New Aspects of Old Equations: Metamaterials and Beyond (Part 2)

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Metamaterial

Near-field Configuration

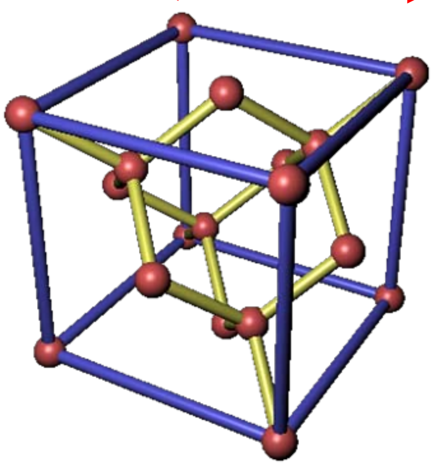
in Periodic Structures

→ New "Material"

Material and Metamaterial

Material

0.5nm $\ll \lambda \sim$ **1** μ**m**

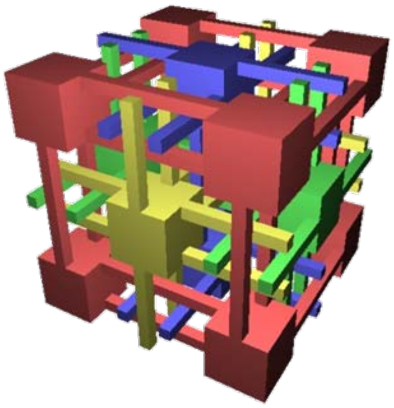


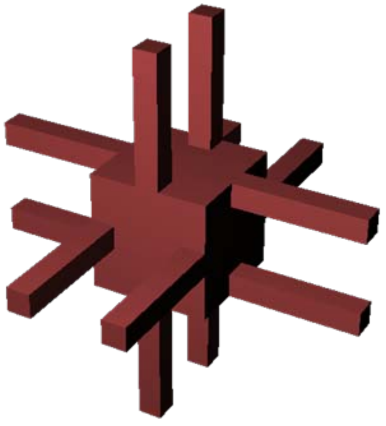
ε (Permittivity)

μ (Permeability)

Metamaterial

100 nm $\ll \lambda \sim$ **1** μ**m**





ε

μ

Photonic Crystal and Metamaterial

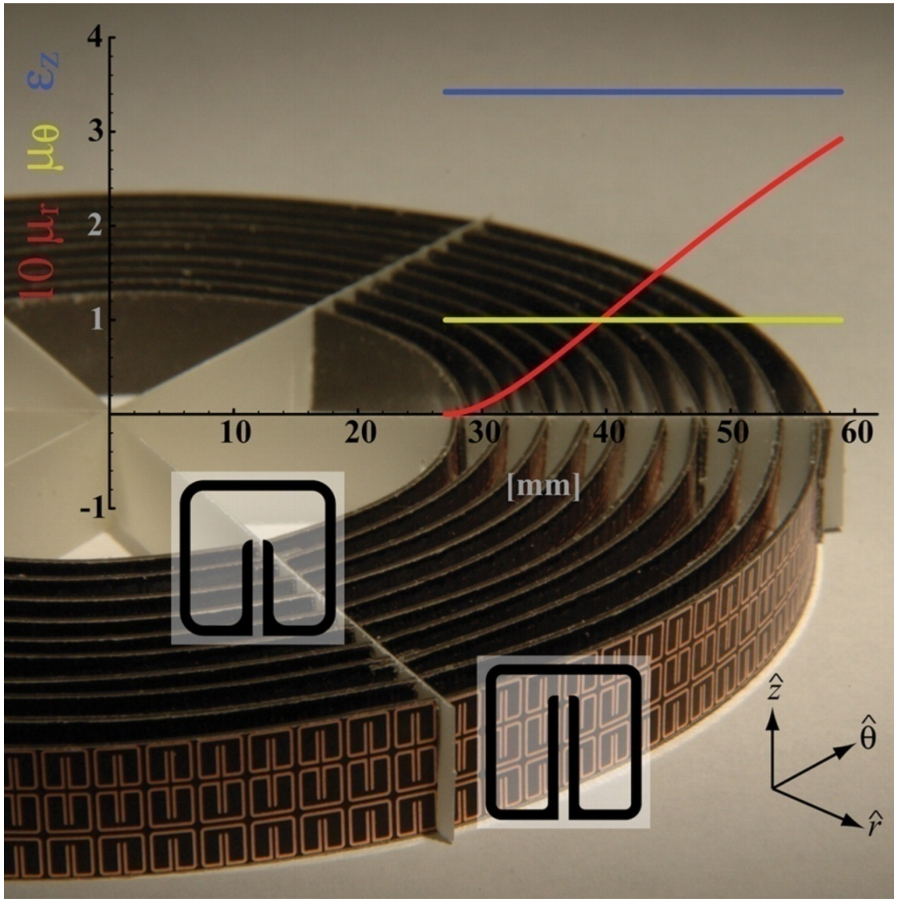
Photonic Crystal

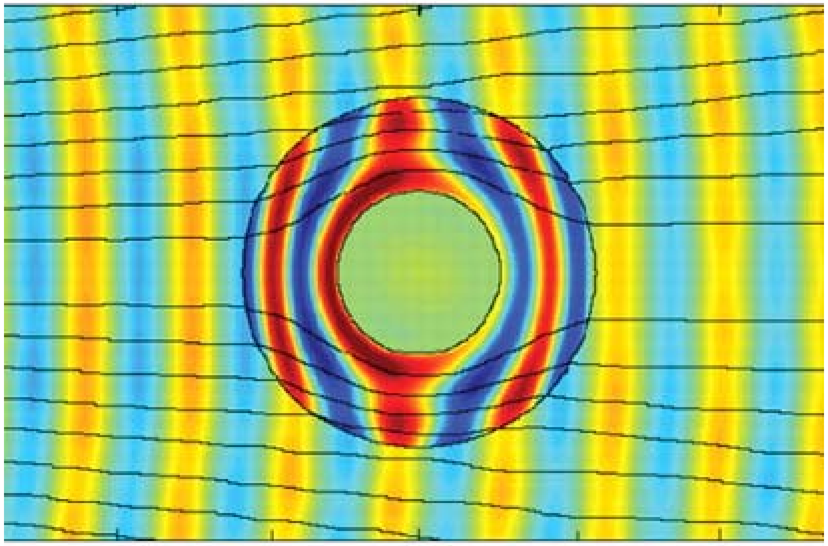
- Periodic
- Dielectric media
- Bragg reflection
- Unit cell size ~ wavelength

Metamaterial

- Periodic
- Metallic elements
- Near-field configuration
- Unit cell size << wavelength

Electromagnetic Metamaterial





D. Schurig et al., *Science* **10**, 5801 (2006)

Effective Material Parameters

(Assuming Maxwellian macroscopic description)

- Electric property

$$\varepsilon_r = 1 + \frac{P}{\varepsilon_0 E}$$

P: Polarization

- Magnetic property

$$\mu_r = 1 + \frac{M}{H}$$

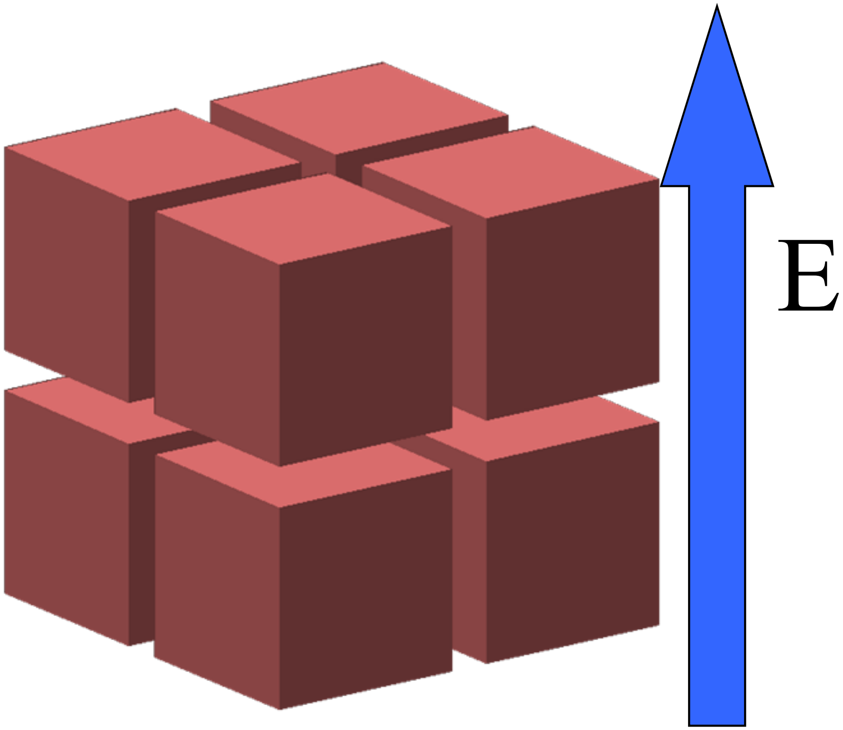
M: Magnetization

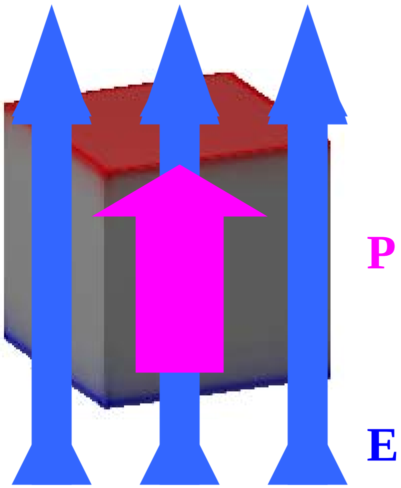
Effective Parameters in the Quasi-static Regime

- Negligible electromagnetic fields inside metal
  - Electric fields terminated by surface charges
  - Magnetic fields fended off by surface currents

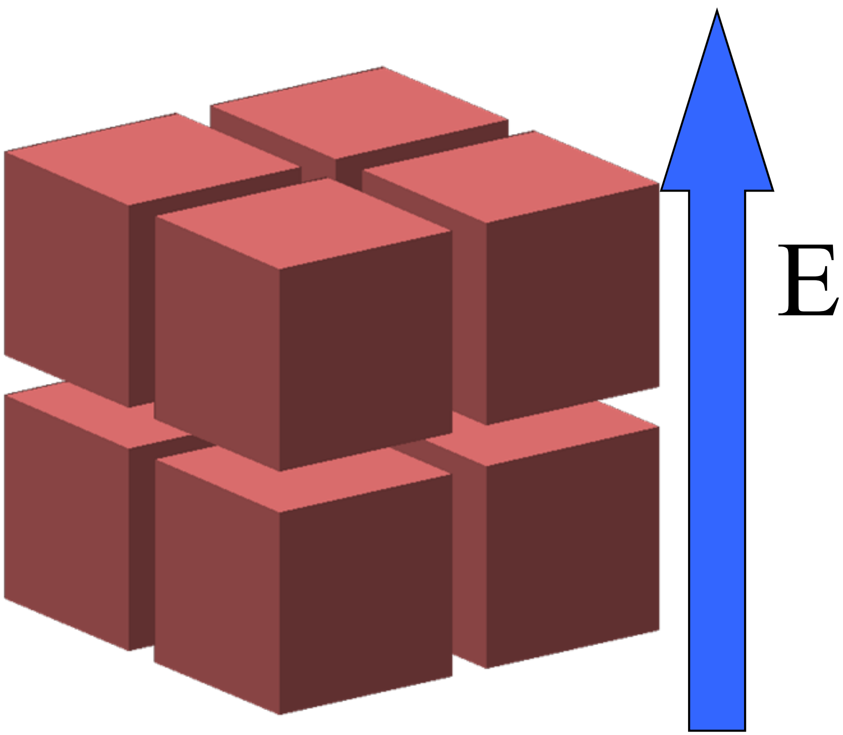
- Shape and size of metal inclusions determines *P* and *M*, hence, ε and μ.

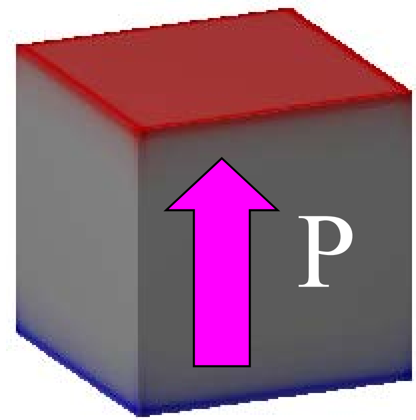
Electric Property





Electric Property



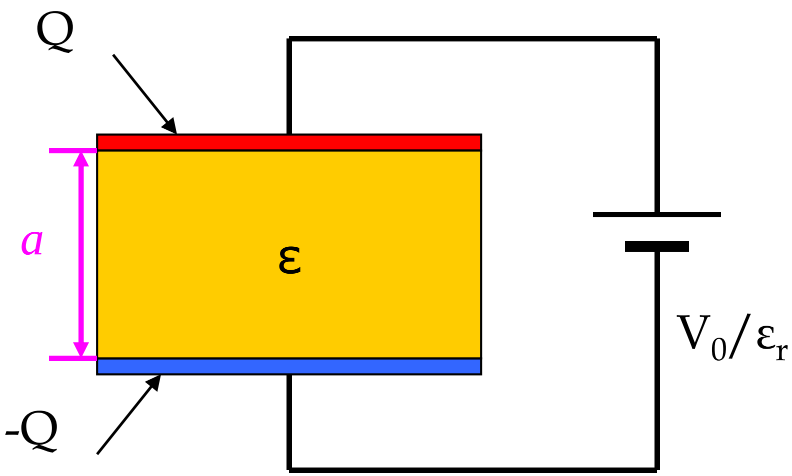


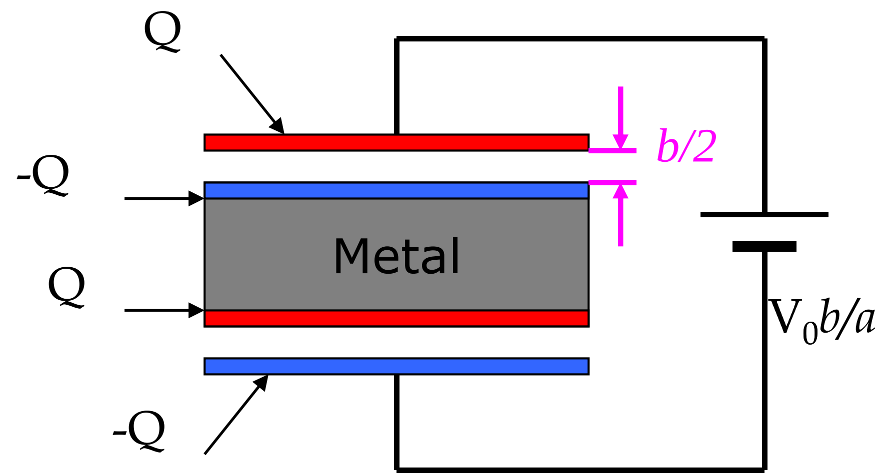
- $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \equiv \varepsilon \mathbf{E}$
- Large P → large ε
- Purely geometrical effect (non-dispersive)

Electric Property
(Alternative Explanation)

- Measurement of ε via capacitance
  of a parallel plate capacitor

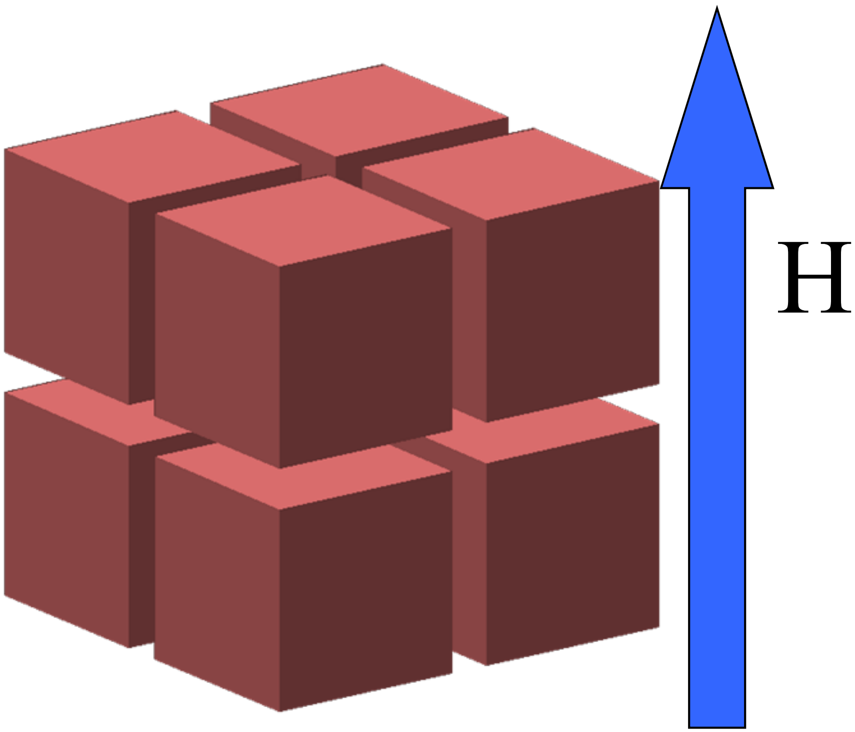
  $C = Q/V = \varepsilon A/a \quad \rightarrow \quad \varepsilon = (Q/V)\,(a/A) = \rho_{av}/E_{av}$

