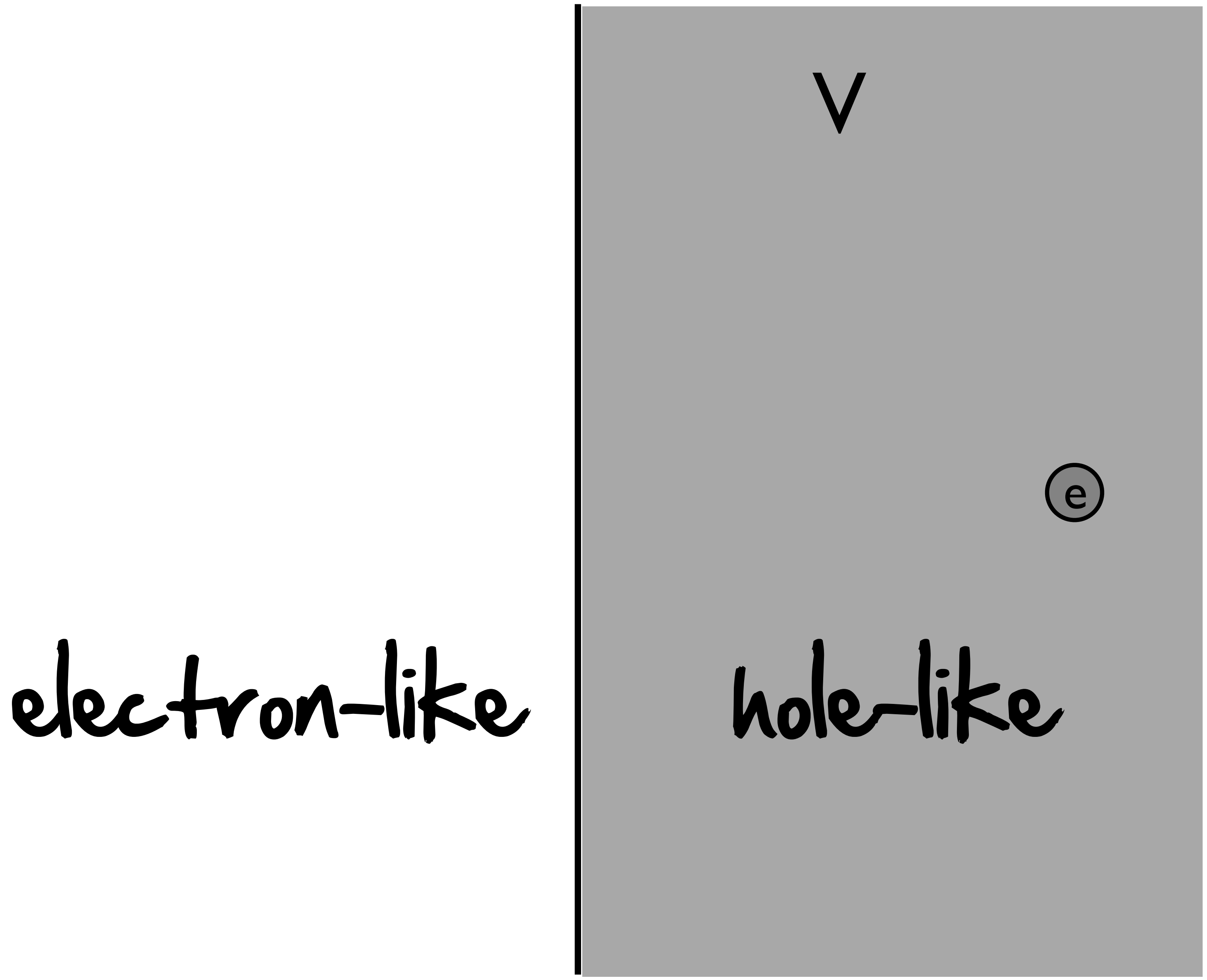

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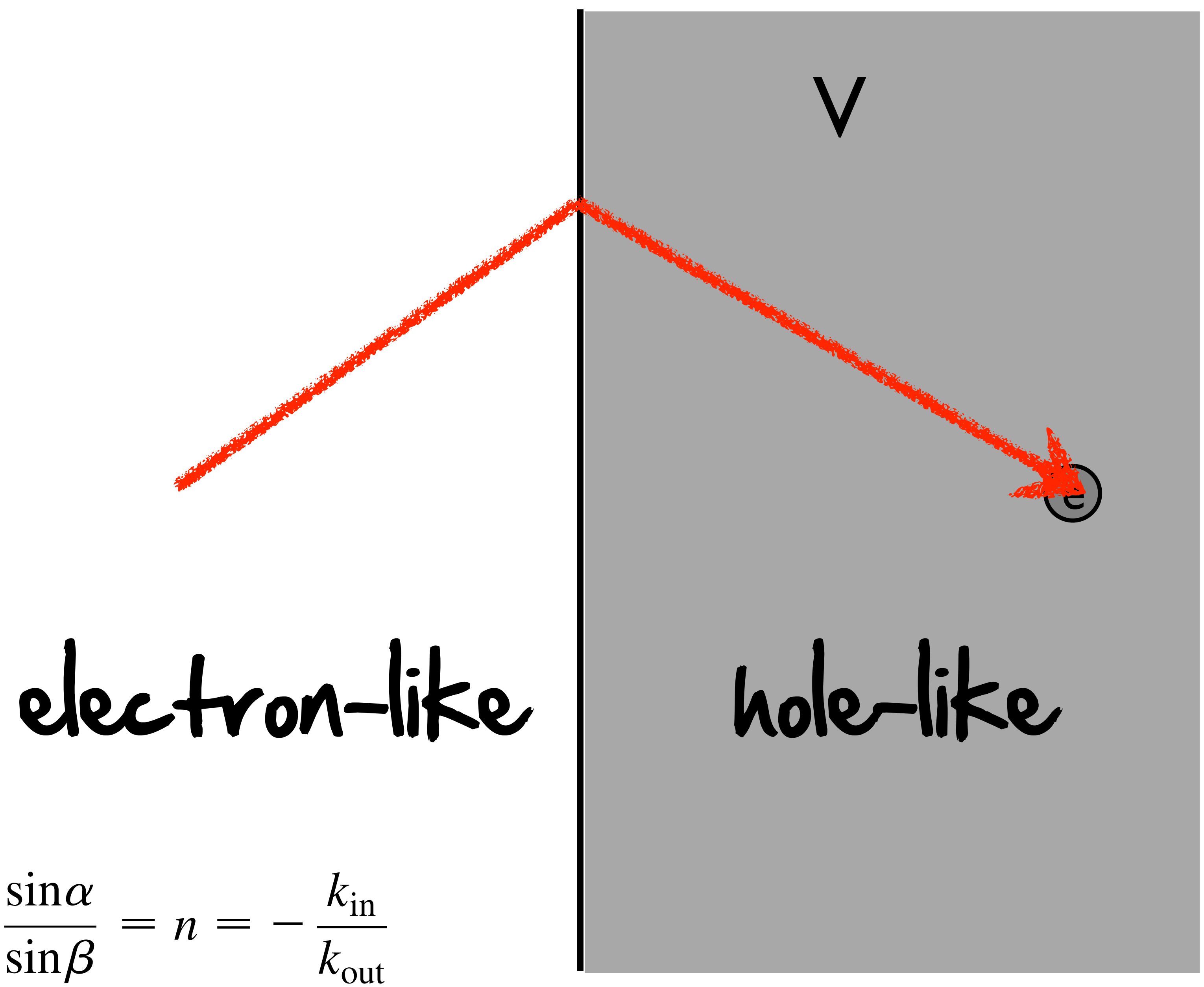

$$\frac{\sin\alpha}{\sin\beta} = n = -\frac{k_{\text{in}}}{k_{\text{out}}}$$

Discussion

The Focusing of Electron Flow and a Veselago Lens in Graphene p-n Junctions Vadim V. Cheianov, et al. Science 315, 1252 (2007);