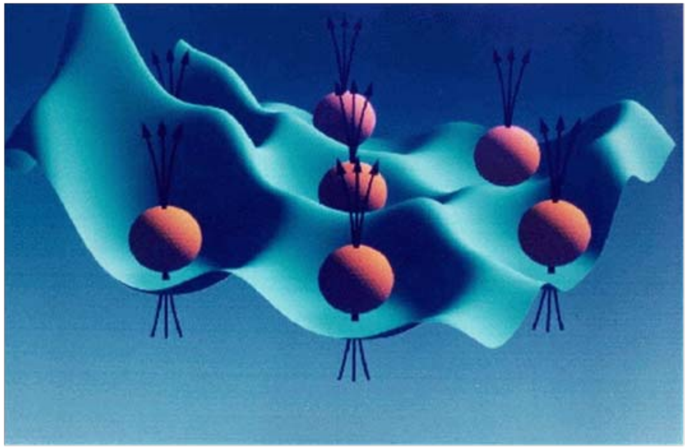
Topological Matter

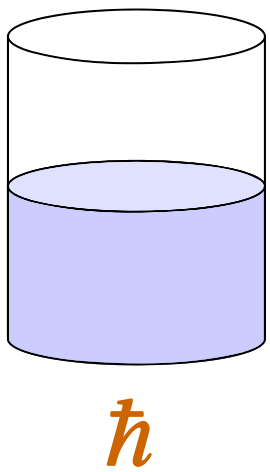


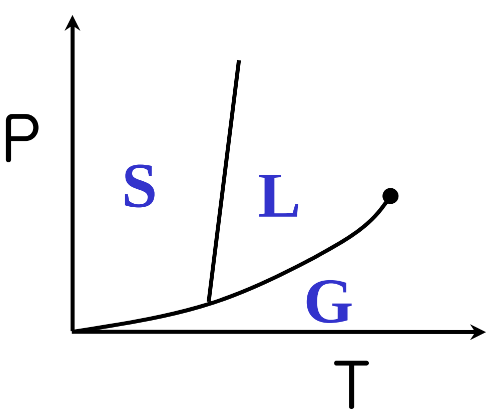
Courtesy of J. Eisenstein

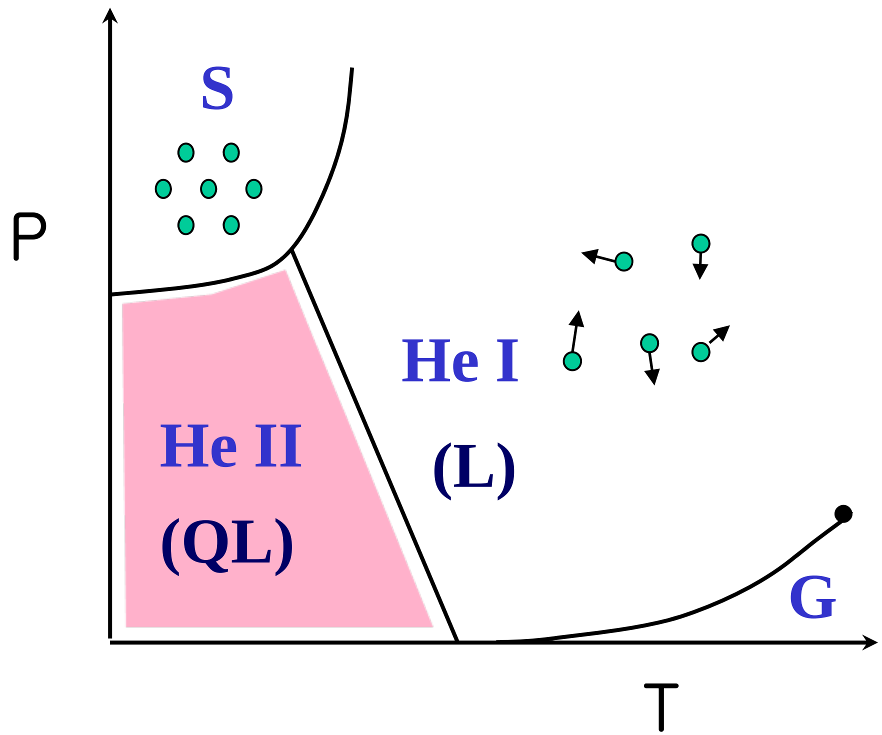
Kyungsun Moon (Yonsei U.)

2010 KIAS-SNU Physics Winter Camp

Quantum Matter and Broken Symmetry

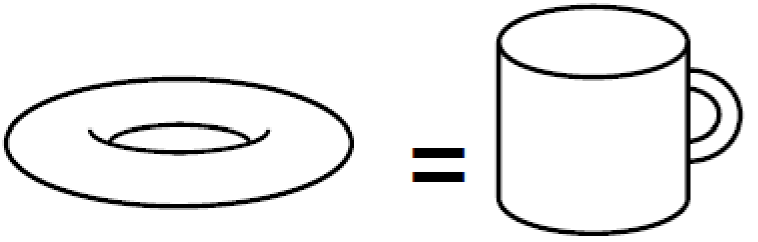






$$\Delta x \cdot \Delta p \geq \hbar$$

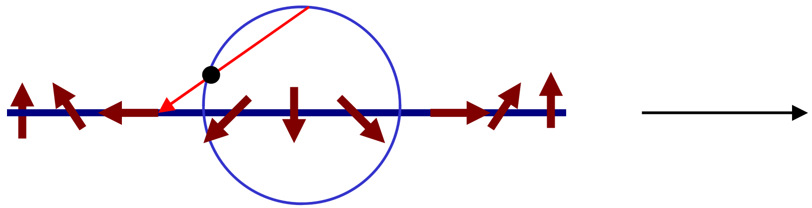
Topology

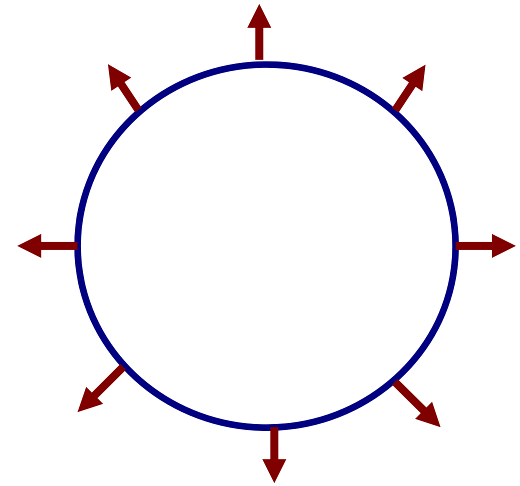


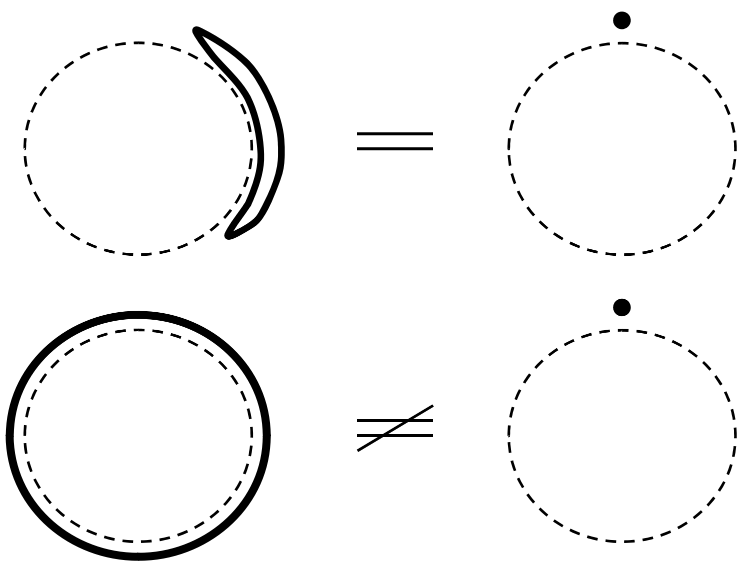
Stable over Smooth Continuous Deformation

Homotopy group: $S^1 \rightarrow S^1$

$\pi_1(S^1) = Z \qquad \vec{m}(x \rightarrow \pm\infty) = \vec{m}_0$







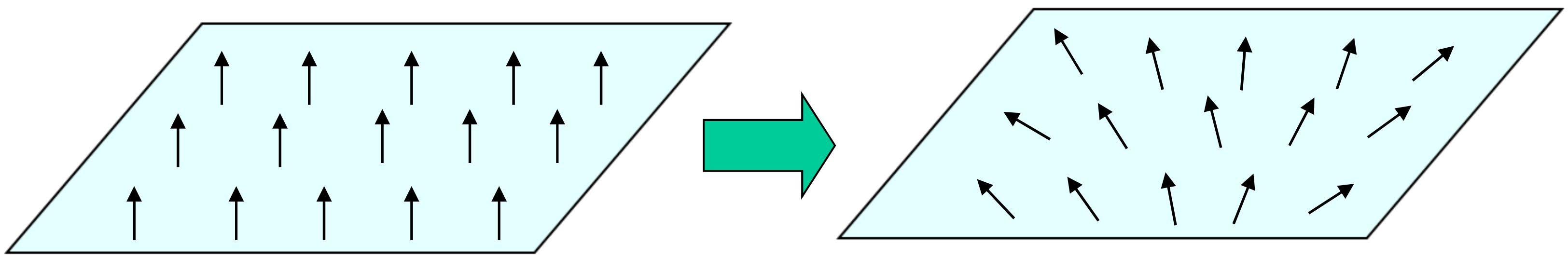
Continuously deformable

Topologically stable

$$Q = \frac{1}{2\pi}\int_0^{2\pi} d\theta \left(\frac{d\Lambda}{d\theta}\right) = \frac{1}{2\pi}\left[\Lambda(2\pi) - \Lambda(0)\right] \qquad \Lambda_n(\theta) = n\theta$$