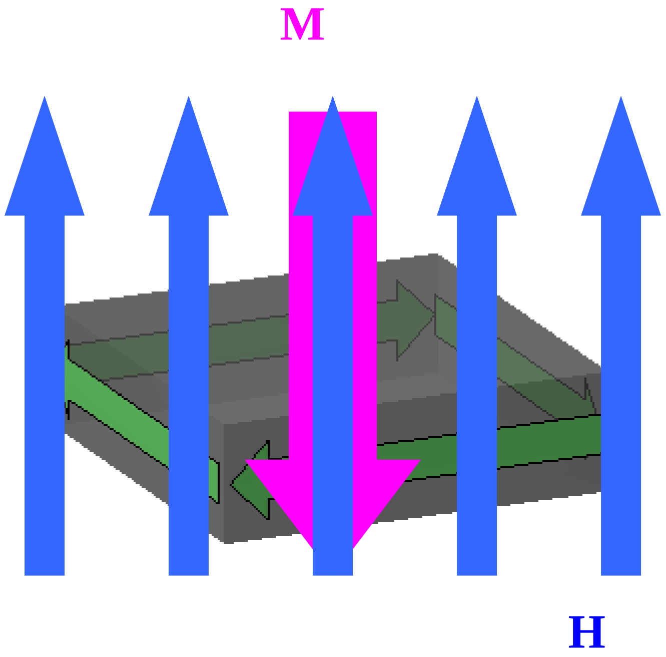




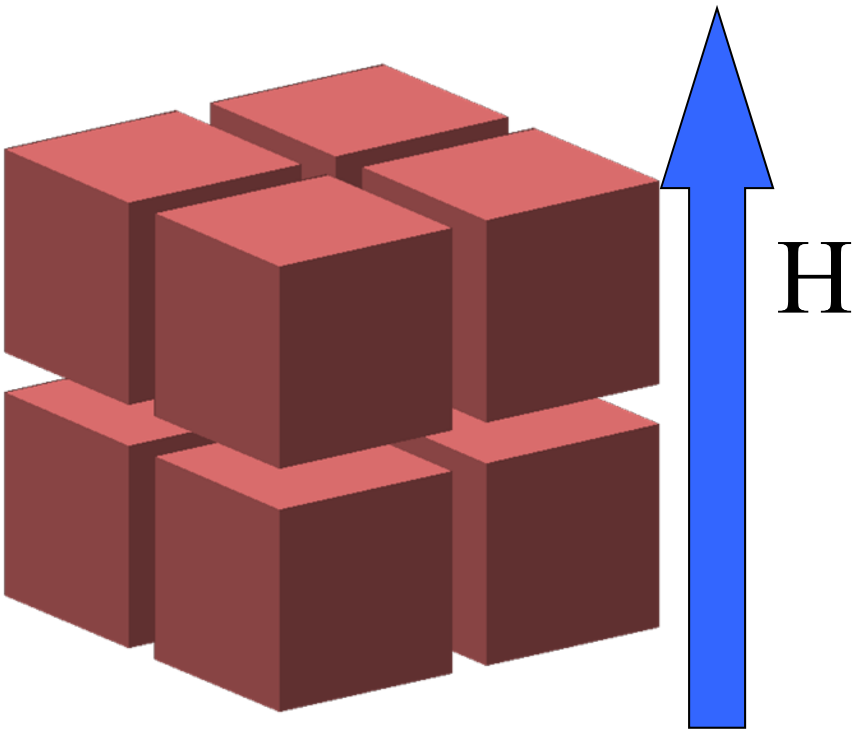
$\therefore\ \varepsilon_r = a/b$ (purely geometric)

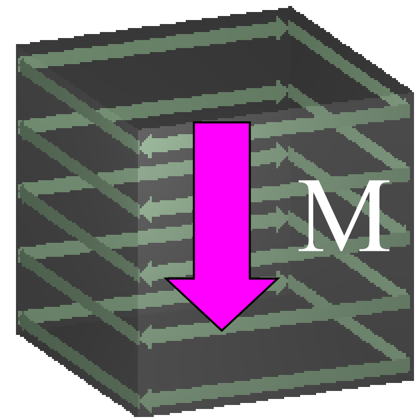
Magnetic Property





Magnetic Property



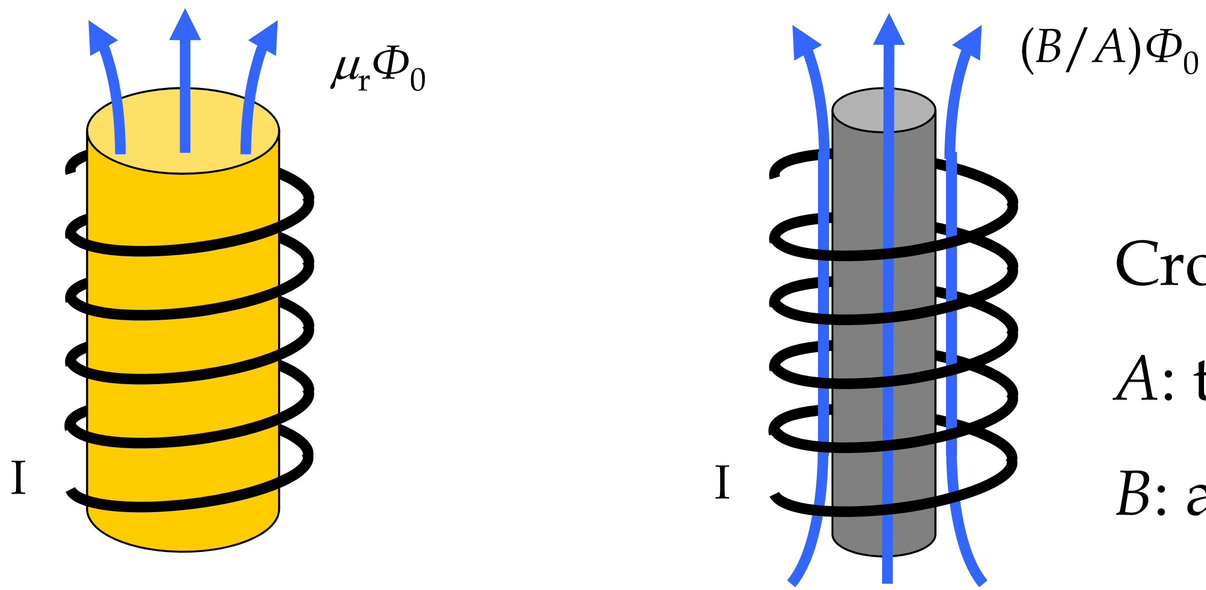


- $\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M}) \equiv \mu\mathbf{H}$
- Negative M → $\mu < \mu_0$
- Purely geometrical efffect (non-dispersive)

Magnetic Property (Alternative Explanation)

- Measurement of μ via inductance

  $L = \Phi/I = \mu N^2 A/l \rightarrow \mu = (\Phi/I)(l/N^2 A)$



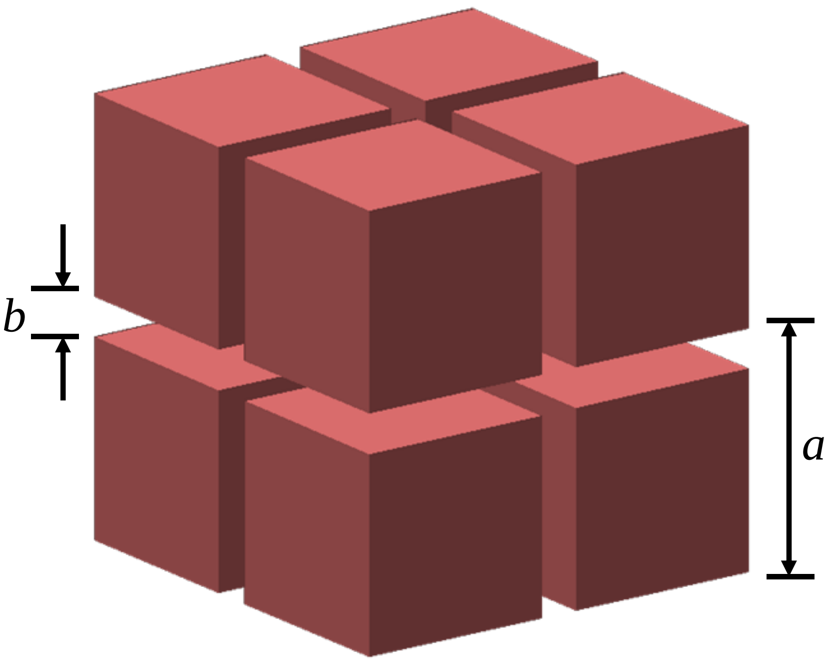
Cross section area

A: total

B: air region

$\therefore\ \mu_r = B/A$ (purely geometric)

Cube Array



(2×2×2 unit cells)

- Large ε

$$\varepsilon_r \sim a/b$$

- Small μ

$$\mu_r \sim 2b/a$$

- Moderate n

$$n = \sqrt{\varepsilon_r \mu_r} \sim \sqrt{2}$$

Broadband, Independent Control of ε and μ

- Is it possible?

- Metal inclusions
  → always large ε, small μ?

Broadband, Independent Control of ε and μ

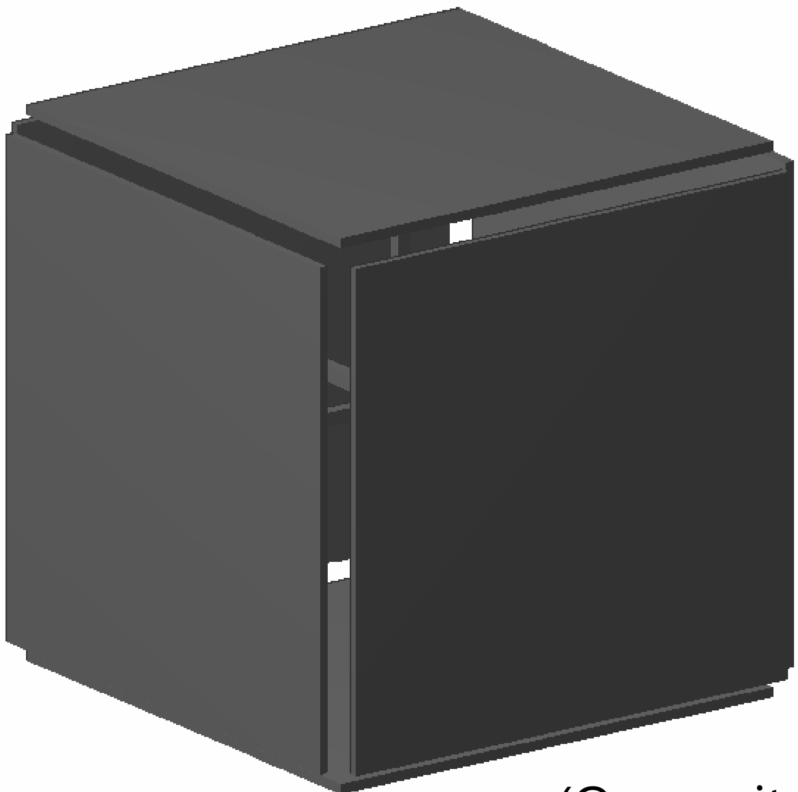
Differences in the boundary conditions

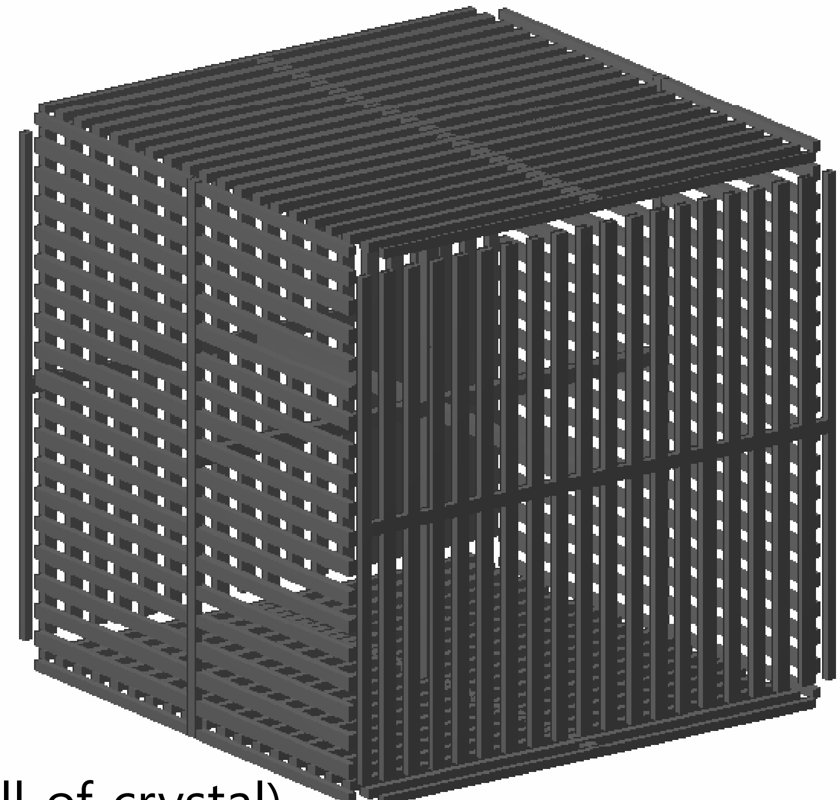
for E and H fields

→Allow an independent control of electrical and magnetic responses

via the engineering of geometry

Isotropic High-Index Structures

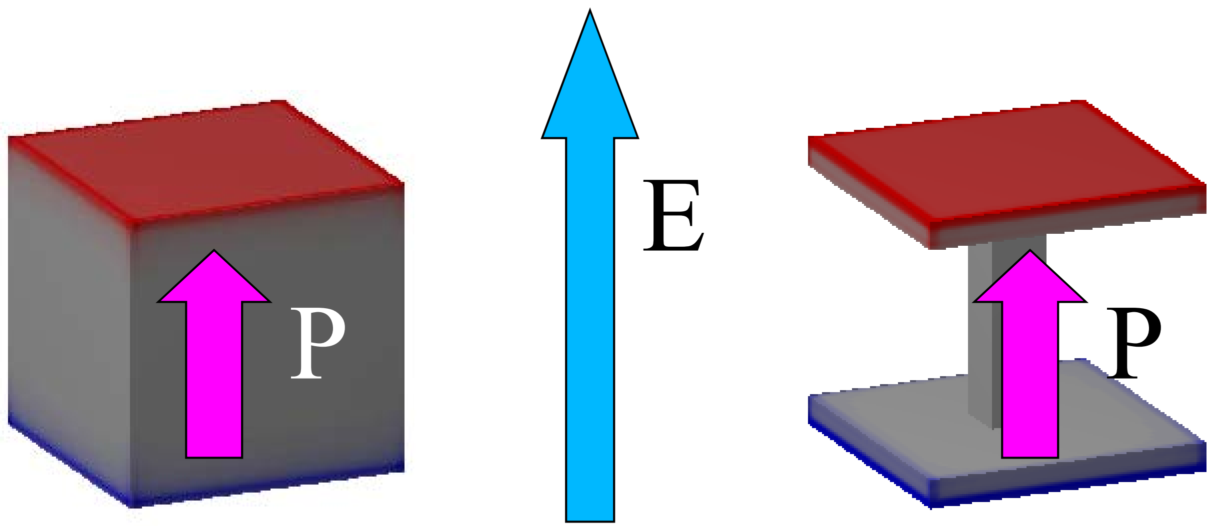




(One unit cell of crystal)

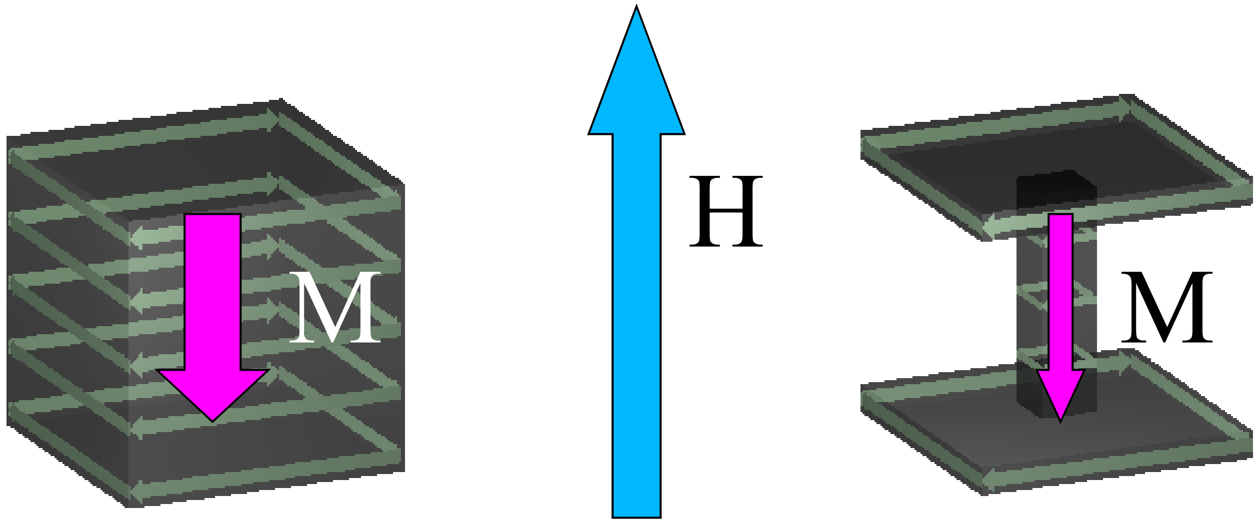
J. Shin, J.-T. Shen, and S. Fan, "Three-dimensional metamaterials with an ultra-high effective refractive index over broad bandwidth," Phys. Rev. Lett. **102**, 093903 (2009)

Electric property



The area and separation of faces normal to E field is important.

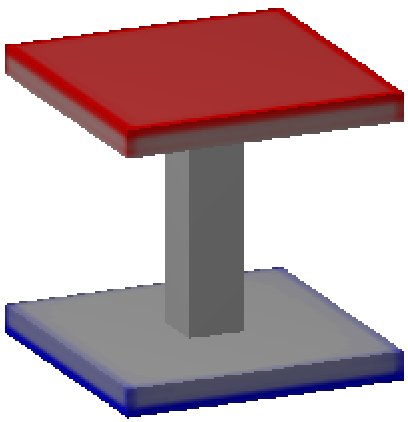
Magnetic property



$\mu_1 \approx 0.1\mu_0$ $\mu_2 \approx 0.5\mu_0$

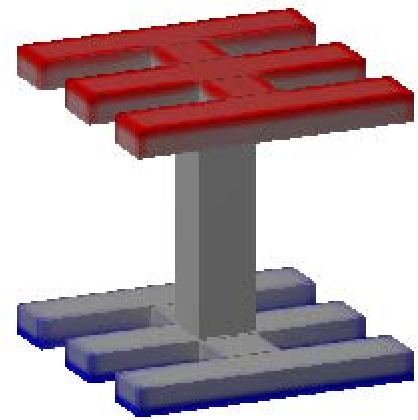
The average area of current loops (volume enclosed by faces parallel to H field) is important.

