Black Hole vs Black Body

이필진

(고등과학원)

KIAS-SNU Physics Wintercamp

휘닉스 파크, 2010년 2월

temperature = energy unit

$$T \equiv k_B T$$

$$1 \, degree \, Kelvin$$

↕ Boltzmann constant

$$\simeq 1.381 \times 10^{-23} \, joule$$

$$\simeq 8.617 \times 10^{-5} \, eV$$

fundamental units / constants

$$c \simeq 2.99792458 \times 10^{8} \; meter \,/\, second$$

$$h \simeq 6.62606896 \times 10^{-34} \; joule \times second$$

$$(\; hc \simeq 1.986 \times 10^{-25} \; joule \times meter \;)$$

fundamental units / constants

$$c \simeq 2.99792458 \times 10^{8} \; meter \, / \, second$$

$$h \simeq 6.62606896 \times 10^{-34} \; joule \times second$$

$$(\, hc \simeq 1.986 \times 10^{-25} \; joule \times meter \,)$$

$$E = Mc^2 \quad \rightarrow \quad c^2 \simeq 8.988 \times 10^{16} \; joule \, / \, kilogram$$

$$\rightarrow \quad h/c^2 \simeq 7.372 \times 10^{-51} \; kilogram \times second$$

$$\rightarrow \quad h/c \simeq 2.210 \times 10^{-42} \; kilogram \times meter$$

Newton's constant in different units

$$G_N \simeq 6.67428 \times 10^{-11} meter^3 \,/\, (kilogram \;\times\; second^2)$$

$$G_N \simeq 6.67428 \times 10^{-11} (meter \,/\, second)^2 \;\times\; (meter \,/\, kilogram)$$

$$G_N/c^2 \simeq 7.425 \times 10^{-28} \; meter \,/\, kilogram$$

Newton's constant in different units

$$G_N \simeq 6.67428 \times 10^{-11} meter^3 \,/\, (kilogram \;\times\; second^2)$$

$$G_N \simeq 6.67428 \times 10^{-11} (meter \,/\, second)^2 \;\times\; (meter \,/\, kilogram)$$

$$G_N/c^2 \simeq 7.425 \times 10^{-28} \; meter \,/\, kilogram$$

$$l^2_{Planck} \equiv G_N h/2\pi c^3 = G_N/c^2 \;\times\; h/2\pi c \simeq 2.612 \times 10^{-70} \; meter^2$$

$$l_{Planck} \simeq 1.616 \times 10^{-35} \; meter$$

many faces of mass in gravitation and in quantum physics

$1\,kilogram$

$\xrightarrow{\times c/h}$

$4.525 \times 10^{41}\ /meter$

= 1 / Compton wavelength

many faces of mass in gravitation and in quantum physics

$1\,kilogram$

$G_N/c^2 \times$ → $7.425 \times 10^{-28}\ meter$

$\times c/h$ → $4.525 \times 10^{41}\ /meter$

~ 1 / Compton wavelength

mass in gravitation

$1\ kilogram$

$G_N/c^2 \times$

$7.425 \times 10^{-28}\ meter$

invert and multiply hc

$267.5\ joule$

mass in gravitation

$$1\,kilogram$$

$$G_N/c^2\times$$

$$7.425\times10^{-28}\ meter \longrightarrow 267.5\ joule$$

$$(1\,kilogram)\times(1\,kilogram)$$

$$\times G_N/hc$$

$$3.36\times10^{14}$$

solar mass in gravitation

$$Solar\ Mass \sim 2 \times 10^{30}\ kilogram$$

$G_N/c^2 \times$

$$1485\ meter$$

invert and multiply hc

$$1.338 \times 10^{-28}\ joule$$

$$(Solar\ Mass) \times (Solar\ Mass)$$

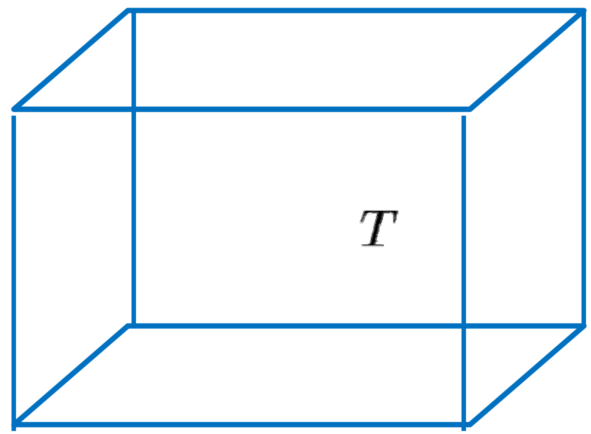
$\times G_N/hc$

$$1.344 \times 10^{75}$$

Thermodynamics (I): Laws of Thermodynamics

$$dE = TdS - pdV + \cdots$$

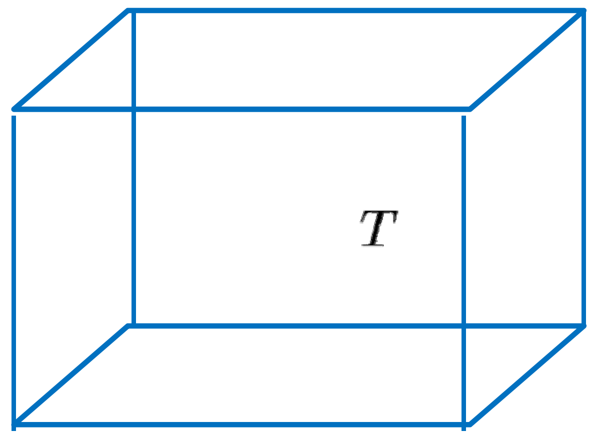
$$\frac{d}{dt}S_{closed} \geq 0$$



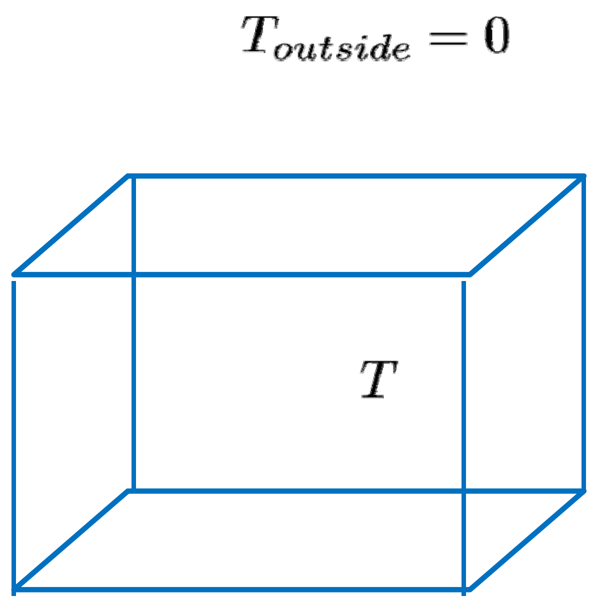
Thermodynamics (I): Laws of Thermodynamics

$$S = \int_{fixed\ V} \frac{1}{T} dE$$

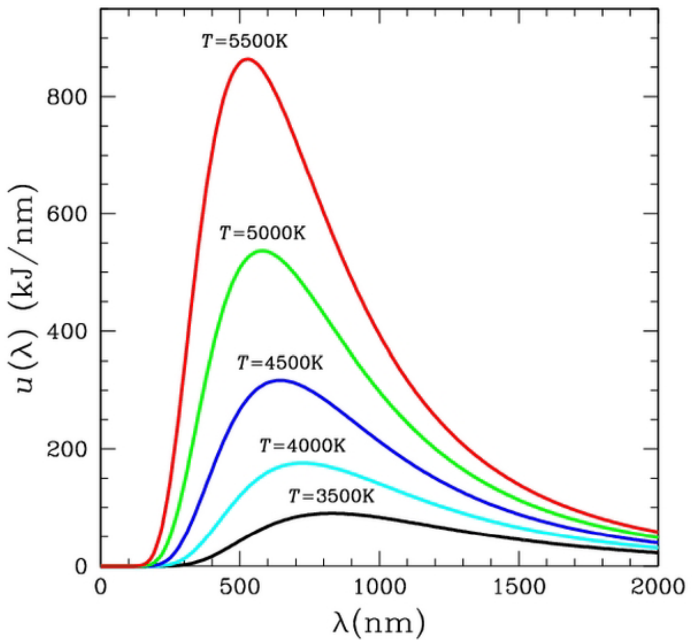
$$\frac{d}{dt} S_{closed} \geq 0$$

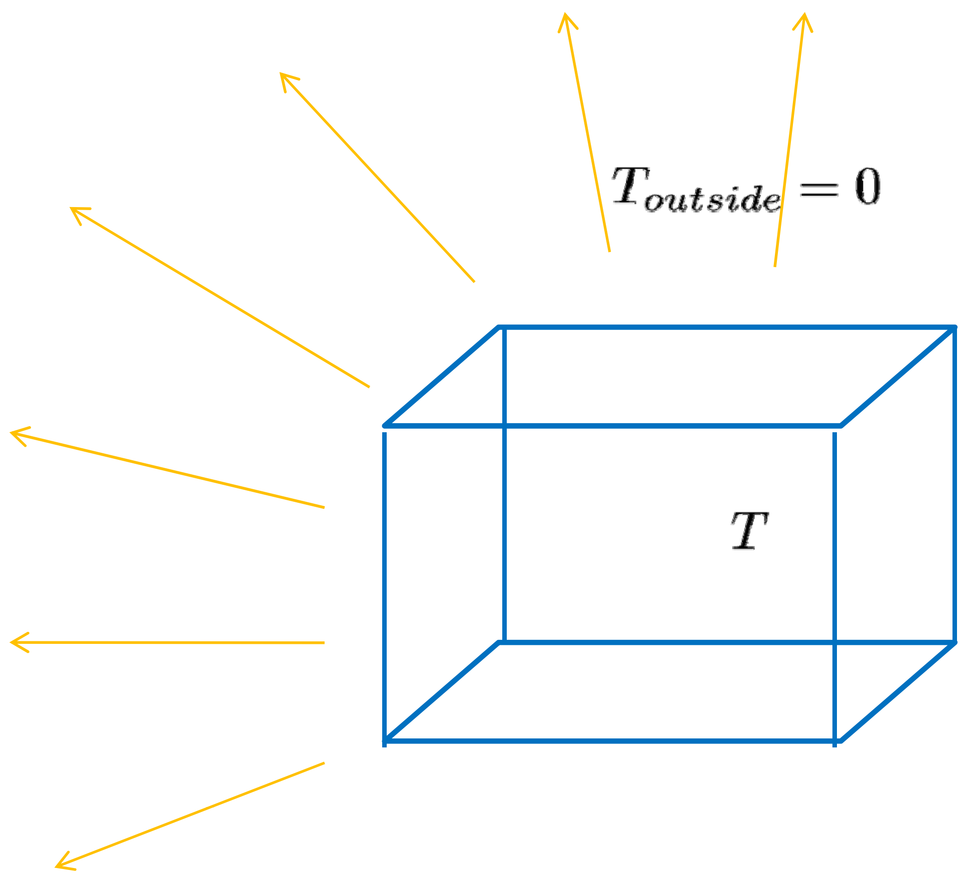


Thermodynamics (II): Blackbody (E&M) Radiation



Thermodynamics (II): Blackbody (E&M) Radiation

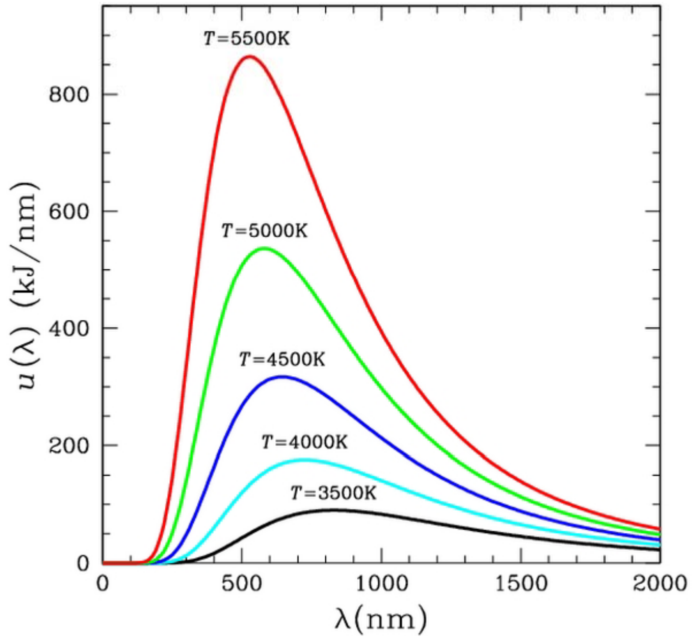




Thermodynamics (II): Blackbody (E&M) Radiation

$$I(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/T}-1}d\nu\, d\Sigma$$

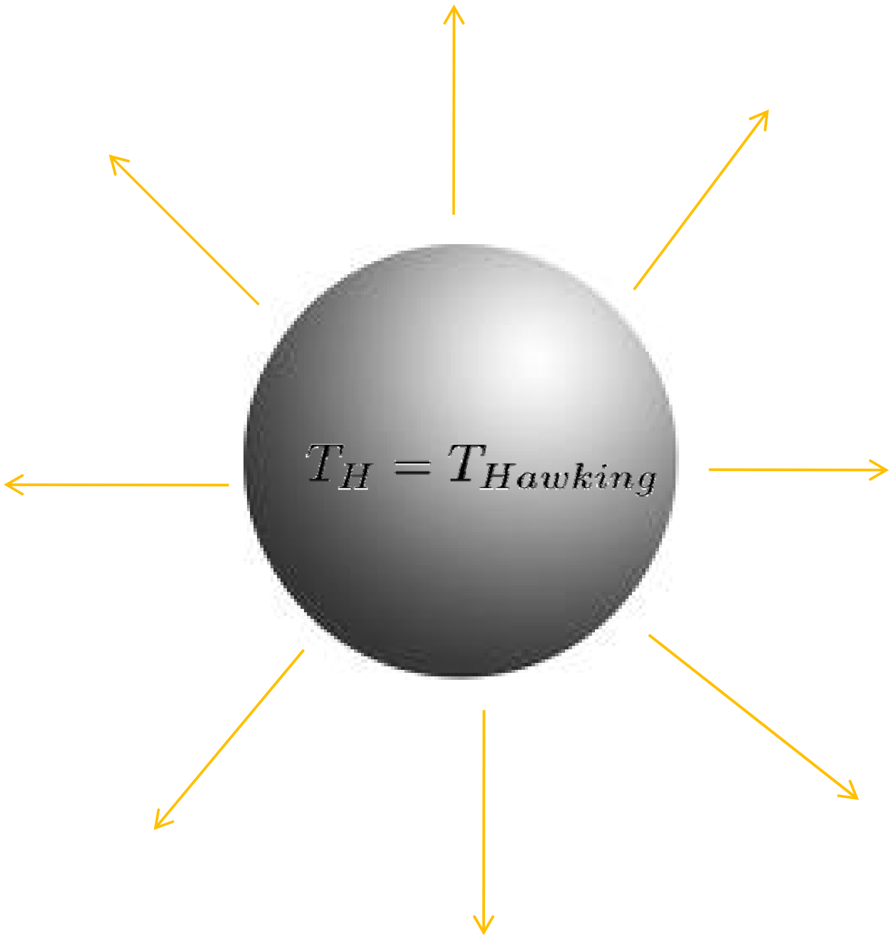
$$\int_\Sigma \int_\nu I(\nu, T) \sim Area \times \frac{T^4}{h^3c^2}$$



Thermodynamics (III): Black Hole is a blackbody ?

$$T_{vacuum} = 0$$

Thermodynamics (III): Black Hole is a blackbody ?



$$T_{vacuum} = 0$$

Thermodynamics (III): Black Hole is a blackbody ?

$$dM_{BH} = T_H dS_{BH}$$

$$\frac{dS_{BH}}{dt} \geq 0$$

$$I(\nu, T)_{Hawking} = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/T_H} - 1} d\nu \, d\Sigma + \cdots\cdots$$

similar radiation from
all particle species

Thermodynamics (III): Black Hole is a blackbody ?

$$T_H = \frac{hc^3}{16\pi^2 G_N M_{BH}}$$

$$S_{BH} = \frac{8\pi^2 G_N M_{BH}^2}{hc}$$

Thermodynamics (III): Black Hole is a blackbody ?

$$T_H = \frac{hc^3}{16\pi^2 G_N M_{BH}} \simeq 8.471 \times 10^{-31}\, joule \times \left(\frac{M_{solar}}{M_{BH}}\right)$$

$$\simeq 5.287 \times 10^{-12}\, eV \times \left(\frac{M_{solar}}{M_{BH}}\right)$$

$$\simeq 6.134 \times 10^{-8}\, degree\, Kelvin \times \left(\frac{M_{solar}}{M_{BH}}\right)$$

$$S_{BH} = \frac{8\pi^2 G_N M_{BH}^2}{hc} \simeq 1.061 \times 10^{75} \times \left(\frac{M_{BH}}{M_{solar}}\right)^2$$

what is a black hole?