Homotopy group: $S^2 \rightarrow S^2 \quad \pi_2(S^2) = Z$

$\vec{m}(\vec{r} \rightarrow \infty) = \vec{m}_0$

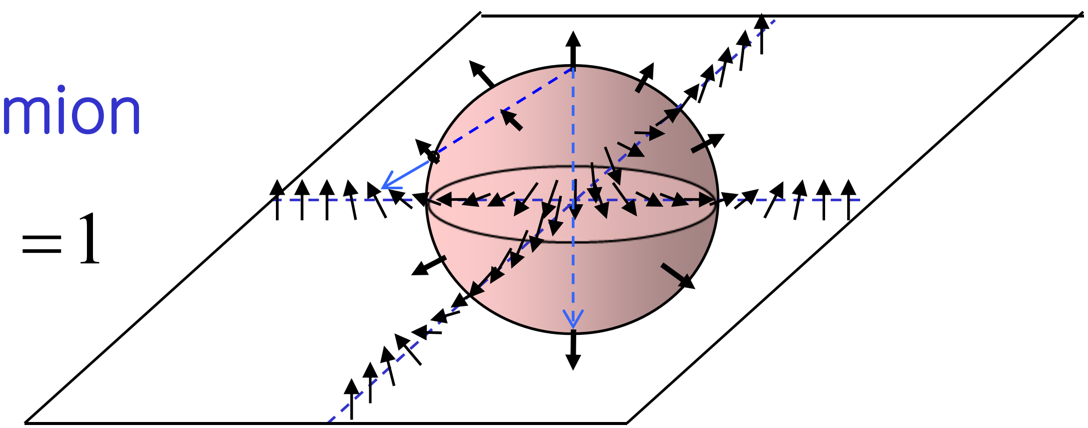


$Q_{\text{top}} = 0$

Spin texture $\vec{m}(\vec{r})$

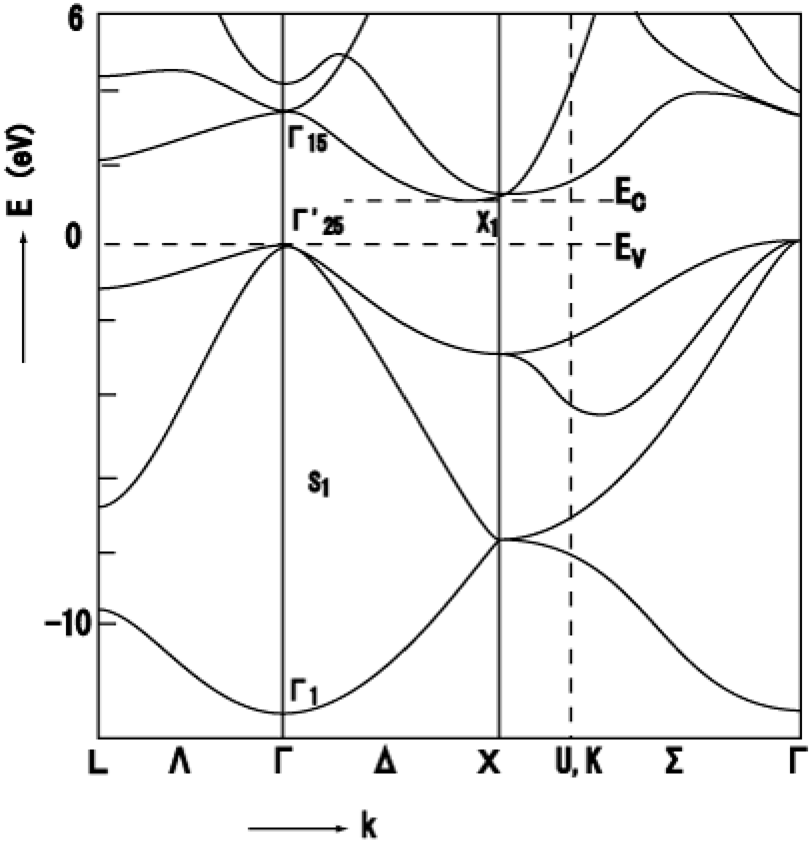
Skyrmion

$Q_{\text{top}} = 1$

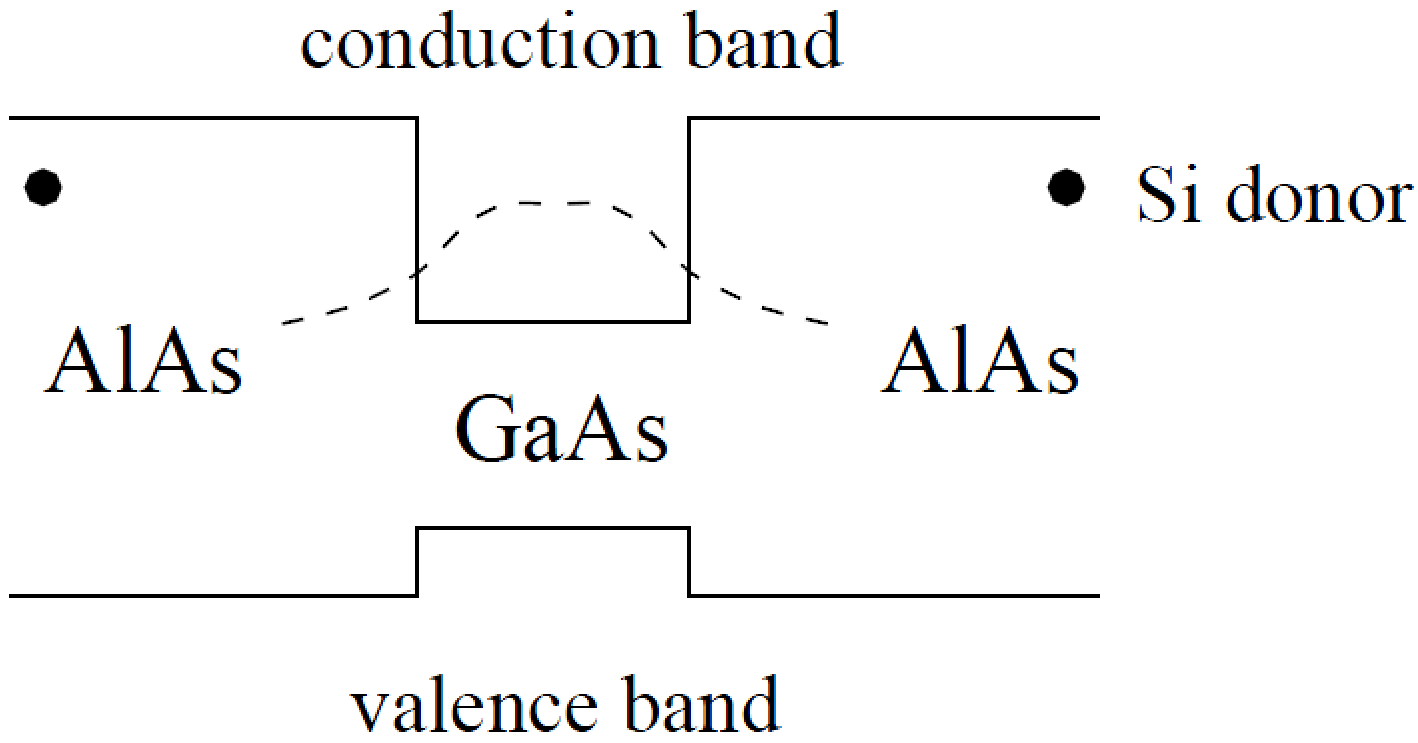


$$Q_{\text{top}} \equiv \frac{1}{8\pi} \int d^2r \, \epsilon^{\alpha\beta} \vec{m} \cdot \partial_\alpha \vec{m} \times \partial_\beta \vec{m}$$

Band Structure of Solid

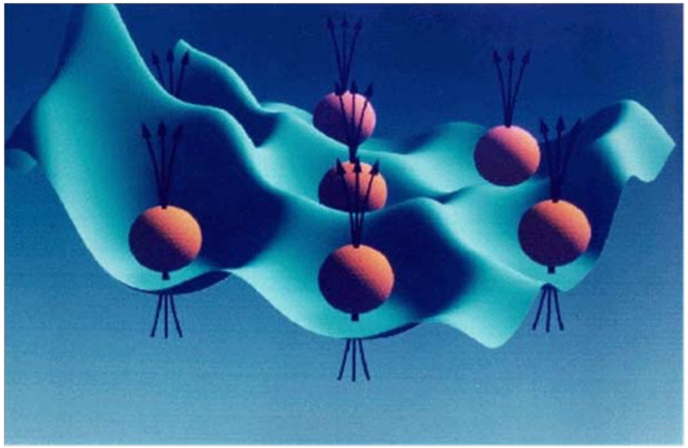


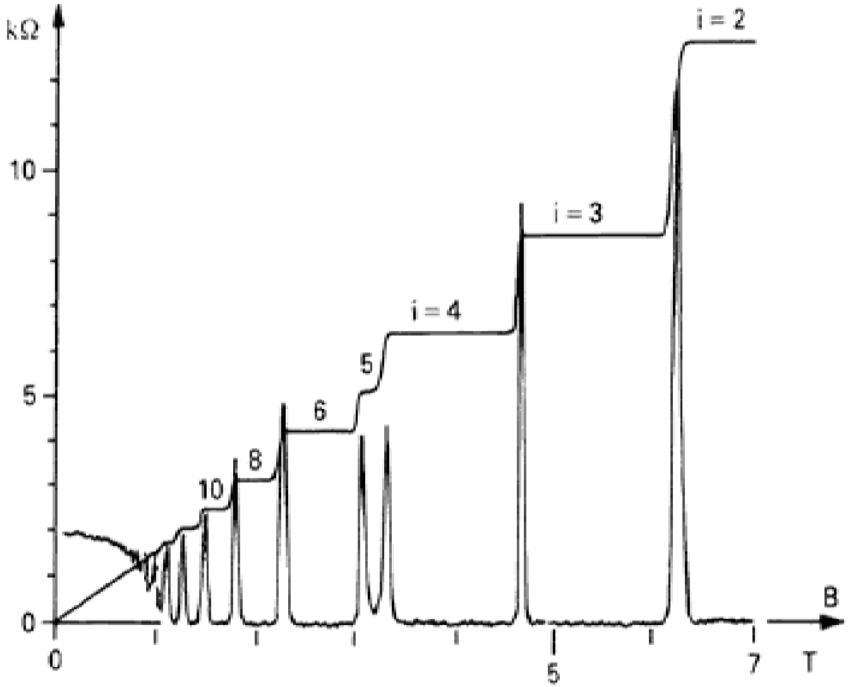
Two-Dimensional Electron Gas (2DEG)

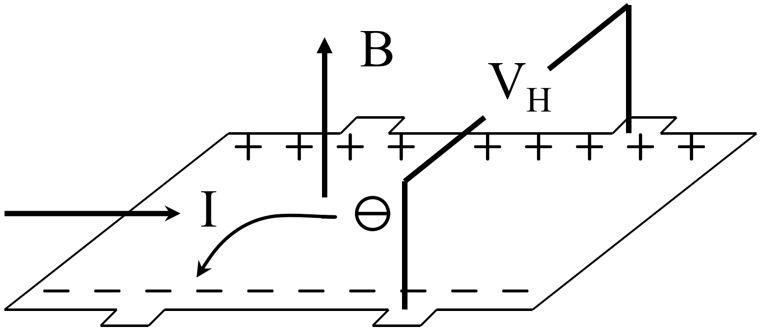


Integer Quantum Hall effect

(von Klitzing,1980)





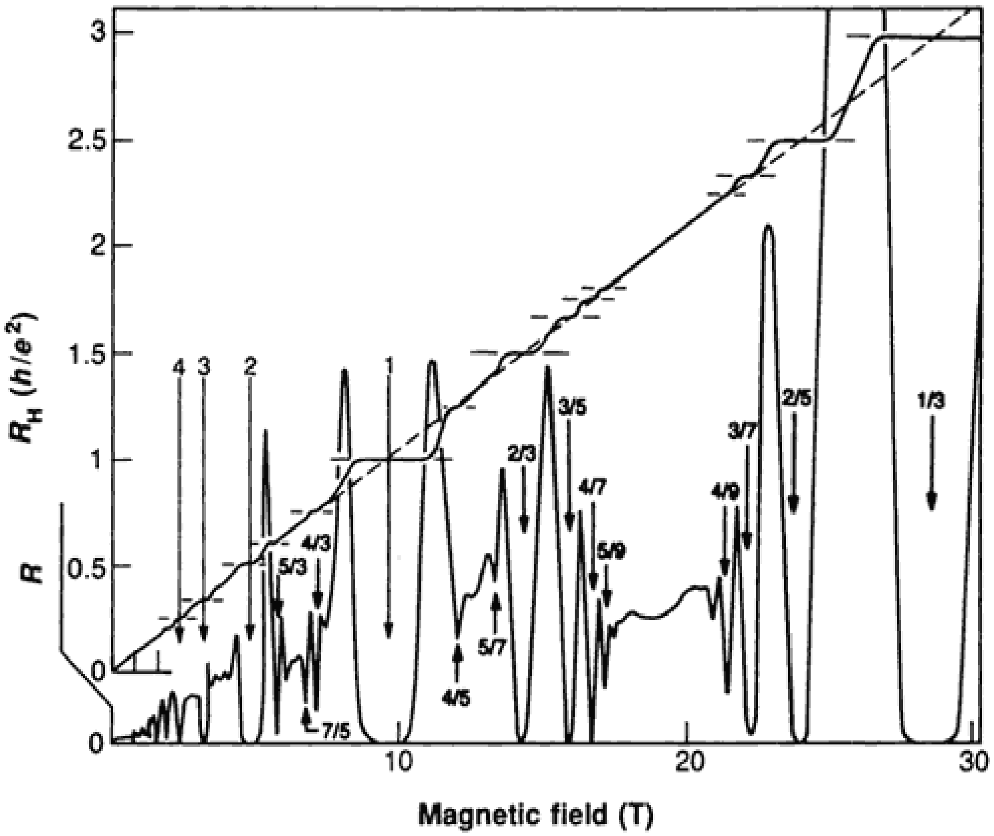


$$\rho_{xy} = \frac{h}{ie^2}, \ i = 1,2,3\cdots$$

$$R_Q = \frac{h}{e^2} \cong 25812.807\Omega$$

Fractional Quantum Hall effect

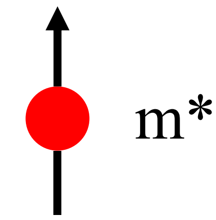
(Tsui, Stormer, Gossard, 1982)



- Cleaner sample
- Cooler temperature
- Higher magnetic field

$$\rho_{xy} = \frac{h}{\nu e^2}, \ \nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7} \cdots$$