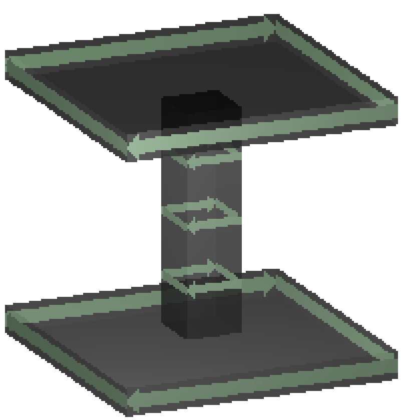
Enhancement of the index

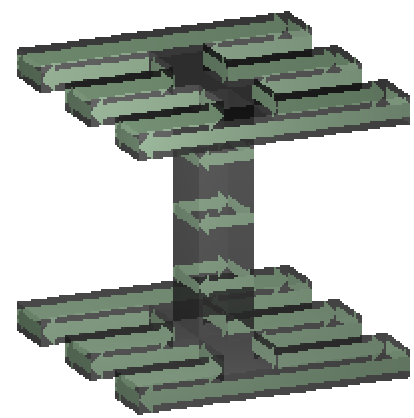


$\varepsilon_2 \approx \varepsilon_3$

(Fringing field effect)



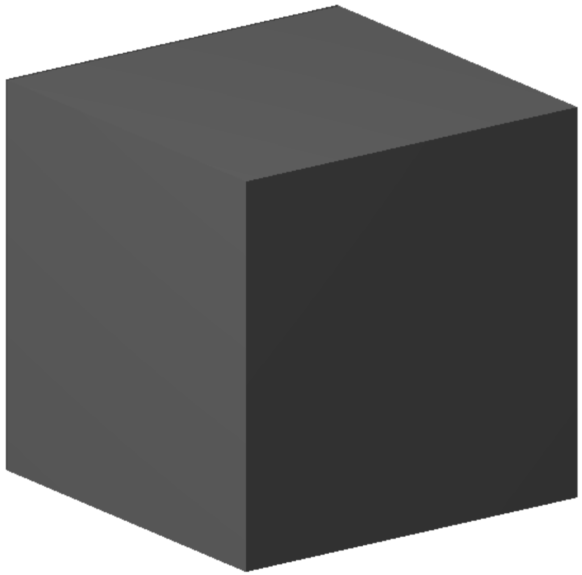


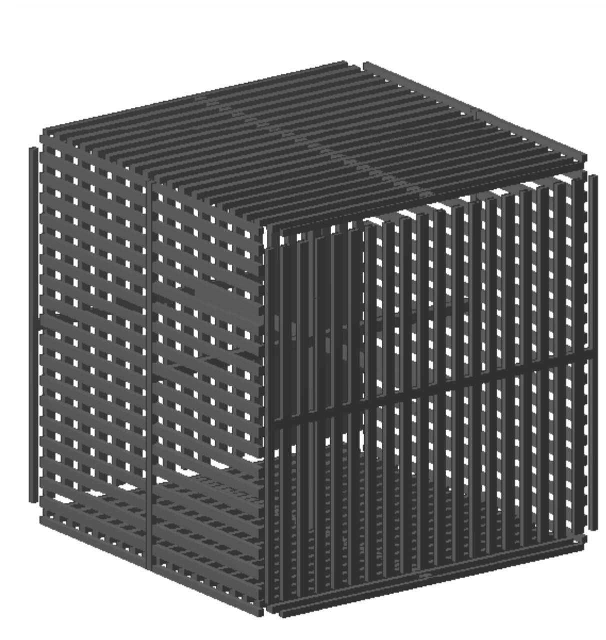


$\mu_2 \approx 0.5\mu_0$

$\mu_3 \approx 0.97\mu_0$

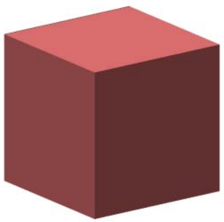
Comparison of Structures

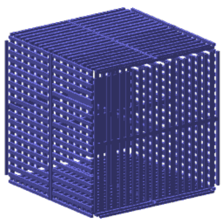


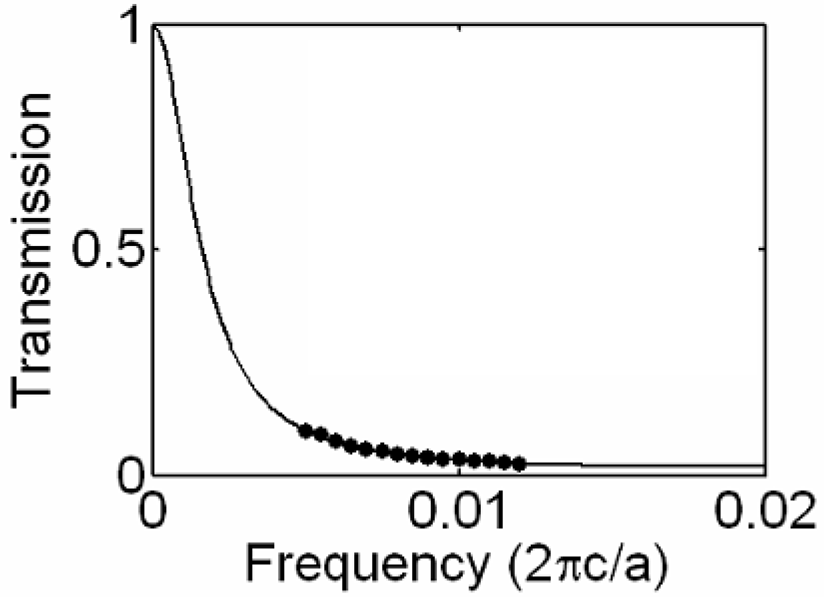


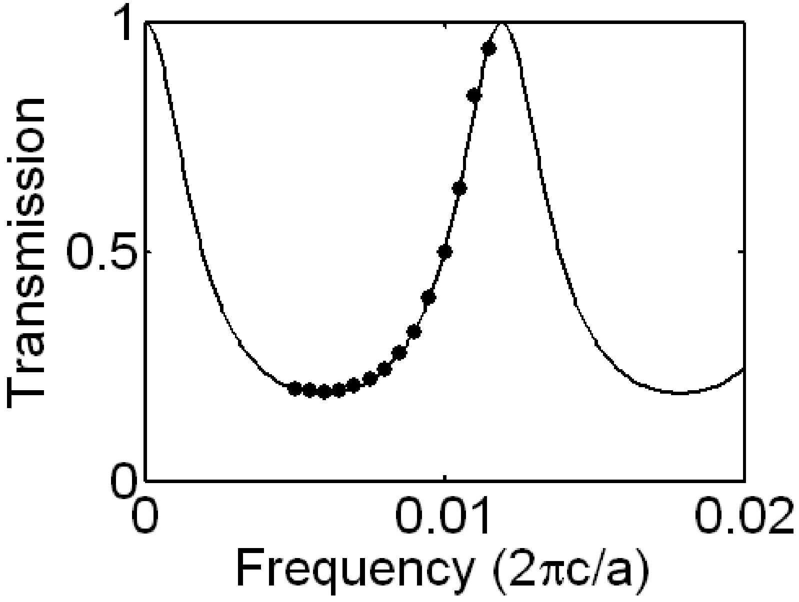
ε_r =	20	19.6	18.3
μ_r =	0.10	0.56	0.96
n =	1.4	3.3	4.2

Numerical Verification









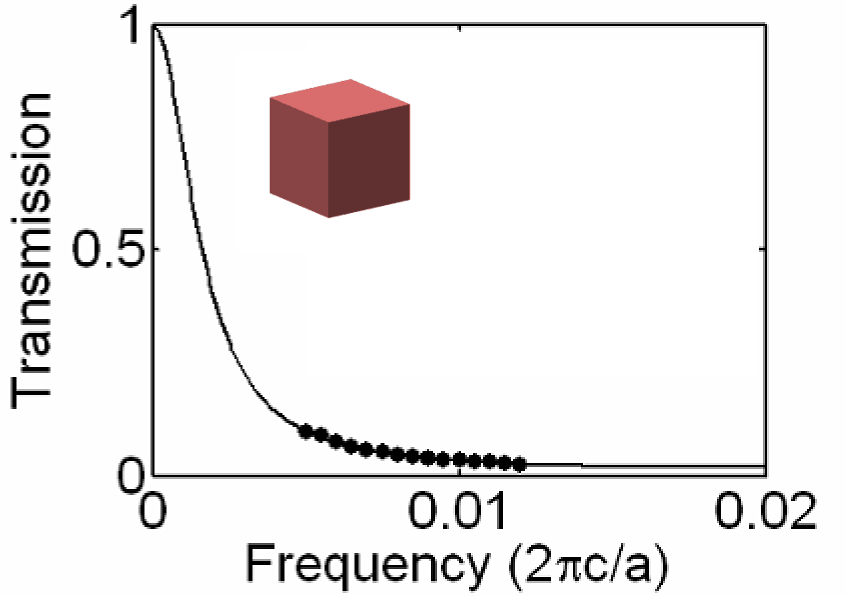
Numerical Verification

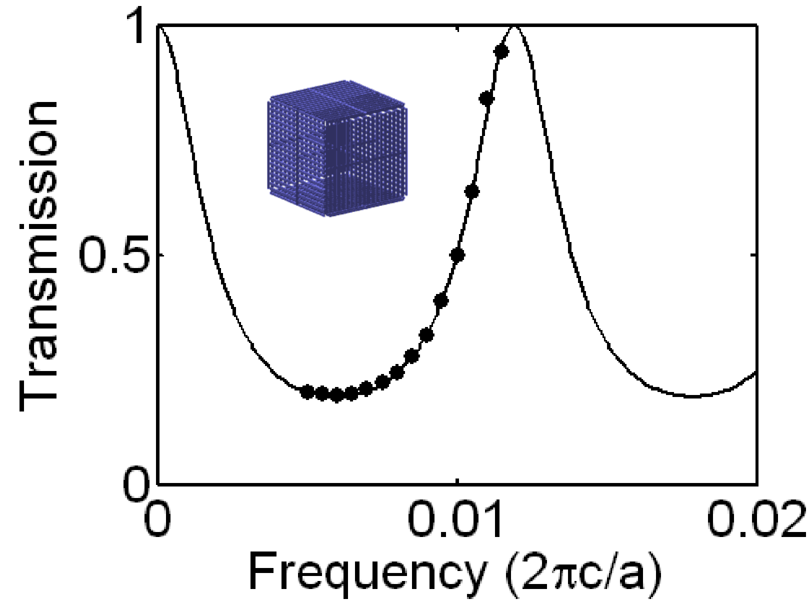
- Fabry-Perot resonance frequencies

  $\Rightarrow$ Index $c\sqrt{\mu\varepsilon}$

- Minimum transmission

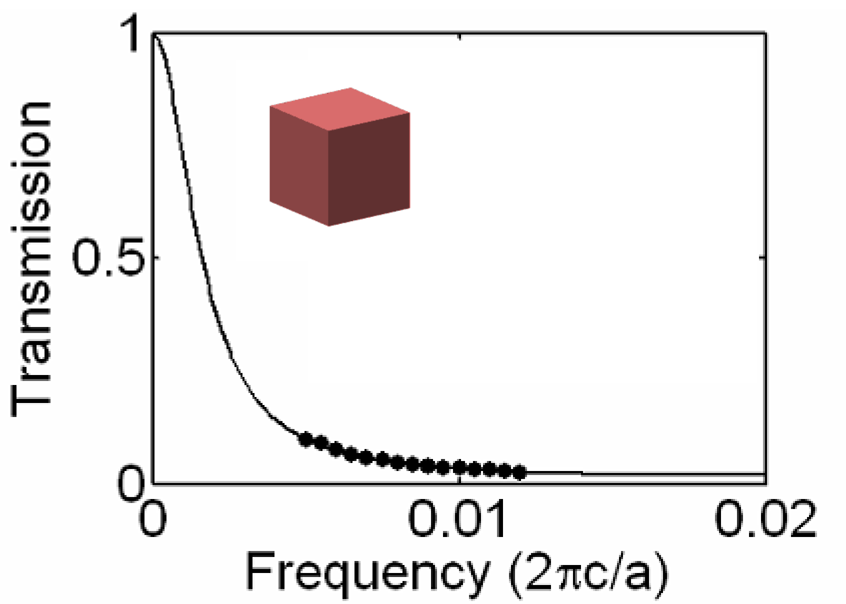
  $\Rightarrow$ Impedance $\sqrt{\mu/\varepsilon}$



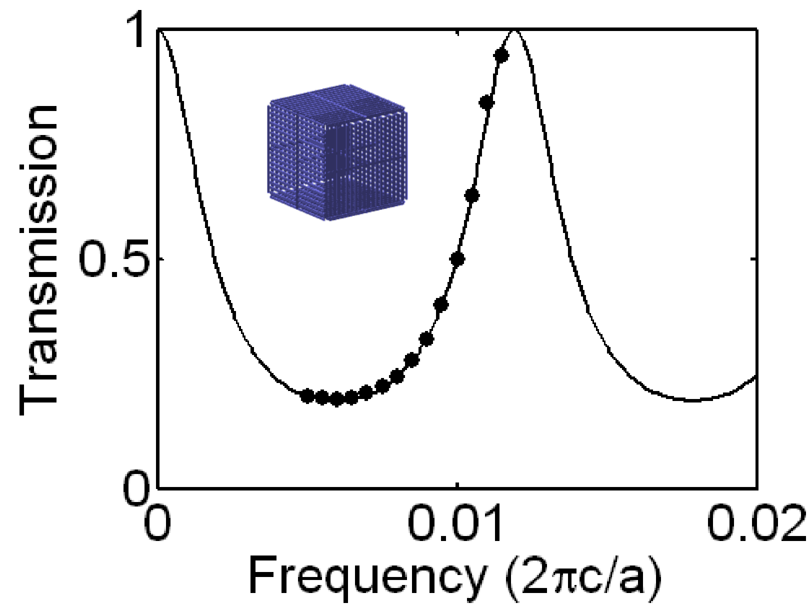


Numerical Verification

$$\varepsilon = 20\varepsilon_0$$
$$\mu = 0.10\mu_0$$
$$n = 1.41$$



$$\varepsilon = 18.3\varepsilon_0$$
$$\mu = 0.97\mu_0$$
$$n = 4.20$$



So far…

- Geometrical design of unit cell

  → (Nearly-) Independent control of ε and μ over a broad wavelength range.

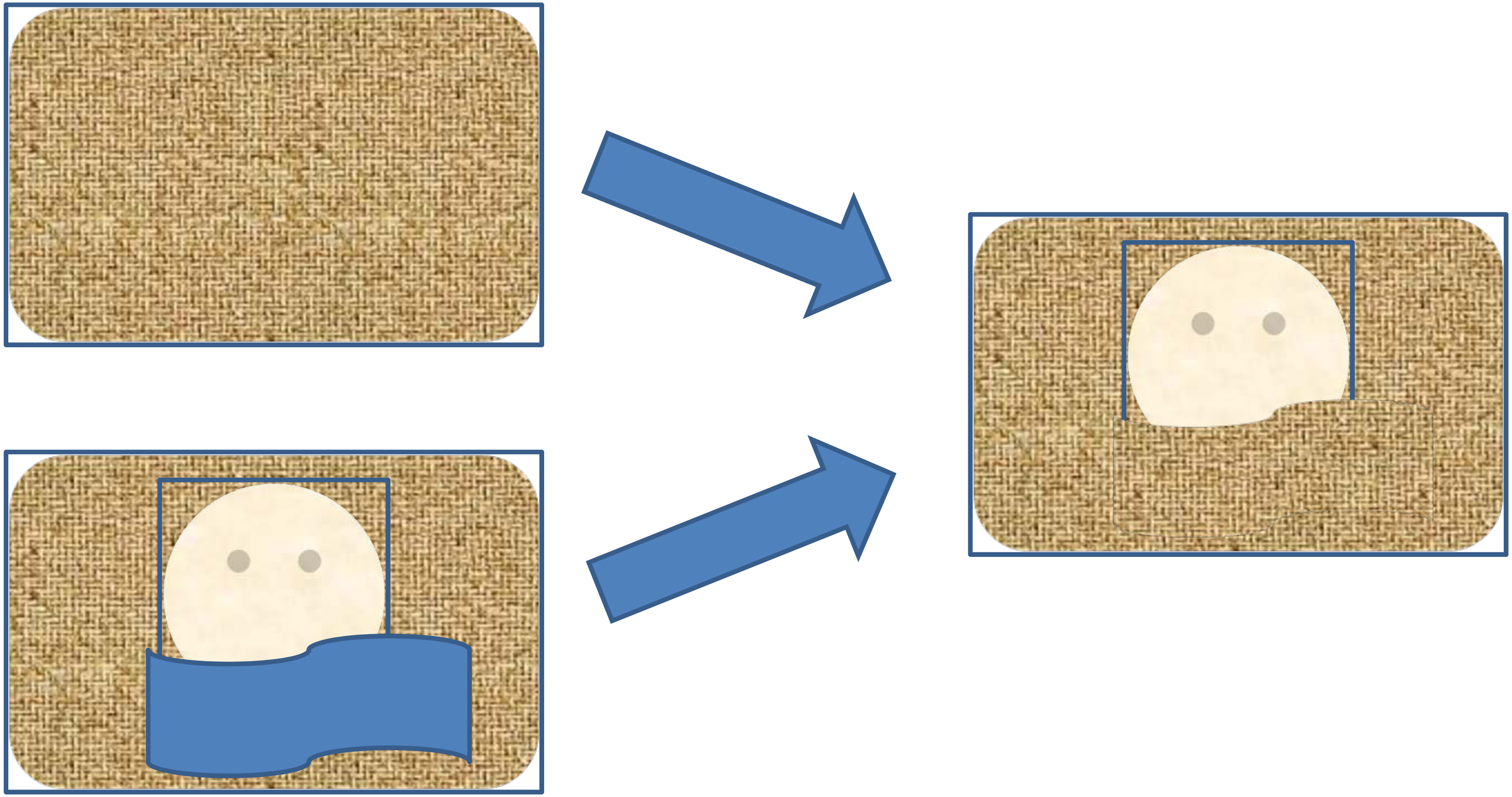
- So, what can we do with it?

Invisibility Cloak?

Cloak Video #1

Invisibility Cloak?

- Just a video trick.