Newtonian Motion around a Star

$$\int dt\, \mathcal{L}_{Newton} = \int dt \left(\frac{m}{2}\dot{\mathbf{x}}^2 - \left(-\frac{G_N M m}{|\mathbf{x}|} \right) \right)$$

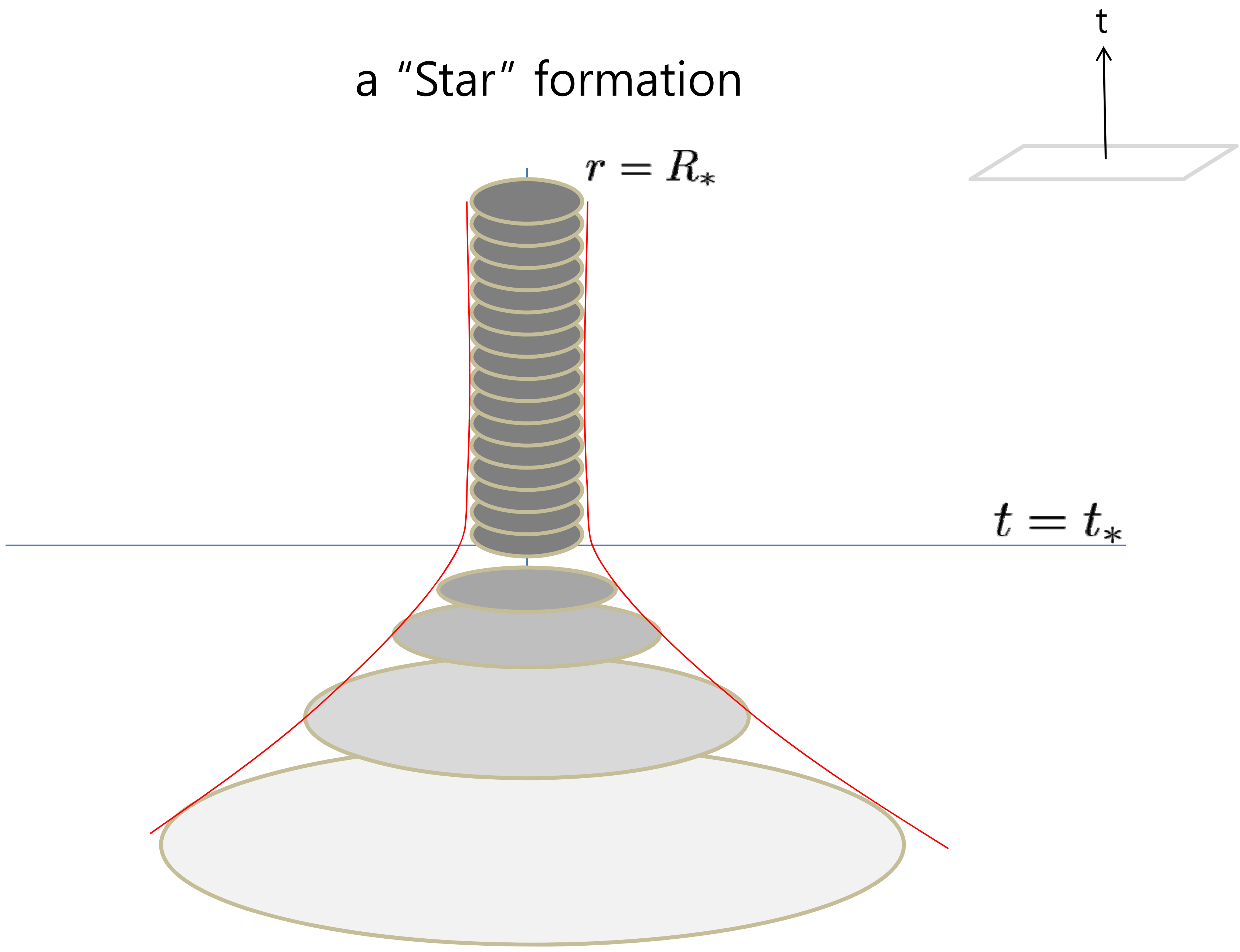
$$\equiv V_{Newton} = m\Phi_{Gravity}$$

we can use such a Lagrangian,

outside the star $r > R_*$

and after the star formation $t > t_*$

a “Star” formation



Constants of Motion

$$E/m = \frac{1}{2}\dot{\mathbf{x}}^2 - \frac{G_N M}{|\mathbf{x}|}$$

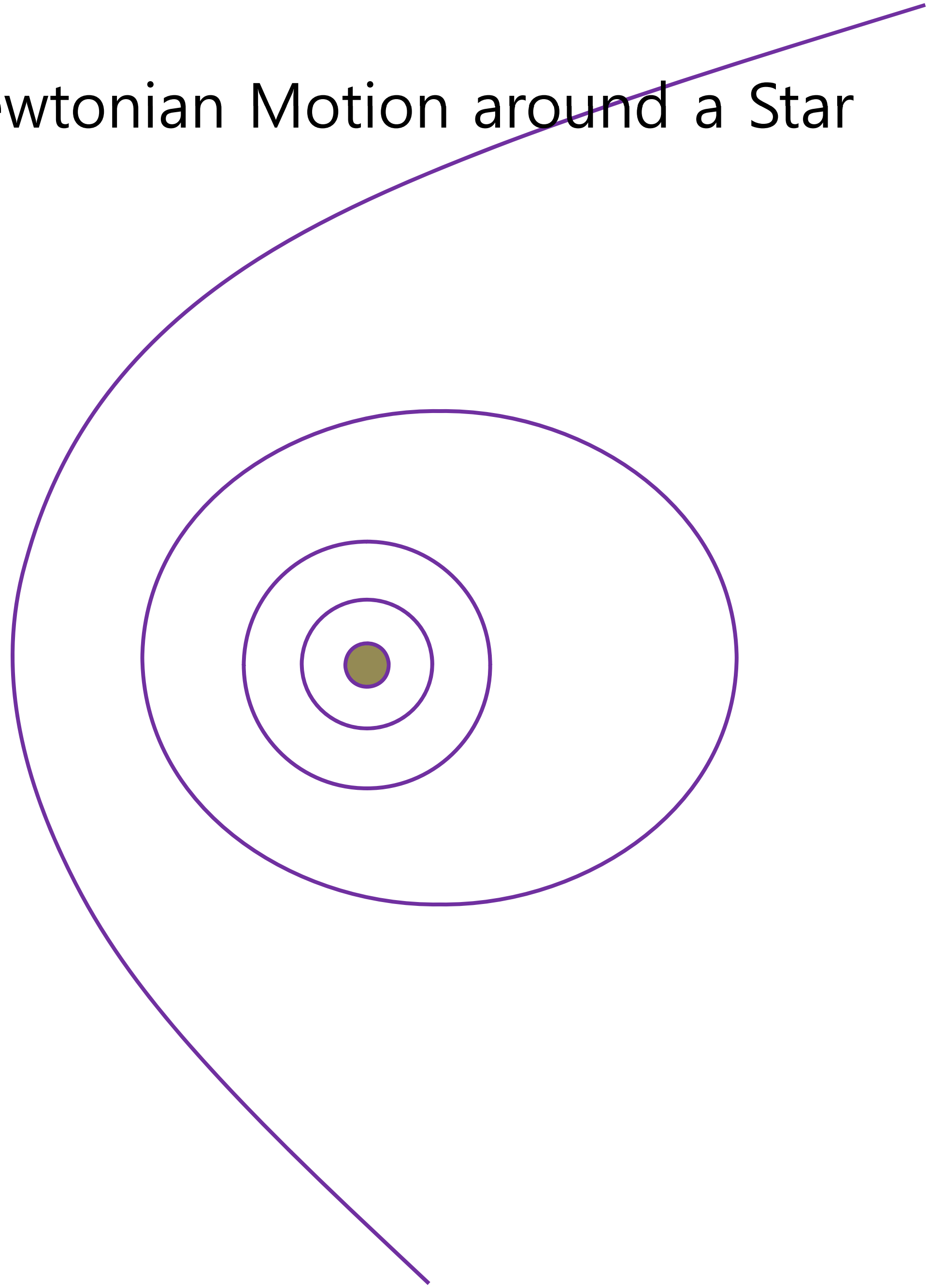
$$\vec{L}/m = \mathbf{x} \times \dot{\mathbf{x}}$$

gravitation pulls in

$$E/m = \frac{1}{2}\dot{r}^2 + \Phi_{Gravity} + \frac{(\vec{L}/m)^2}{2r^2}$$

Angular momentum barrier pushes out

Newtonian Motion around a Star



Relativistic Motion around a "Star"

$$\int dt\,\mathcal{L} = -\int dt\, mc^2 \sqrt{f(r) - \frac{\dot{r}^2}{c^2 f(r)} - \frac{\dot{\mathbf{x}}^2_{angular}}{c^2}}$$

$$f(r) = 1 + 2\Phi_{Gravity}/c^2 = 1 - \frac{2G_N M}{c^2|\mathbf{x}|}$$

Relativistic Motion around a "Star"

expanding for small velocity and small $\Phi_{gravity}/c^2$

$$\int dt\, \mathcal{L} \simeq -\int dt\, mc^2 \left(1 - \frac{1}{2c^2}\left(\dot{r}^2 + \dot{\mathbf{x}}^2_{anglular}\right) + (f-1) + \cdots\right)$$

$$= -\int dt\, mc^2 + \boxed{\int dt\, \frac{m}{2}\dot{\mathbf{x}}^2 - (m\Phi_{Gravity})} + \cdots$$

$$= \int dt\, \mathcal{L}_{Newton}$$

Relativistic Motion around a "Star"

$$\int dt\,\mathcal{L} = -\int dt\, mc^2 \sqrt{f(r) - \frac{\dot{r}^2}{c^2 f(r)} - \frac{\dot{\mathbf{x}}^2_{angular}}{c^2}}$$

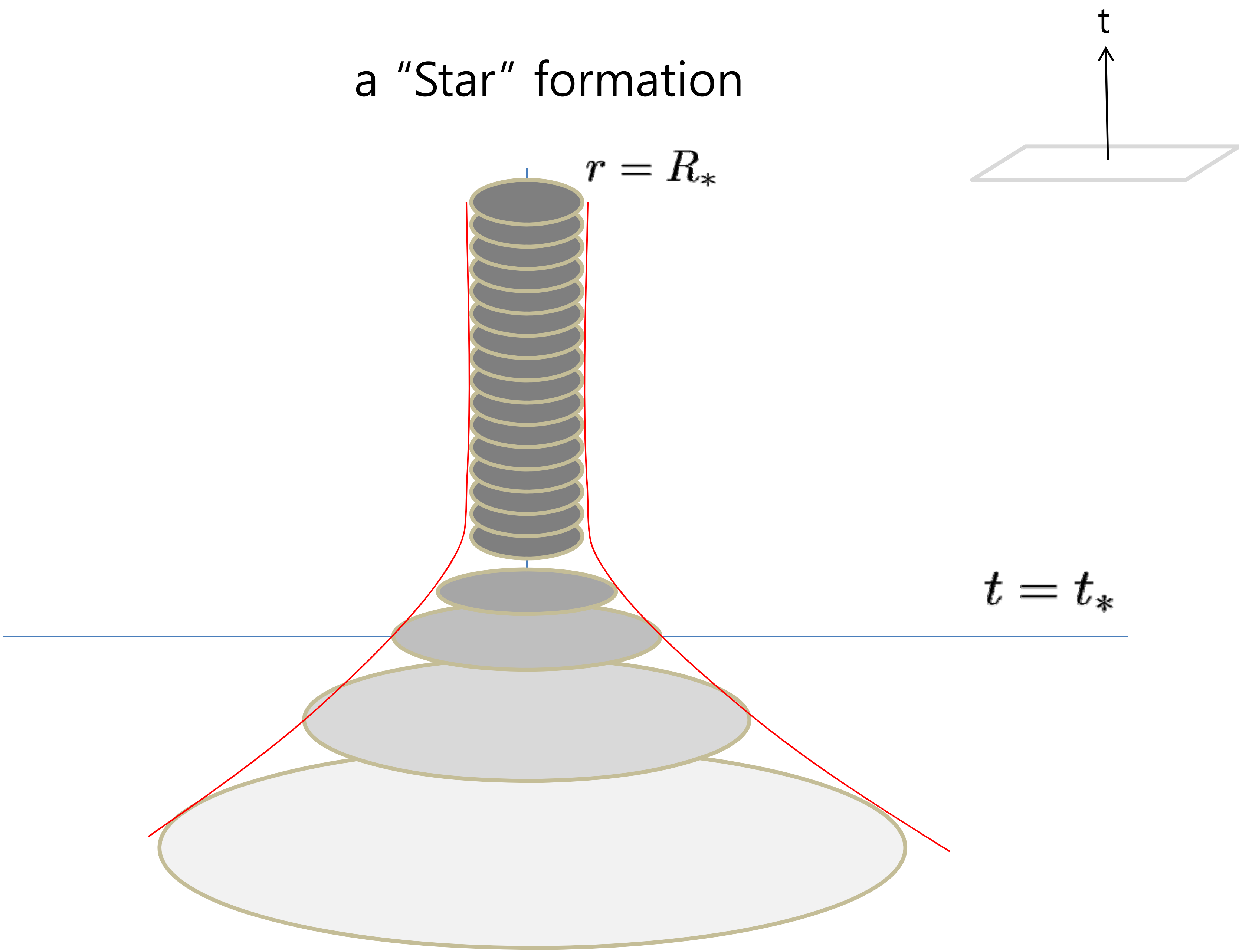
$$f(r) = 1 + 2\Phi_{Gravity}/c^2 = 1 - \frac{2G_N M}{c^2|\mathbf{x}|}$$

we can use such a Lagrangian,

outside the star $r > R_*$

and after $t > t_* + r/c$

a “Star” formation



Constants of Motion

$r > R_*$ $\qquad t > t_*$

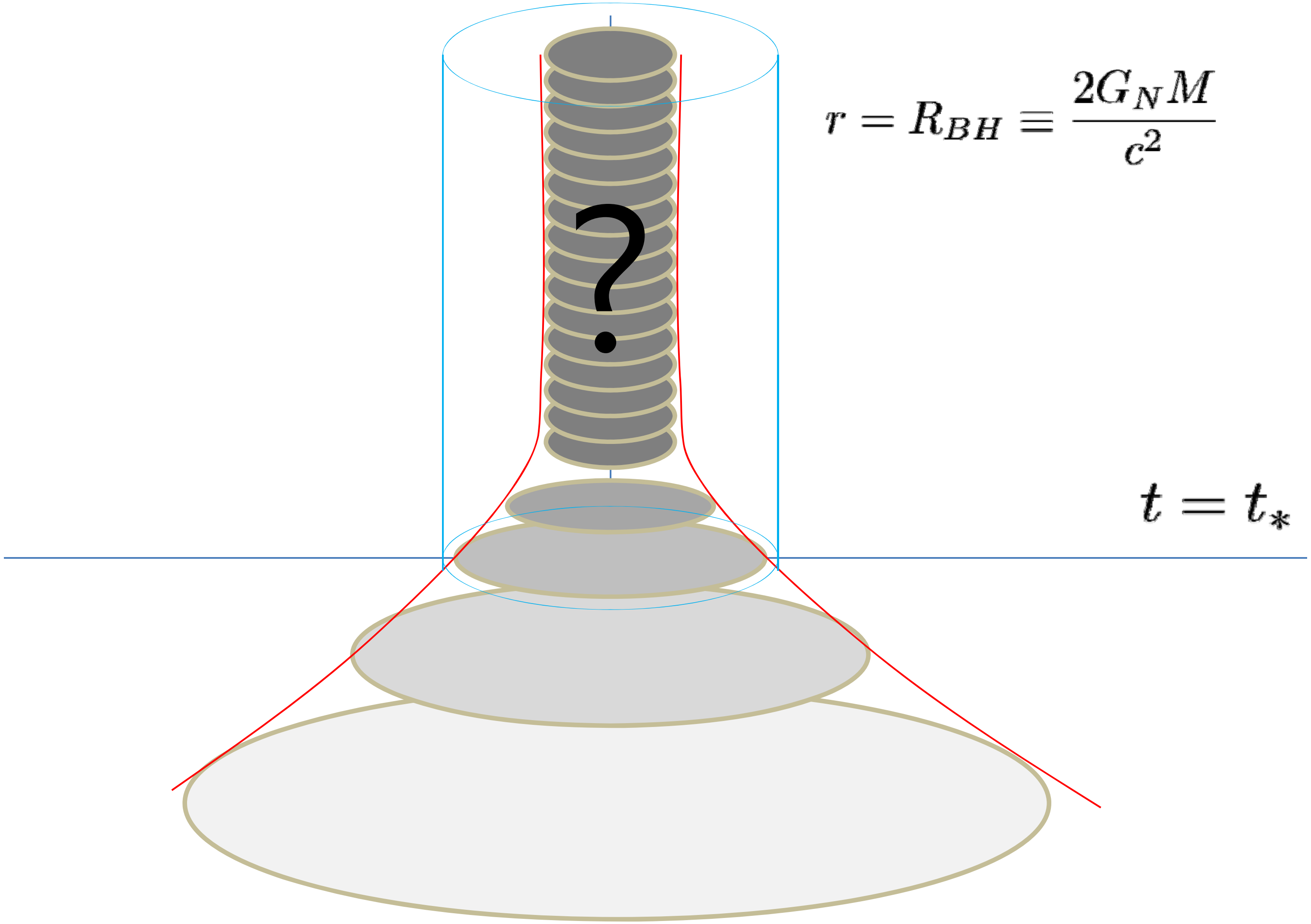
$$\mathcal{E}/mc^2 = \frac{f(r)}{\sqrt{f(r) - \dot{r}^2/c^2 f(r) - \dot{\mathbf{x}}^2_{angular}/c^2}}$$