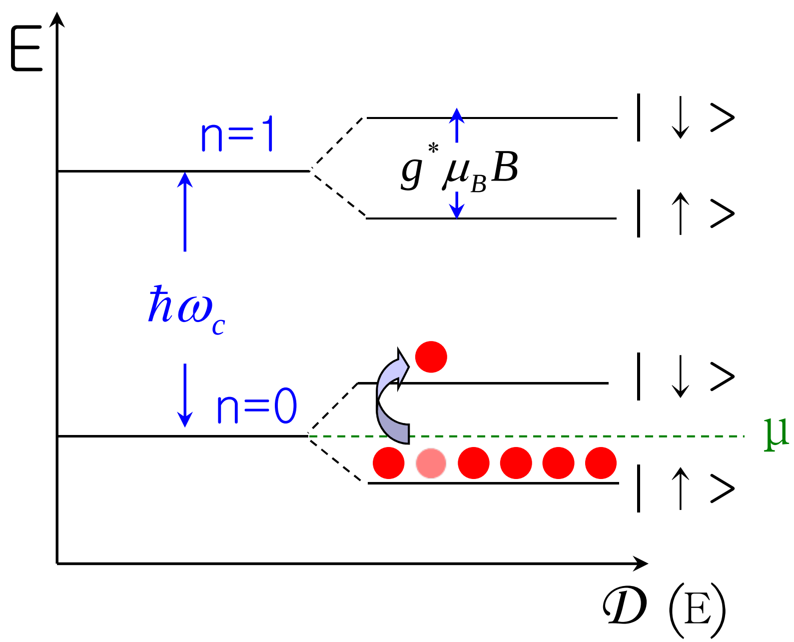
2DEG in a Strong Magnetic Field

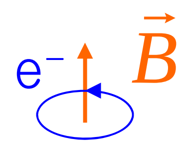


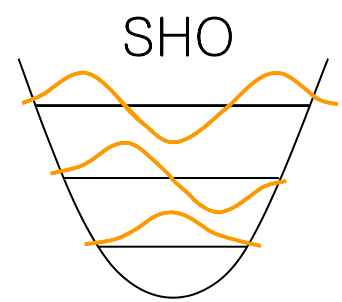
$$H = \sum_{j=1}^{N} \frac{1}{2m^*}\left(\vec{p}_j + \frac{e}{c}\vec{A}\right)^2 + g^* \mu_B B \sum_{j=1}^{N} S_j^z + V$$

K.E. Zeeman E Coulomb

'Landau Level'





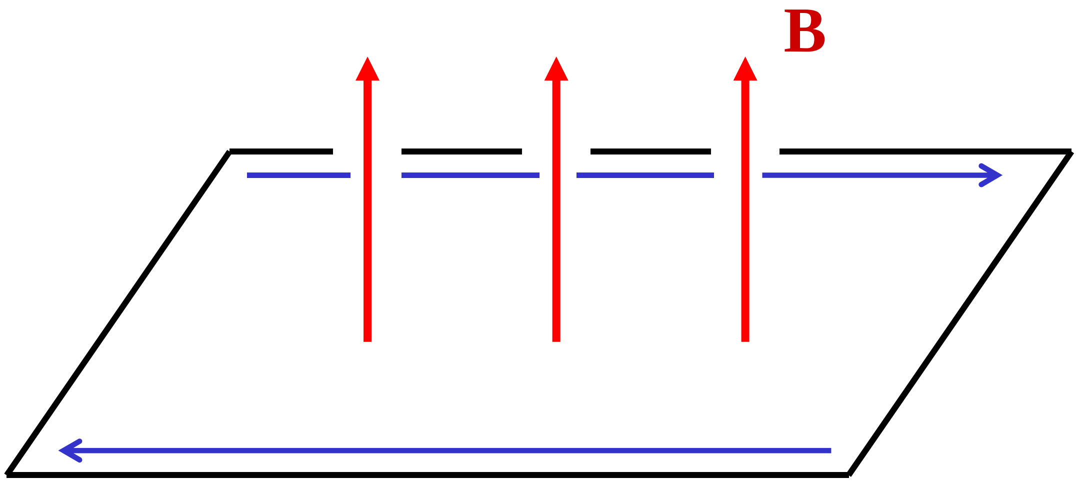


$$E_n = (n + \frac{1}{2})\hbar\omega_c$$

$$\omega_c = \frac{eB}{m^* c}$$

$$l = \sqrt{\frac{\hbar c}{eB}}$$

Quantum Hall Edge States



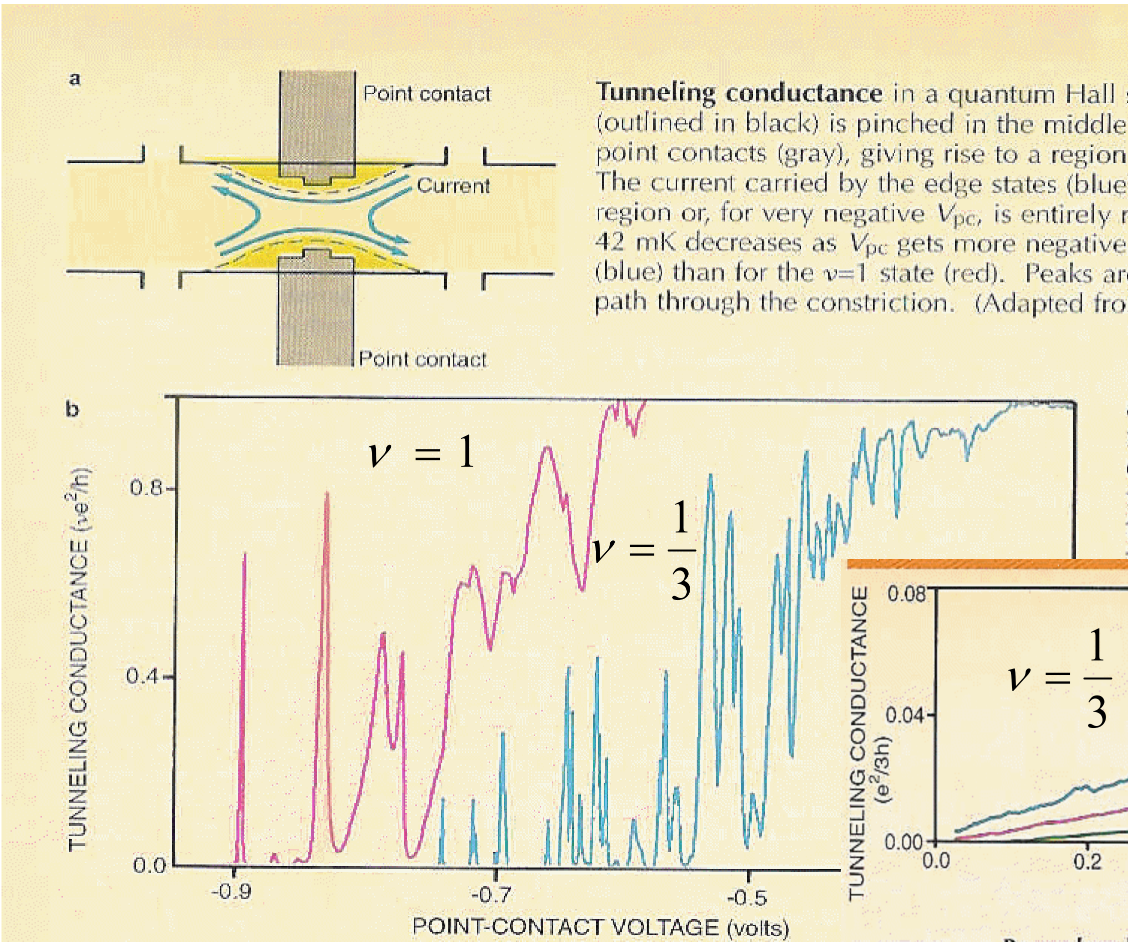
Chiral Edge states due to Broken time reversal symmetry

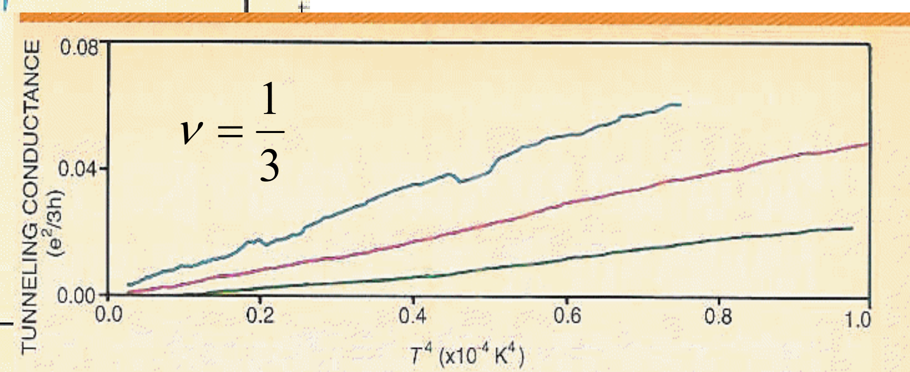
Topologically Stable Conducting States

Disorder: Back-scattering is strongly suppressed

Coulomb Interaction: Chiral Luttinger liquid

Transport through a quantum point contact





Power-law dependence of tunneling conductance on temperature is a hallmark of the Luttinger liquid. The curves shown here are measured for a quantum Hall state with $\nu = 1/3$ at three different values of point-contact voltage V_{pc}, corresponding to three different minima of the curve shown on page 22. (Adapted from ref. 1.)

Quantum Hall States (Topological Matter)

Characterized by an integer N: Number of chiral edge states

Quantized Hall conductance

TKNN number (Thouless-Kohmoto-Nightingale-den Nijs)

$$\sigma_{\mathrm{xy}} = -\frac{e^2}{h} C \qquad \psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_{\vec{k}}(\vec{r})$$

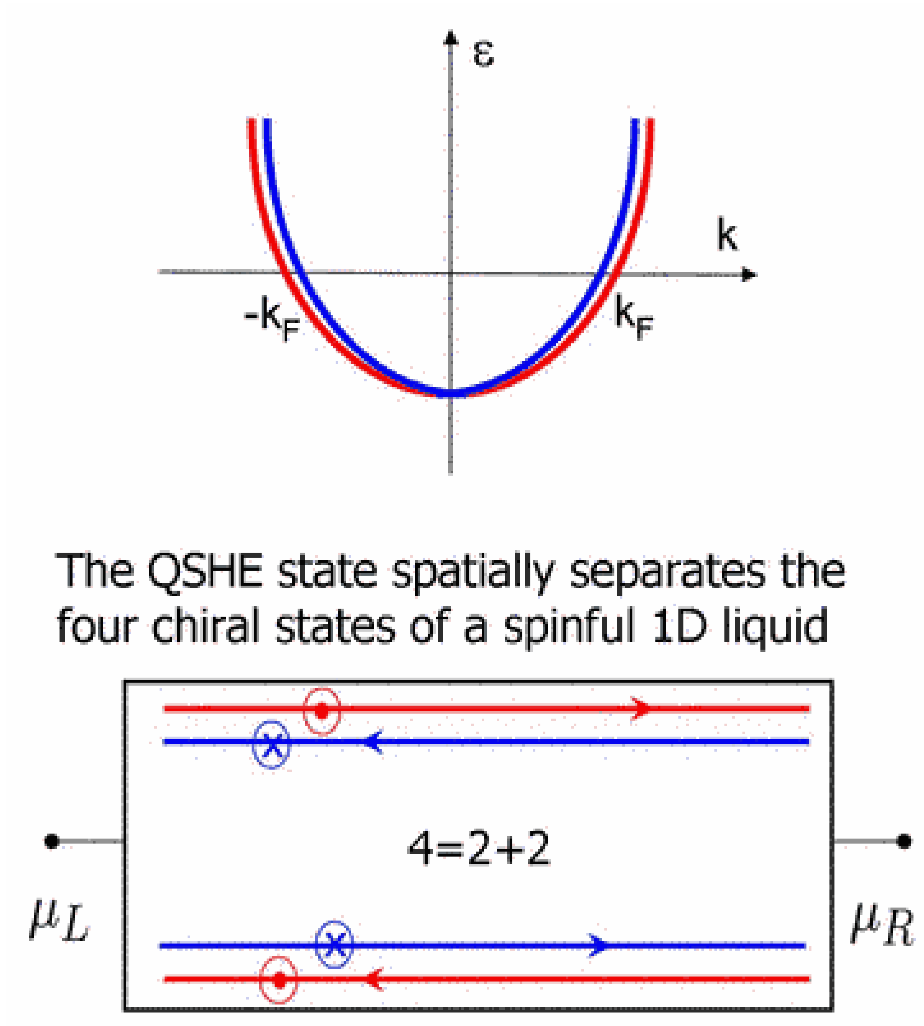
$$C = \frac{1}{2\pi i} \int_{\vec{k}\in \mathrm{FB}} d^2k \int d^2r \left(\frac{\partial u^*}{\partial k_y} \frac{\partial u}{\partial k_x} - \frac{\partial u^*}{\partial k_x} \frac{\partial u}{\partial k_y} \right) \in Z$$

First Chern number (Topologically invariant)

Quantum Spin Hall States (Helical Edge states)