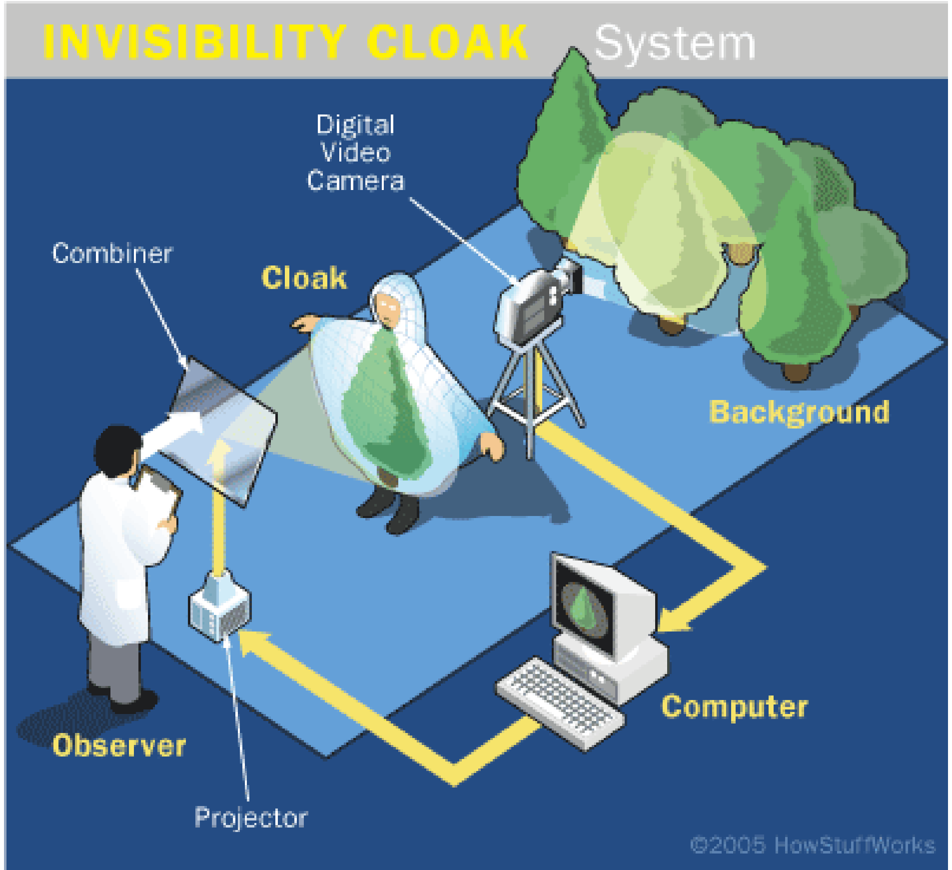


Invisibility Cloak?

Cloak Video #2

Invisibility Cloak?

- Optical camouflage.



Invisibility Cloak?

- Optical camouflage

  → Good engineering problem but not much new physics in it.

  (Animals have done it for millions of years!)

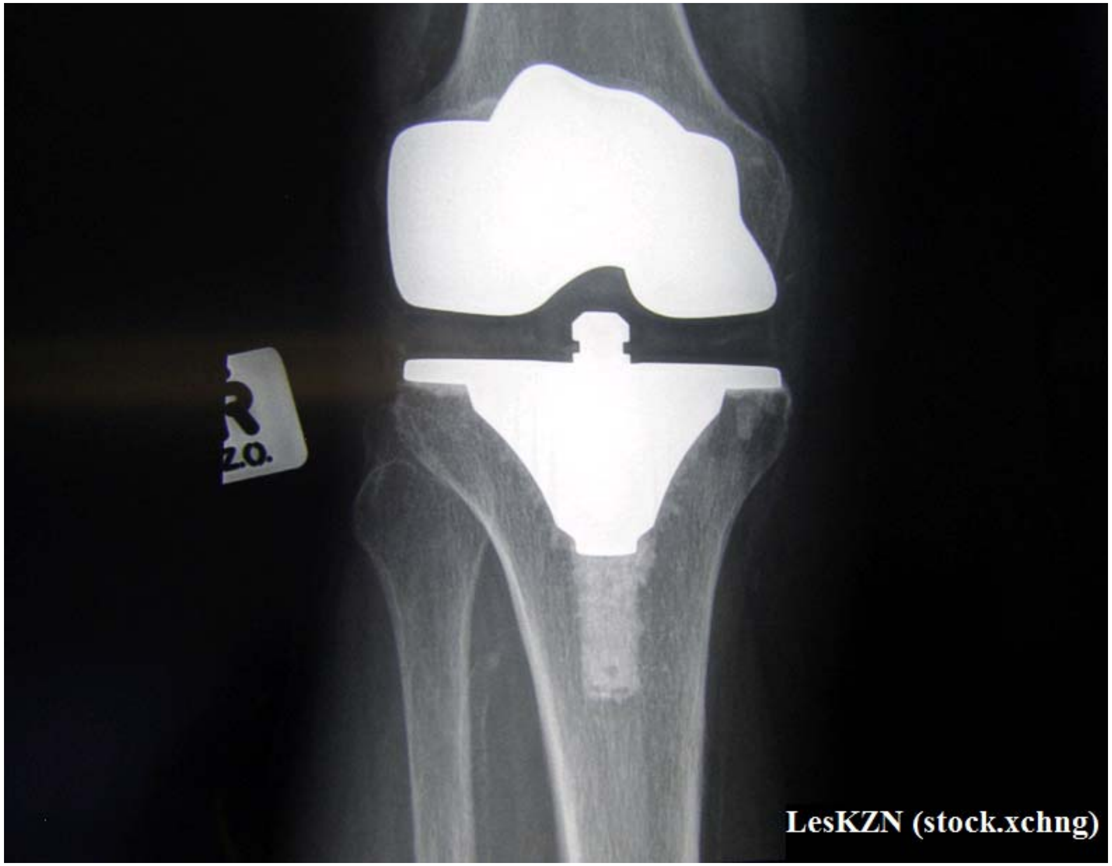
Physics-based Invisibility Cloak?

- Visibility

  ←Mainly from absorption and scattering of light (exceptions:)

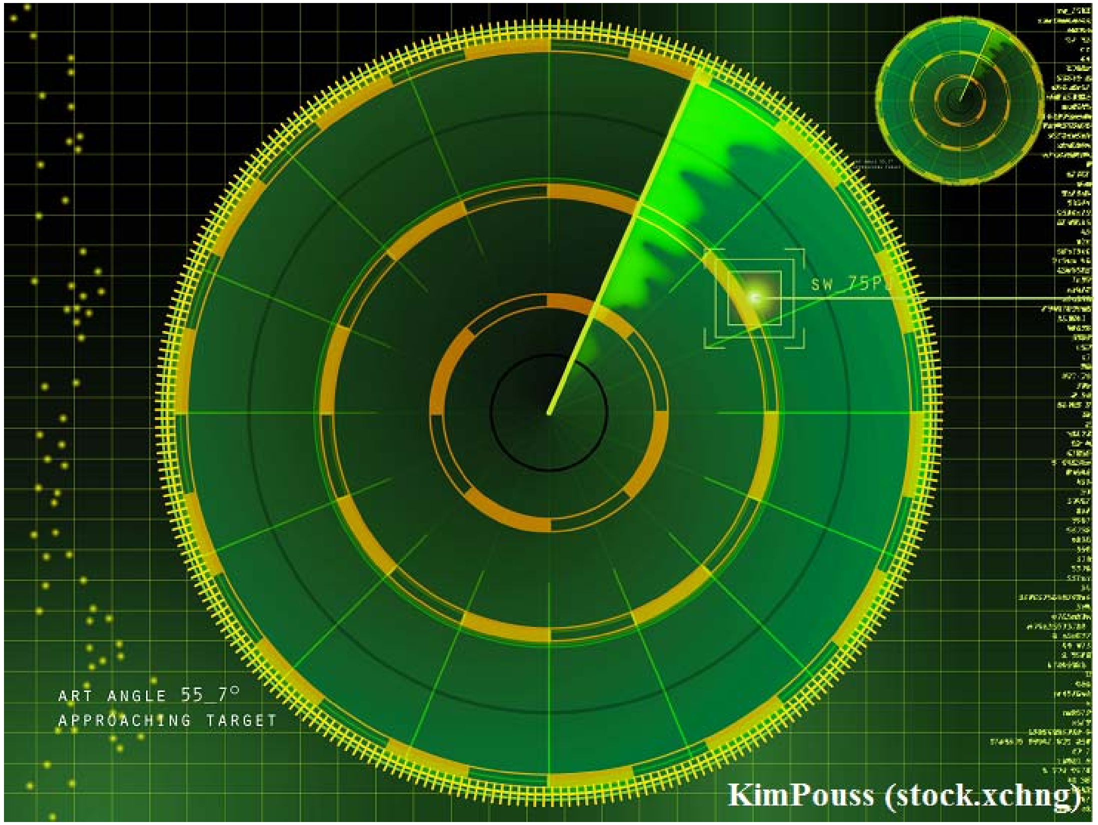
Visibility

- X-ray: absorption



Visibility

- Radar: scattering



Visibility

- Sonar: scattering

Physics-based Invisibility Cloak?

- Visibility

  ←Mainly from absorption and scattering of light

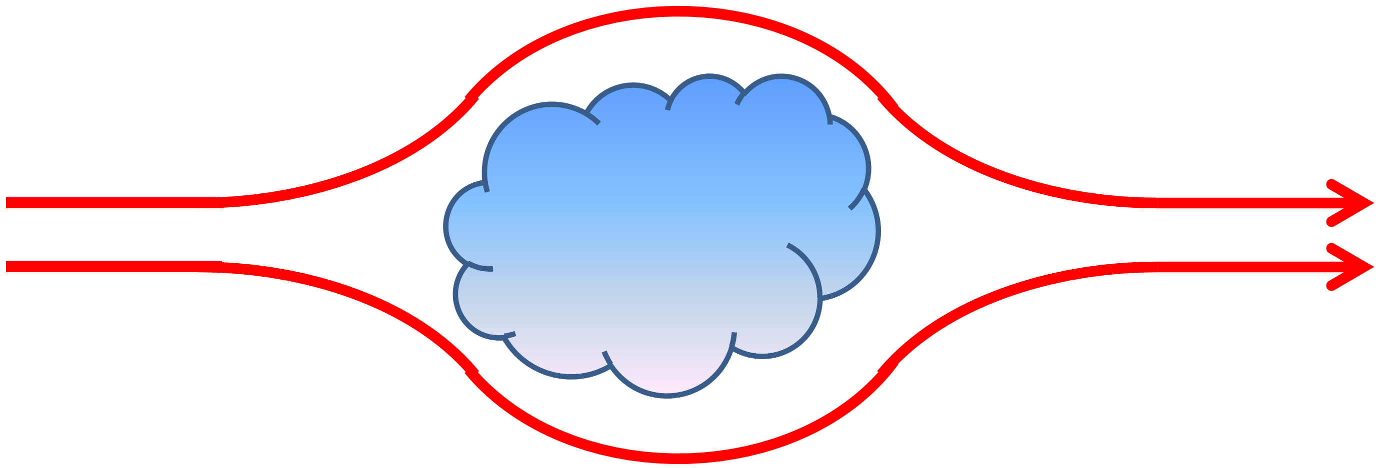
- Invisibility

  ←No absorption, no scattering

  But, how?

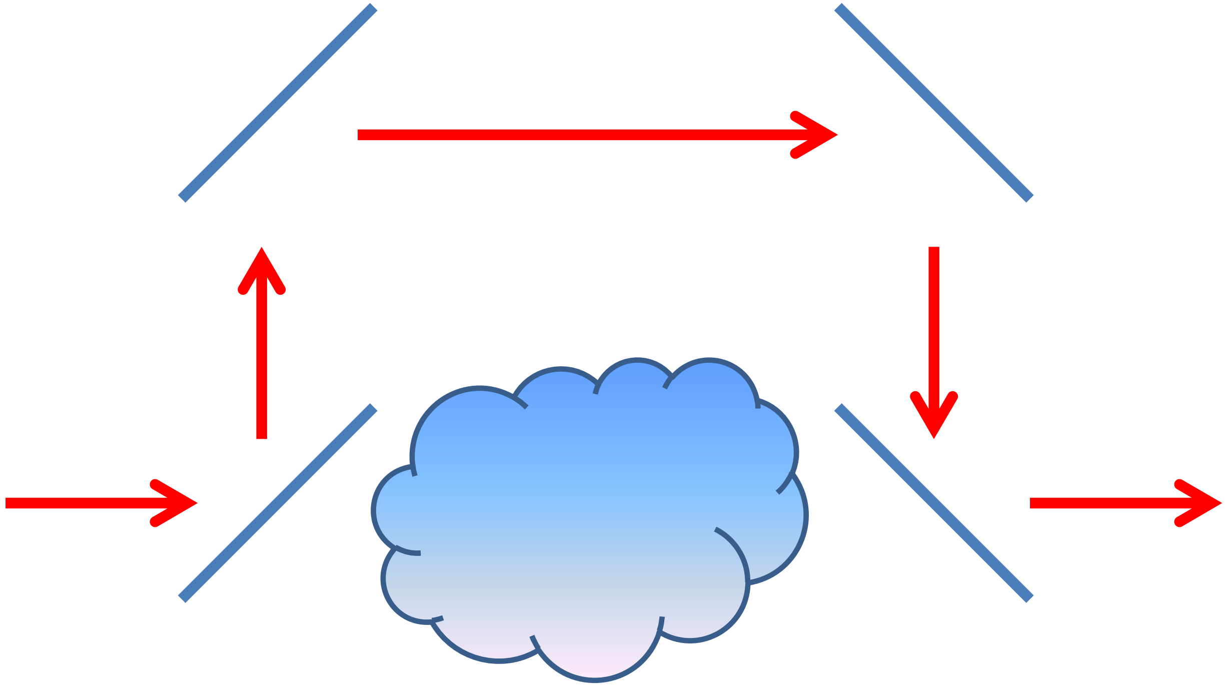
No Absorption, No Scattering?

- Make the light to circle around the object.



Ray optics picture

Periscopes?

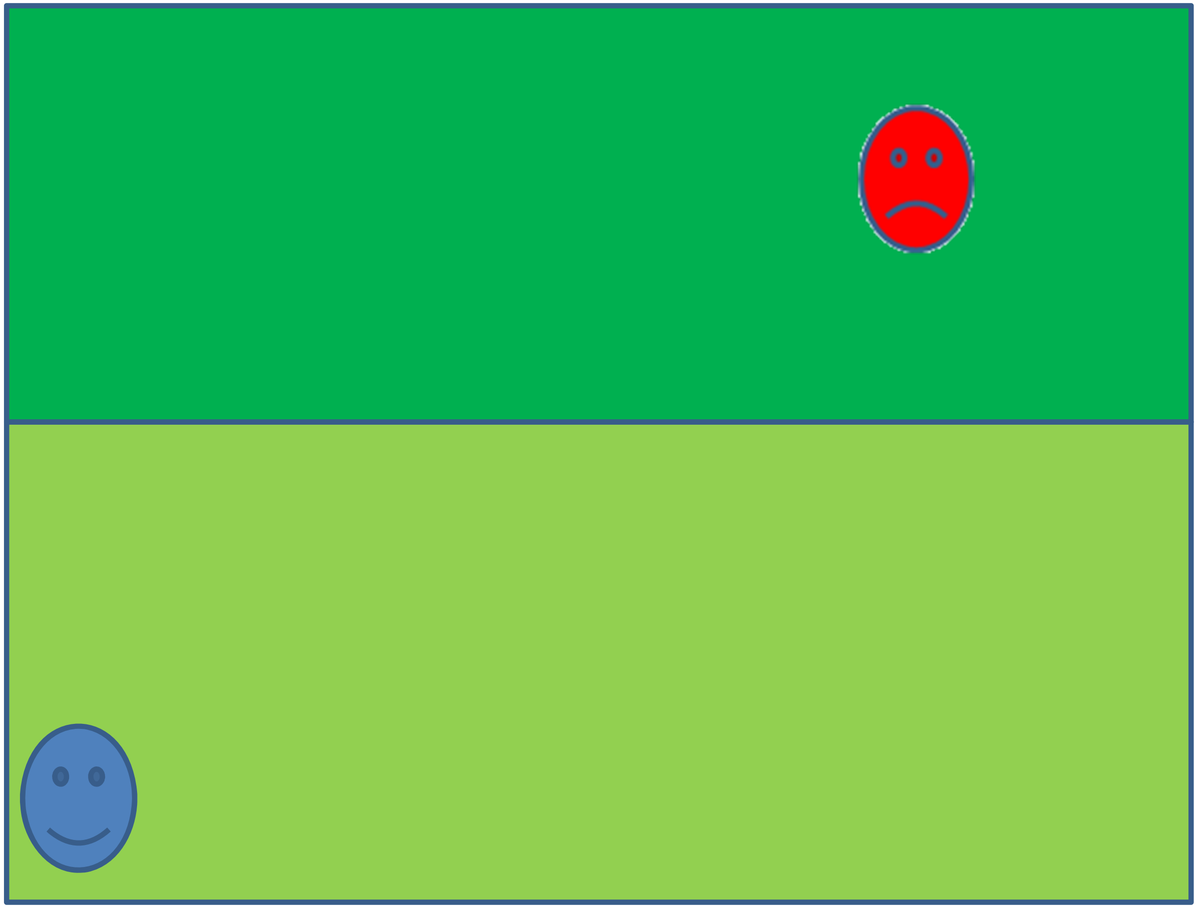


→Only in one direction, at the right distance.