$$f(r) = 1 + 2\Phi_{Gravity}/c^2 = 1 - \frac{2G_N M}{c^2 r}$$

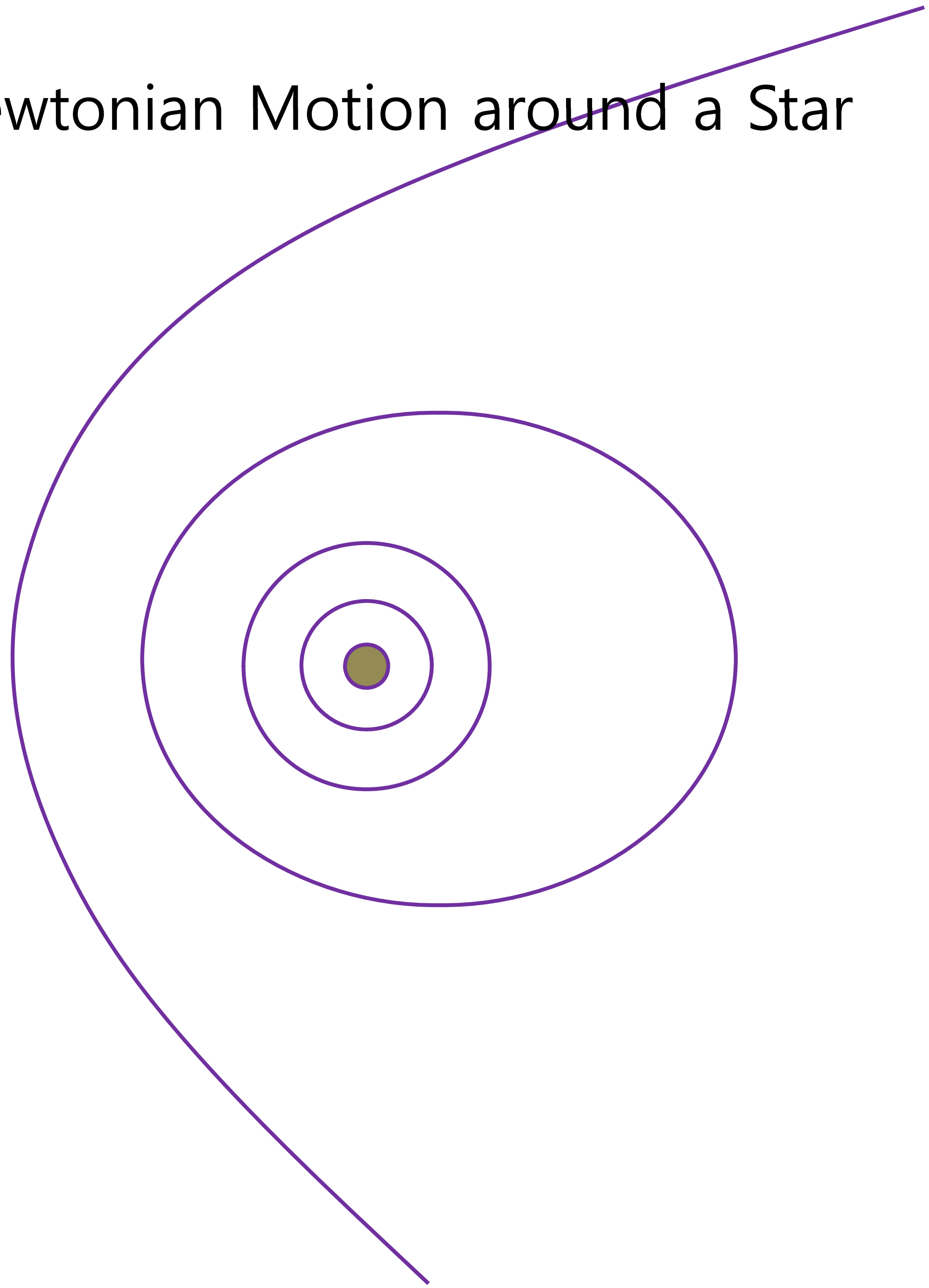
something strange happens if

$$R_* < R_{BH} \equiv \frac{2G_N M}{c^2}$$

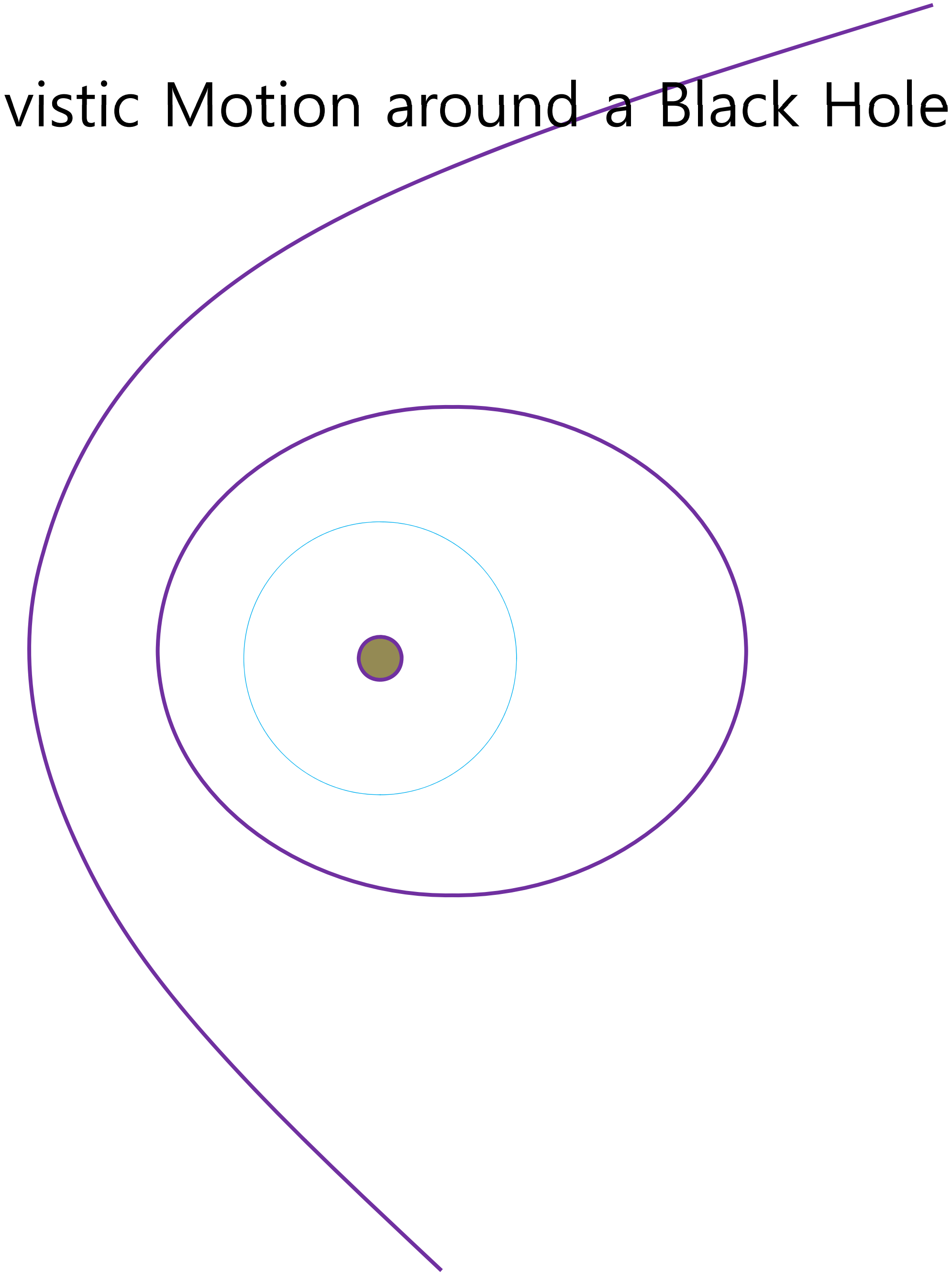
a Black Hole formation ?



Newtonian Motion around a Star



Relativistic Motion around a Black Hole



Constants of Motion

$r > R_*$ $\qquad t > t_*$

$$\mathcal{E}/mc^2 = \frac{f(r)}{\sqrt{f(r) - \dot{r}^2/c^2 f(r) - \dot{\mathbf{x}}^2_{angular}/c^2}}$$

$$f(r) = 1 + 2\Phi_{Gravity}/c^2 = 1 - \frac{2G_N M}{c^2 r}$$

$r(t) \to R_{BH} \equiv \frac{2G_N M}{c^2}$ → $f(r(t)) \to 0$ → $\dot{r}(t) \to 0$

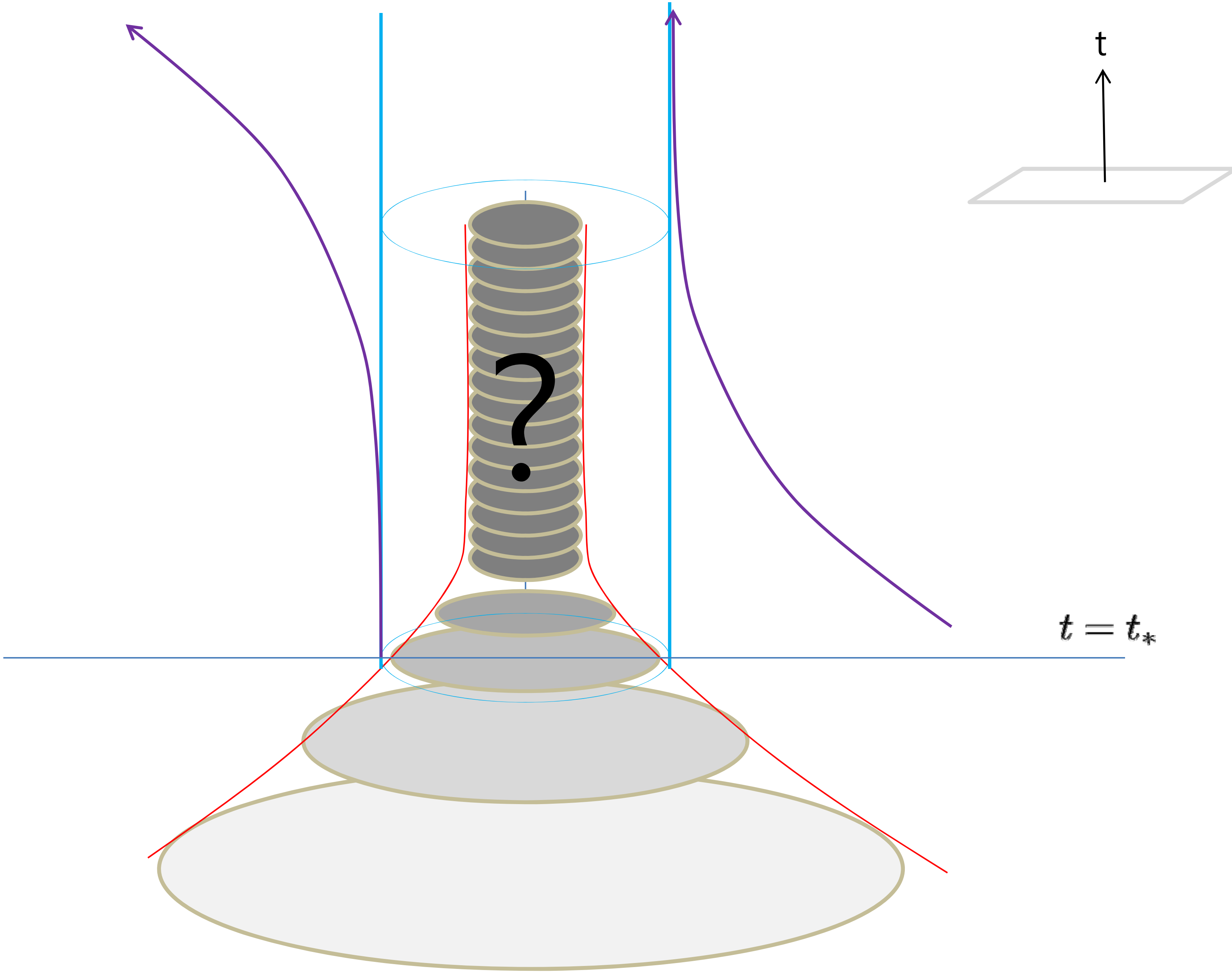
Radial Motion

$$\dot{\mathbf{x}}_{angular} = \mathbf{0}$$

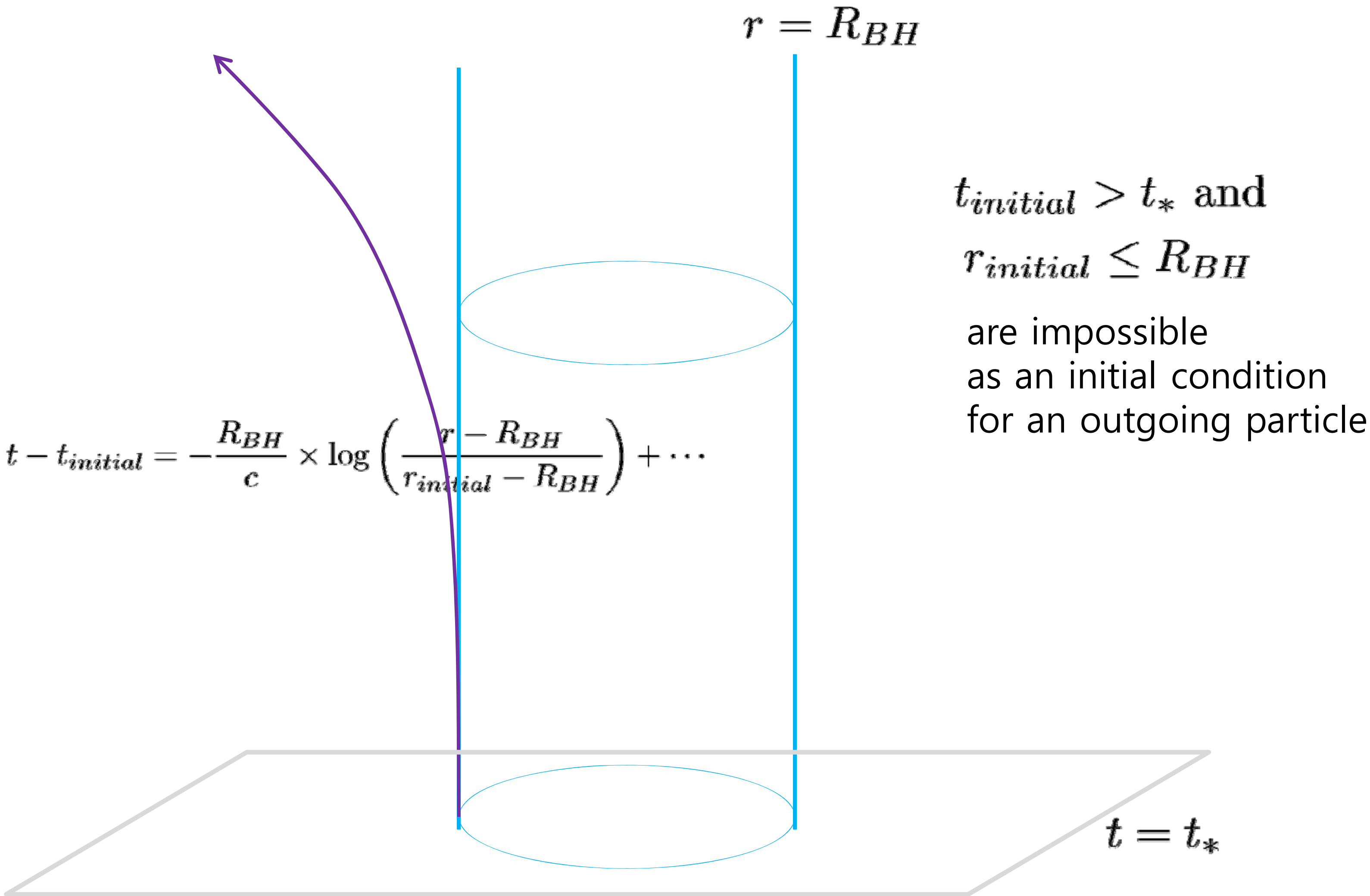
$$\mathcal{E}/mc^2 = \frac{f(r)}{\sqrt{f(r) - \dot{r}^2/c^2 f(r)}}$$

$$\int dt = \int dr \left(\frac{1}{cf(r)} + mc/\mathcal{E} \times O(1) \right) = -\frac{R_{BH}}{c} \times \log(r/R_{BH} - 1) + \cdots$$

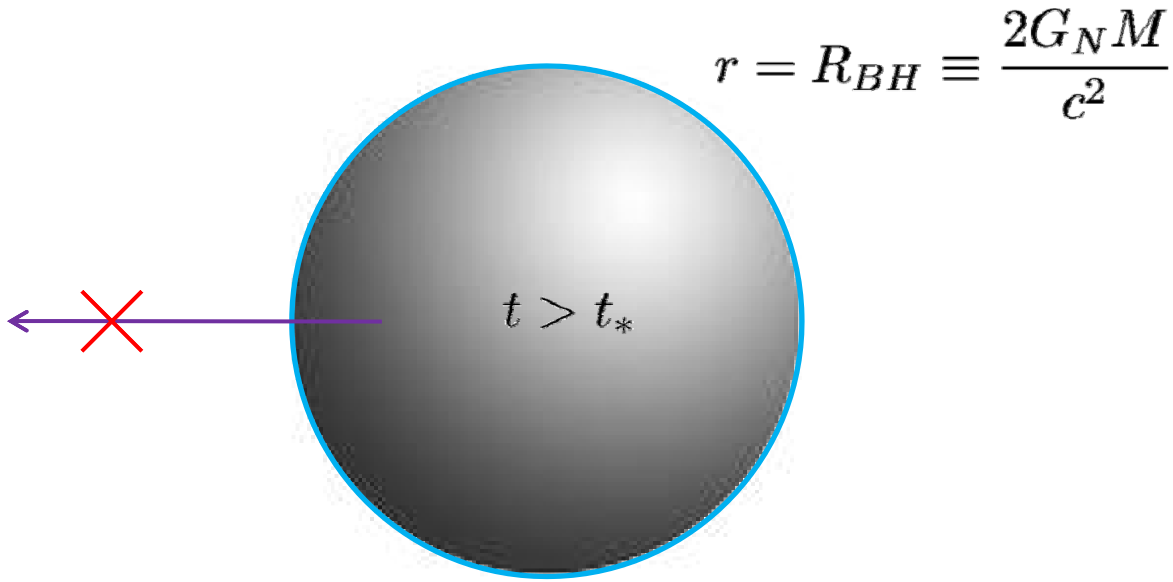
$$t - t_{initial} = -\frac{R_{BH}}{c} \times \log\left(\frac{r - R_{BH}}{r_{initial} - R_{BH}} \right) + \cdots$$



t
$t = t_*$
?



$r = R_{BH}$
$t_{initial} > t_*$ and
$r_{initial} \leq R_{BH}$
are impossible
as an initial condition
for an outgoing particle
$t - t_{initial} = -\frac{R_{BH}}{c} \times \log\left(\frac{r - R_{BH}}{r_{initial} - R_{BH}}\right) + \cdots$
$t = t_*$

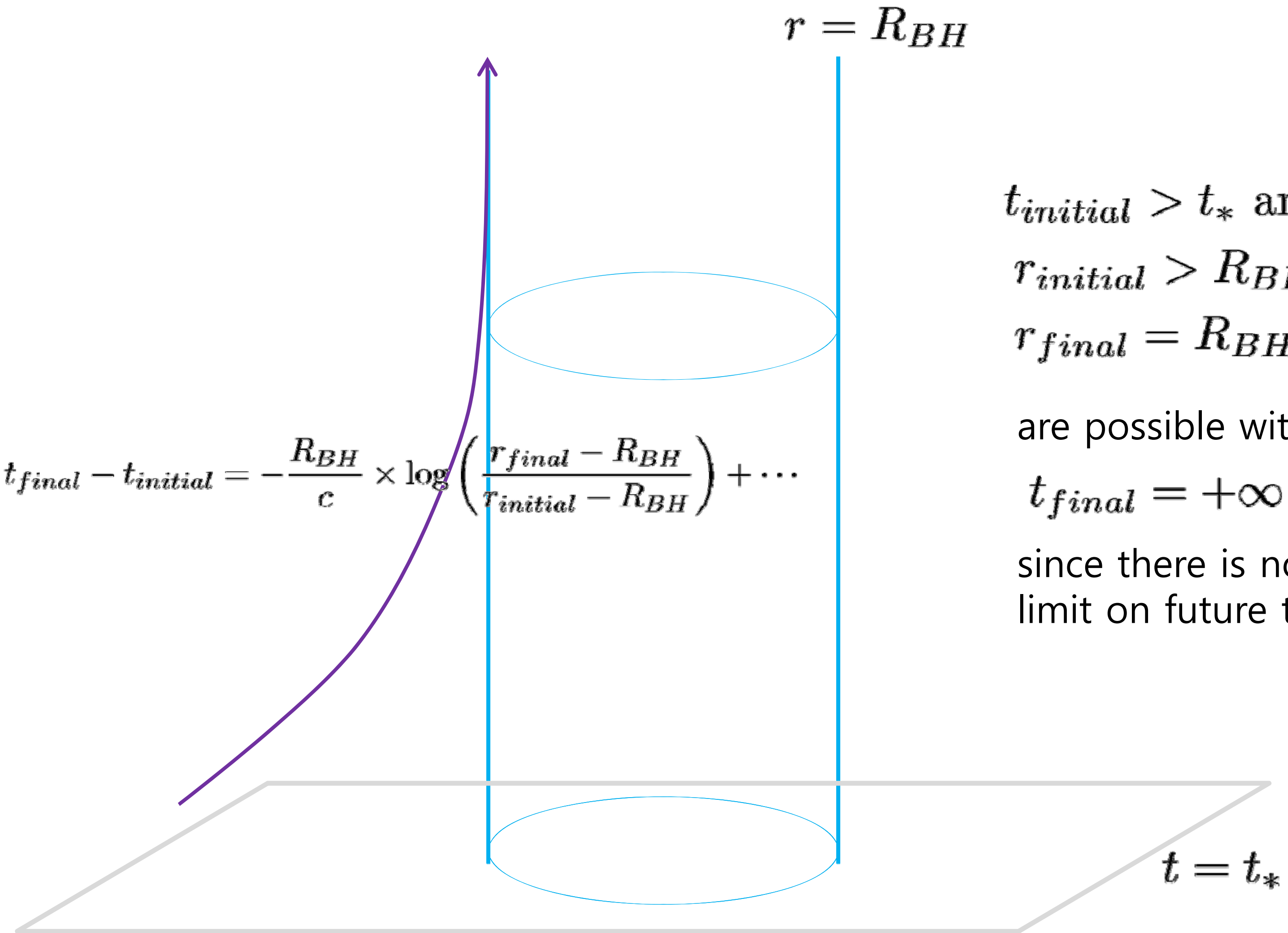


things located inside $r = R_{BH}$ after $t = t_*$

cannot be seen from outside because

they cannot pass through $r = R_{BH}$

this barrier is called “Black Hole Horizon”