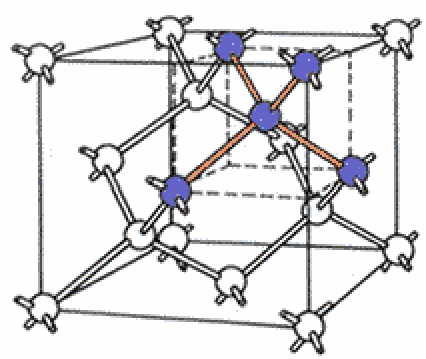
[S.C. Zhang]

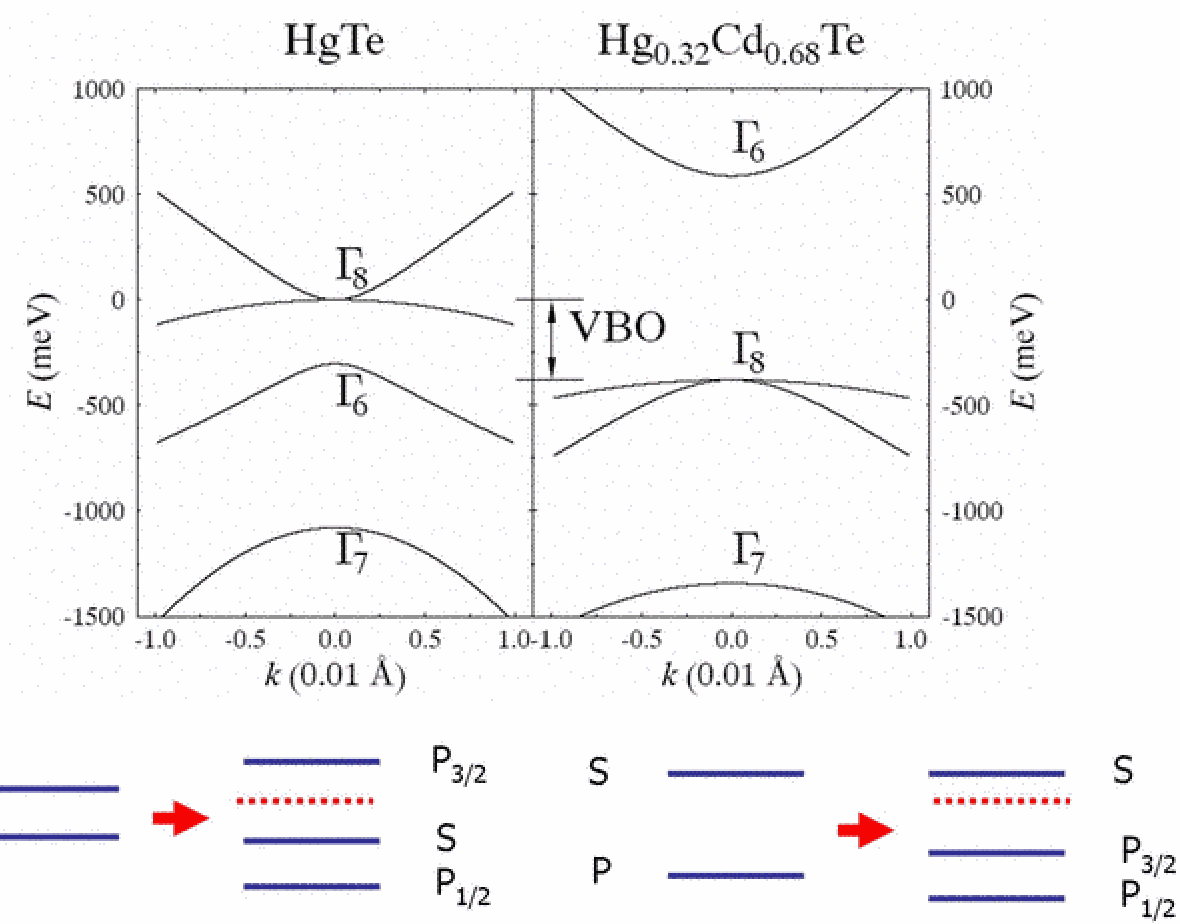
Time reversal invariance



Band Structure of HgTe

B. Bernevig, T. Hughes, S.C. Zhang, Science 314, 1757 (2006).

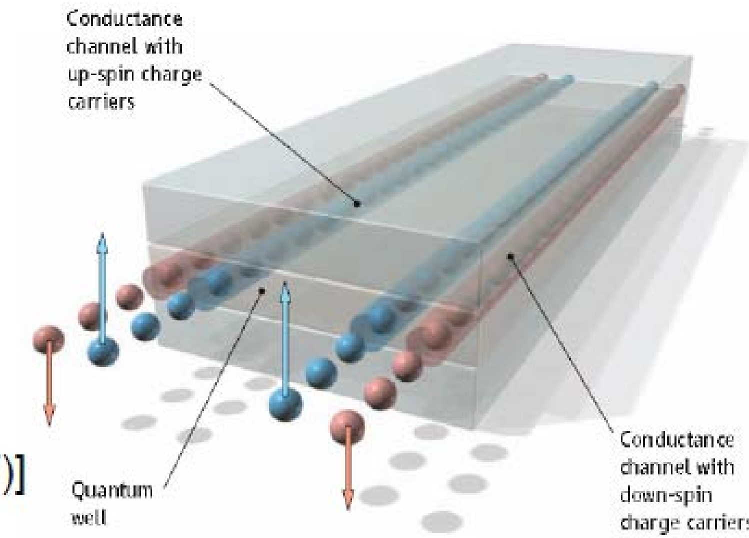




Experiment

HgTe/(Hg,Cd)Te quantum wells

Konig et al. [Science 318, 766 (2007)]



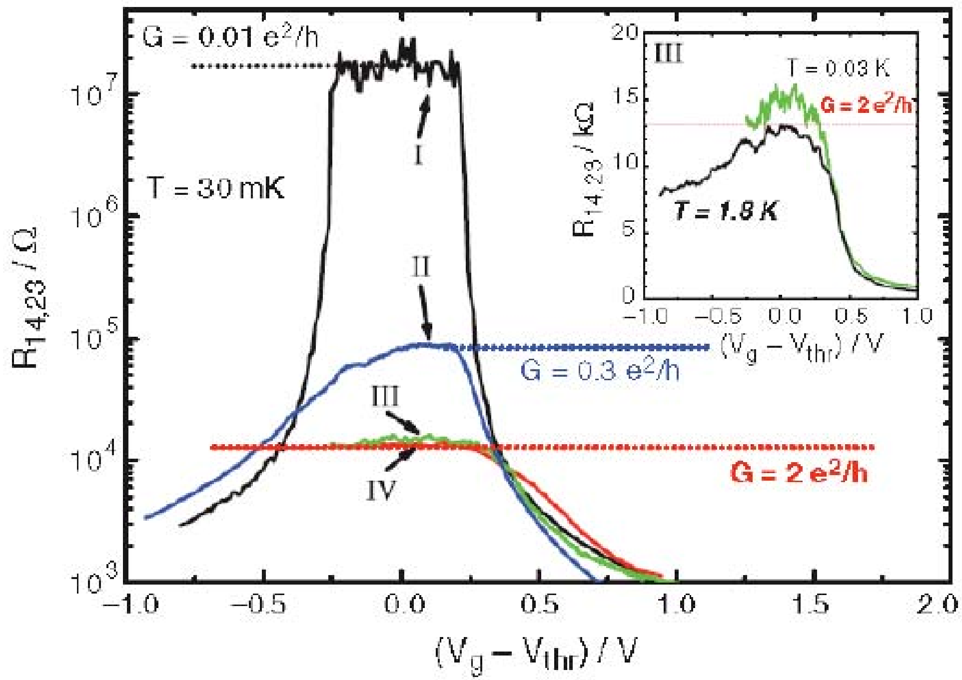
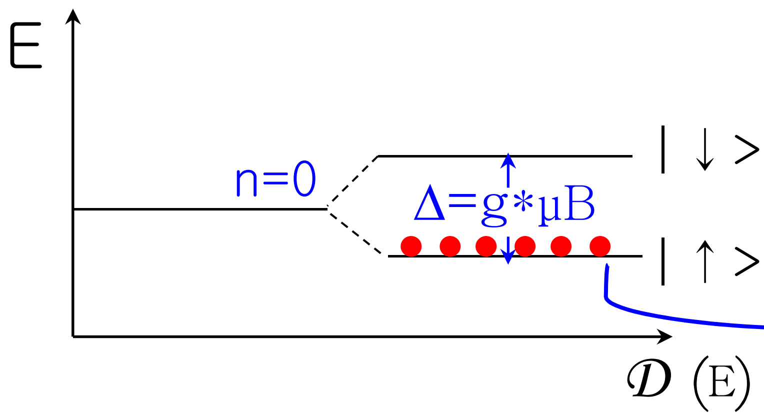


Fig. 4. The longitudinal four-terminal resistance, $R_{14,23}$, of various normal (d = 5.5 nm) (I) and inverted (d = 7.3 nm) (II, III, and IV) QW structures as a function of the gate voltage measured for B = 0 T at T = 30 mK. The device sizes are (20.0 × 13.3) μm² for devices I and II, (1.0 × 1.0) μm² for device III, and (1.0 × 0.5) μm² for device IV. The inset shows $R_{14,23}(V_g)$ of two samples from the same wafer, having the same device size (III) at 30 mK (green) and 1.8 K (black) on a linear scale.

Role of Coulomb Interactions: Topological Excitations

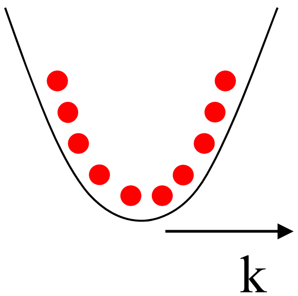
- Spin texture states: Skyrmion
- Bilayer quantum Hall system (BQHS): Meron
- BQHS with parallel magnetic field: Domain wall

Quantum Hall Ferromagnet: $\nu = 1$



2DEG : B=0

$$E_k = \frac{\hbar^2 k^2}{2m}$$



$$\Delta \cong 3K << \frac{e^2}{\varepsilon l} \cong 100K$$

Coulomb interaction

When Δ=0, one-body gap disappears.

V≠0 Coulomb interaction

Ground state : Quantum Hall ferromagnet

$$\Psi_s = \left|\uparrow\uparrow\uparrow\cdots\uparrow\right\rangle$$: SU(2) symmetry

spatial (→ Ψ_r), spin (→ Ψ_s)

$$\Psi = \Psi_r(\vec{r}_1, \vec{r}_2, \cdots, \vec{r}_N) \otimes \Psi_s$$

Antisymmetric (↑ Ψ_r) Symmetric (↑ Ψ_s)

$$KE = \frac{1}{2}\hbar\omega_c N$$

↑ degenerate

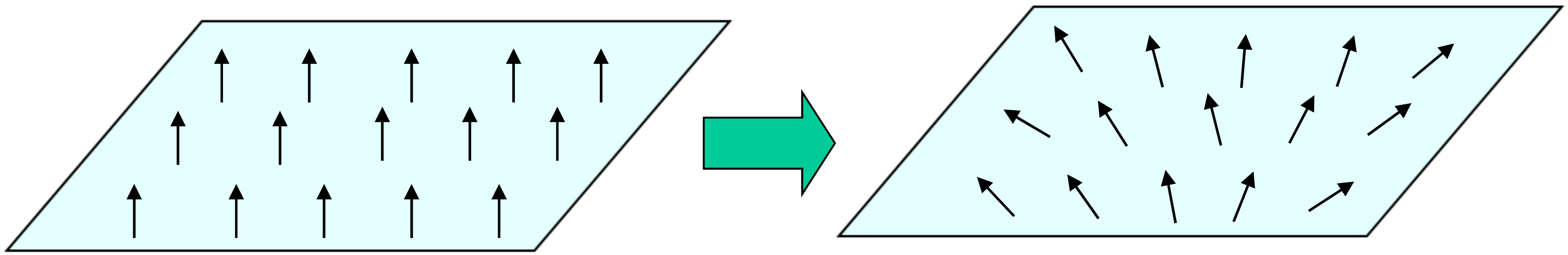
'Rigidity'

Coulomb interaction minimized!!