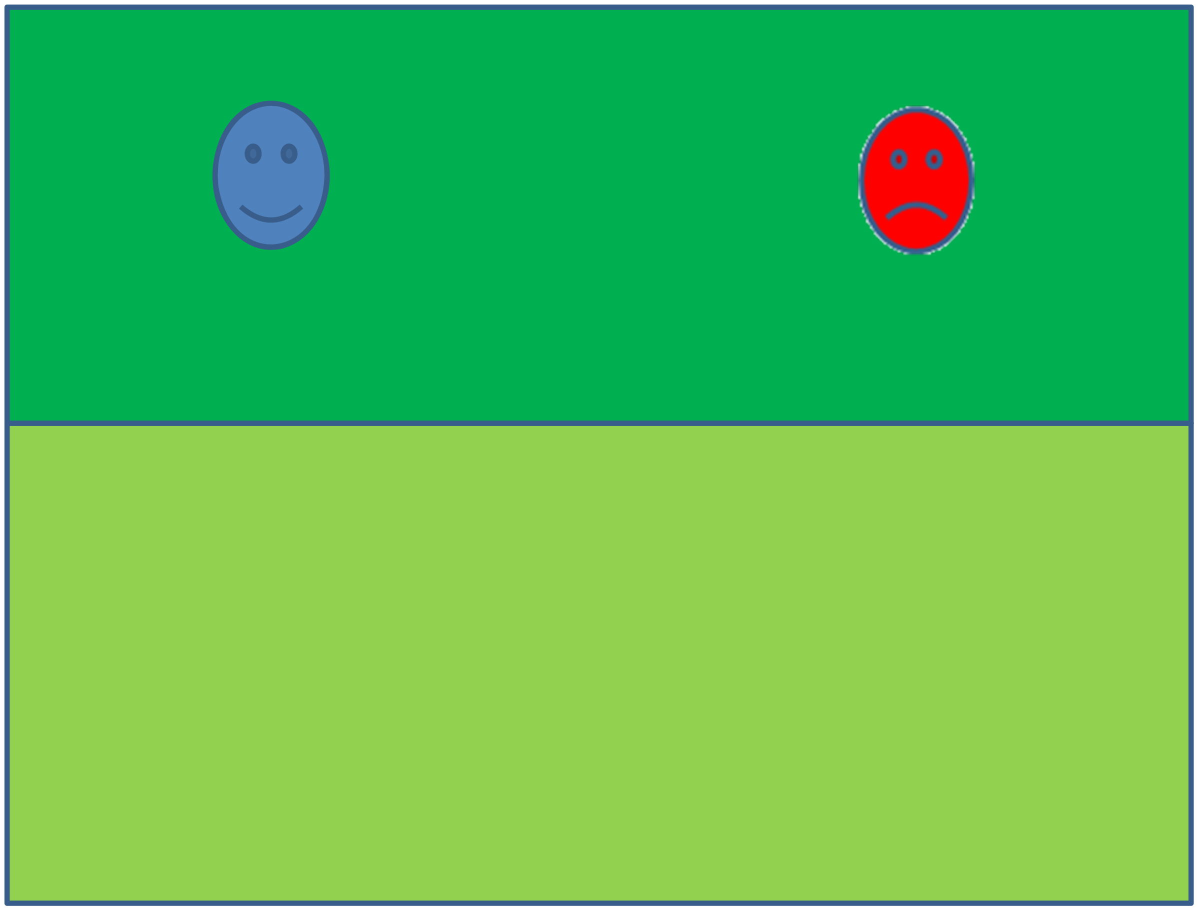
Mirrors visible.

Lifeguard Problem

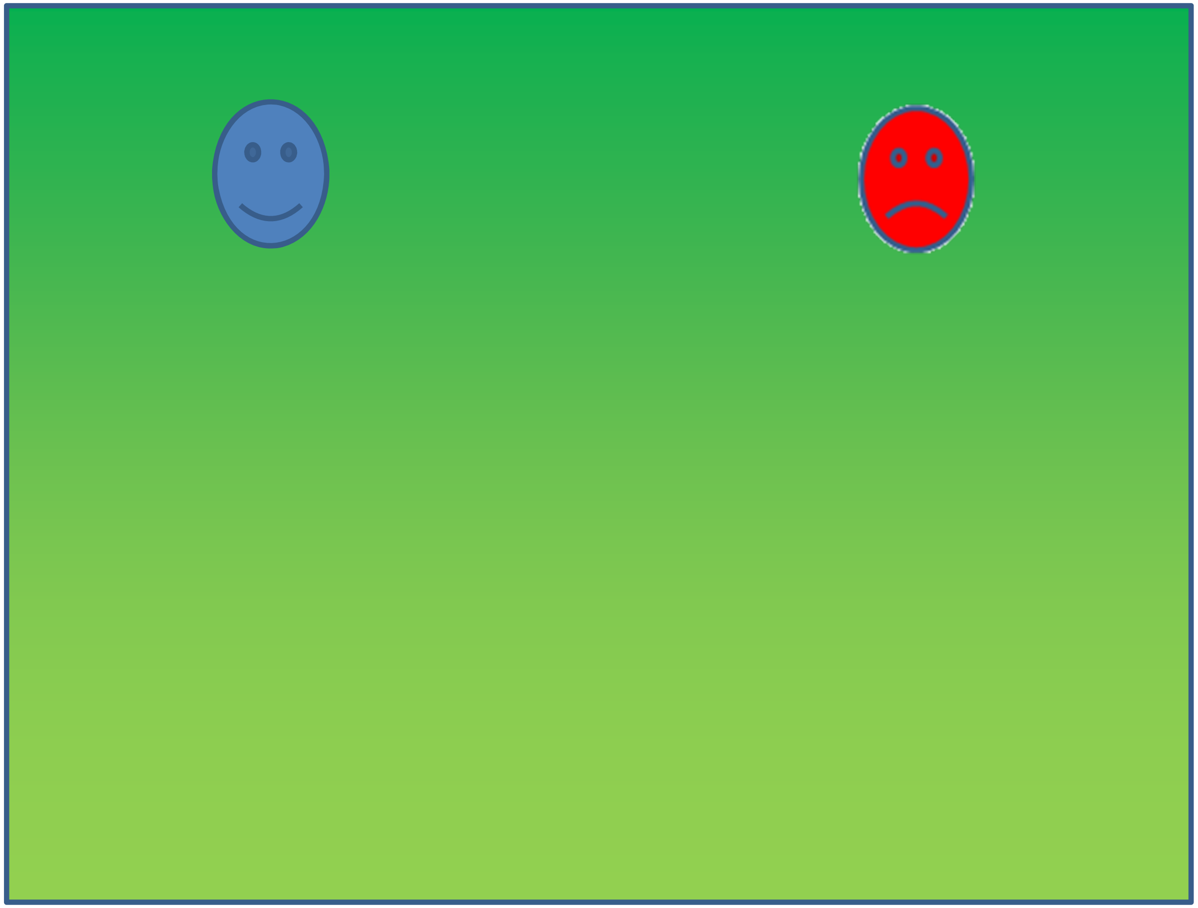
Lifeguard Problem



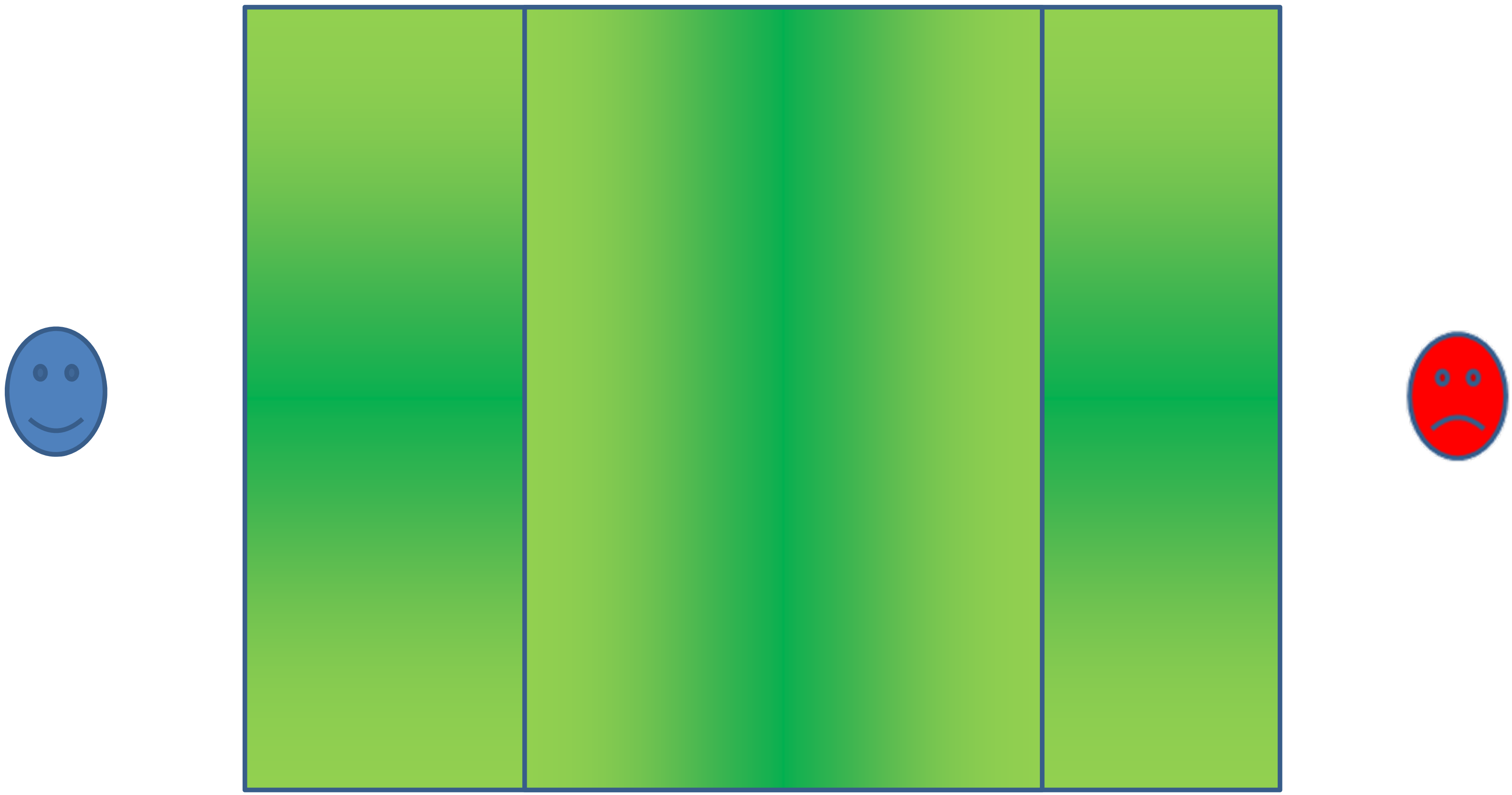
Lifeguard Problem



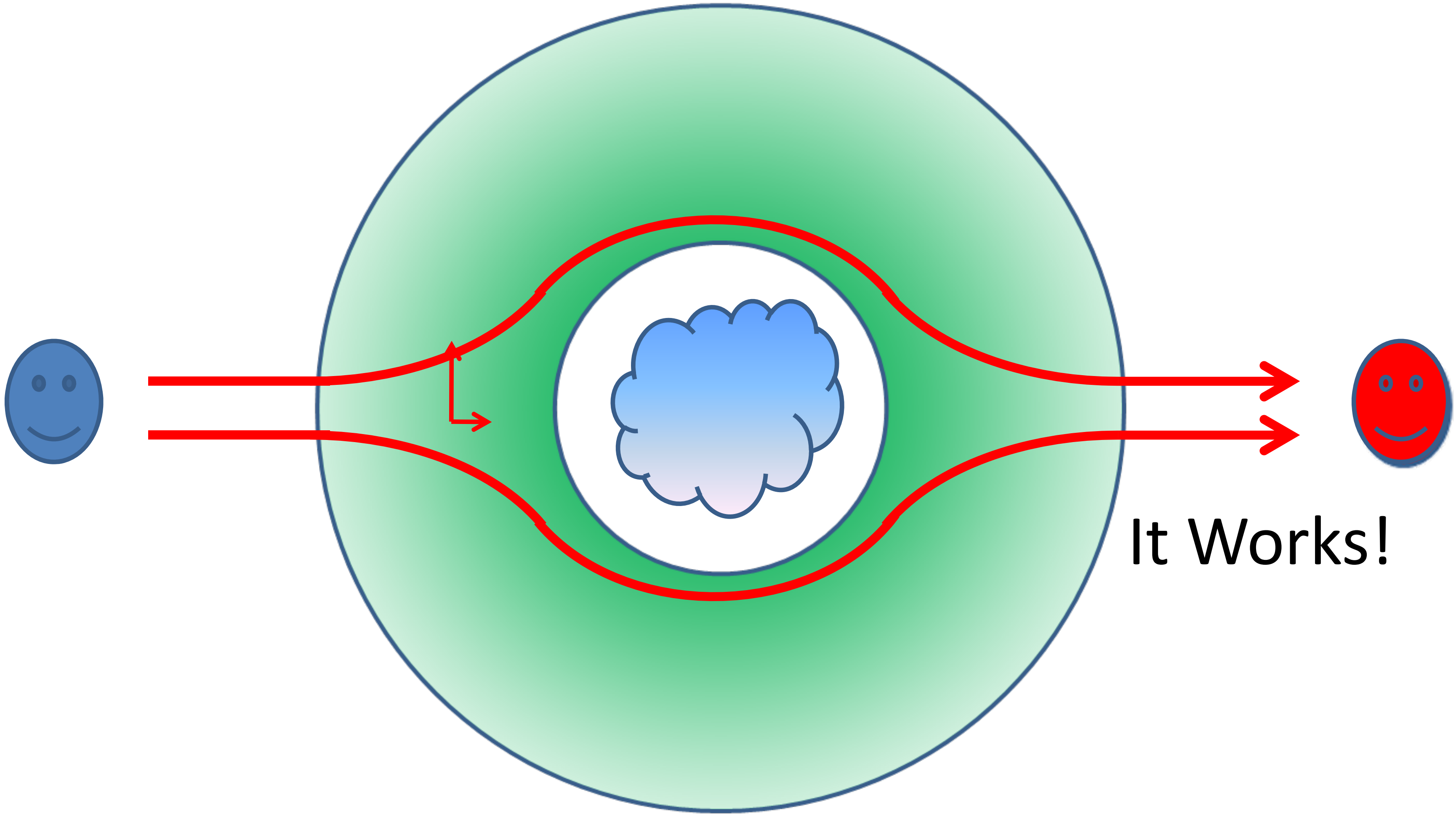
Lifeguard Problem



Invisibility Cloak Using Graded-Index Lens?