$t_{initial} > t_*$ and

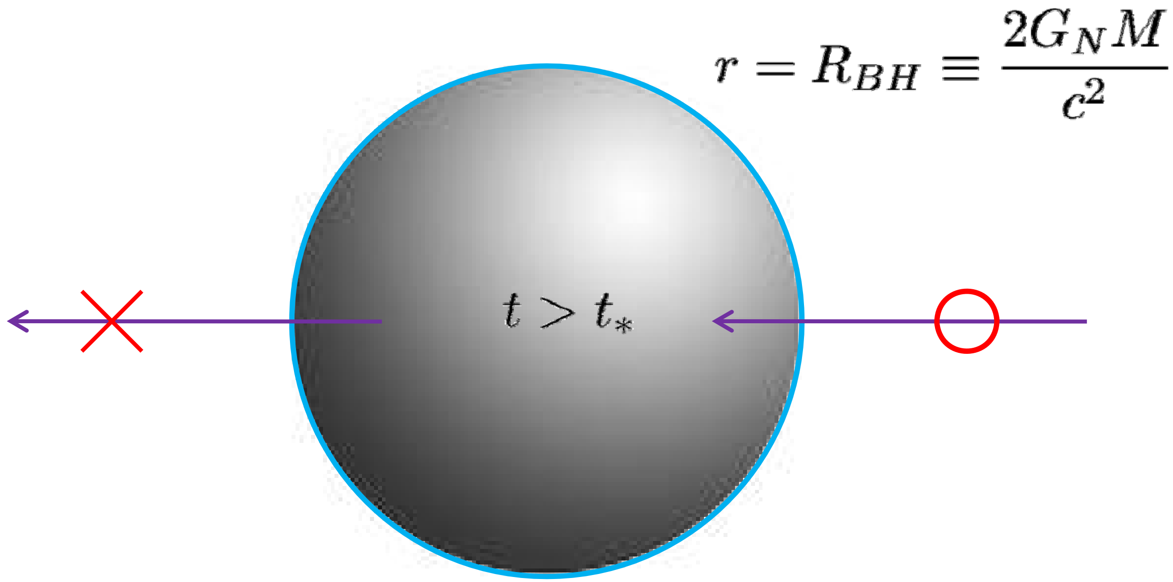
$r_{initial} > R_{BH}$

$r_{final} = R_{BH}$

are possible with

$t_{final} = +\infty$

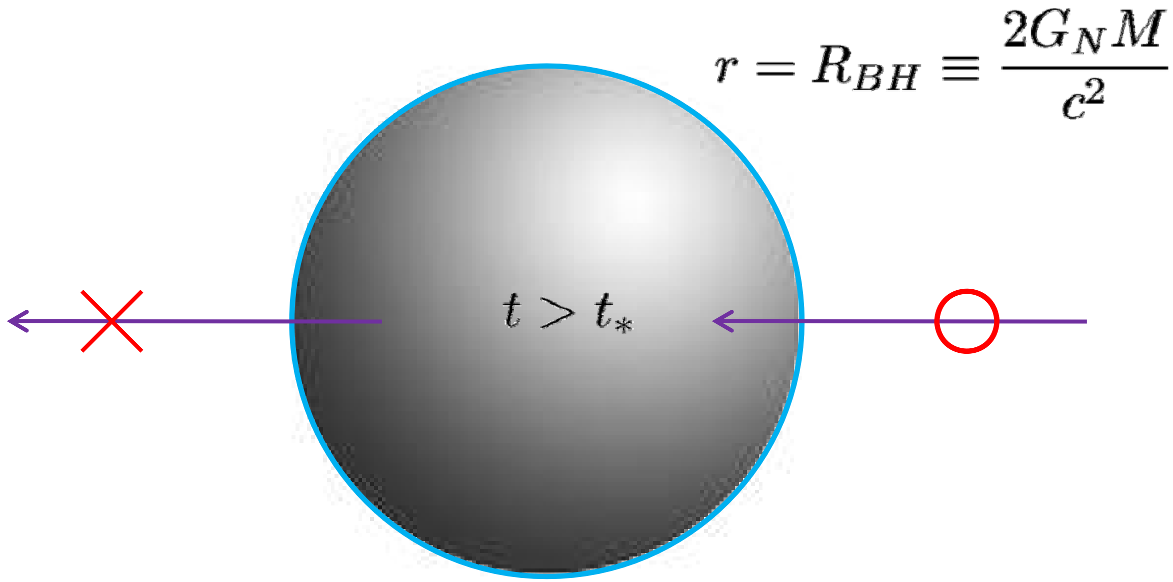
since there is no upper limit on future time



in fact, time measured by a clock falling into a black hole is finite

$$\int d\tau_{proper} = \int dt\ \mathcal{L}/mc^2 = \int dr \left(\frac{1}{cf(r)^{1/2}} + \cdots \right) \propto \frac{\sqrt{(r - R_{BH})R_{BH}}}{c} < \infty$$

and a rocket falling-in would not feel any resistance to its path



the black hole continues to accept incoming particles and energy

and can grow its mass, and thus the size, indefinitely

but cannot shrink since it cannot release its energy by emission of anything is a black hole also a blackbody ?

it seems to contradict with what we learned so far

since it would require

energy / mass that crosses horizon outward,

which is mechanically forbidden

however, there is no contradiction because

blackbody radiation of a black hole occurs only at quantum level

$$T_H = \frac{hc^3}{16\pi^2 G_N M_{BH}}$$

Newtonian → Relativistic

$$\Phi_{Gravity} \quad \rightarrow \quad f(\mathbf{x}) = 1 + 2\Phi_{Gravity}/c^2$$

$$f(r = R_{BH}) = 0$$

$$\mathbf{F}_{Newton} = -m\nabla\Phi_{Gravity} \quad \rightarrow \quad \Gamma^{\mu}_{\nu\lambda} \sim \partial f, \ \cdots$$

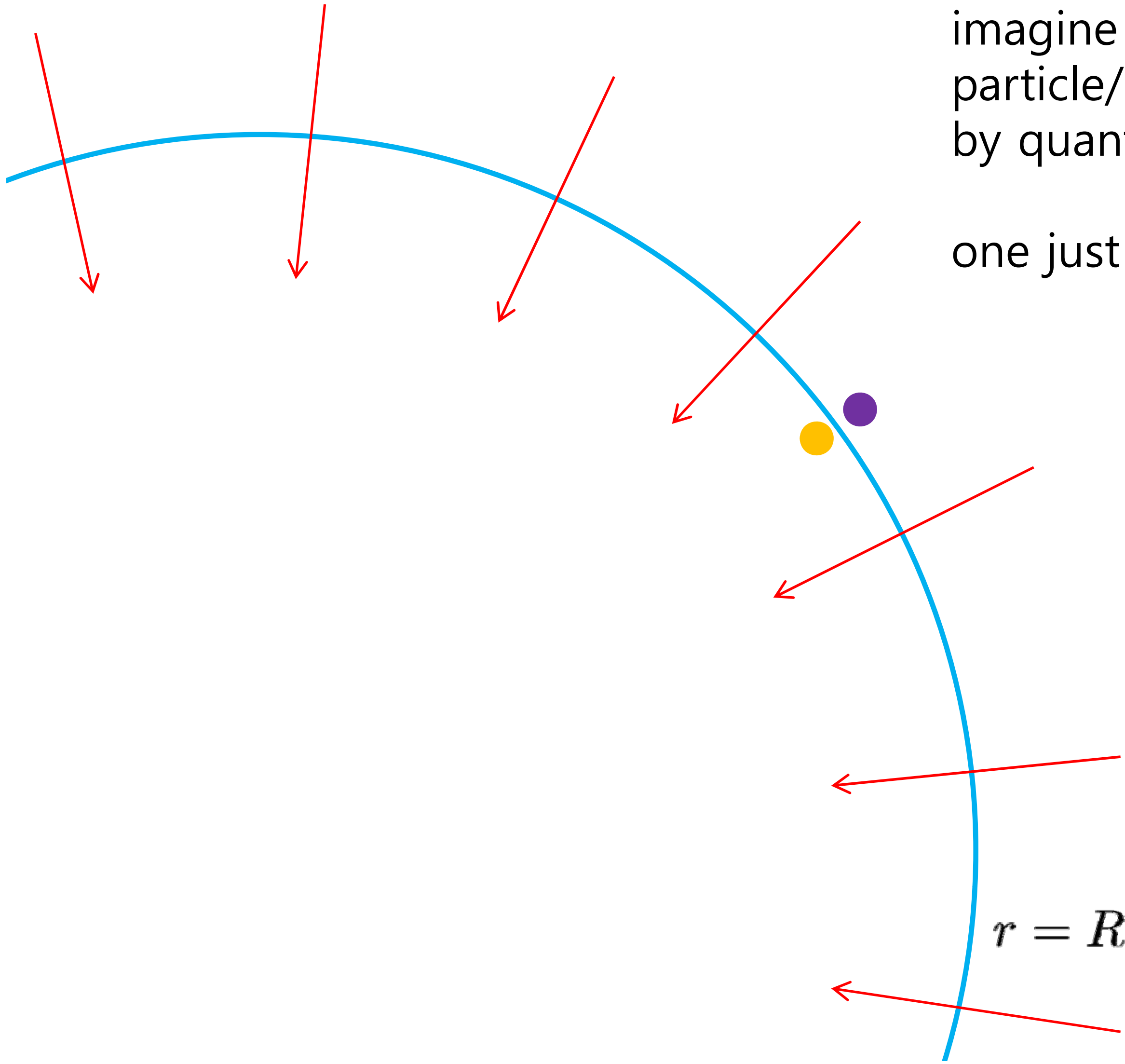
→ as far as "force" felt by particle goes, nothing drastic happens at BH horizon

provided that R_{BH} is much larger than $l_{Planck} \simeq 10^{-35}\ meter$

$$\mathbf{F} \sim \mathbf{F}_{Newton} = -G_N M m \frac{\mathbf{x}}{r^3}$$

imagine creating a pair of
particle/anti-particle near the horizon,
by quantum tunneling process

one just outside, and the other inside



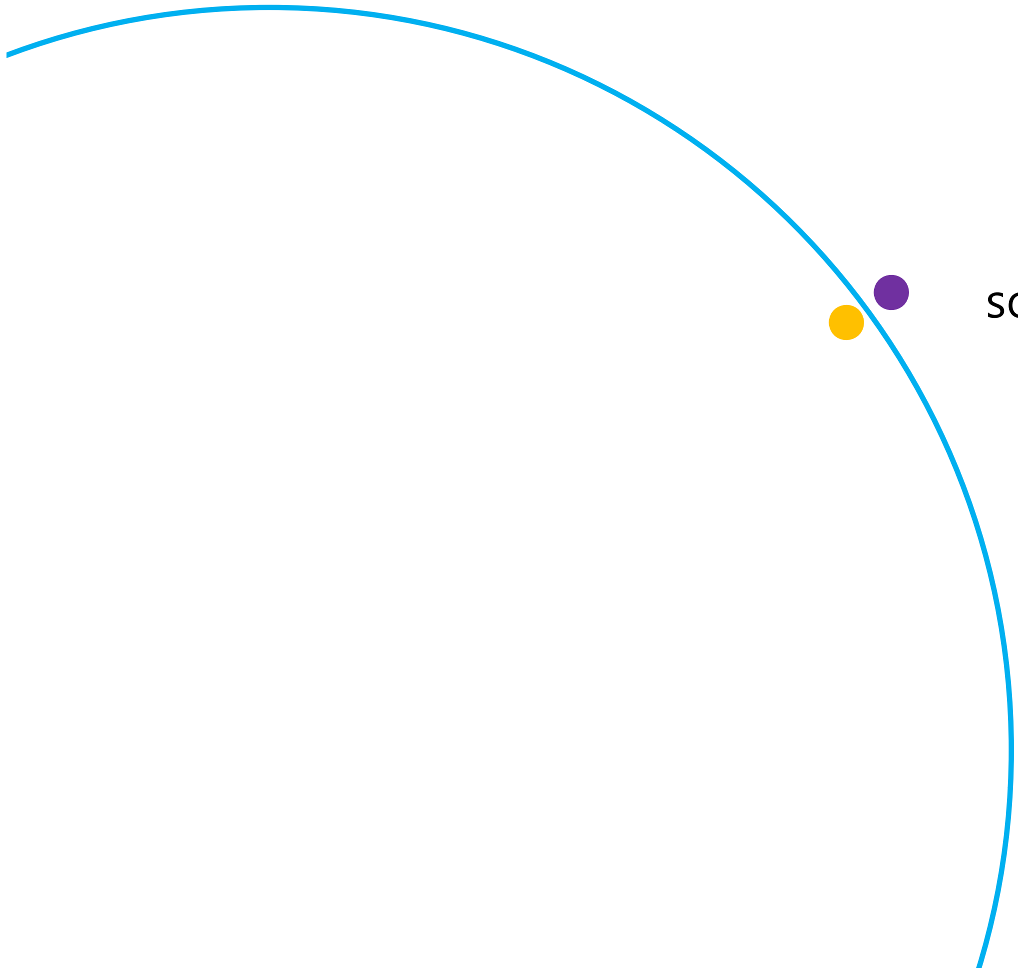
$$r = R_{BH} \equiv \frac{2G_N M}{c^2}$$

$$\mathbf{F} \sim \mathbf{F}_{Newton} = -G_N M m \frac{\mathbf{x}}{r^3}$$

the net energy of the configuration is

$$\sim 2 \times mc^2 + V_{Gravity}$$

$$\simeq 2mc^2 \times \left(1 - \frac{G_N M}{c^2 R_{BH}}\right) = mc^2 \quad > \quad 0$$



so it does not happen

$$r = R_{BH} \equiv \frac{2G_N M}{c^2}$$

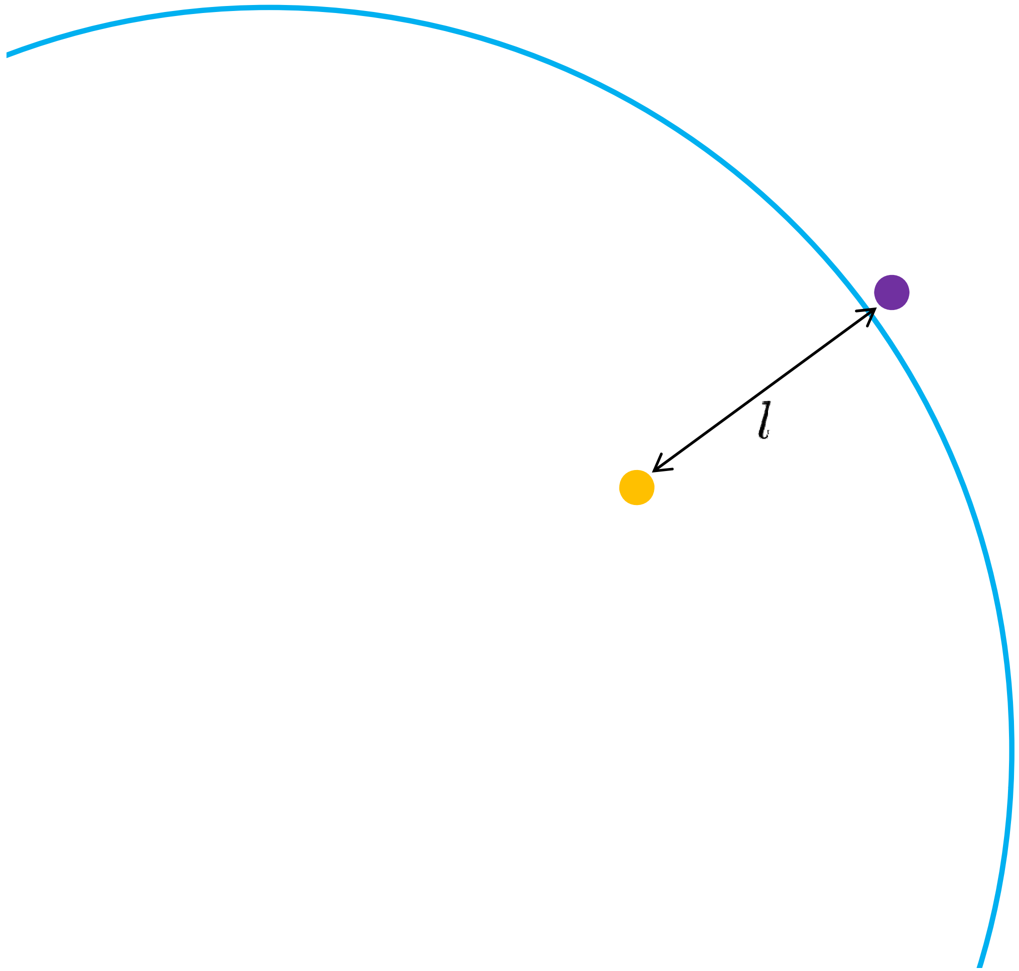
$$\mathbf{F} \sim \mathbf{F}_{Newton} = -G_N M m \frac{\mathbf{x}}{r^3}$$

if one particle is a bit further inside

$$\sim 2 \times mc^2 + V_{Gravity}$$

$$\simeq 2mc^2 \times \left(1 - \frac{G_N M}{2c^2 R_{BH}} - \frac{G_N M}{2c^2 (R_{BH} - l)}\right)$$

$$\simeq mc^2 \times \left(\frac{R_{BH} - 2l}{R_{BH} - l}\right)$$



$$r = R_{BH} \equiv \frac{2G_N M}{c^2}$$

$$\mathbf{F} \sim \mathbf{F}_{Newton} = -G_N M m \frac{\mathbf{x}}{r^3}$$

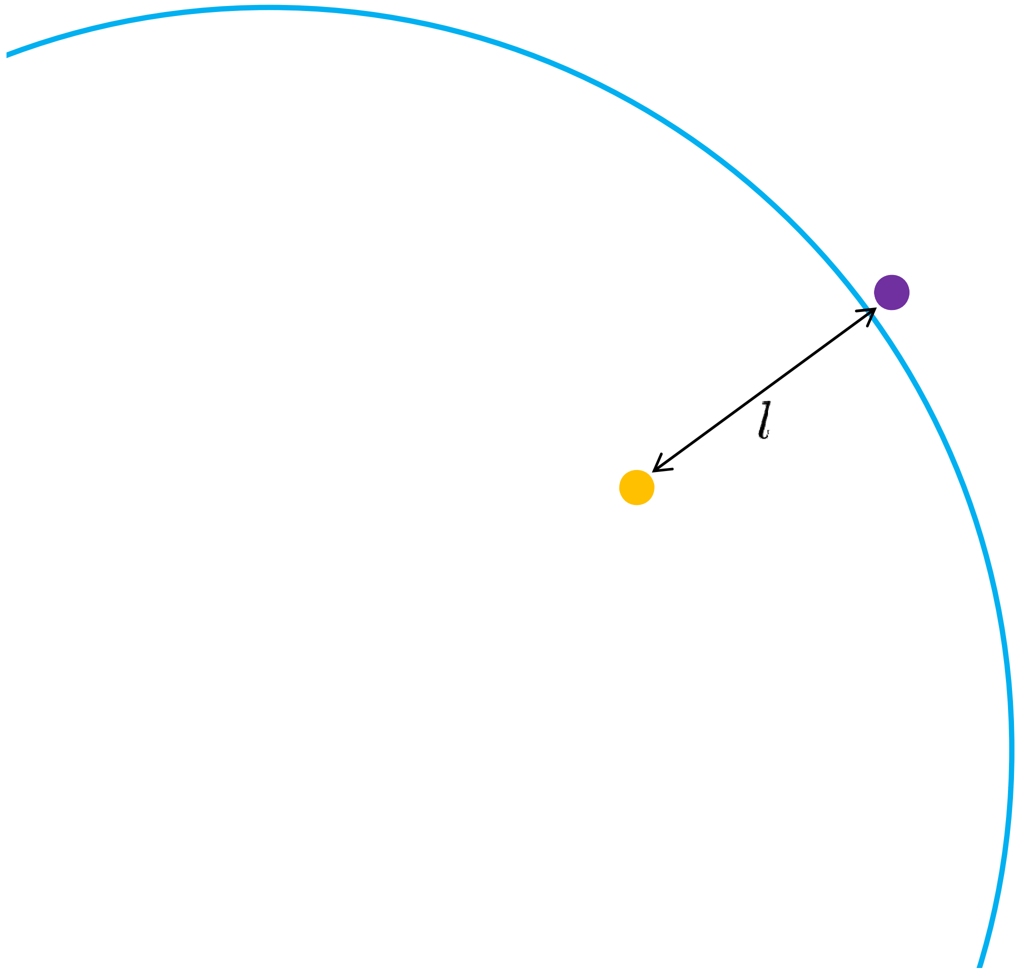
a WKB quantum tunneling is possible !!!