Strongly Correlated Insulator

"Interaction is crucial to select out the ground states"

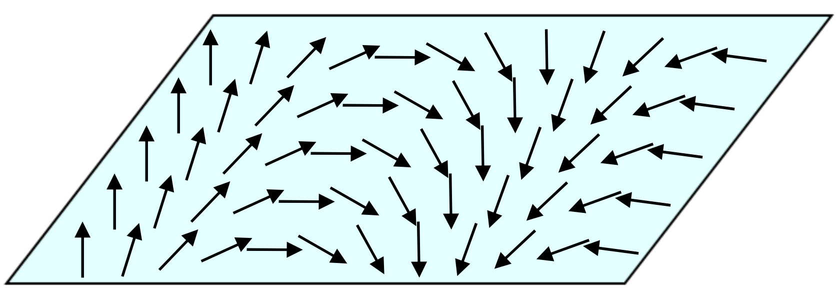
Ground state : Quantum Hall ferromagnet



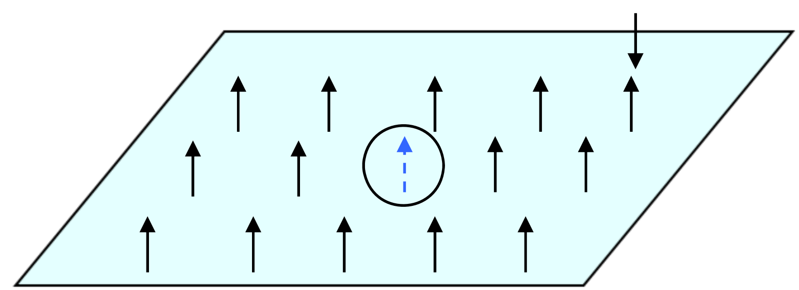
|G.S.>

Spin texture $\vec{m}(\vec{r})$

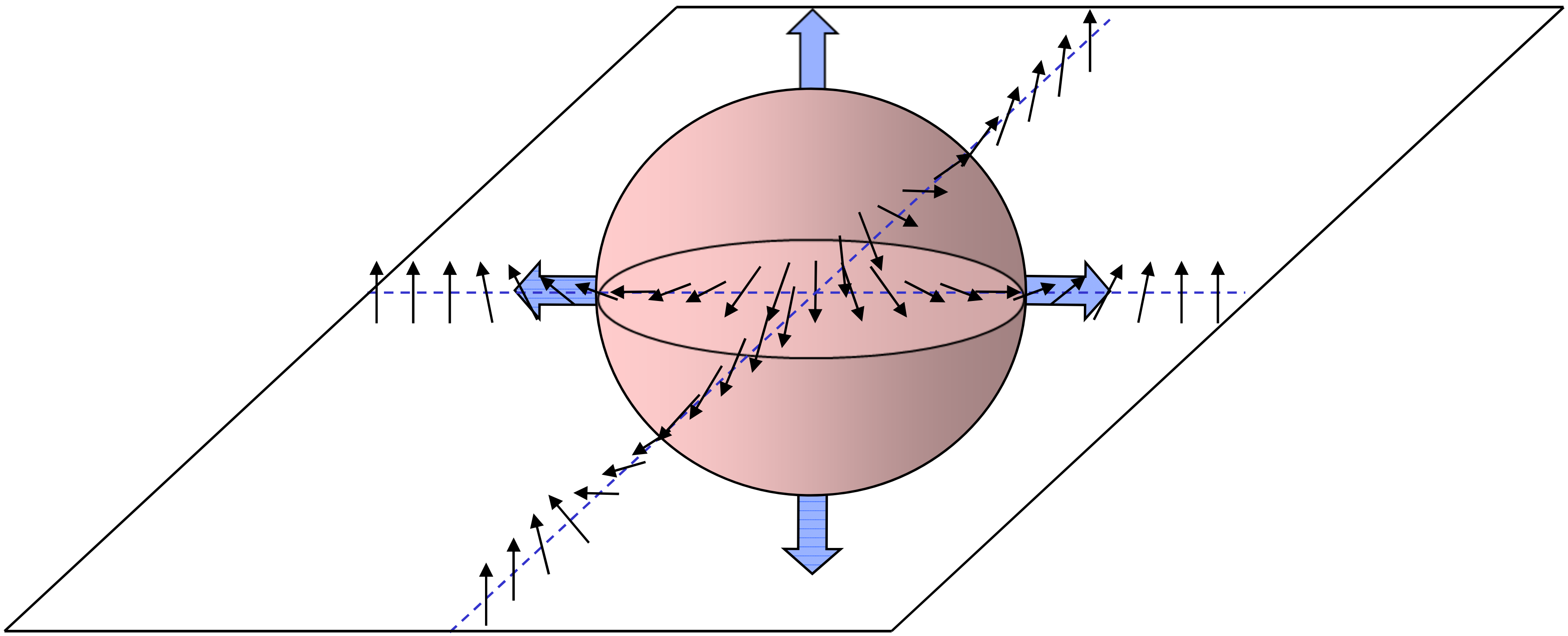
Charge neutral: Spin wave



Particle−hole pair



Topological excitation: Skyrmion



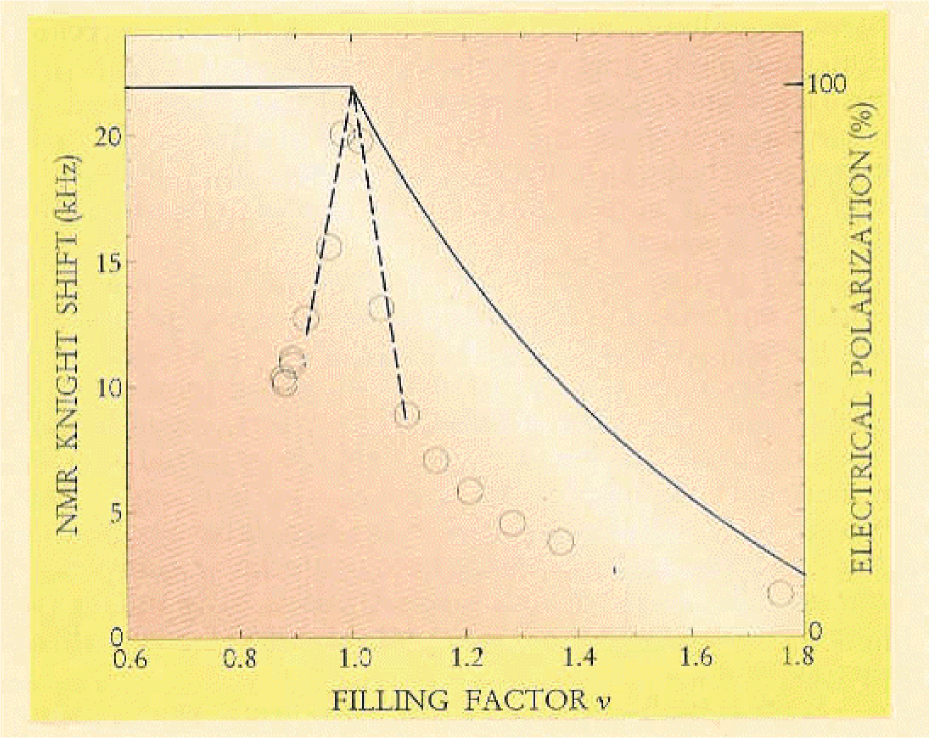
Quantized charge: $q_{sk} = Q_{\text{top}}[e] = \pm e$

Spin: $\Delta S \approx 7$ for GaAs sample

$$Q_{\text{top}} \equiv \frac{1}{8\pi} \int d^2 r \, \epsilon^{\alpha\beta} \vec{m} \cdot \partial_\alpha \vec{m} \times \partial_\beta \vec{m}$$

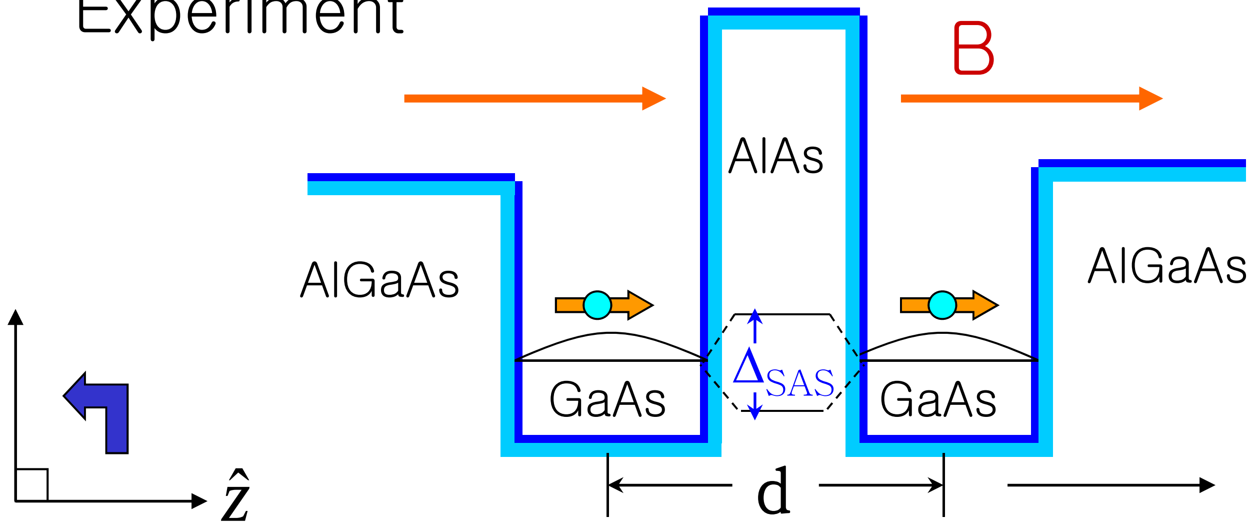
NMR Knight shift measurement : Skyrmion

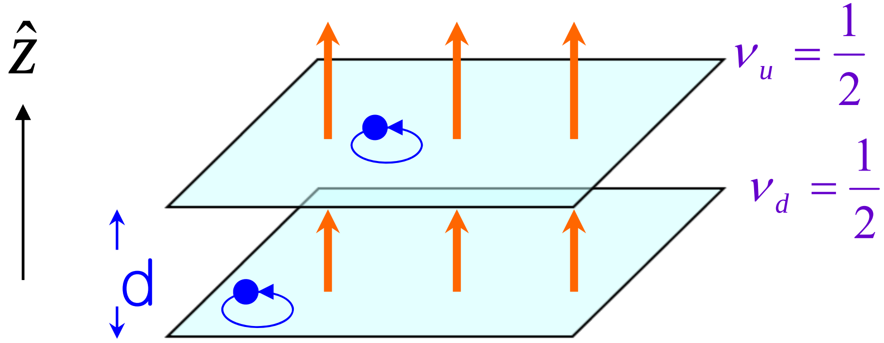
[Barrett *et al.*]



Bilayer Quantum Hall System

Experiment





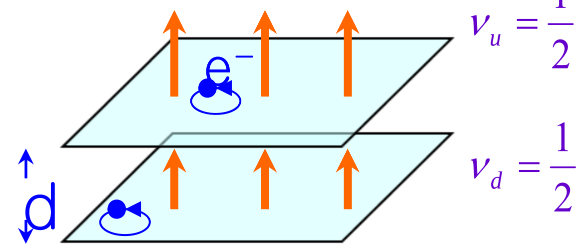
i) $\Delta_{SAS} \sim 1\mathrm{K}$

ii) $d \approx l \cong 130\,\mathring{A}$ at $\nu = 1$

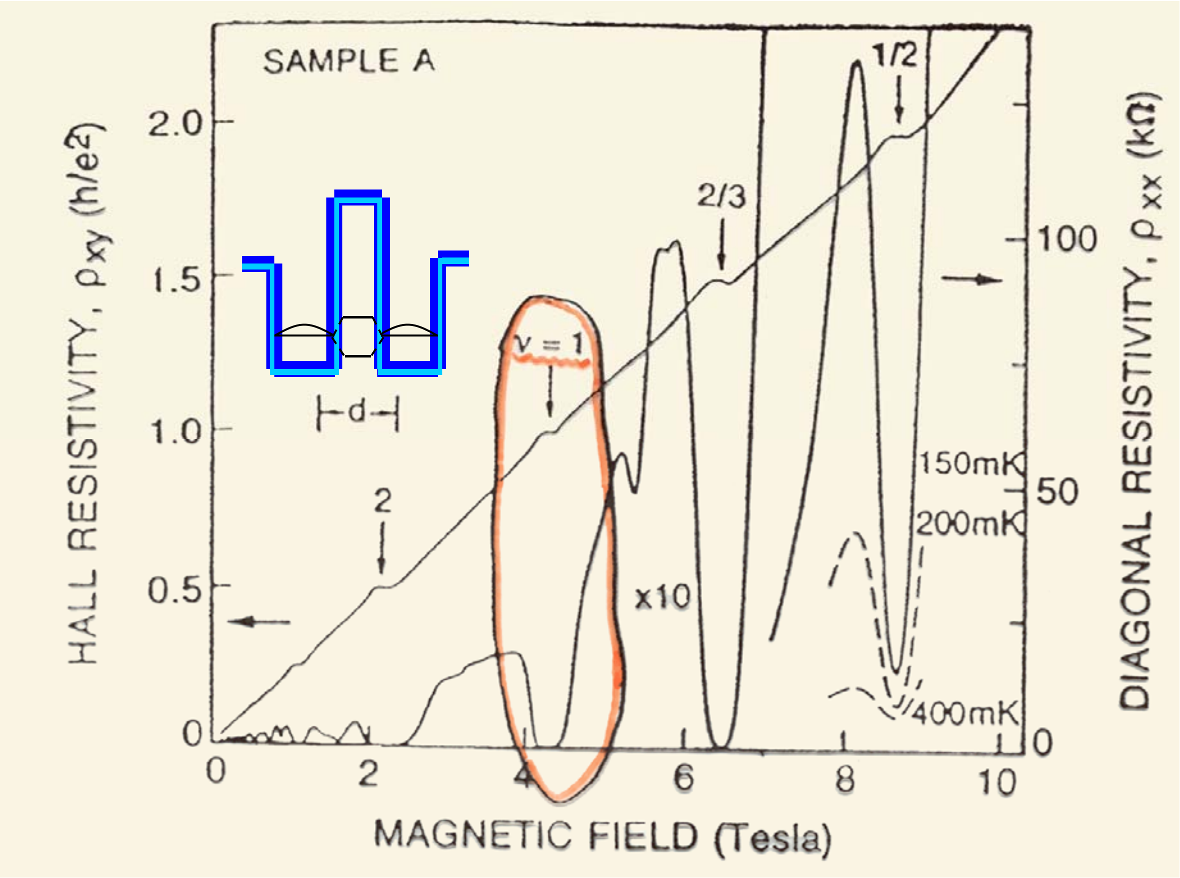
Spin is fully polarized due to Zeeman gap.