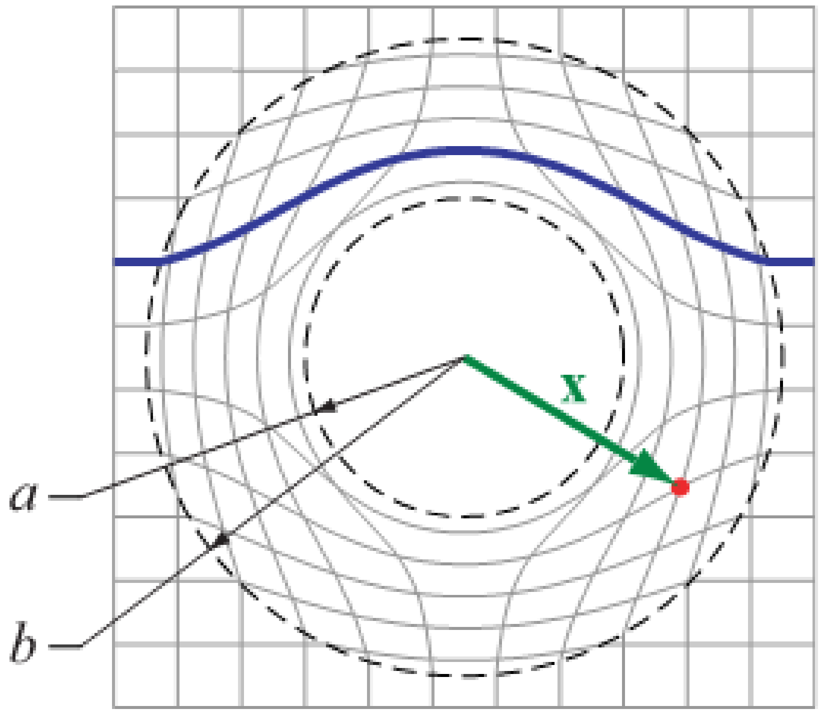


Anisotropic, Inhomogeneous Medium

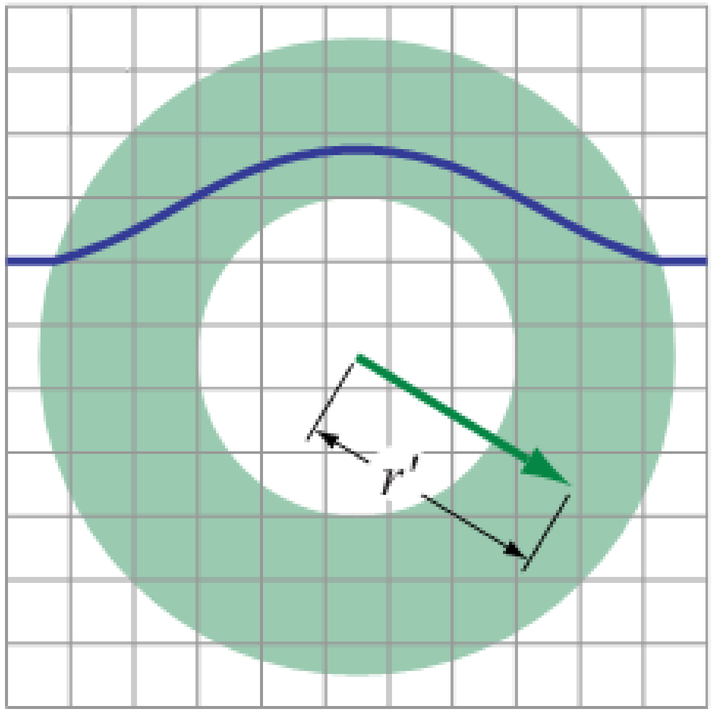


Two Pictures

Topological interpretation



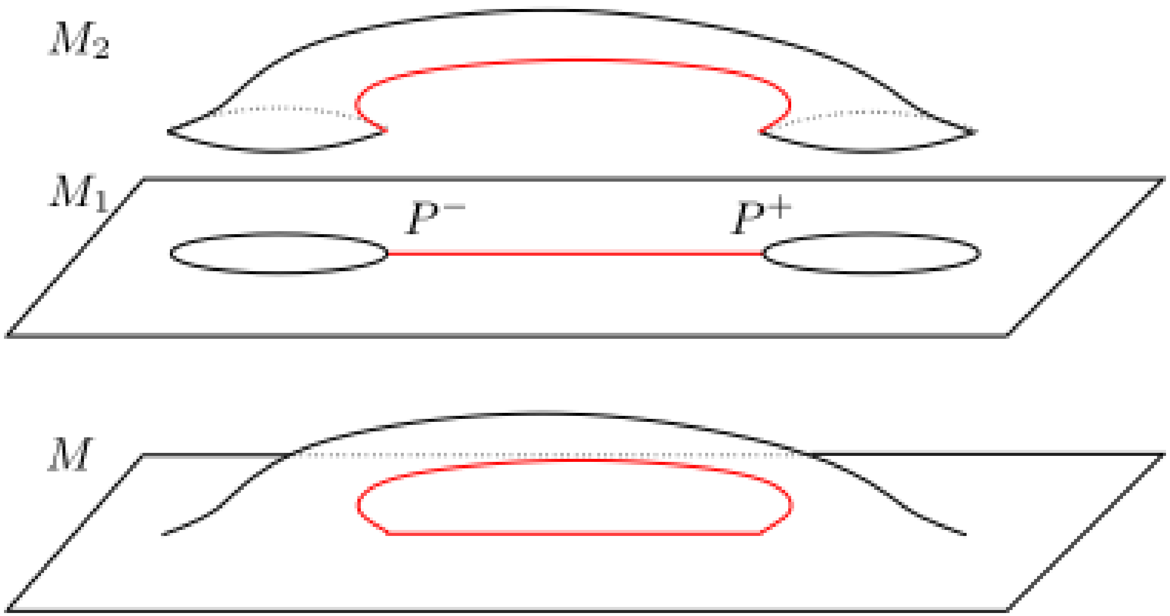
Material interpretation



D. Schurig *et al.*, Opt. Express **14**, 9794 (2006)

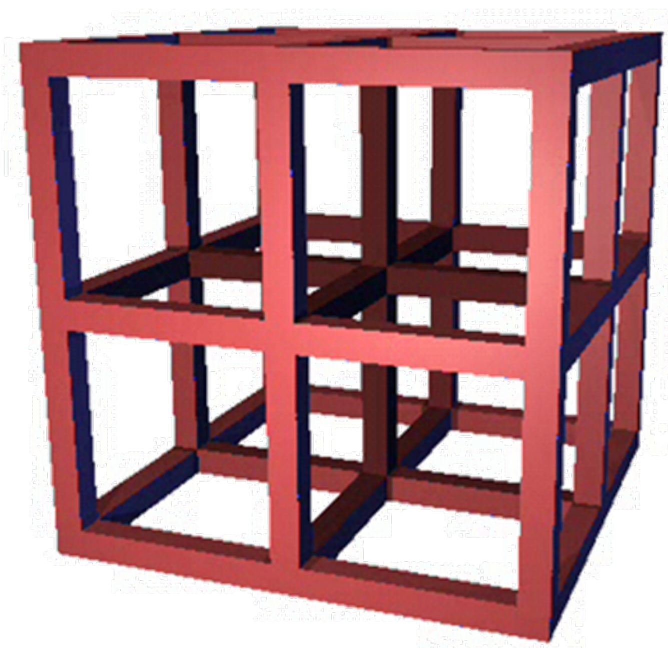
Transformation Optics

- Optical blackhole, wormhole, waveguide, …



A. Greenleaf *et al*., Phys. Rev. Lett **99**, 183901 (2007)

Metamaterials Beyond ε and μ



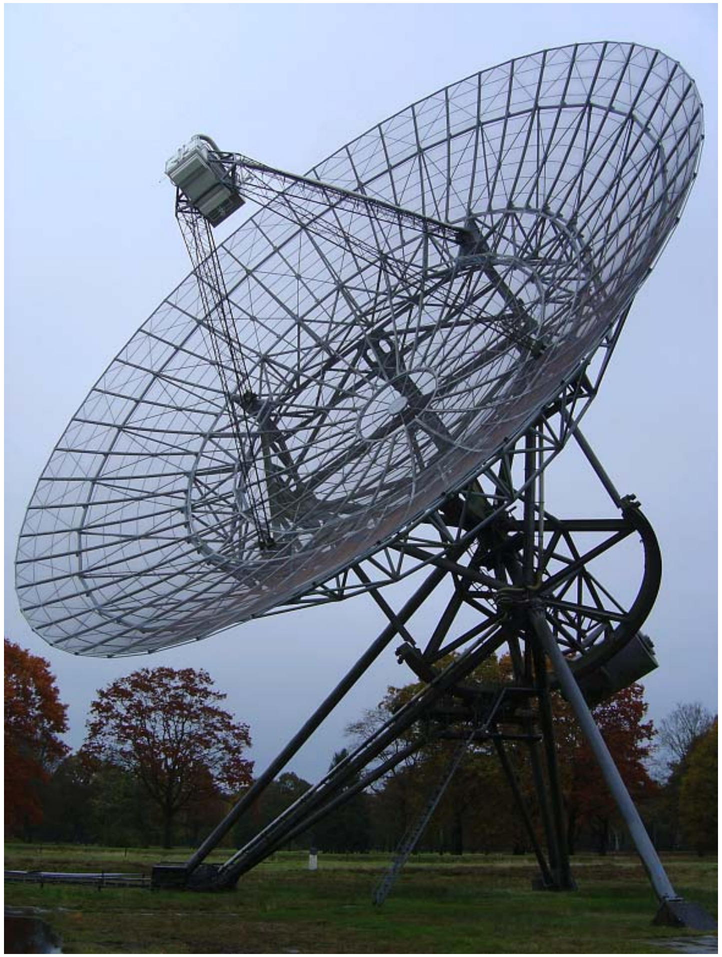


Photo: stock.xchng ID 001099

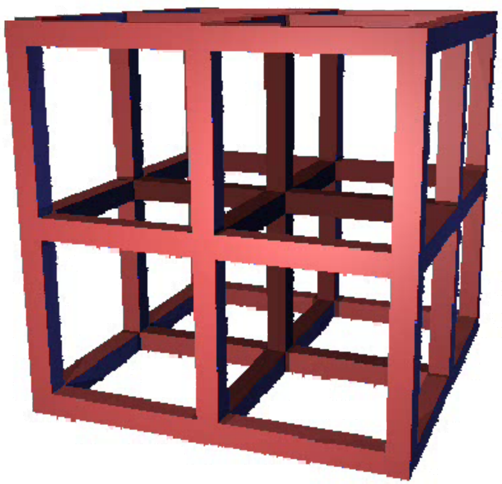
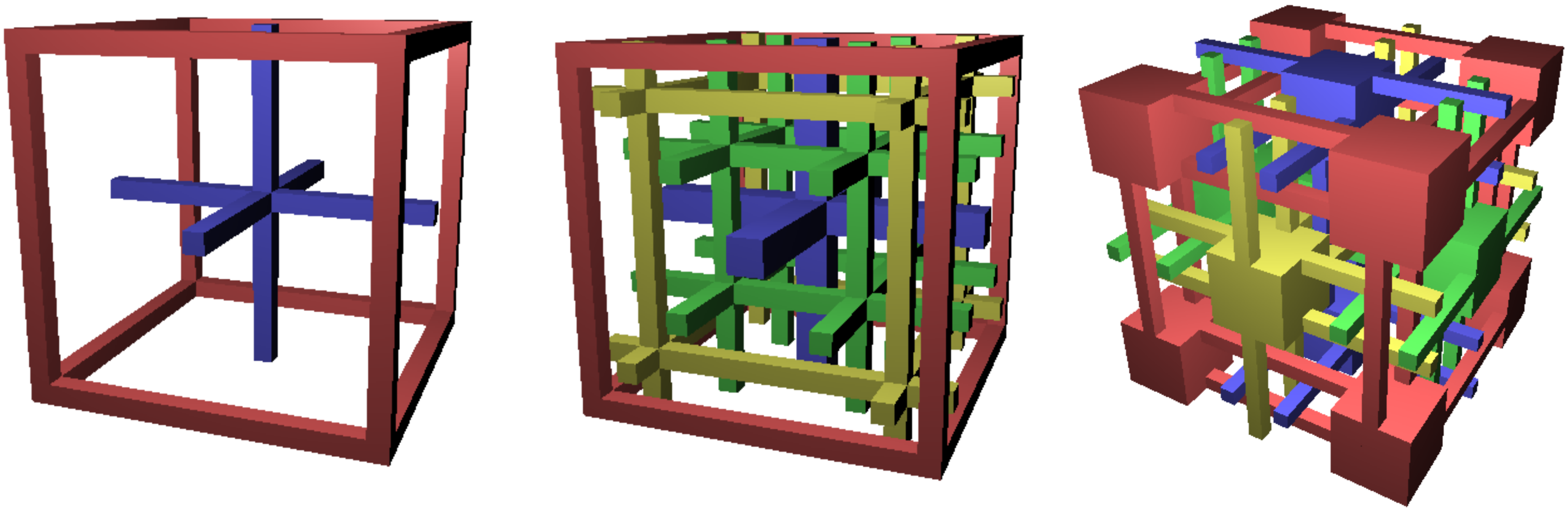


Photo: stock.xchng ID sjtoh

Interweaving Conductor Metamaterial (ICM)

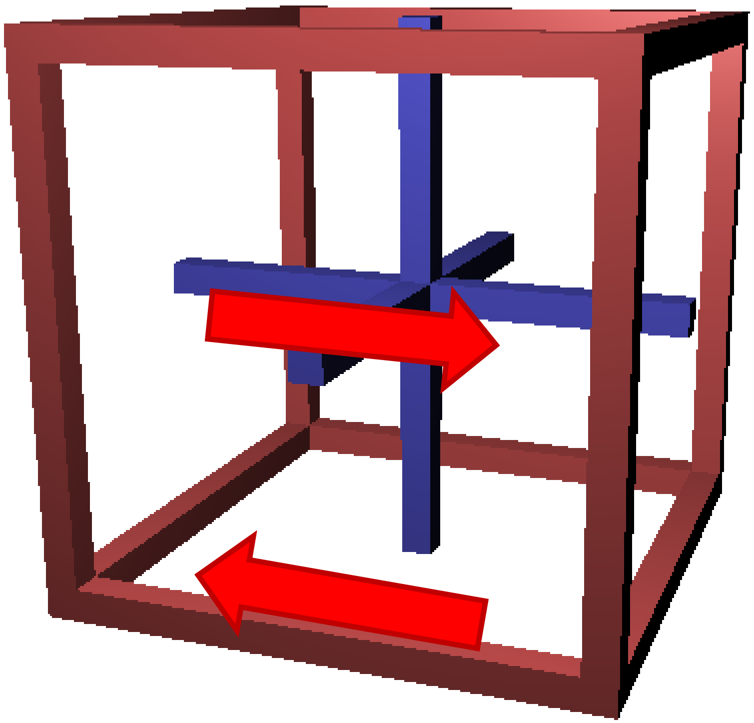


Best described by a non-Maxwellian effective medium

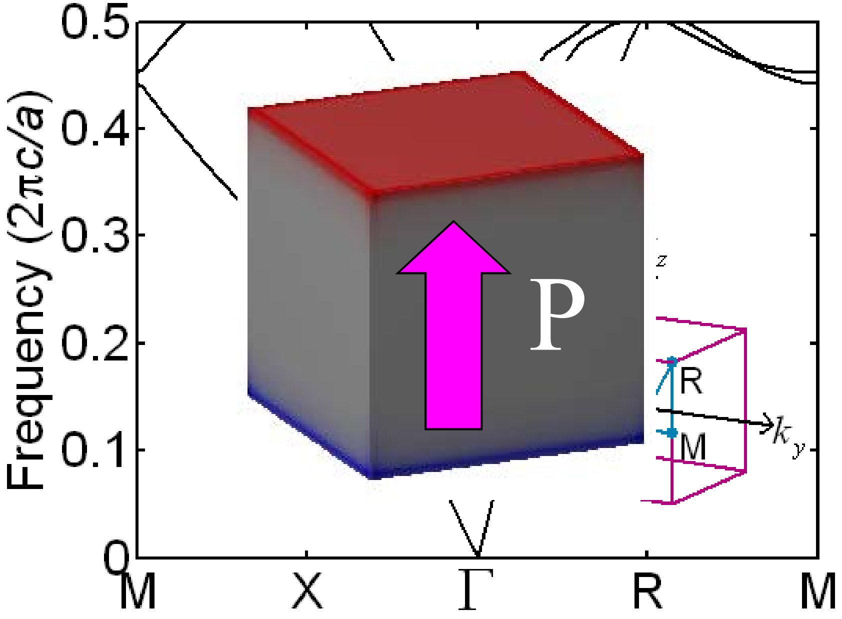
$$\nabla \mathbf{V} = -\frac{\partial \mathbf{A}}{\partial t}, \quad S\nabla \cdot U\mathbf{A} = -\frac{\partial \mathbf{V}}{\partial t}.$$

J. Shin, J.-T. Shen, and S. Fan, "Three-dimensional electromagnetic metamaterials that homogenize to uniform non-Maxwellian media," Phys. Rev. B **76**, 113101 (2007)

Quasi-static Mode



P?



Constraints of Maxwellian Model Assuming Local Parameters

Cubic symmetry of the structure

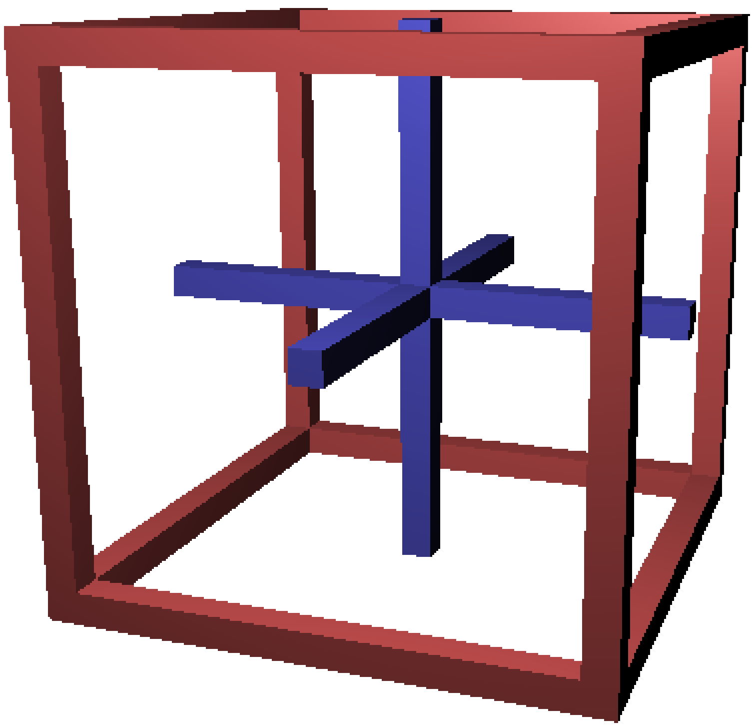
→ $$\varepsilon = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_1 \end{pmatrix}, \quad \mu = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_1 & 0 \\ 0 & 0 & \mu_1 \end{pmatrix}.$$

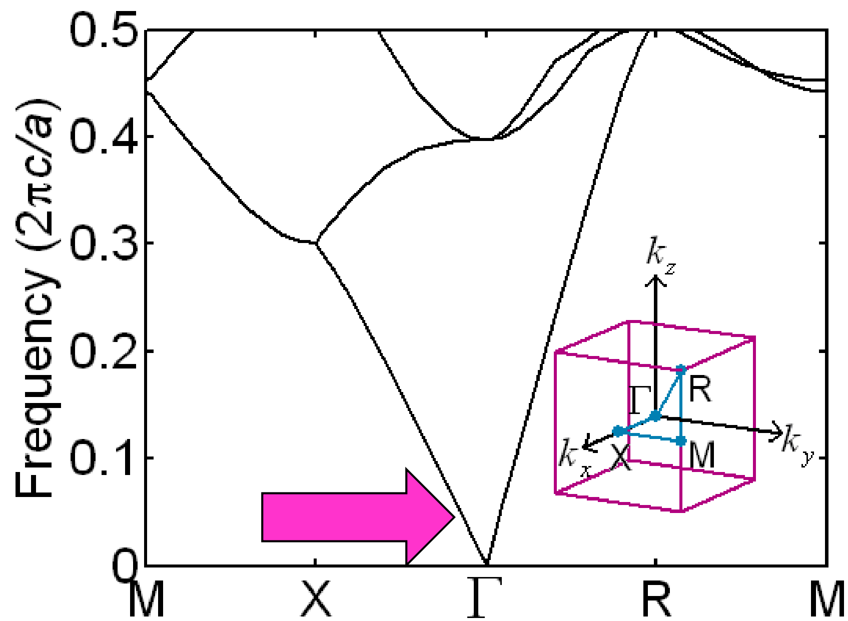
→ zero or two propagating modes

→ modes are isotropic

→ modes are degenerate

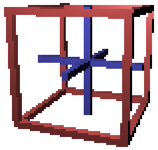
Two-Network Example

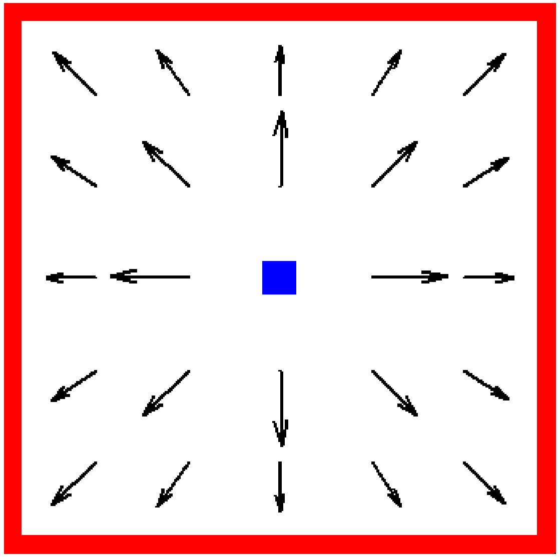




Single mode

Scalar Field in Full 3D



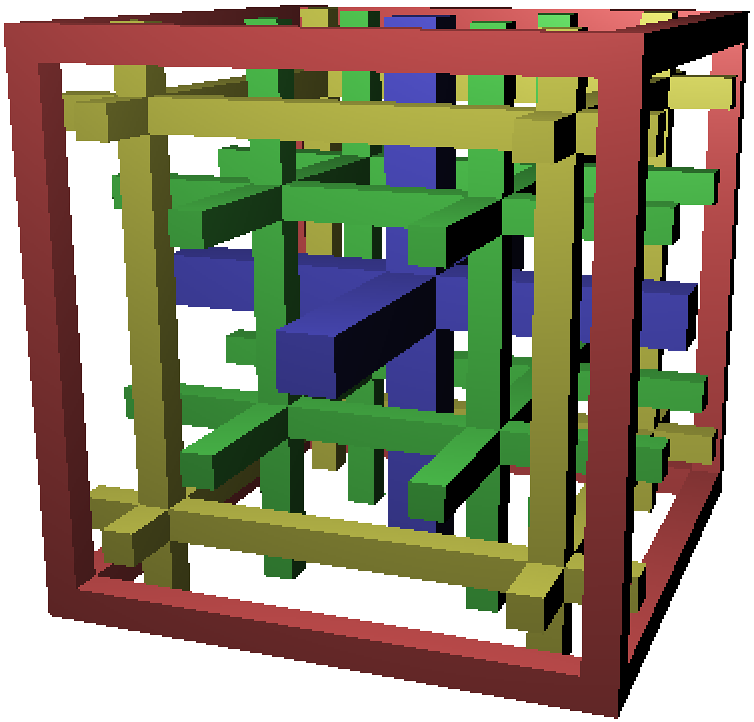


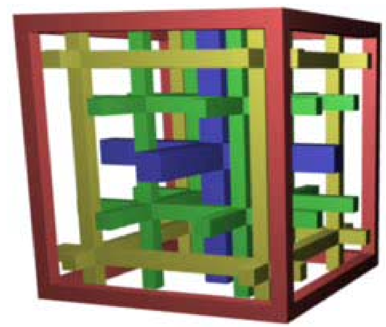
Electric field, $z = 0$ plane.