$$\sim 2 \times mc^2 + V_{Gravity}$$

$$\simeq 2mc^2 \times \left(1 - \frac{G_N M}{2c^2 R_{BH}} - \frac{G_N M}{2c^2 (R_{BH} - l)}\right)$$

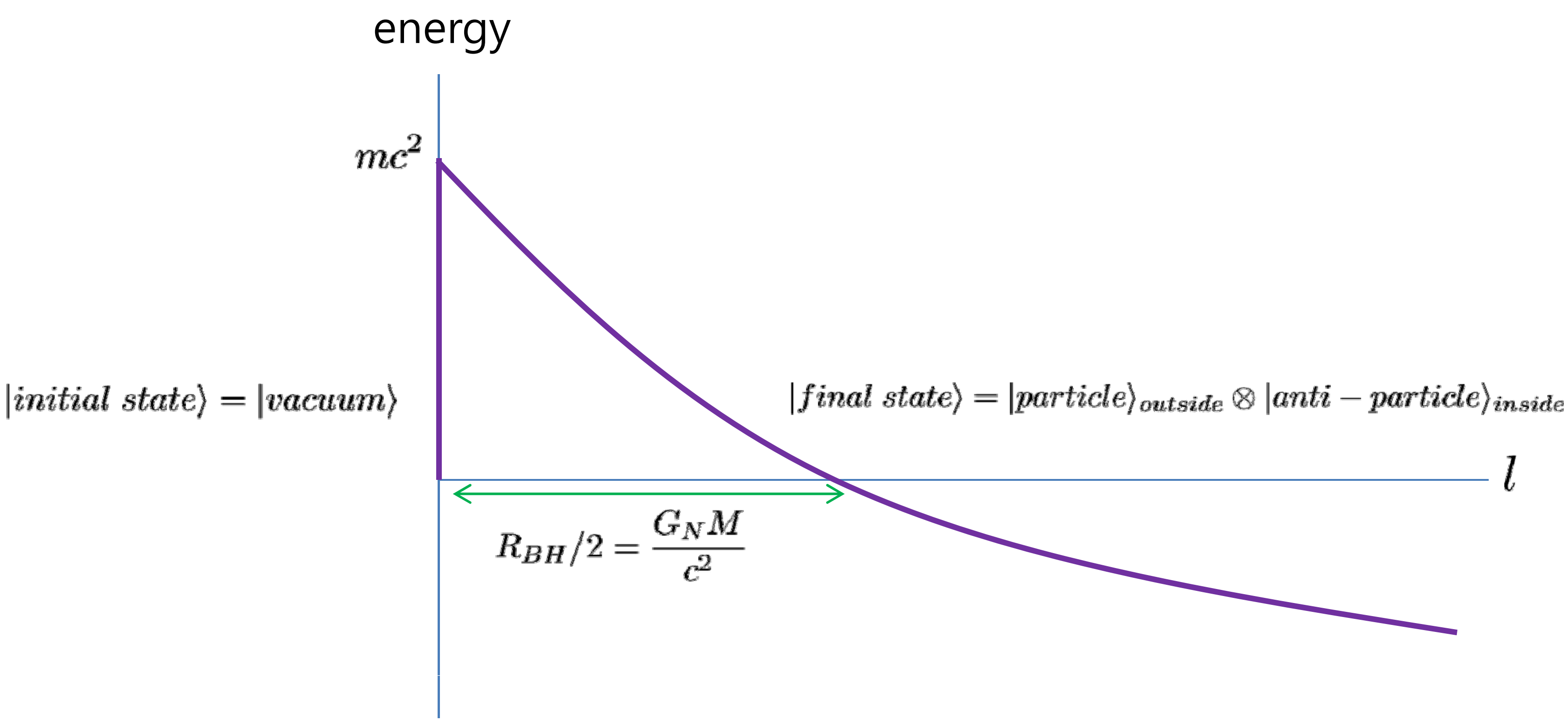
$$\simeq mc^2 \times \left(\frac{R_{BH} - 2l}{R_{BH} - l}\right)$$

$$= 0 \quad \text{if} \quad l = R_{BH}/2$$



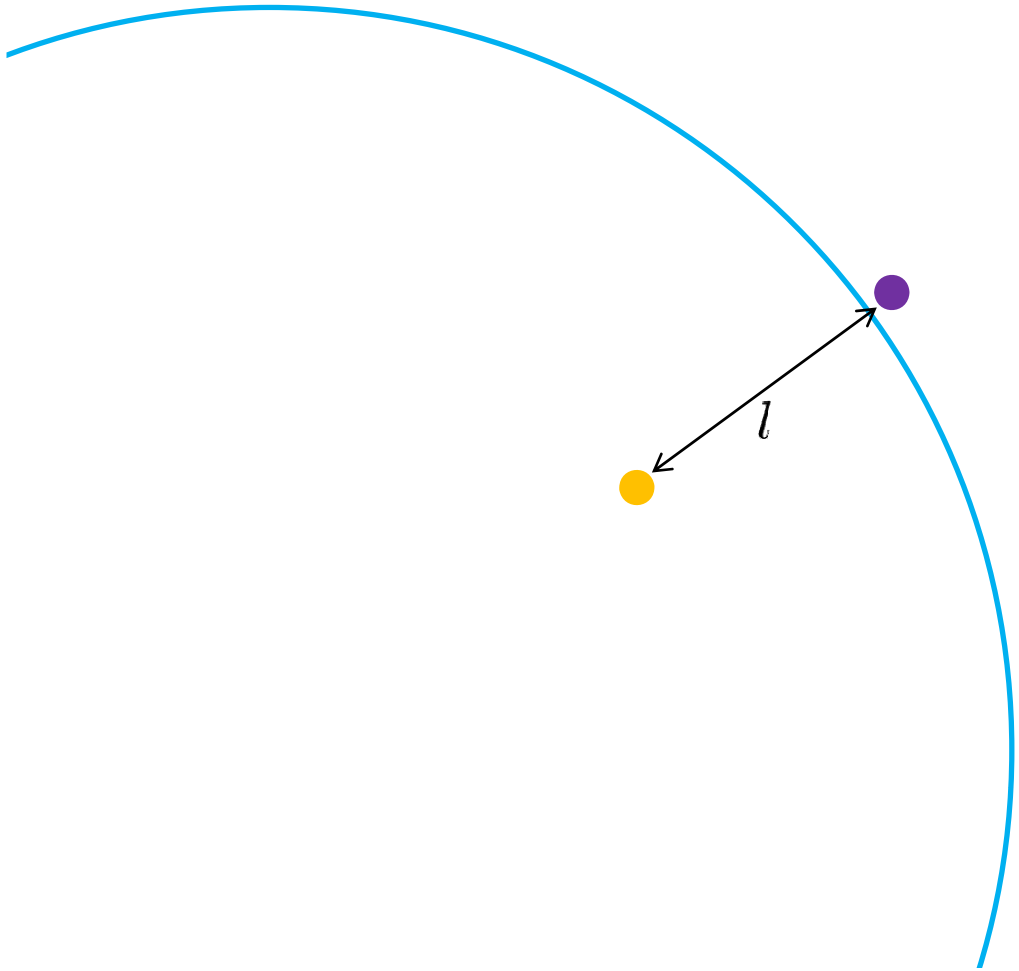
$$r = R_{BH} \equiv \frac{2G_N M}{c^2}$$

WKB amplitude ?



a WKB tunneling amplitude must be

$$\simeq e^{-\# m R_{BH} c/h}$$



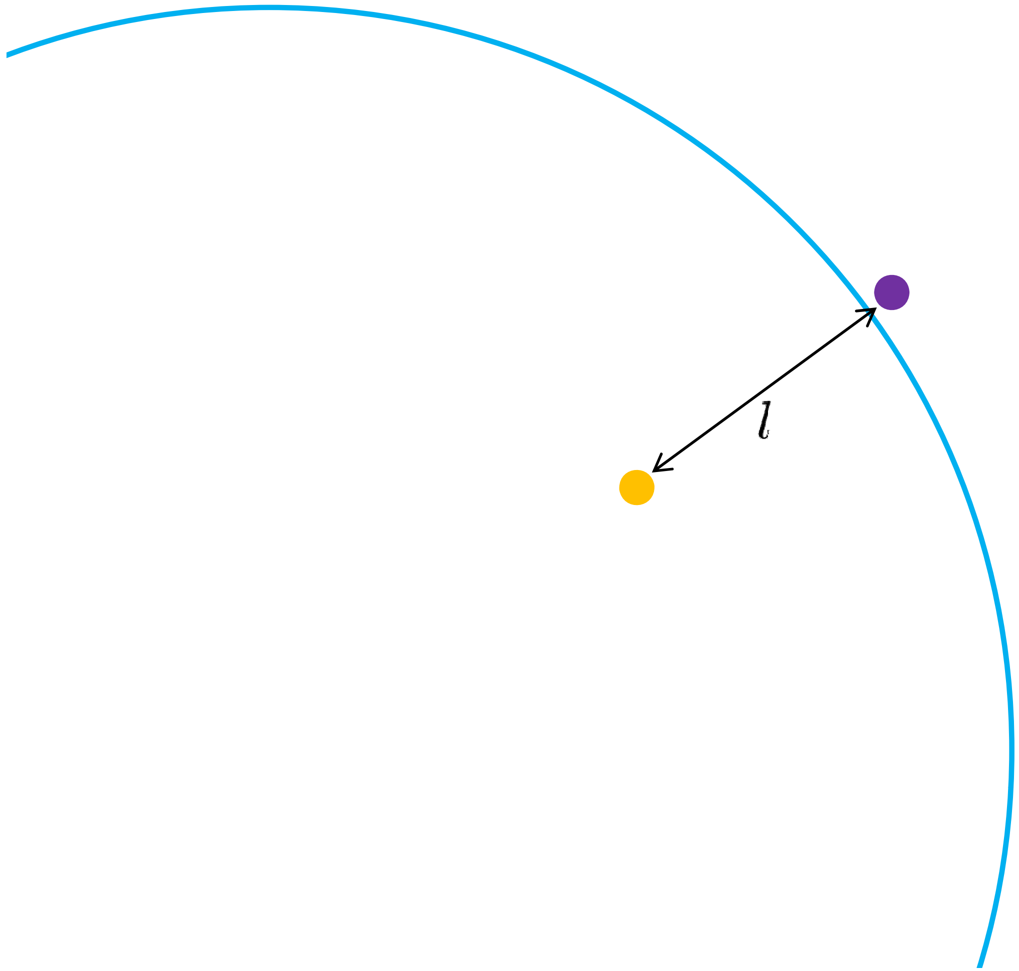
$$r = R_{BH} \equiv \frac{2G_N M}{c^2}$$

a WKB tunneling probability must be

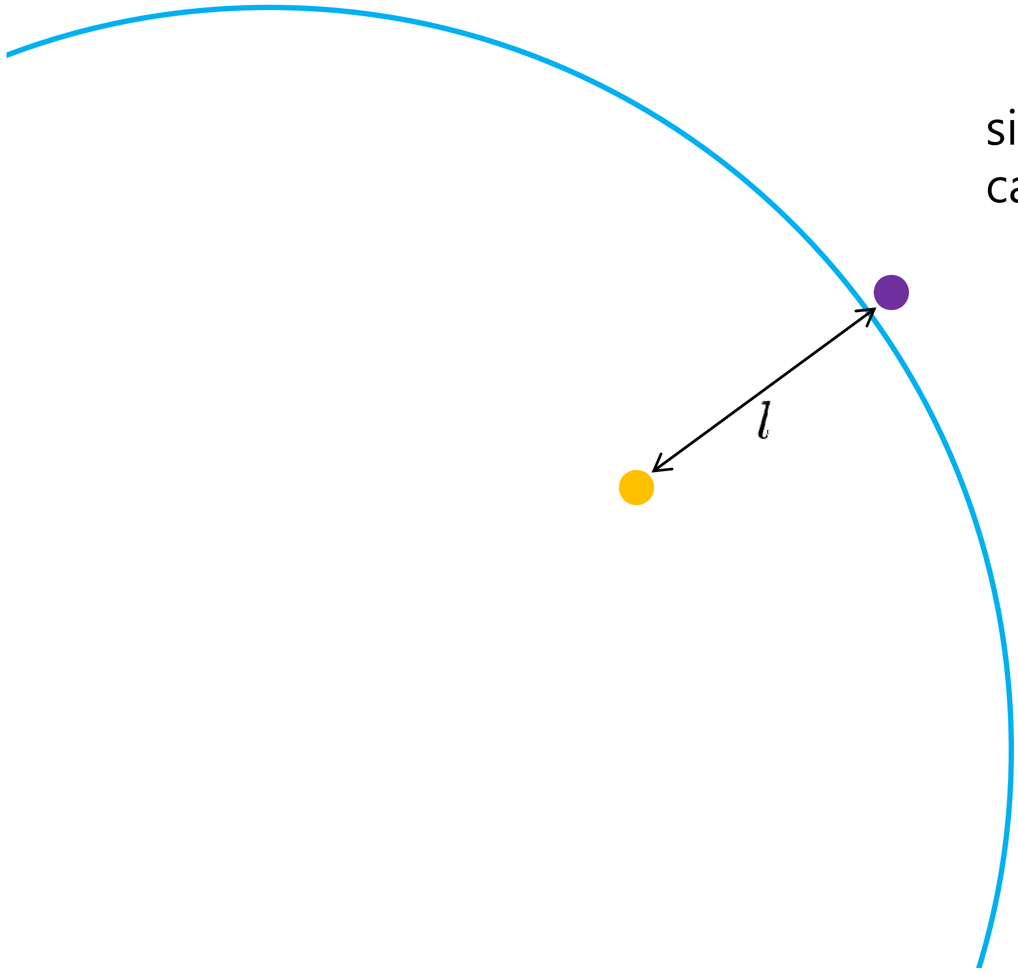
$$\simeq e^{-\#2mR_{BH}c/h}$$

$$\simeq e^{-mc^2/T_*}$$

$$T_* = \#\frac{hc}{2R_{BH}}$$



$$r = R_{BH} \equiv \frac{2G_N M}{c^2}$$



a WKB tunneling probability
must be more generally,

$$\simeq e^{-E/T_*}$$

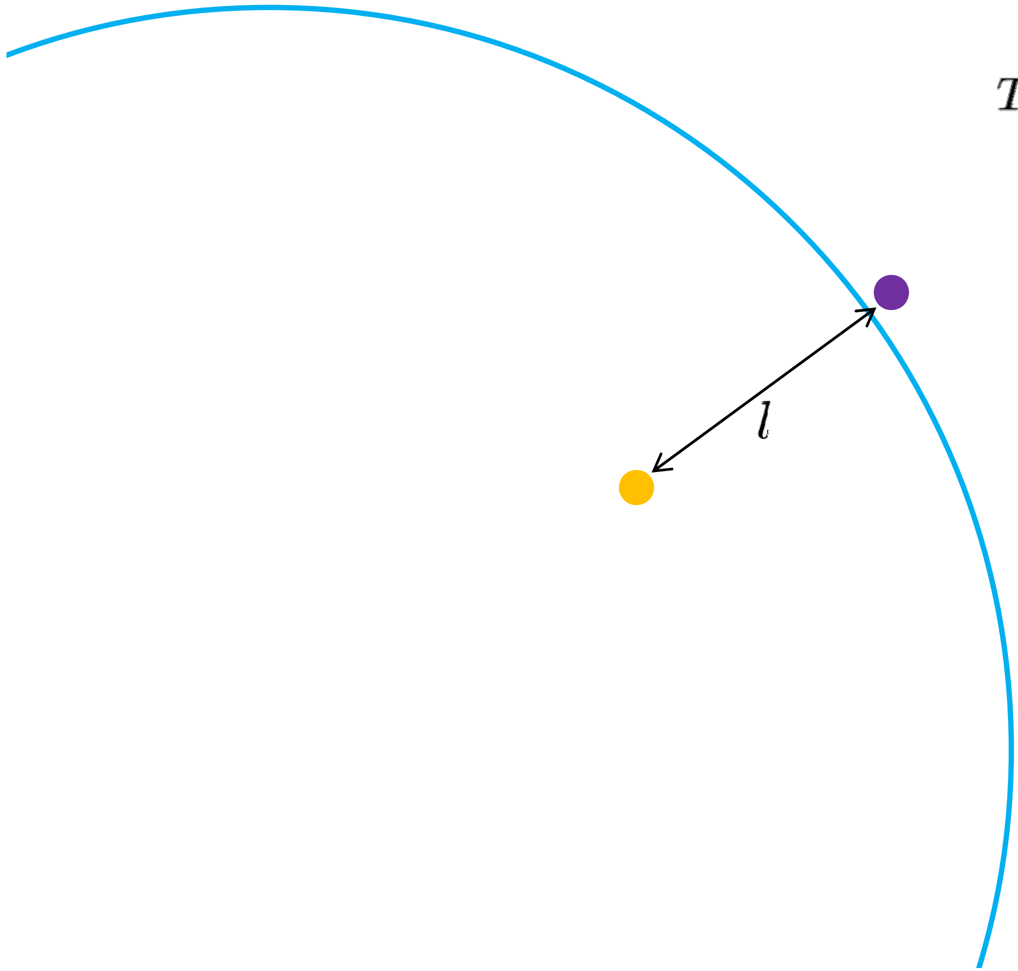
since the particles
can have kinetic energy as well,