$g^*\mu B \gg \Delta_{SAS}$

Experiment

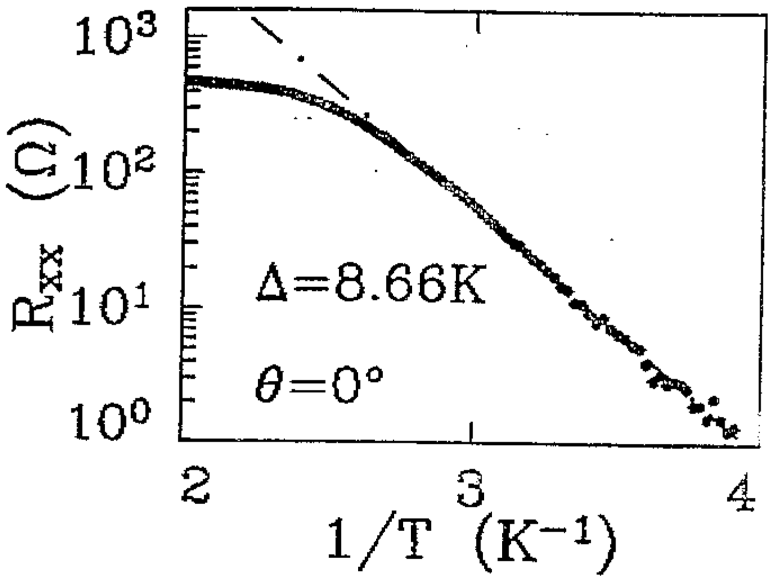


$d \cong l \cong 130\ \mathring{A}$



$\nu = 1$ means $\nu = \frac{1}{2}$ in each layer.

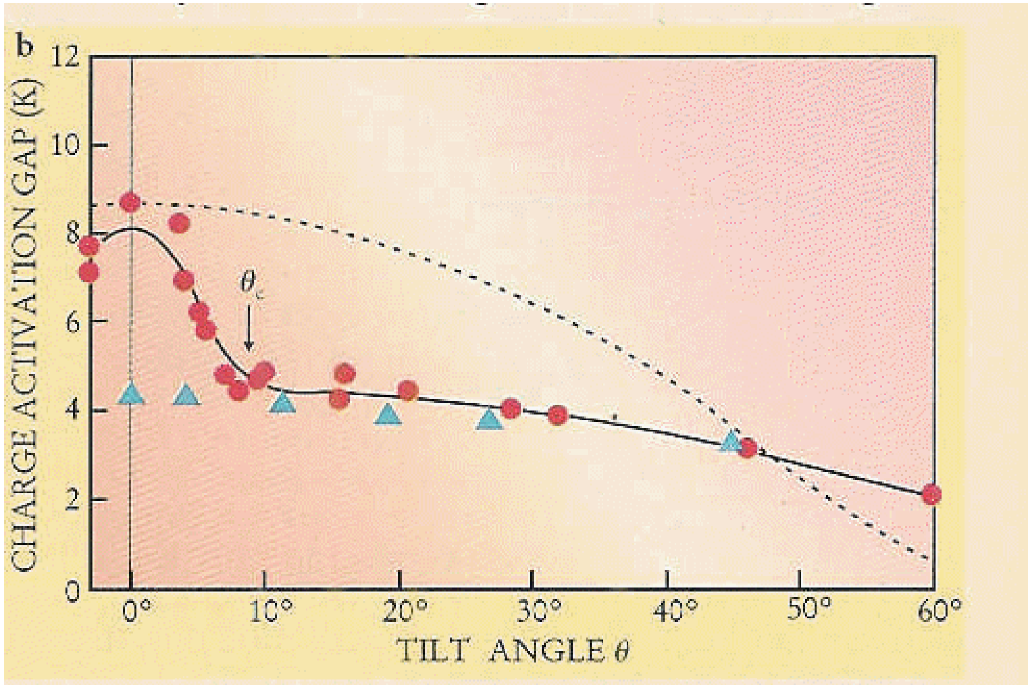
Experiment-1

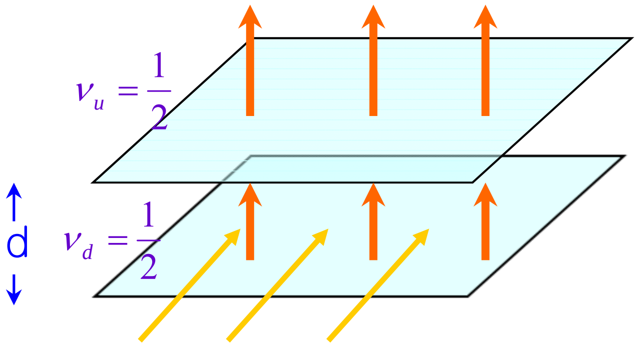


Gap collapses at **T=0.4K**

$$\Delta_{SAS} = 2t = 1K$$

Experiment-2

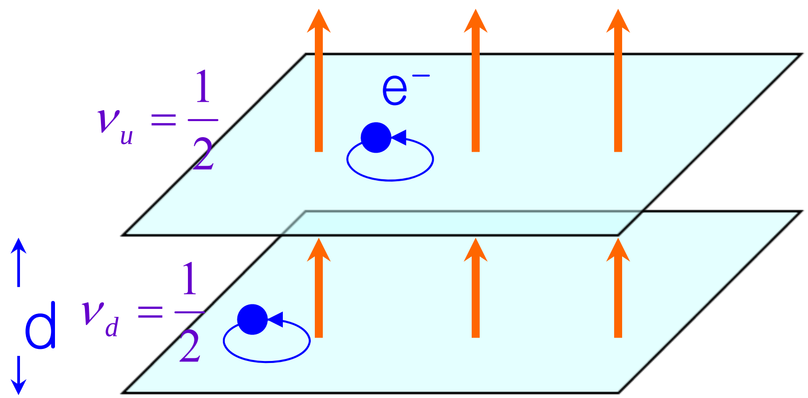




$$L_{\parallel} = \frac{\Phi_0}{d} \cong 20\,l$$

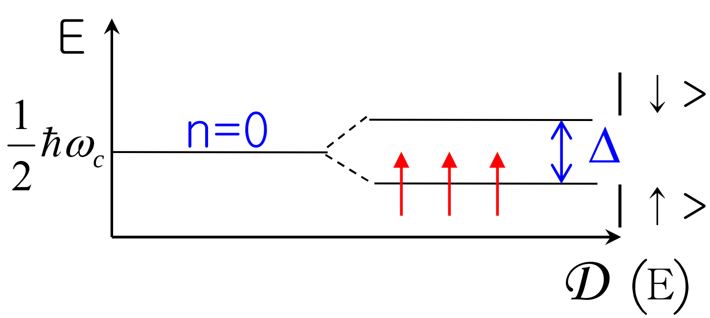
Bilayer Quantum Hall System

Theory *



$$\nu_{tot} = \nu_u + \nu_d = 1$$

Real spin polarized $|\uparrow>$



1. LLL approxm.
2. $\Delta = g^* \mu_B B \rightarrow \infty$

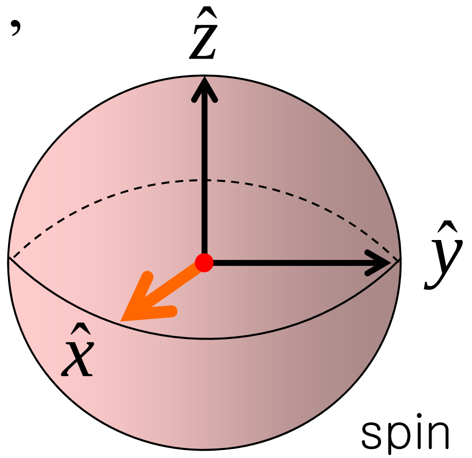
Pseudo-spin

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$ 'Proton'

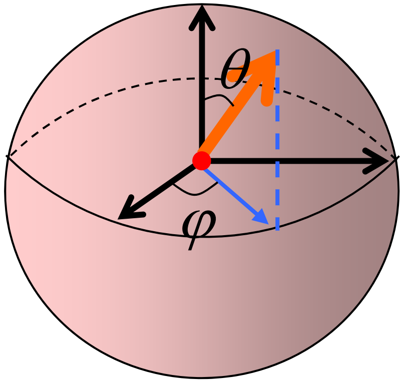
$$|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$ 'Neutron'

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

Q.M.

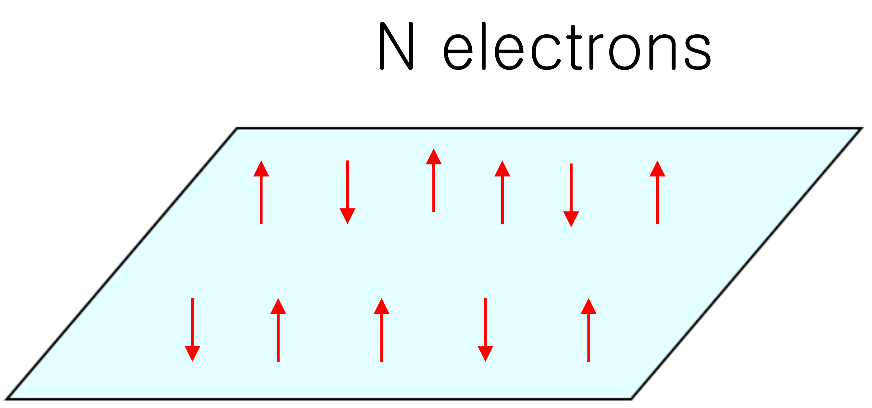


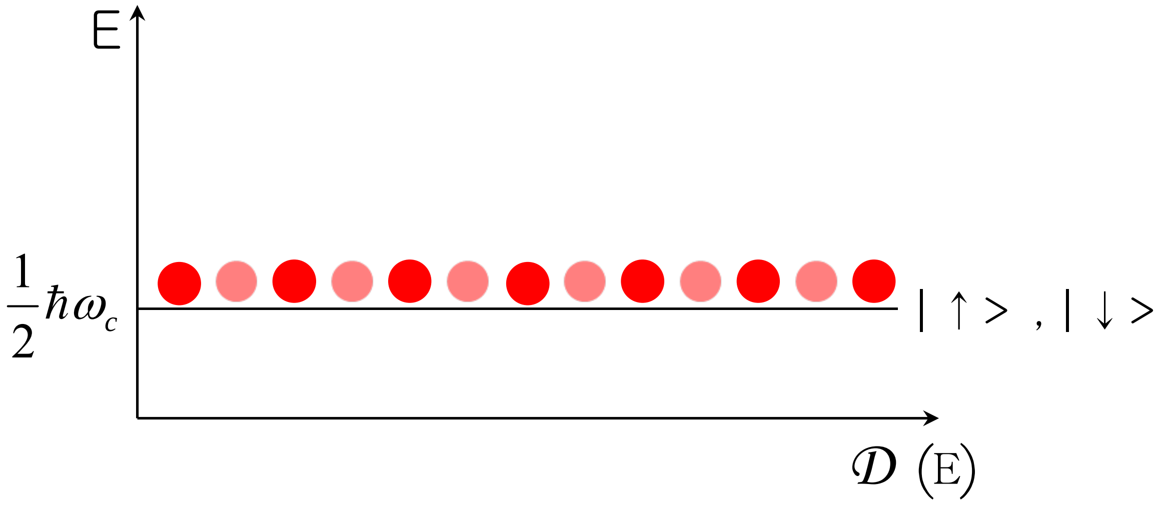
$$|\vec{m}\rangle = e^{i\varphi/2}\cos\frac{\theta}{2}|\uparrow\rangle + e^{-i\varphi/2}\sin\frac{\theta}{2}|\downarrow\rangle$$



$$'d = 0' \ (t \simeq 0)$$

$$K.E = \frac{1}{2}\hbar\omega_c N$$





Interaction V minimization : SU(2)