Full symmetry in the field:
imply a non-degenerate state

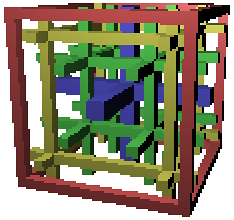
No polarization dependency in full three-dimension

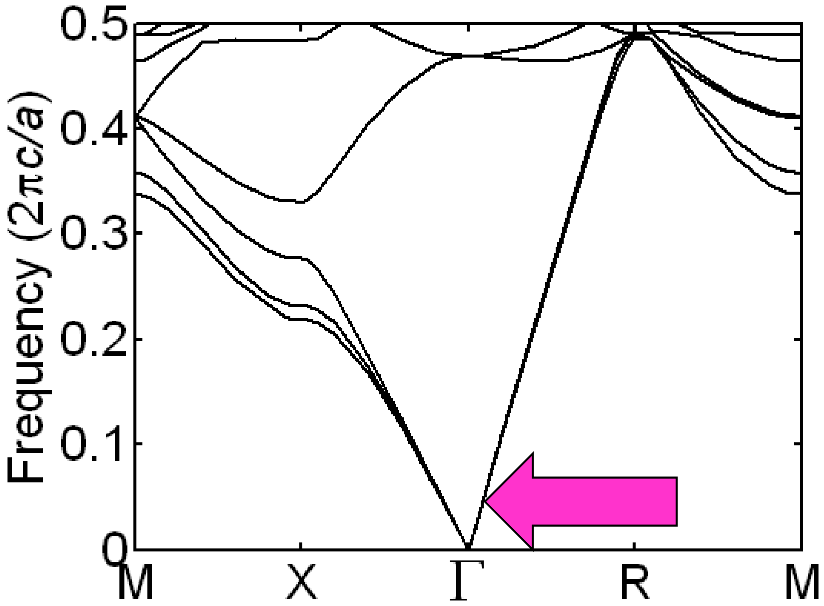
N-Network Example





Bandstructure



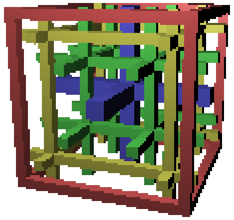


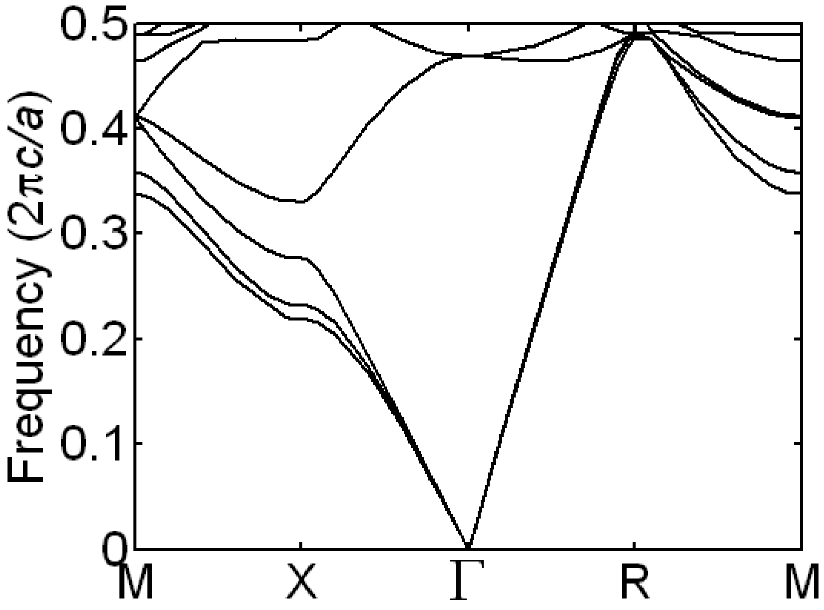
- *N*-1 modes

  (*N* = 4, in this case)

- Non-degenerate
  - despite the full cubic symmetry of the system

Bandstructure

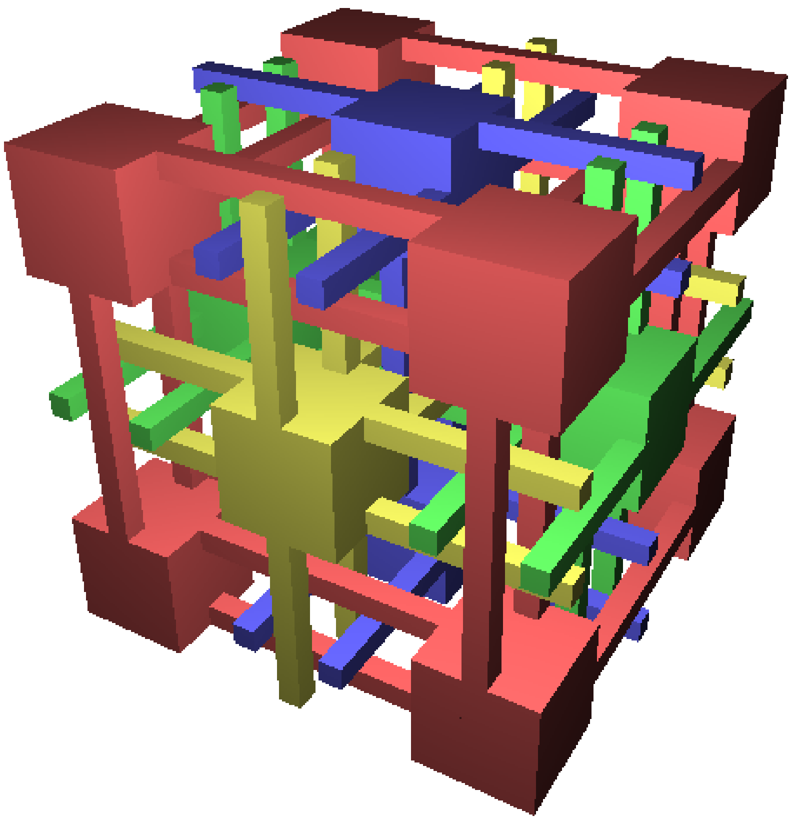


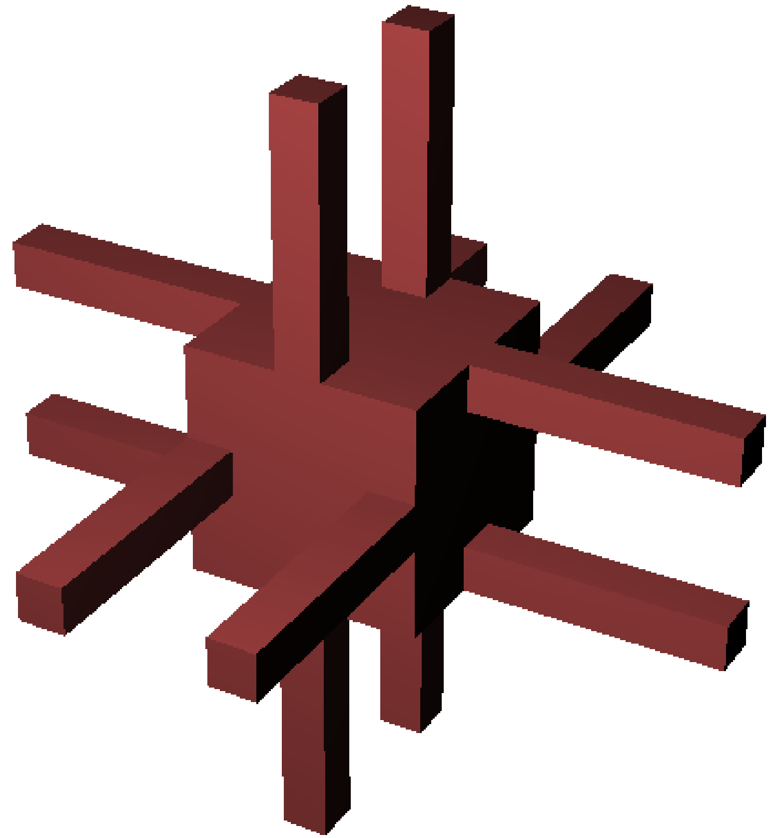


- *N*-1 modes
  (*N* = 4, in this case)

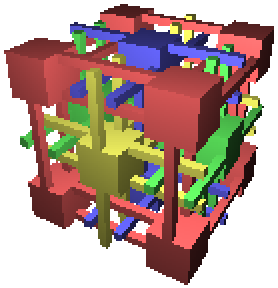
Density-of-state enhancement over a very broad frequency range

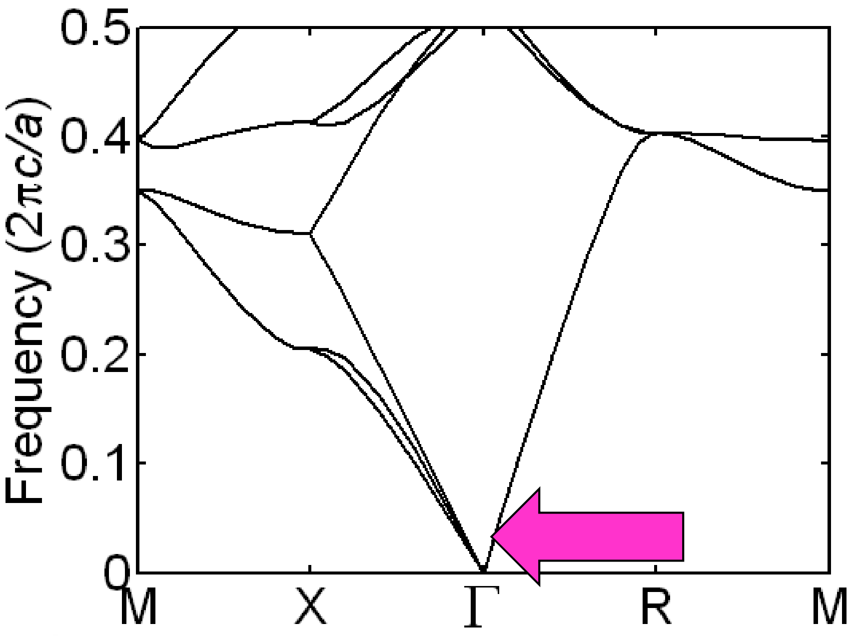
Four-Network FCC Example

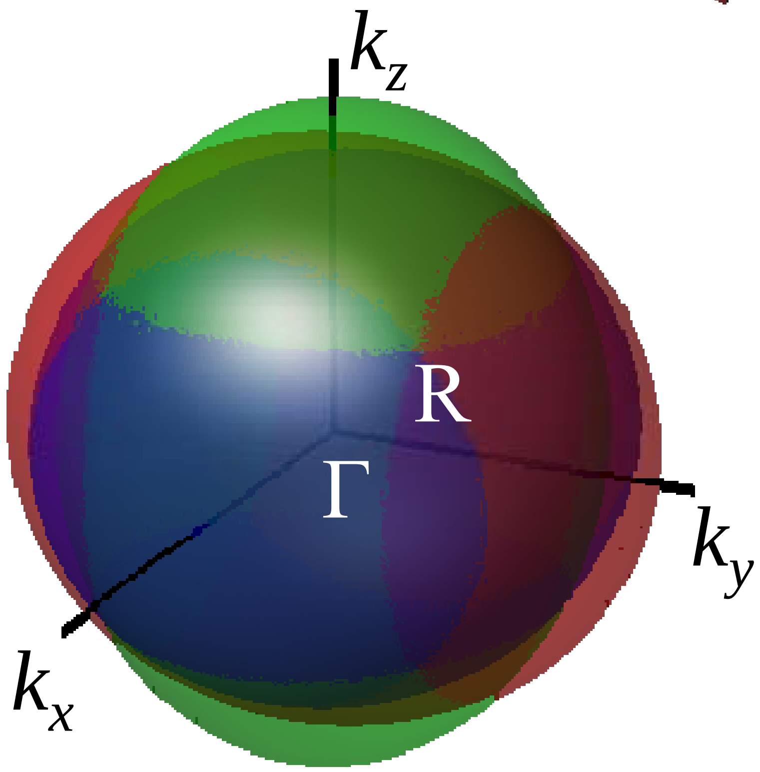




Bandstructure







Iso-frequency surfaces

- Triply-degenerate
  - in [1 1 1] direction
- Anisotropic
  - even with pyritohedral symmetry

Effective Analytic Theory

$$\frac{\partial^2 \mathbf{V}}{\partial t^2} = S\nabla \cdot \left(U\nabla\mathbf{V}\right), \qquad \mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}.$$

- V_i: Spatially
- *S*, *U*

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = \varepsilon^{-1}\nabla\times\left(\mu^{-1}\nabla\times\mathbf{E}\right),\ \mathbf{E} = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}.$$

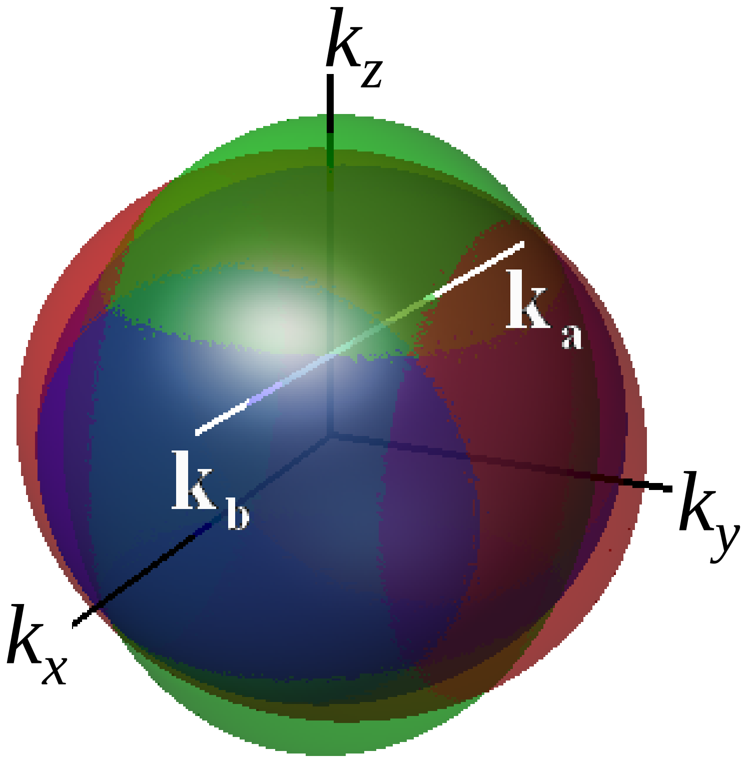
Analog

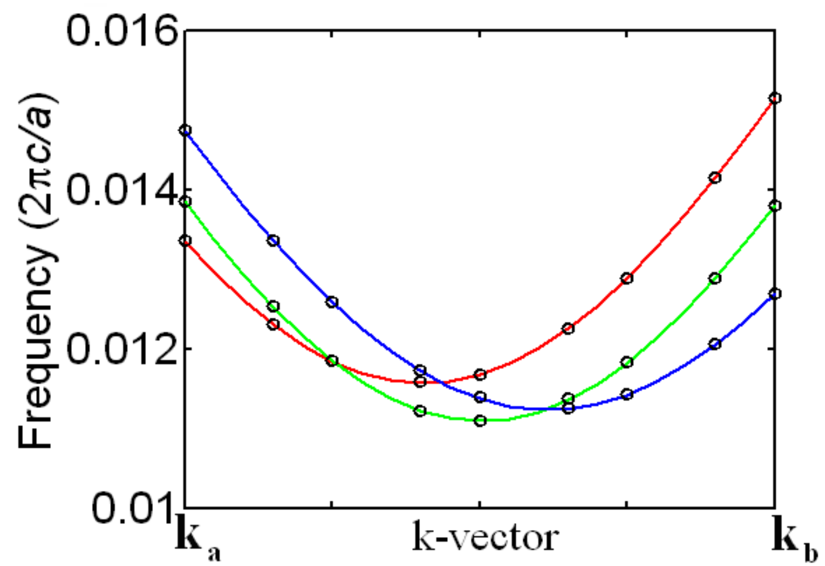
Effective Analytic Theory

$$\frac{\partial^2 \mathbf{V}}{\partial t^2} = S\nabla \cdot \left(U \nabla \mathbf{V} \right), \qquad \mathbf{V} = S\mathbf{Q}, \quad \mathbf{I} = U\mathbf{A}.$$

- S, U : Determined by the capacitance and inductance between networks.
- Dimenion of S and U → Number of modes.
- Symmetry of S and U → Anisotropy of modes.
- Eigenvalues of S U equation → Degeneracy of modes.