$$T_* = \# \frac{hc}{2R_{BH}}$$

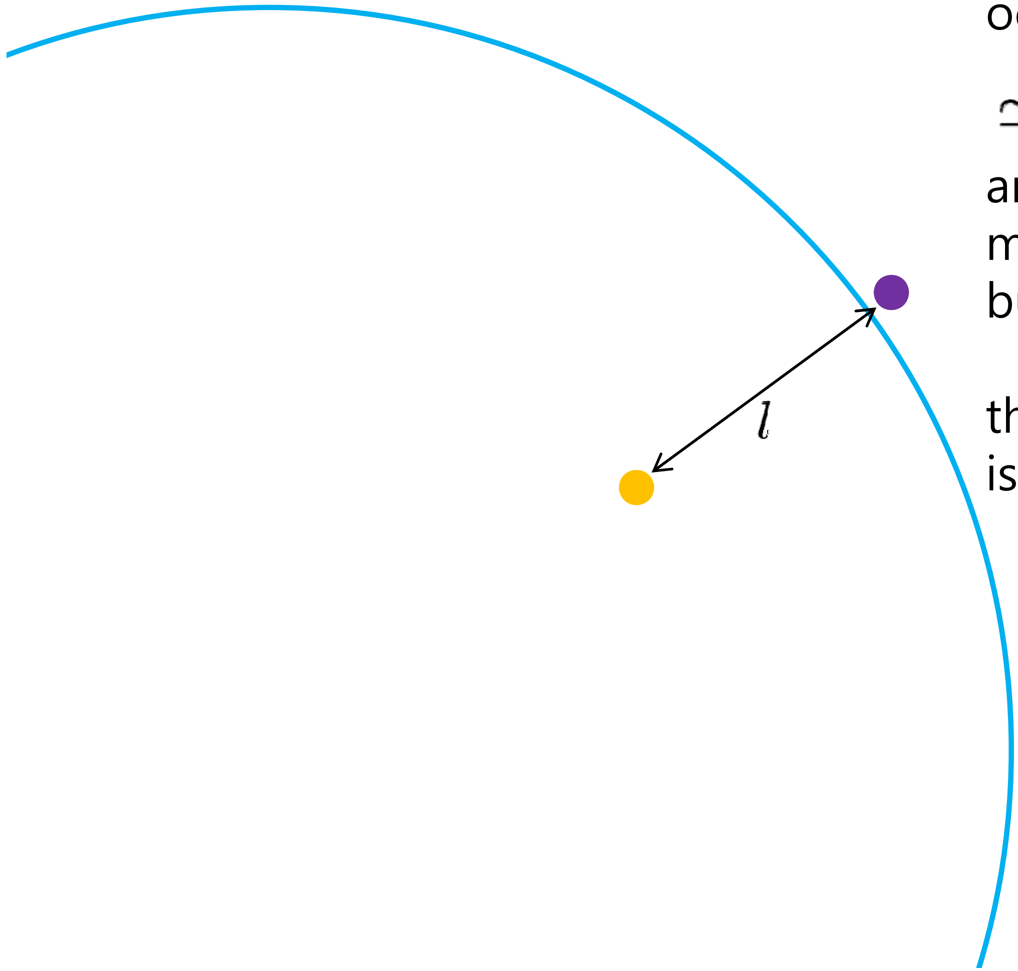
$$r = R_{BH} \equiv \frac{2G_N M}{c^2}$$

precise estimate of the temperature
is due to S. Hawking

$$T_* = T_{H=Hawking} = \frac{hc^3}{16\pi^2 G_N M}$$



$$r = R_{BH} \equiv \frac{2G_N M}{c^2}$$



pair-creation by tunneling
per unit area and per unit time
occur at the rate

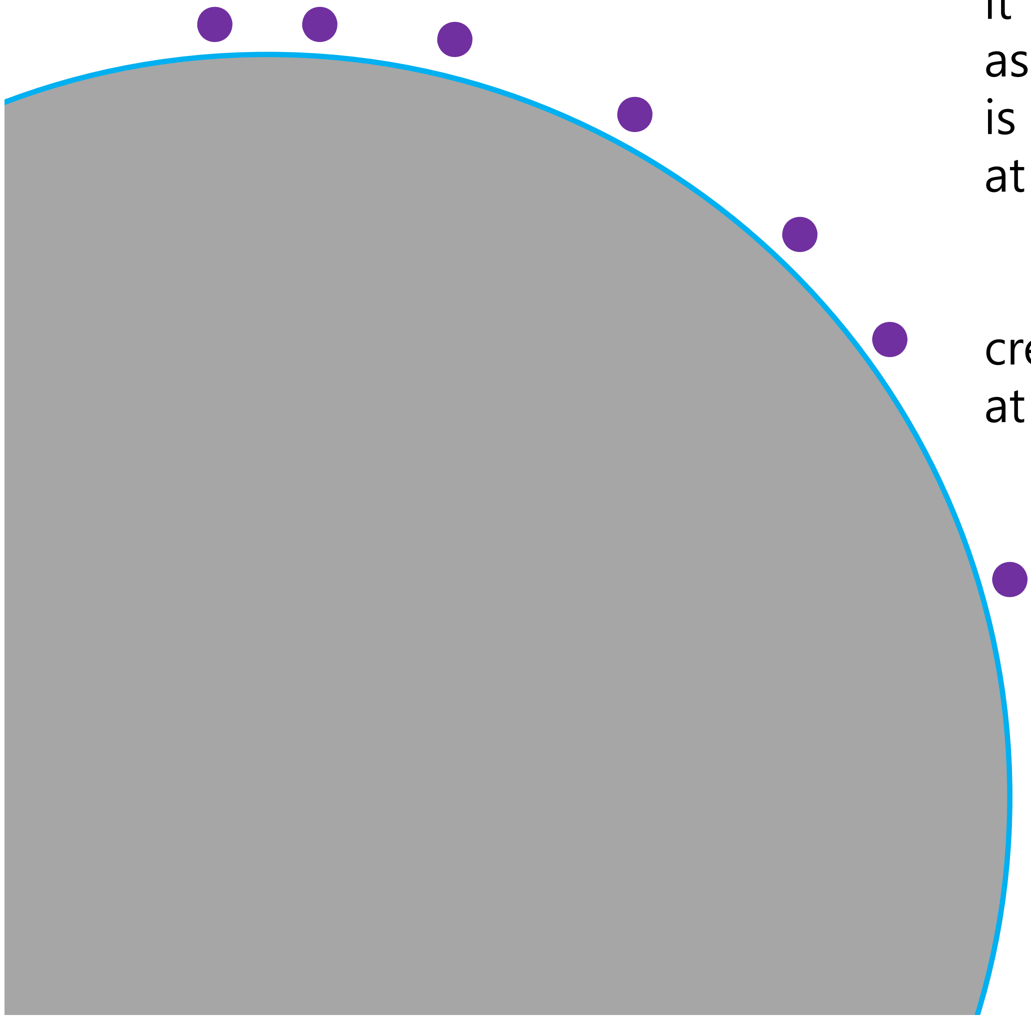
$$\simeq e^{-E/T_H}$$

and some of the particles outside
may escape the black hole attraction
but none inside can

the resulting outward radiation
is called the Hawking radiation

$$R_{BH} = \frac{2G_N M}{c^2}$$

$$T_H = \frac{hc^3}{16\pi^2 G_N M}$$



as far as outside observers go,
it appears quantum mechanically
as if the black hole horizon
is a surface of a blackbody
at temperature

$$T_H = \frac{hc^3}{16\pi^2 G_N M}$$

creating /emitting particles
at thermal rate

$$R_{BH} = \frac{2G_N M}{c^2}$$

$$T_H = \frac{hc^3}{16\pi^2 G_N M}$$

$$T_H = \frac{hc^3}{16\pi^2 G_N M} \simeq 8.471 \times 10^{-31}\, joule \times \left(\frac{M_{solar}}{M}\right)$$

$$\simeq 5.287 \times 10^{-12}\, eV \times \left(\frac{M_{solar}}{M}\right)$$

$$\simeq 6.134 \times 10^{-8}\, degree\, Kelvin \times \left(\frac{M_{solar}}{M}\right)$$

$$T_{CMB} = 2.728\ degree\ Kelvin$$

$$T_{freezing} = 273.2\ degree\ Kelvin$$

$$T_{SUN} = 5778\ degree\ Kelvin$$

if a black hole acts like a blackbody,

can we treat it as if it is a thermal object ?

if a black hole acts like a blackbody,

can we treat it as if it is a thermal object ?

$$d(E \rightarrow Mc^2) = (T \rightarrow T_H)d(S \rightarrow S_{BH}?)$$

$$\frac{d(S \rightarrow S_{BH})}{dt} \geq 0$$

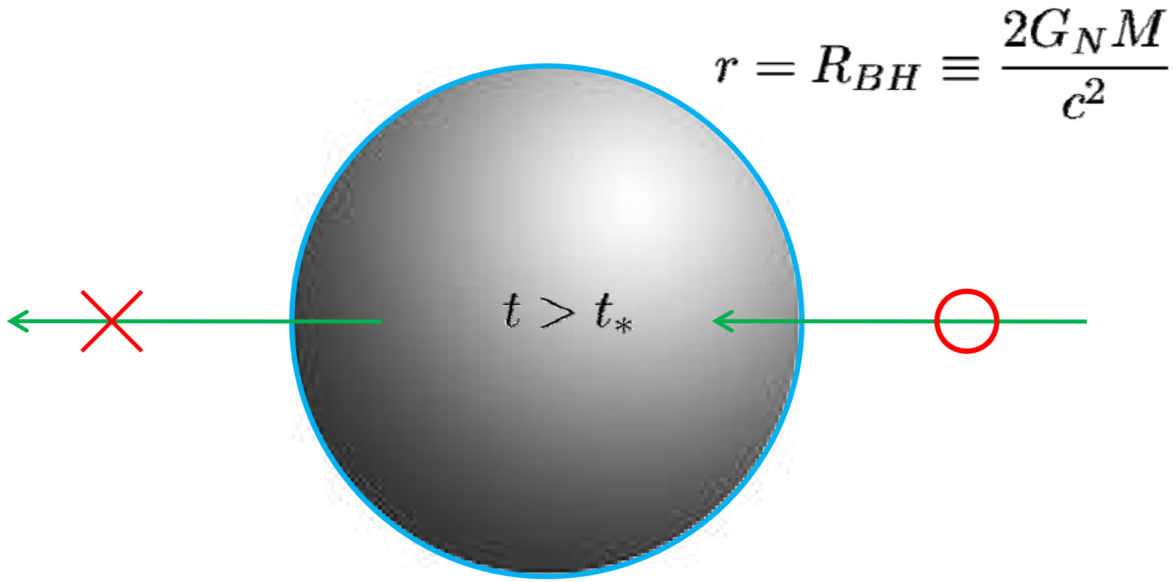
$$S_{BH} = \int \frac{1}{T_H} d(Mc^2) = \int \frac{16\pi^2 G_N M}{hc} dM = \frac{8\pi^2 G_N M^2}{hc}$$

$$= \frac{4\pi R_{BH}^2}{4(G_N/c^3)(h/2\pi)} = \frac{Horizon\ Area}{4(G_N/c^3)(h/2\pi)}$$

what about the 2nd law of the thermodynamics ?

classically,

$$\frac{d}{dt}S_{BH} = \frac{16\pi^2 G_N M}{hc}\frac{dM}{dt} \quad > \quad 0$$

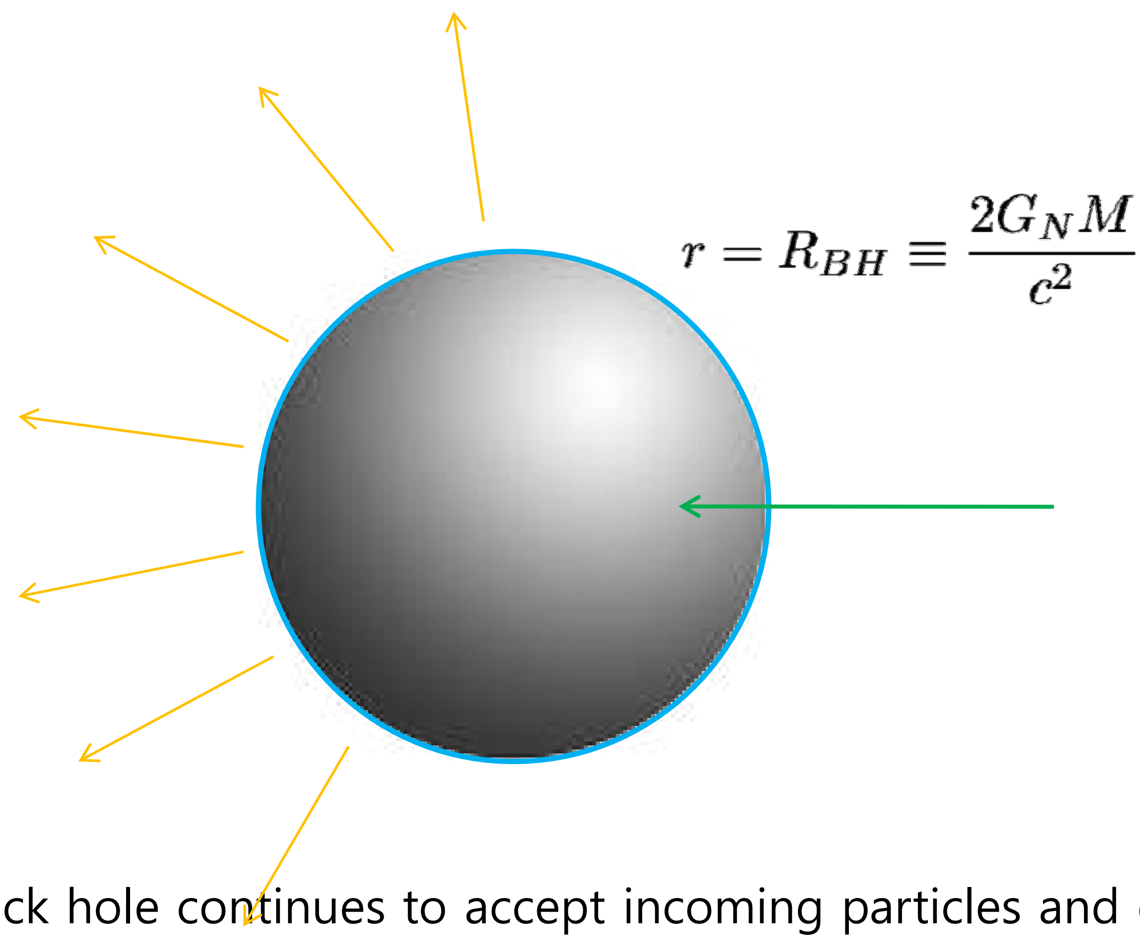


the black hole continues to accept incoming particles and energy

and can grow its mass, and thus the size, indefinitely

but cannot shrink since it cannot release its energy by emission of anything at quantum level

$$\frac{d}{dt}S_{BH} = \frac{16\pi^2 G_N M}{hc}\frac{dM}{dt} \not> 0$$



$$r = R_{BH} \equiv \frac{2G_N M}{c^2}$$

the black hole continues to accept incoming particles and energy

and can grow its mass, and thus the size, indefinitely

but it also lose energy due to Hawking radiation at quantum level at quantum level

$$\frac{d}{dt}S_{BH} = \frac{16\pi^2 G_N M}{hc}\frac{dM}{dt} \not> 0$$

$$\frac{d}{dt}(S_{BH} + S_{radiation}) = \frac{16\pi^2 G_N M}{hc}\frac{dM}{dt} + \dot{S}_{radiation} \quad > \quad 0$$

why BH entropy and why area ?

is BH entropy small or large ?

do we lose quantum information ?

what is entropy ?

Entropy = a Measure of Unavailable Information (Shannon)

$$S = -\mathrm{Tr}\,\rho \log \rho$$

↑

density matrix

Entropy = a Measure of Unavailable Information (Shannon)

if the system is in a pure quantum state, $\rho = |\Psi\rangle\langle\Psi|$

$$S = -\mathrm{Tr}\,\rho\log\rho$$

$$\rho = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

$$= -1 \times \log(1)$$

$$= 0$$

Entropy = a Measure of Unavailable Information (Shannon)

if the density matrix is proportional to the identity matrix over the Hilbert space with N number of states, i.e., if all N states are equally probable

$$S = -\mathrm{Tr}\, \rho \log \rho$$

$$\downarrow \quad \rho = \frac{1}{N} I_{N\times N}$$

$$= -N \times \frac{1}{N} \log(1/N)$$

$$= \log N$$

entropy is a measure of information which
we choose to, or are forced to, ignore.

for example, a two-state quantum system can have
a maximum entropy of log(2)

knowing the entropy = log(N), backwardly
we do learn that the system has at least
N states available inside.

which begs for the question:

what is the meaning of

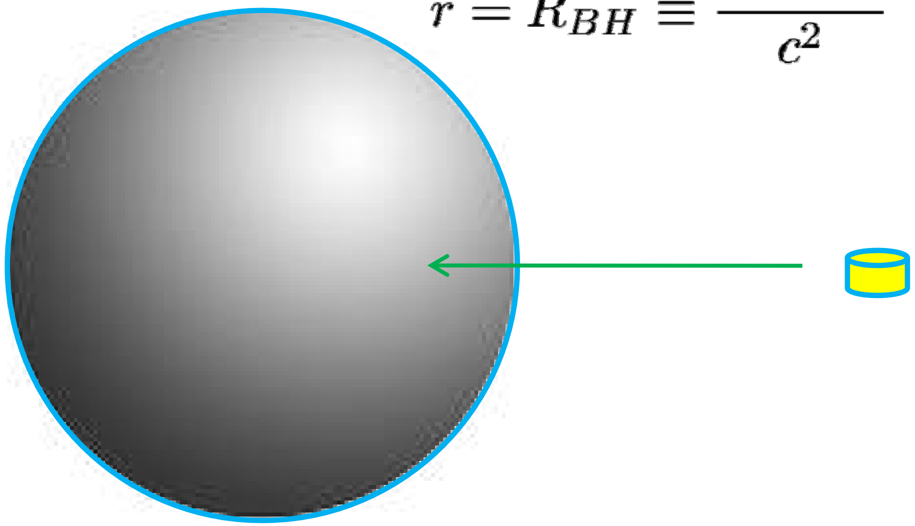
$$N_{BH} = e^{S_{BH}}$$

?

let us first consider, however,

why BH entropy should have entropy

and why it is proportional to the area?