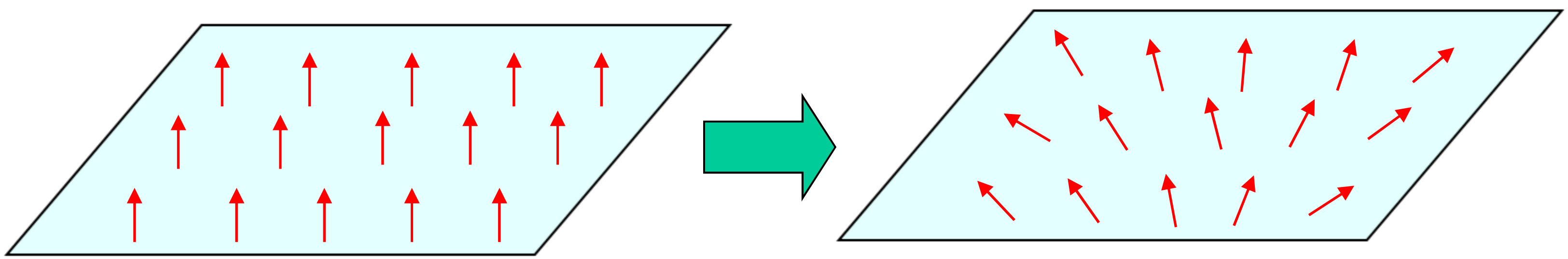
Pseudo-spin fully polarized

(i) z=x+iy

$$\Psi[z_1,\cdots,z_N]=\prod_{i>j}^{N}(z_i-z_j)e^{-\sum_k \frac{|z_k|^2}{4}}\cdot|\uparrow\uparrow\uparrow\cdots\uparrow\rangle$$

Anti-symmetric Slater Det. — symmetric

(ii) Hund's rule : Ground state → Maximum pseudo-spin



|G.S.>

Pseudo-spin texture $\vec{m}(\vec{r})$

Effective Energy Functional

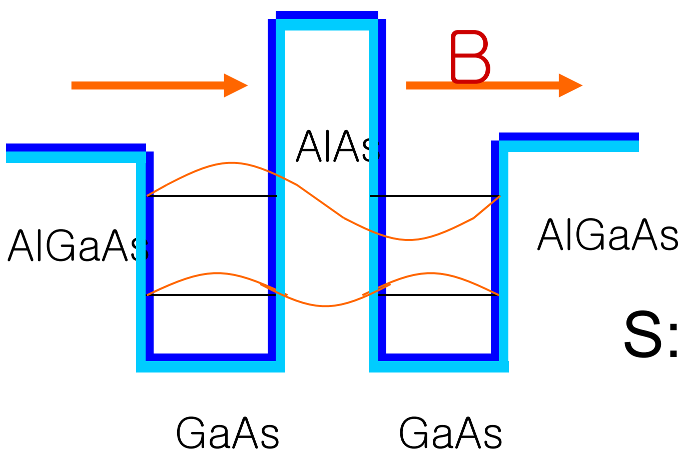
Exchange E

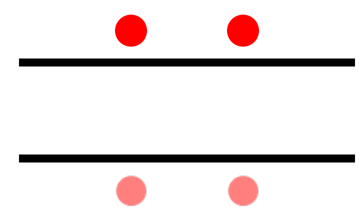
$$H_{eff}[\vec{m}] = \int d^2 r \left\{ \frac{1}{2}\rho_s \left|\vec{\nabla}\vec{m}\right|^2 - \frac{1}{2}\Delta_{SAS}\cdot m_x + U m_z^2 \right\}$$

Tunneling

Charging E

$$\frac{\Delta Q^2}{2C} : \Delta Q \sim m_z$$





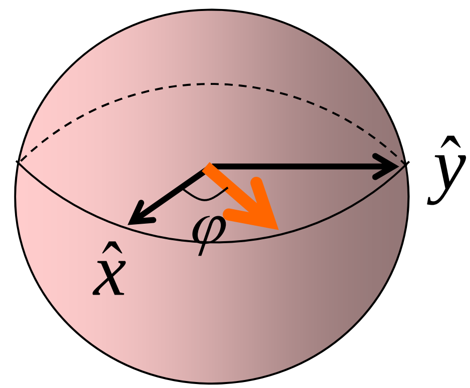
S: $\frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle + \left|\downarrow\right\rangle\right)$

$\mathrm{U m_z}^2 = 0 \rightarrow \mathrm{m_z} = 0$: Pseudo−spins lie in XY spin space.

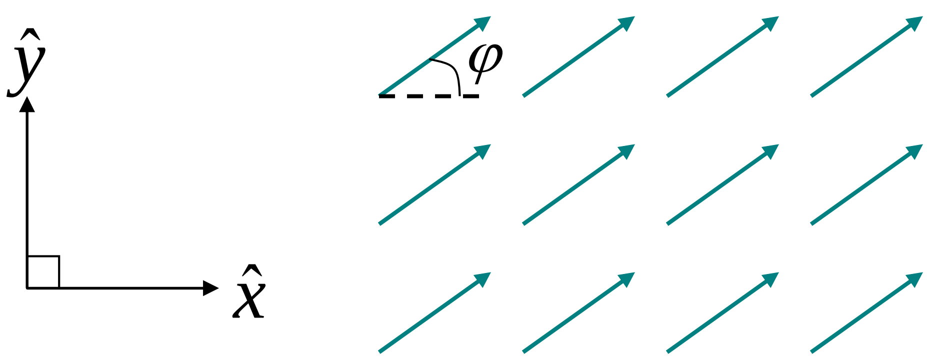
For Δ_{SAS} =0

$$|\varphi\rangle = \frac{1}{\sqrt{2}}\left(e^{i\varphi/2}|\uparrow\rangle + e^{-i\varphi/2}|\downarrow\rangle \right)$$

$$\mathrm{H}_{eff}[\varphi] = \int d^2 r \frac{1}{2}\rho_s \left|\vec{\nabla}\varphi\right|^2$$

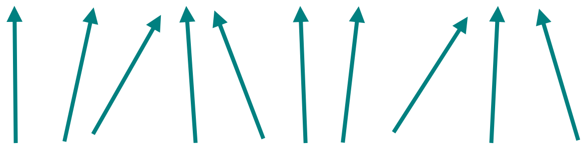


XY-ferromagnet

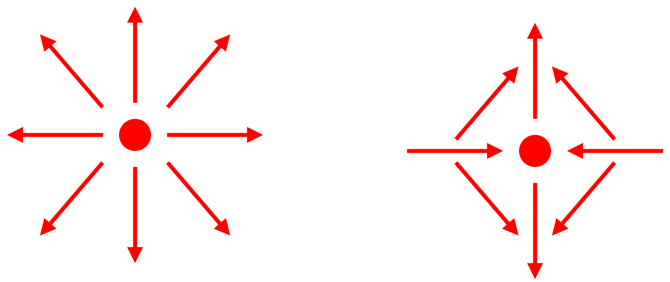


|G.S.>

Excitations



Spin waves

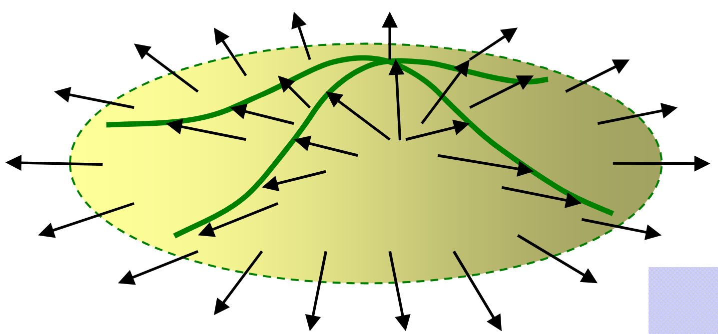


Vortices

Singularity ‘NO’

Core spin // $\hat{z}$

Escape into the 3rd dimension.