Verification of the Model





Metamaterial (Summary)

- Deep-subwavelength metallic unit cell

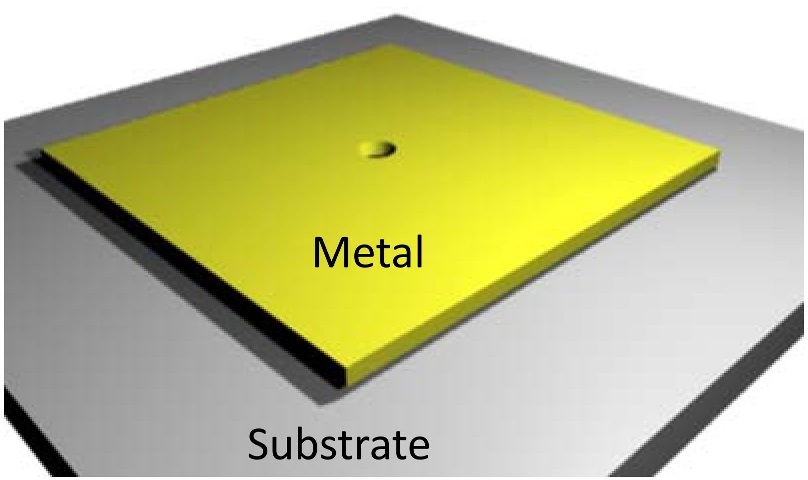
→Designable near-field configuration

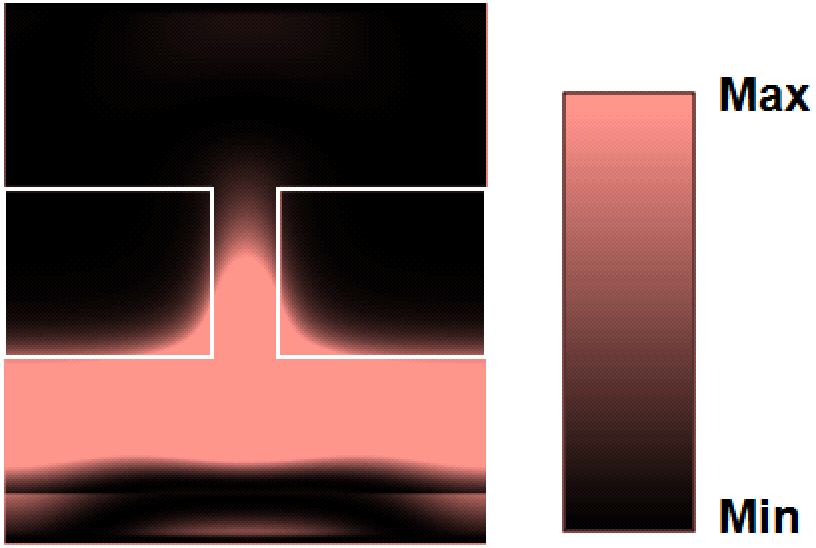
→Artificial “materials” with novel optical properties

Duality, Babinet's Principle

Field Localization and Field Enhancement

- Related, but not the same
- Localized-but-not-enhanced case: subwavelength hole





Electric Field Enhancement

- Applications:

  Nonlinear devices, emission enhancements, optical tweezing, etc.

- Occurs in metal structures with a sharp tip or narrow gap

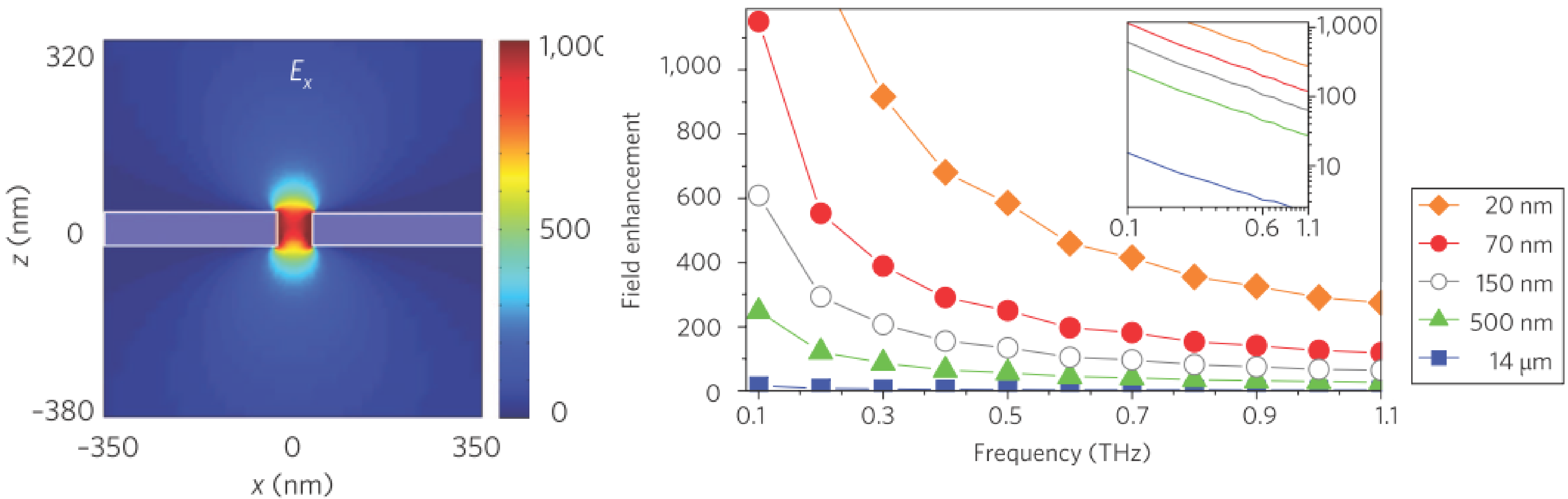
  - r, a << wavelength
  - e.g., SNOM tip, bow-tie antenna, metal slits

Example: Metal Slit

- Narrow (~ 70 nm) slit on a thin gold film

Example: Metal Slit

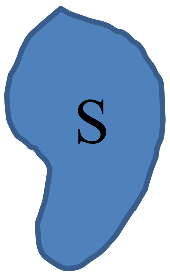
- Narrow (~ 70 nm) slit on a thin gold film
- Large (~ 1,000) electric field enhancement in THz regime.



M.A. Seo, Nature Photon. **22**, 152 (2009)

What about magnetic fields?

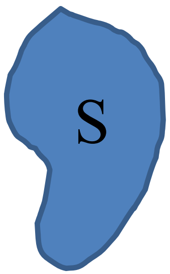
Babinet's Principle (Simplest Version)



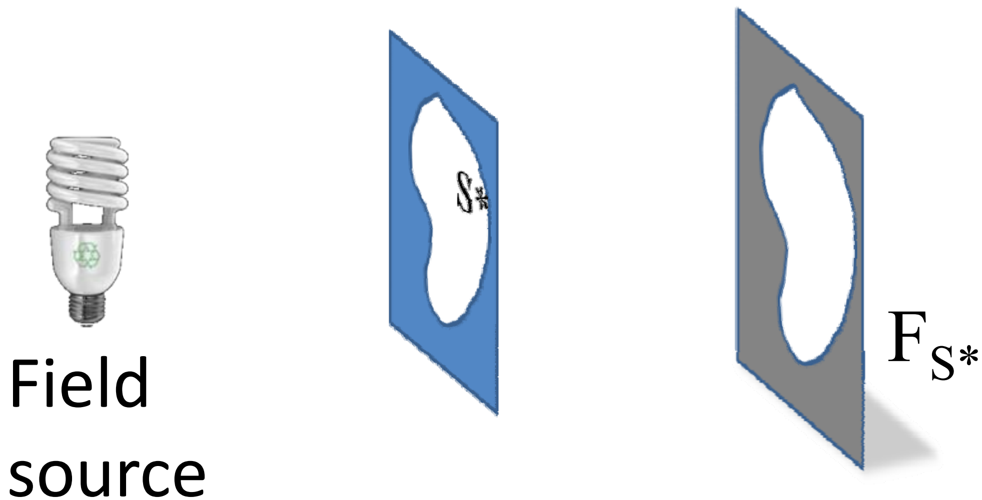
(Infinitesimally thin, absorbing screen)

Field source

Babinet's Principle (Simplest Version)

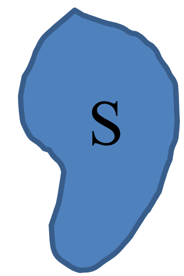


(Infinitesimally thin, absorbing screen)

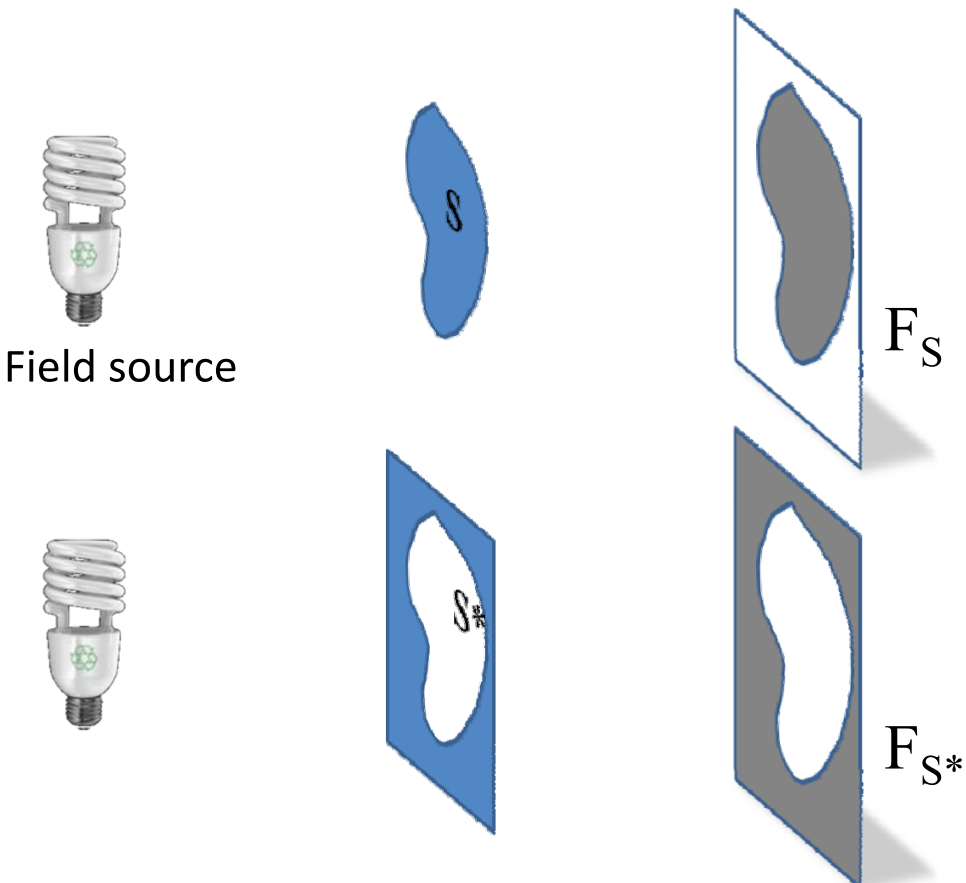


Babinet's Principle (Simplest Version)

- (Field behind S) + (Field behind S*) = Field without any screen



(Infinitesimally thin, absorbing screen)

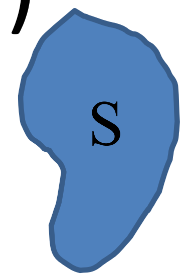


$$F_S + F_{S*} = F_o$$

R. Harrington, *Time-Harmonic Electromagnetic Fields*

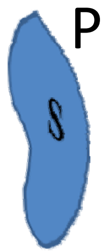
Babinet’s Principle (with Booker’s Extension)

- (Field behind S) + (Field behind S*) = Field without any screen



(Infinitesimally thin, PEC screen)

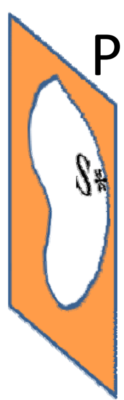
$\uparrow \mathbf{J}$



PEC

$(\mathbf{E}_S, \mathbf{H}_S)$

$\uparrow \mathbf{J}$



PMC

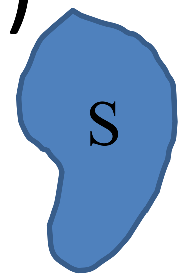
$(\mathbf{E}_{S*}, \mathbf{H}_{S*})$

$$\mathbf{E}_S + \mathbf{E}_{S*} = \mathbf{E}_o$$
$$\mathbf{H}_S + \mathbf{H}_{S*} = \mathbf{H}_o$$

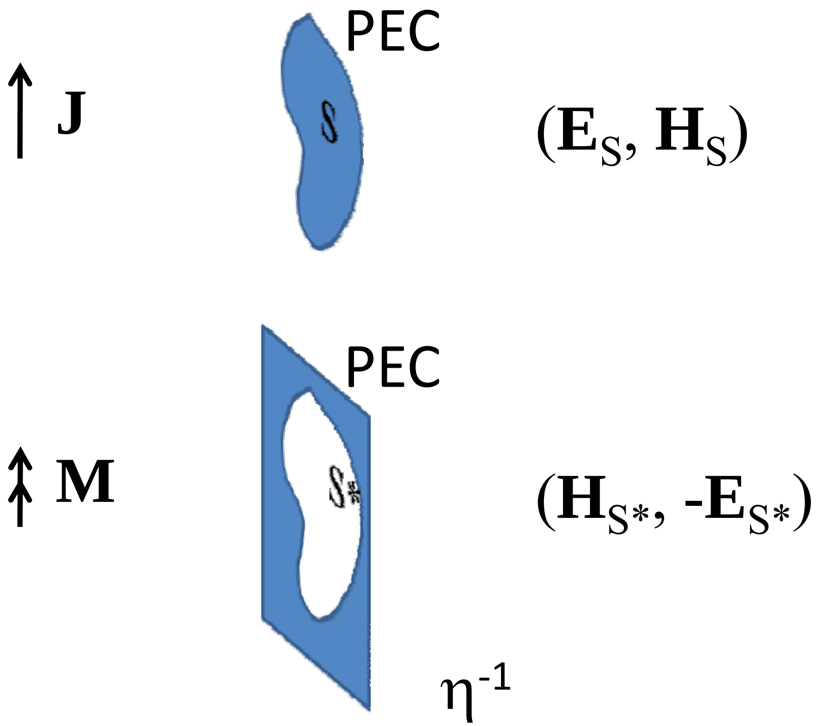
R. Harrington, *Time-Harmonic Electromagnetic Fields*

Babinet's Principle (with Booker's Extension)

- (Field behind S) + (Field behind S*) = Field without any screen



(Infinitesimally thin, PEC screen)



$$\mathbf{E}_S + \mathbf{H}_{S*} = \mathbf{E}_o$$
$$\mathbf{H}_S - \mathbf{E}_{S*} = \mathbf{H}_o$$

R. Harrington, *Time-Harmonic Electromagnetic Fields*

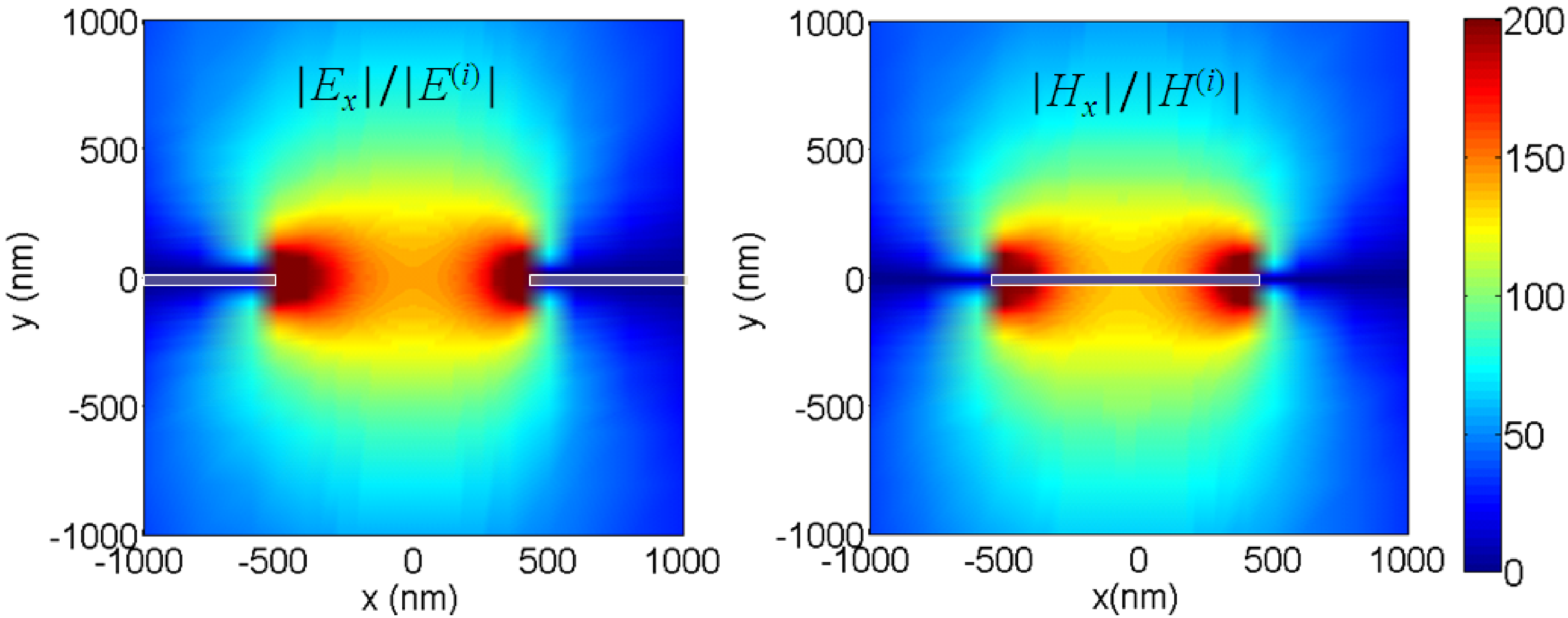
Example: Metal Slit vs. Metal Wire

Metal slit

Metal wire

Example: Metal Slit vs. Metal Wire

If PEC is assumed,
the enhancement factor is almost the same.



Sukmo Koo *et al*. Phys. Rev. Lett. **103**, 263901 (2009)

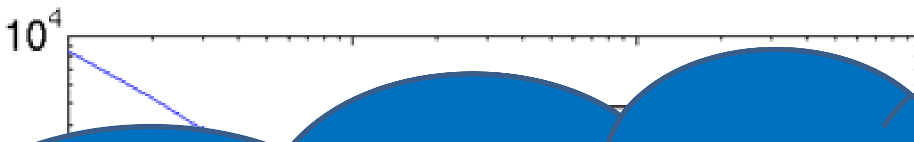
Breakdown of Babinet's Principle

- Violation of the fundamental assumptions
  - Perfect electric conductor → lossy metal (Finite permittivity , non-zero skin-depth)
  - Infinitesimally thin → measurable thickness

- No exact complementary magnetic structure for a given electric structure
  → Different enhancement factors

Example: Metal Slit vs. Metal Wire

- With realistic metal parameters,
  enhancement reduces by orders of magnitudes.



Need to study the effects of finite ε!

→ Graduate studies