$$r = R_{BH} \equiv \frac{2G_N M}{c^2}$$

$$\mathcal{E}$$

$$S = \log(2)$$

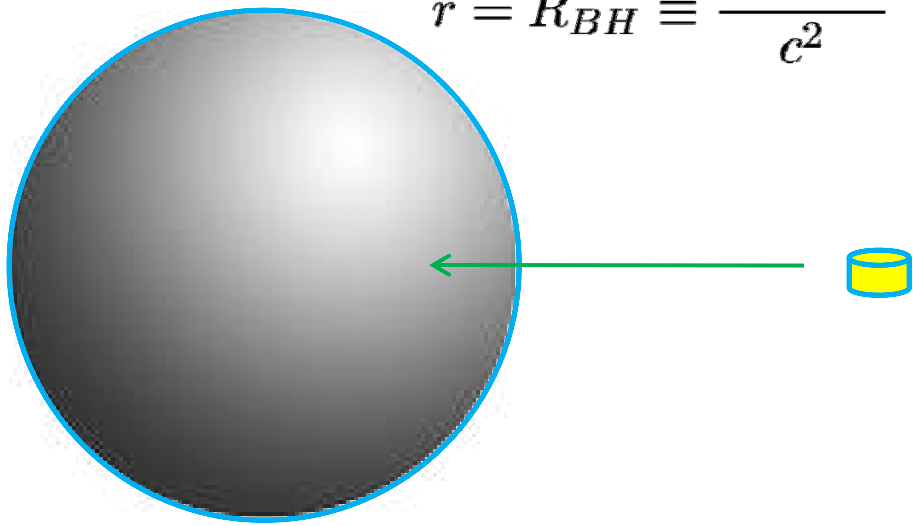
black holes eat up anything that come across its path;

consider a small box containing some small energy and also small entropy;

after a BH eats the box up,

we would seem to have lost the entropy associated with that box

$$r = R_{BH} \equiv \frac{2G_N M}{c^2}$$



$\mathcal{E}$

$S = \log(2)$

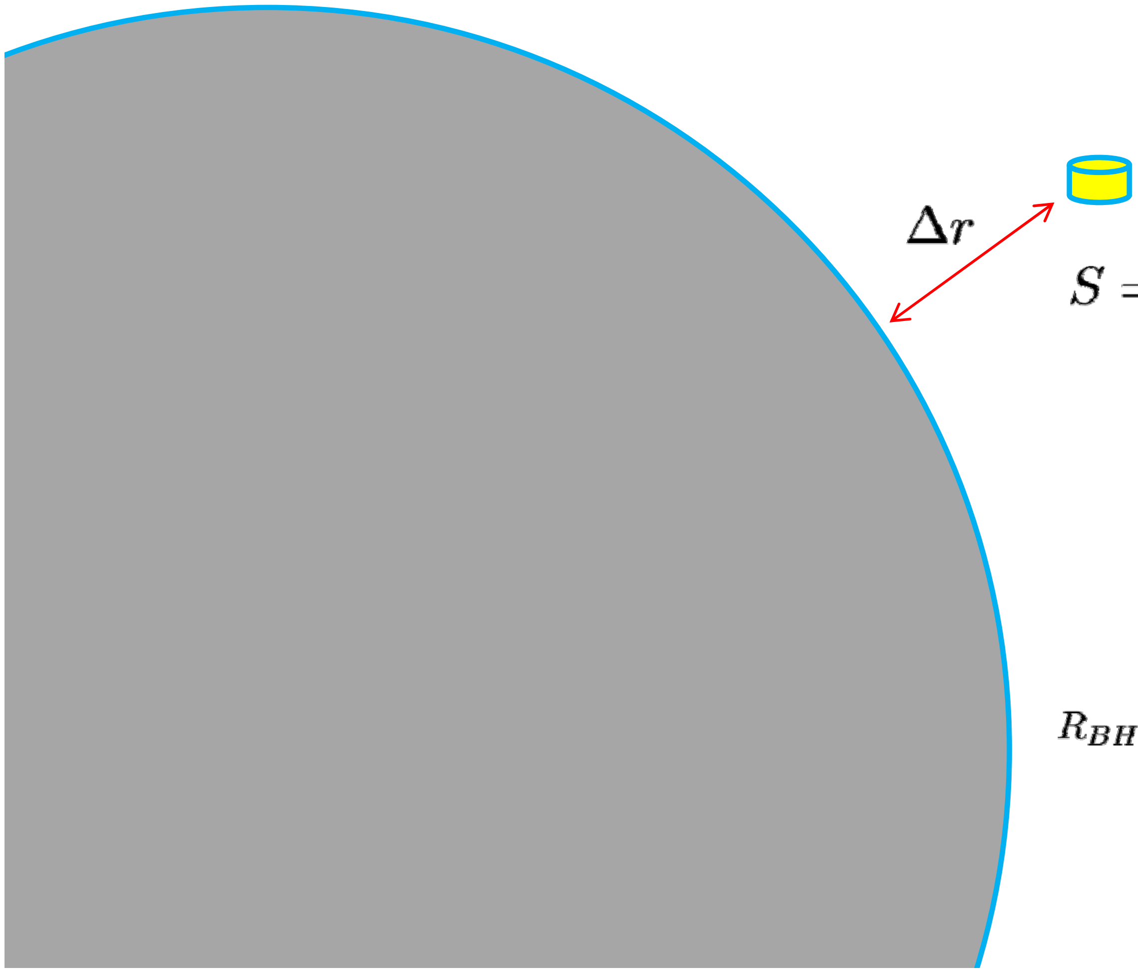
if there is no concept of entropy associated with black holes,

the net entropy can decrease, contrary to the 2^{nd} law.

what would then be the most economical entropy we can assign to the black hole,

in order for the combined entropy to obey the 2^{nd} law ?

consider a small box that contains a single particle of energy $\mathcal{E}$
with the smallest unit of entropy = log(2)
bring it near the black hole horizon and hold it there



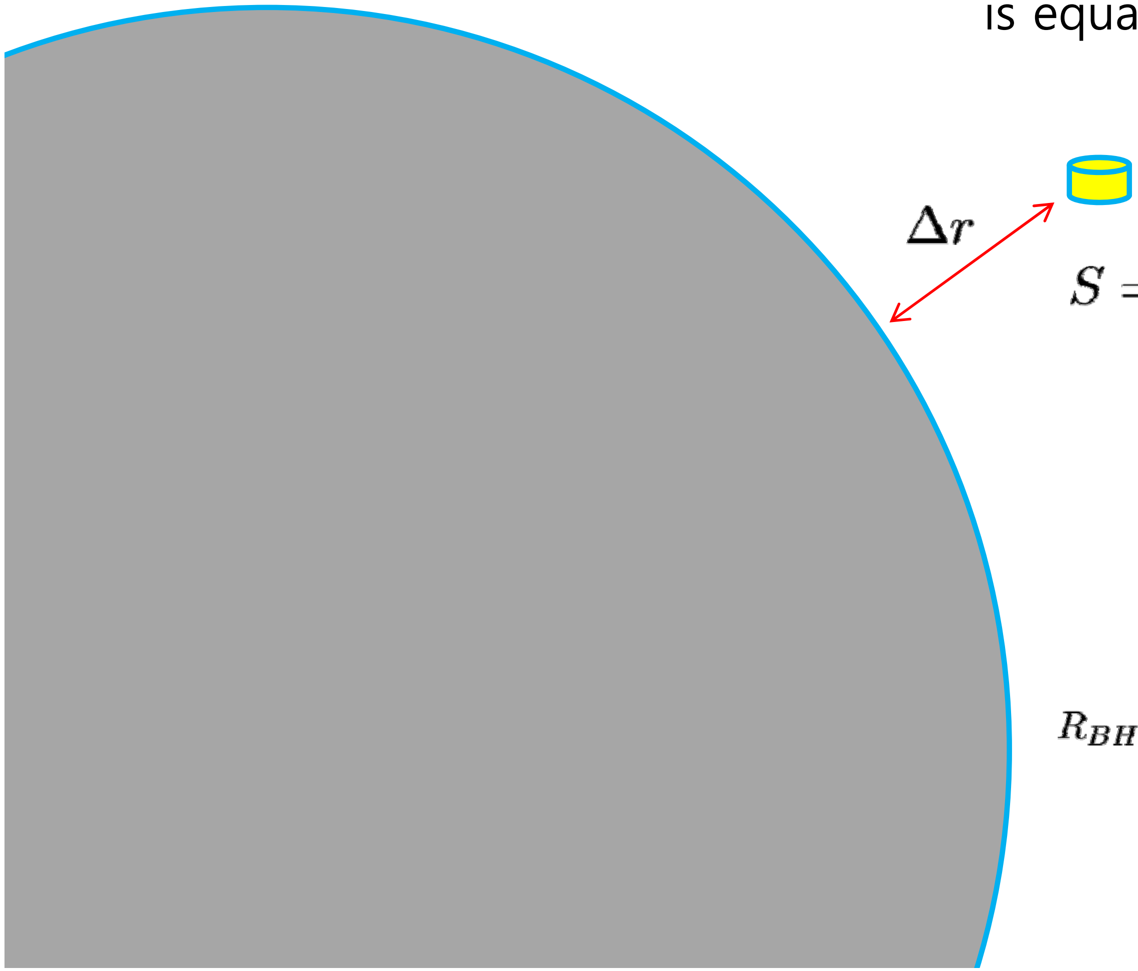
$$\mathcal{E}' = \mathcal{E} \times \sqrt{\frac{\Delta r}{R_{BH}}}$$

$$S = \log(2)$$

gravitational redshift

$$R_{BH} = \frac{2G_N M}{c^2}$$

Consider the box to be part of BH
if the proper distance to the black hole
is equal to the Compton wavelength

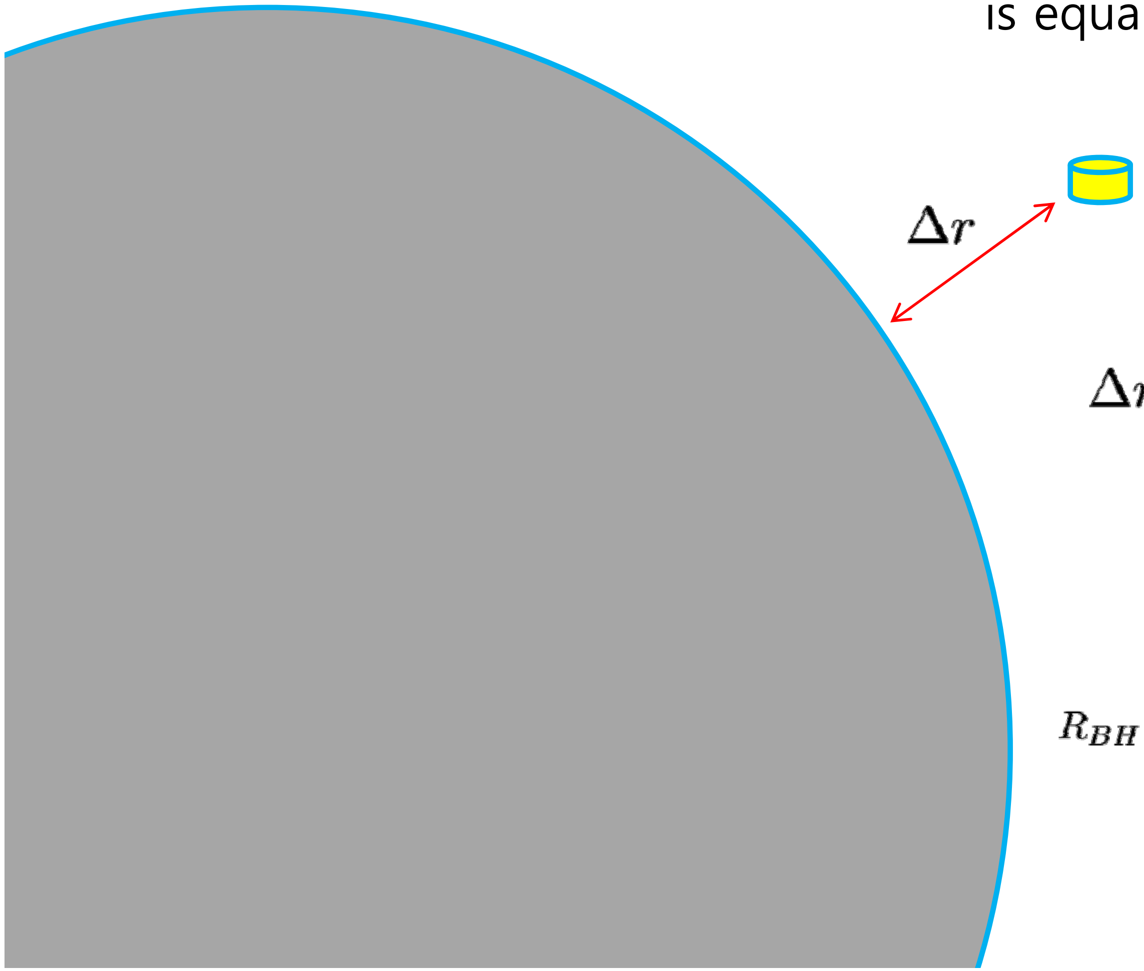


$$\mathcal{E}' = \mathcal{E} \times \sqrt{\frac{\Delta r}{R_{BH}}}$$

$$S = \log(2)$$

$$R_{BH} = \frac{2G_N M}{c^2}$$

Consider the box to be part of BH
if the proper distance to the black hole
is equal to the Compton wavelength

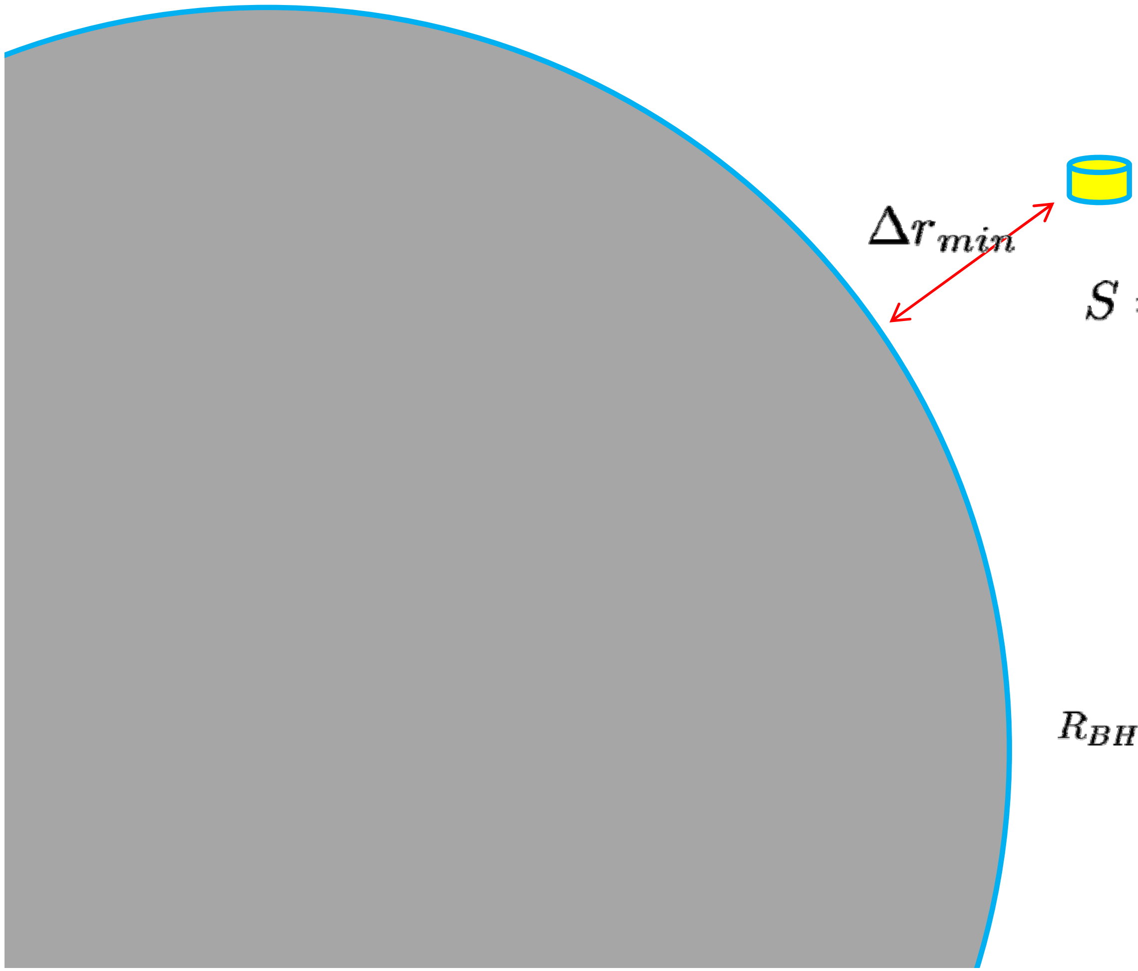


$$\mathcal{E}'_{min} = \mathcal{E} \times \sqrt{\frac{\Delta r_{min}}{R_{BH}}}$$

$$\Delta r_{min} \sim \frac{hc}{\mathcal{E}} \times \sqrt{\frac{\Delta r_{min}}{R_{BH}}}$$

gravitational redshift

$$R_{BH} = \frac{2G_N M}{c^2}$$

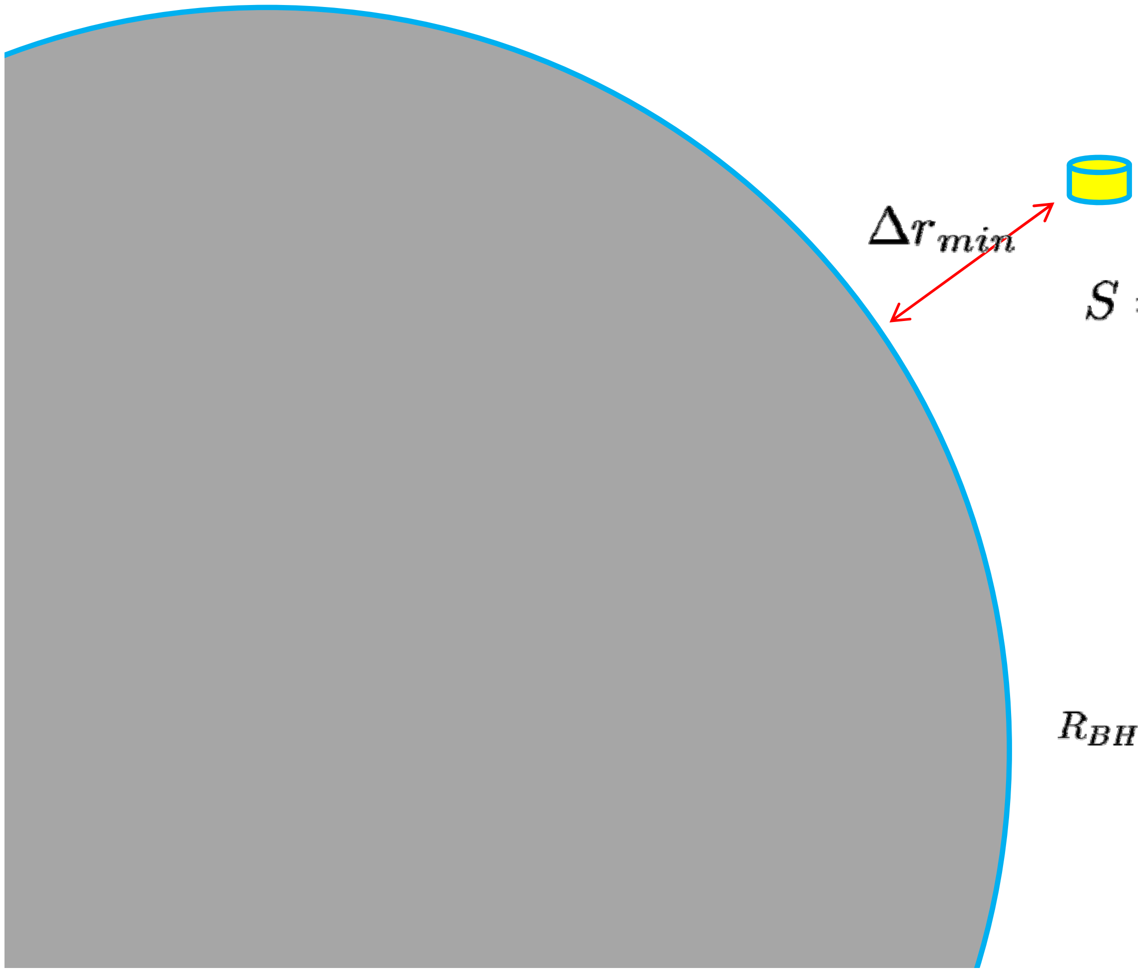


$$\mathcal{E}'_{min} = \mathcal{E} \times \sqrt{\frac{\Delta r_{min}}{R_{BH}}} \sim \frac{hc}{R_{BH}}$$

$$S = \log(2)$$

$$R_{BH} = \frac{2G_N M}{c^2}$$

$$\delta S_{BH} = \frac{16\pi^2 G_N M}{hc^3}\delta(Mc^2) \sim \frac{R_{BH}}{hc}\mathcal{E}'_{min} \sim \frac{R_{BH}}{hc}\frac{hc}{R_{BH}} \sim 1$$



$$\mathcal{E}'_{min} = \mathcal{E} \times \sqrt{\frac{\Delta r_{min}}{R_{BH}}} \sim \frac{hc}{R_{BH}}$$

$$S = \log(2)$$

$$R_{BH} = \frac{2G_N M}{c^2}$$

BH entropy ~ area is exactly right for preserving 2nd law of thermodynamics in the presence of black holes.