Meron

Um_z^2 cost

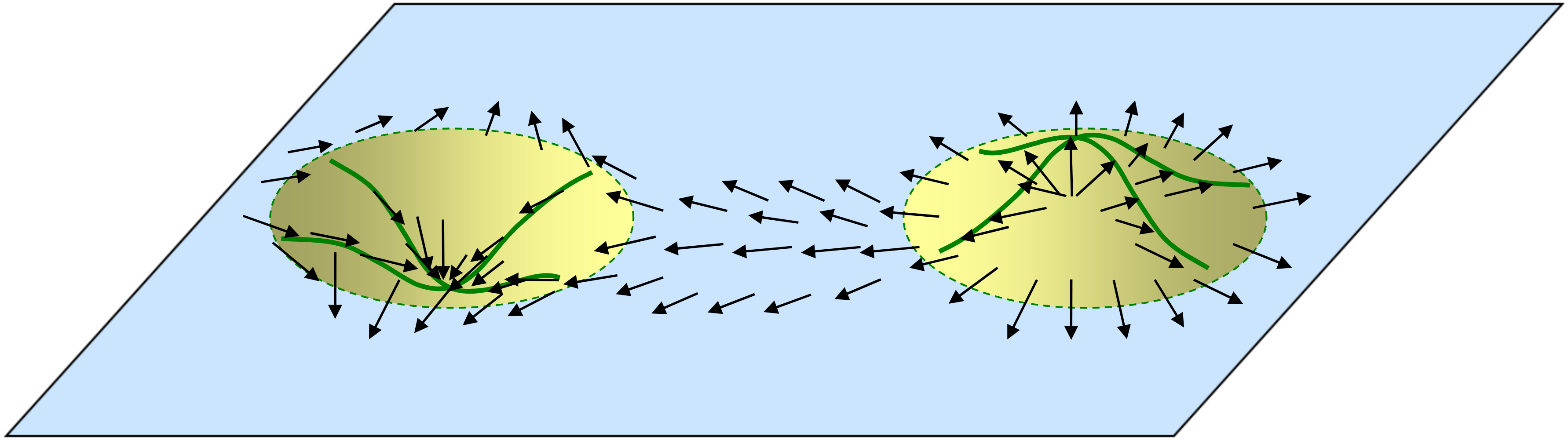
$$q_m = \pm \frac{1}{2} e$$

4 different flavors

$$2 \otimes 2$$

Vorticity   Core spin

Charge Excitation: Meron-pair



Meron-pair charge = $q_{sk} = \pm e$

$$q_m = vorticity \otimes m_z(0) \quad \left[\frac{e}{2}\right]$$

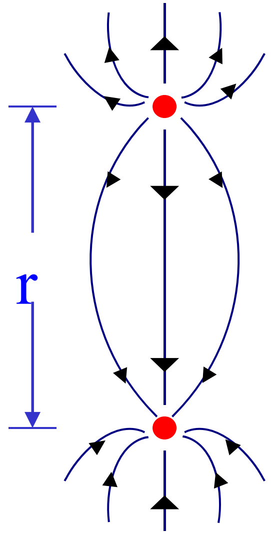
KT Phase Transition

$$F = E - TS = \left(\rho_s - \frac{\pi}{2} k_B T \right) \ln L$$

$\underline{\qquad\qquad}$ = 0

$$T_{KT} \cong \frac{2}{\pi} \rho_s \sim 0.5K$$

Meron-pair excitation



$$E_A(r) = 2E_c + 2\pi\rho_s \log \frac{r}{l} + \frac{(e/2)^2}{\varepsilon r}$$

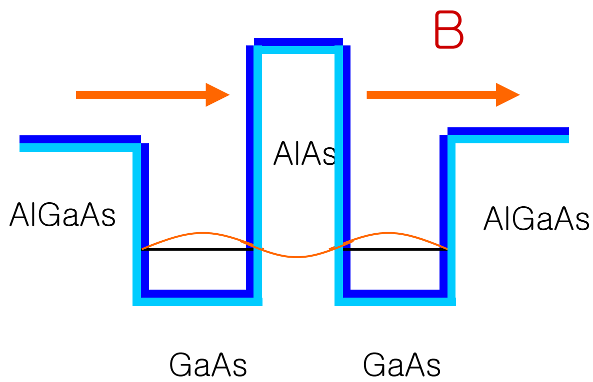
$$E_A{}^* = 2E_c + 7K$$

$$r^* \cong 10l$$

$T > T_{KT}$: Free vortices $\rightarrow$ gap = 0

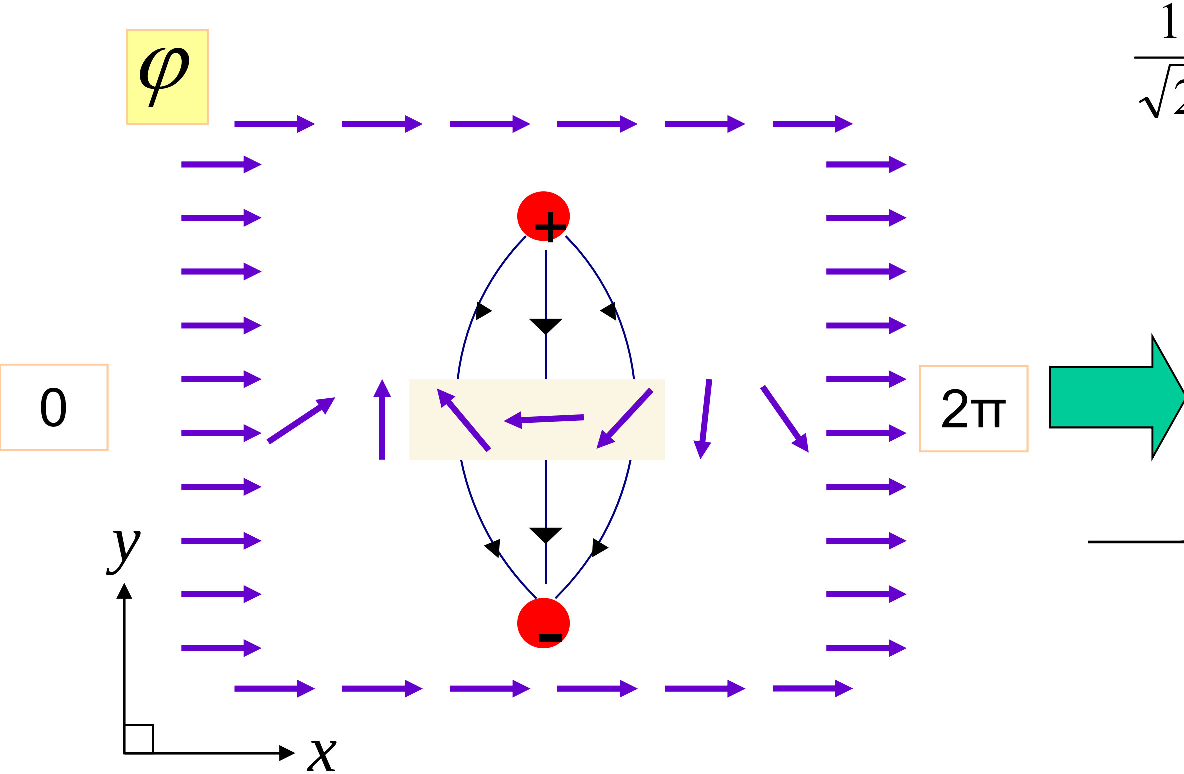
Finite tunneling $\Delta_{SAS} \neq 0$

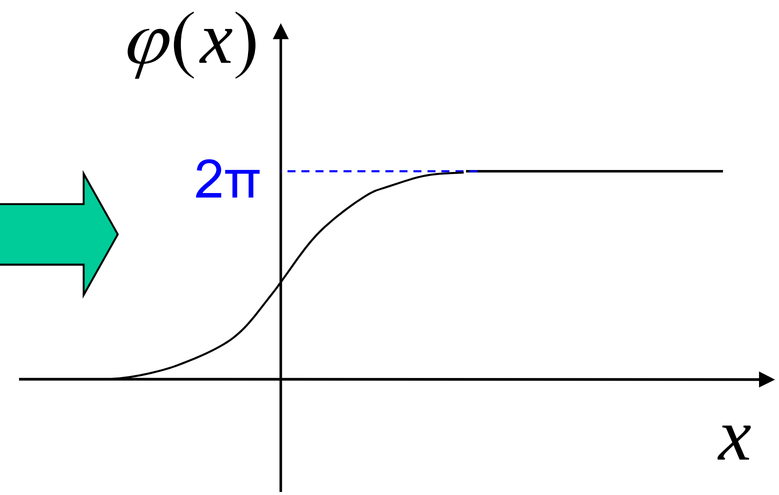
$$H_{eff}[\varphi] = \int d^2 r \left\{ \frac{1}{2} \rho_s \left| \vec{\nabla} \varphi \right|^2 - t \cos \varphi \right\}$$



$\varphi = 0$ preferred

$$\frac{1}{\sqrt{2}} \left(|\uparrow\rangle + |\downarrow\rangle \right)$$





Domain Wall

DW Energy

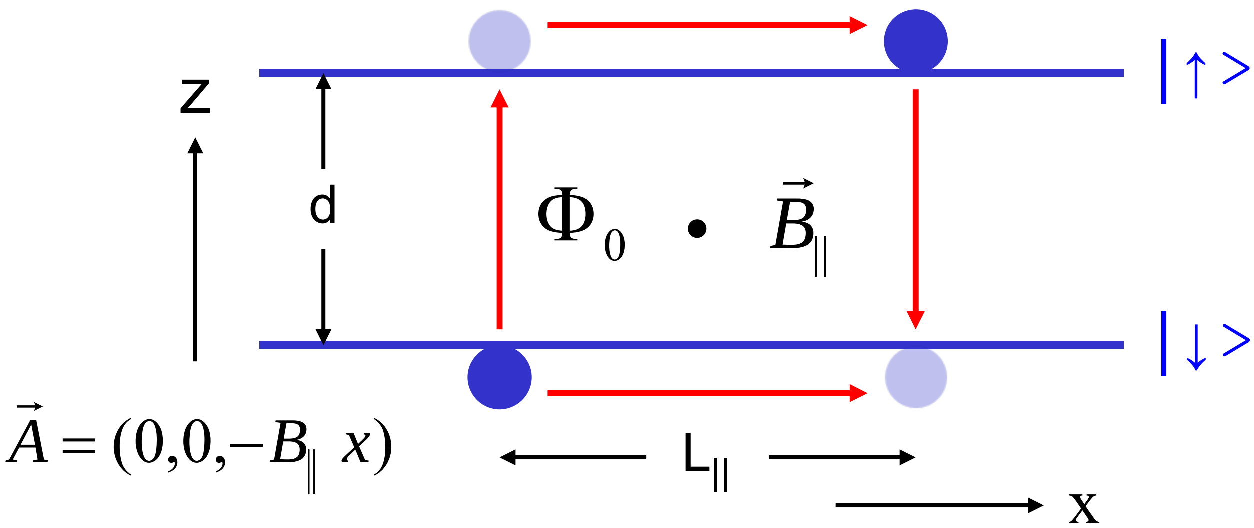
Linear confinement $T \approx \varepsilon_{DW} \cdot r$

(cf. quark – antiquark)

$$E_A(r) = 2E_C + \varepsilon_{DW} \cdot r + \frac{(e/2)^2}{\varepsilon r}$$

Finite D.W. ≡ Meron – pair excitation

The role of parallel $B_{||}$ field



$$L_{||} \cdot dB_{||} = \Phi_0$$

$$L_{||} = \frac{\Phi_0}{dB_{||}}$$

$$t(x) = te^{i\frac{2\pi}{\Phi_0}\int d\vec{r}\cdot\vec{A}} = te^{iQ\cdot x}$$

$$Q = \frac{2\pi}{L_{||}} \propto B_{||}$$

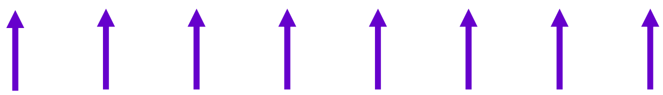
'Aharonov – Bohm phase'

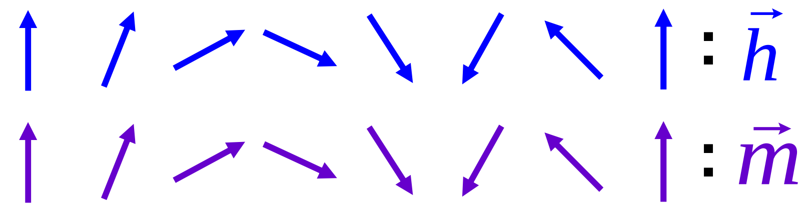
$$\mathrm{H}_{eff}[\varphi] = \int dr^2 \left\{ \frac{1}{2}\rho_s \left|\vec{\nabla}\varphi\right|^2 - \vec{h}(x)\cdot\vec{m} \right\}$$

$$\vec{h}(x) = t(\cos Qx, \sin Qx)$$

Fake magnetic field

$$\varphi = const$$

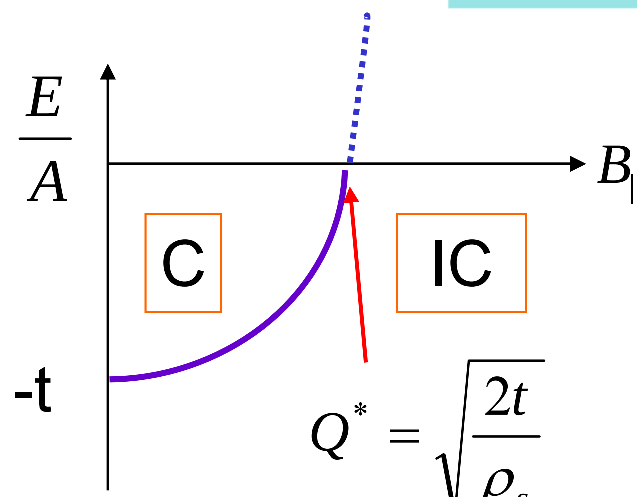




Incommensurate

Commensurate phase

$$\frac{E}{A} = 0$$



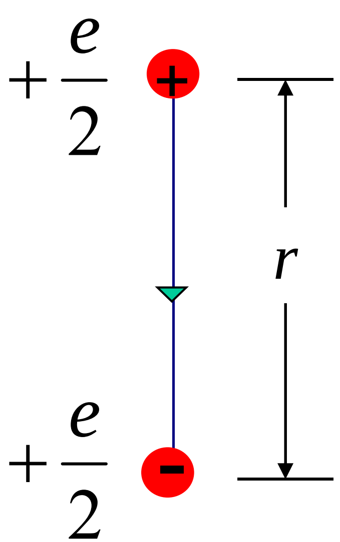
$$Q^* = \sqrt{\frac{2t}{\rho_s}}$$

$$\varphi = Qx$$

$$\frac{E}{A} = \frac{1}{2}\rho_s Q^2 - t$$

Meron-pair Excitation Energy

$$T(B_{\parallel}) = T_0 \cdot \left[1 - \frac{B_{\parallel}}{B_{\parallel}^{c}} \right]$$

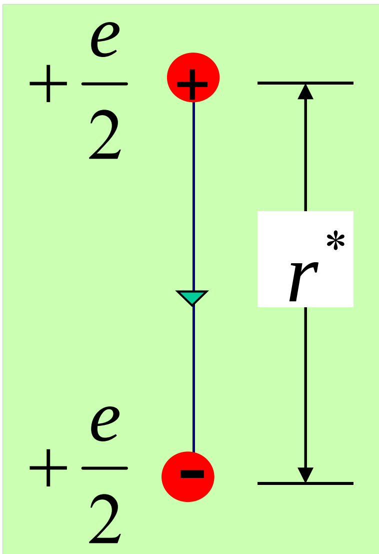


$$E_A(r) = 2E_C + T(B_{\parallel}) \cdot r + \frac{(e/2)^2}{\varepsilon r}$$

$$\frac{\partial E_A}{\partial r} = 0$$

$$r^{*}(B_{\parallel}) \approx \left(1 - B_{\parallel} / B_{\parallel}^{c} \right)^{-1/2}$$

Meron-pair Excitation Energy



$$E_A(B_{\parallel}) \approx 2E_c + \alpha \cdot \left(1 - B_{\parallel} / B_{\parallel}^c\right)^{1/2}$$