This is due to Bekenstein, well before Hawking's discovery,

$$S_{Bekenstein} = \frac{1}{2} \log 2 \times \frac{Horizon\ Area}{(G_N/c^3)(h/2\pi)}$$

$$S_{BH} = \frac{1}{4} \frac{Horizon\ Area}{(G_N/c^3)(h/2\pi)}$$

is BH entropy small or large ?

$$Planck\ Length \sim 10^{-34}\ meter$$

$$S_{BH} = \frac{Horizon\ Area}{4(G_N/c^3)(h/2\pi)} = \frac{Horizon Area}{4 \times (Planck\ Length)^2}$$

$$\simeq 1.061 \times 10^{75} \times \left(\frac{M}{M_{solar}}\right)^2$$

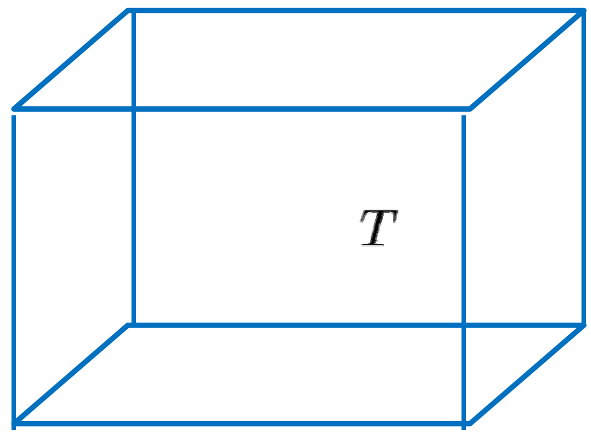
the entropy is unusual in that it is proportional to the surface area, instead of to the volume.

nevertheless, actual value seem incredibly large conventional thermodynamics system

$$\rho \sim T^4/h^3c^3$$

$$E \sim \rho V \sim L^3T^4/h^3c^3$$

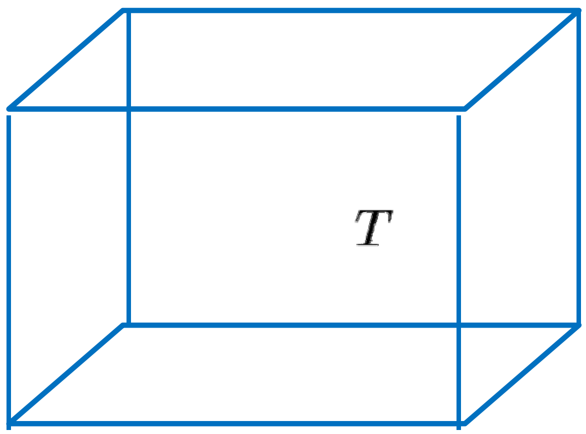
$$S \sim L^3T^3/h^3c^3$$



what is the maximum (conventional) entropy one can squeeze in a box of size $L \sim R_{BH}$ and at temperature $T \sim T_H$?

$$L > R_{BH}(E/c^2) = \frac{2EG_N}{c^4}$$

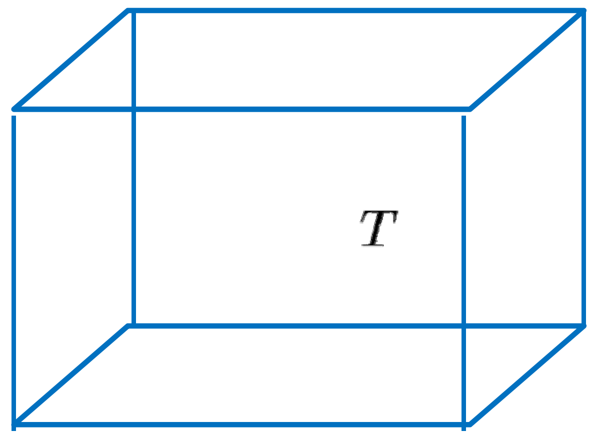
otherwise, the box collapses gravitationally to become a black hole



what is the maximum (conventional) entropy one can squeeze in a box of size $L \sim R_{BH}$ and at temperature $T \sim T_H$?

$$L > R_{BH}(E/c^2) = \frac{2EG_N}{c^4} \sim \frac{L^3 T^4 G_N}{h^3 c^7} \sim S_{box} \times \frac{TG_N}{c^4}$$

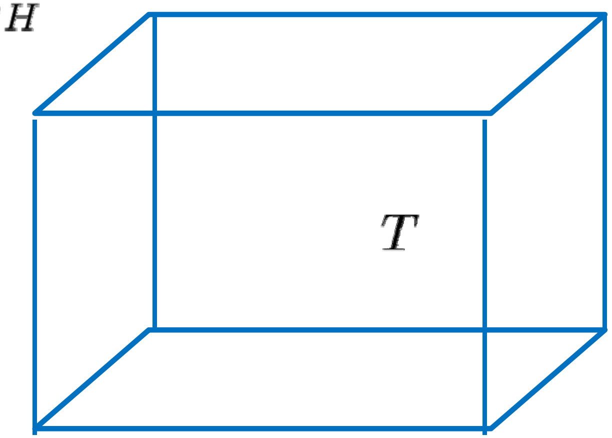
otherwise, the box collapses gravitationally to become a black hole



what is the maximum (conventional) entropy one can squeeze in a box of size $L \sim R_{BH}$ and at temperature $T \sim T_H$?

$$L > R_{BH}(E/c^2) = \frac{2EG_N}{c^4} \sim \frac{L^3T^4G_N}{h^3c^7} \sim S_{box} \times \frac{TG_N}{c^4}$$

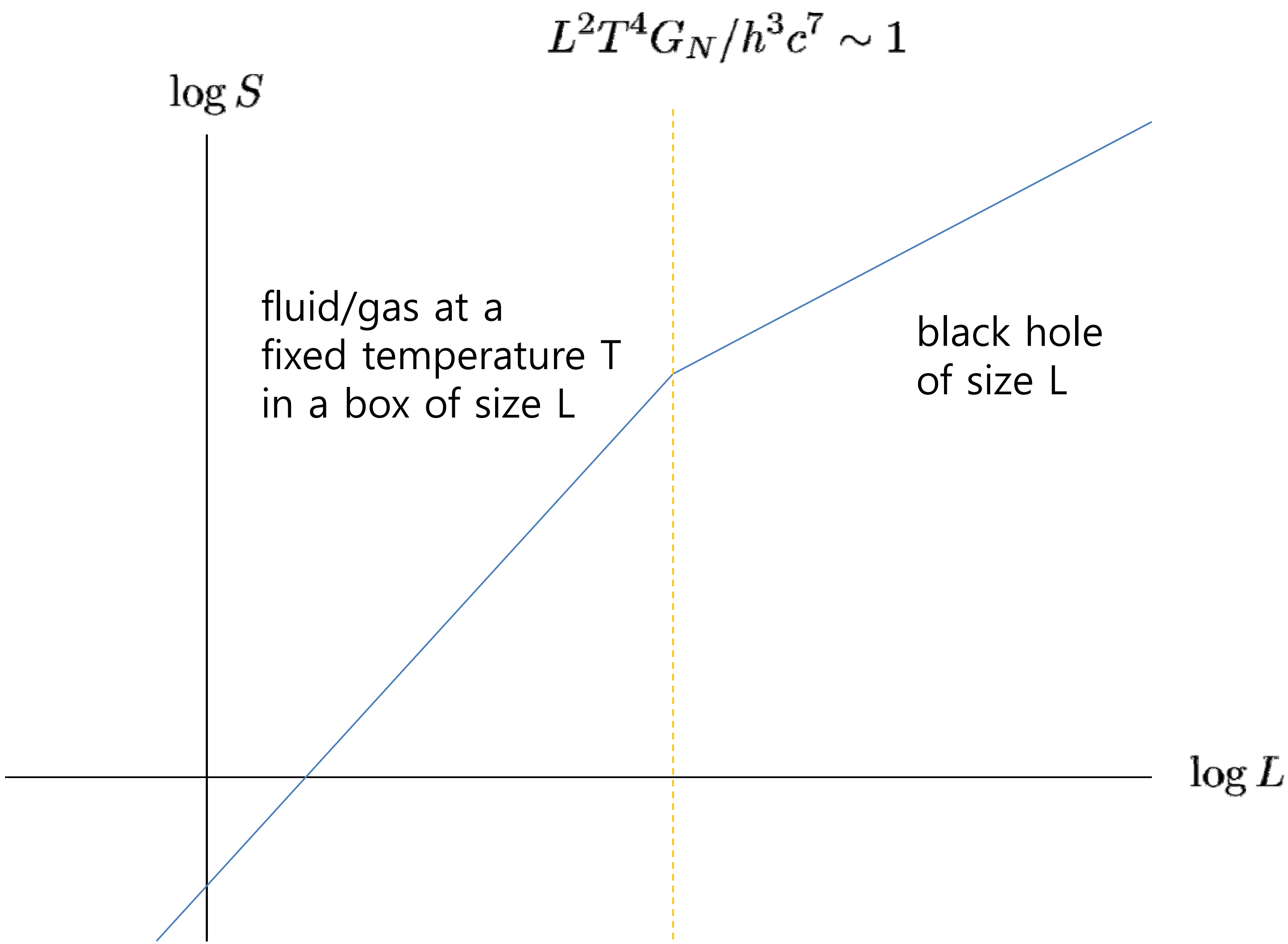
$$S_{box} < \frac{c^4 L}{TG_N} \sim \frac{c^4 R_{BH}}{G_N T_H} \sim \frac{R_{BH}^2 c^3}{hG_N} \sim S_{BH}$$



although black hole entropy scales
only with the square of the box size,
it still represents an absolute upper maximum
on conventional entropy,

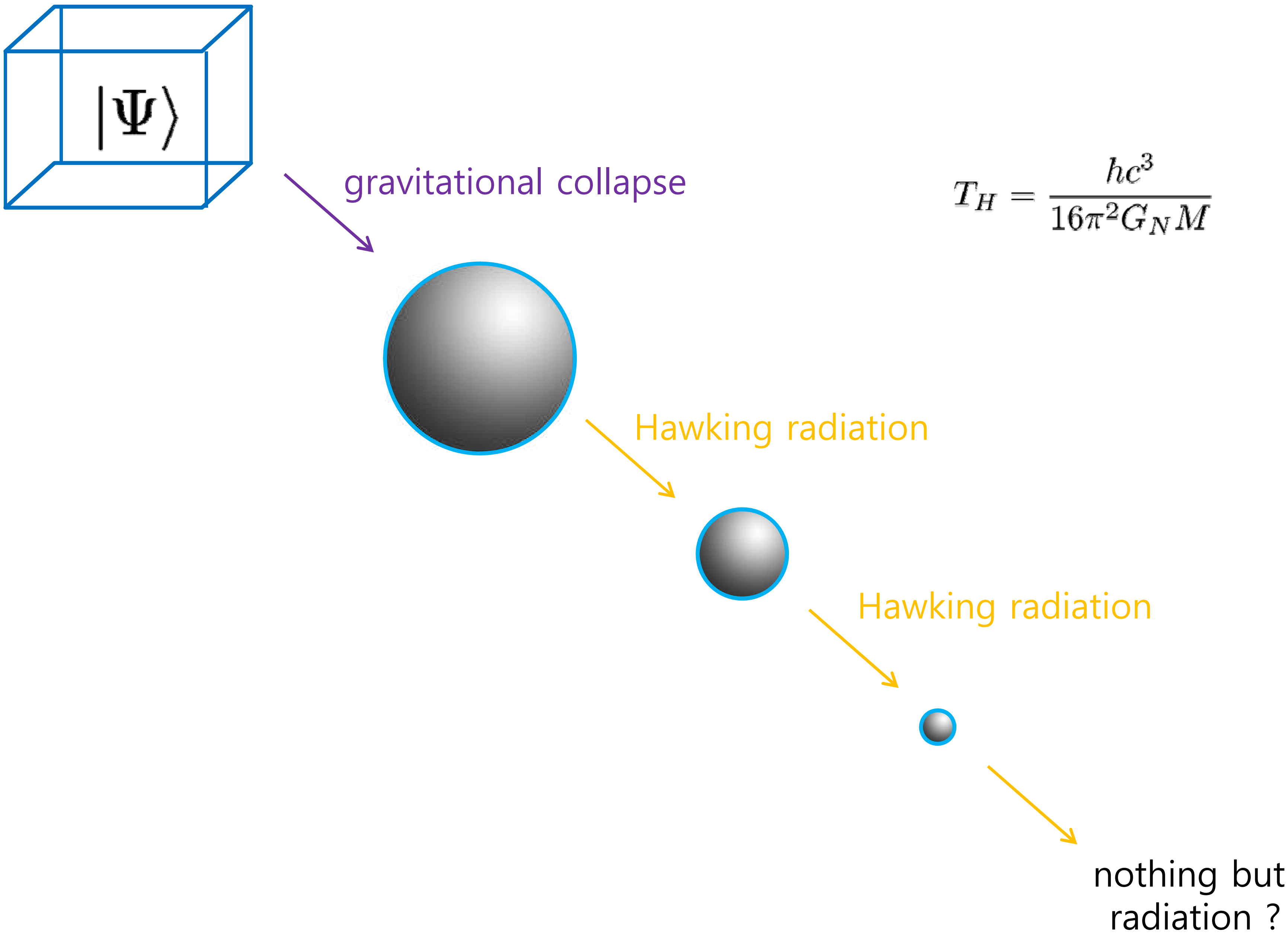
simply because

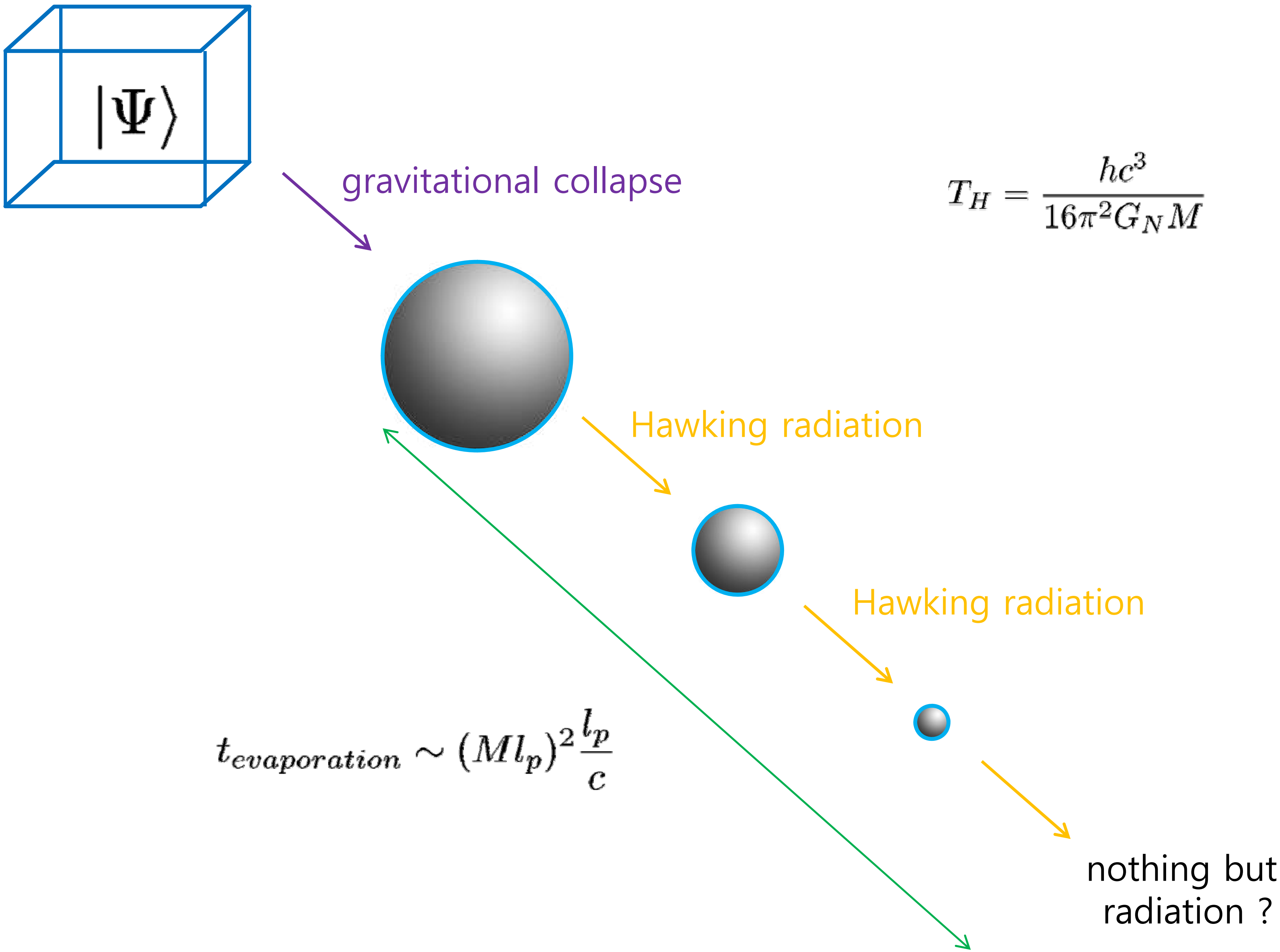
if one tries to put in more entropy,
either by making the box larger or elevating temperature,
too much energy accumulates in the box and
the system collapses gravitationally to a black hole.



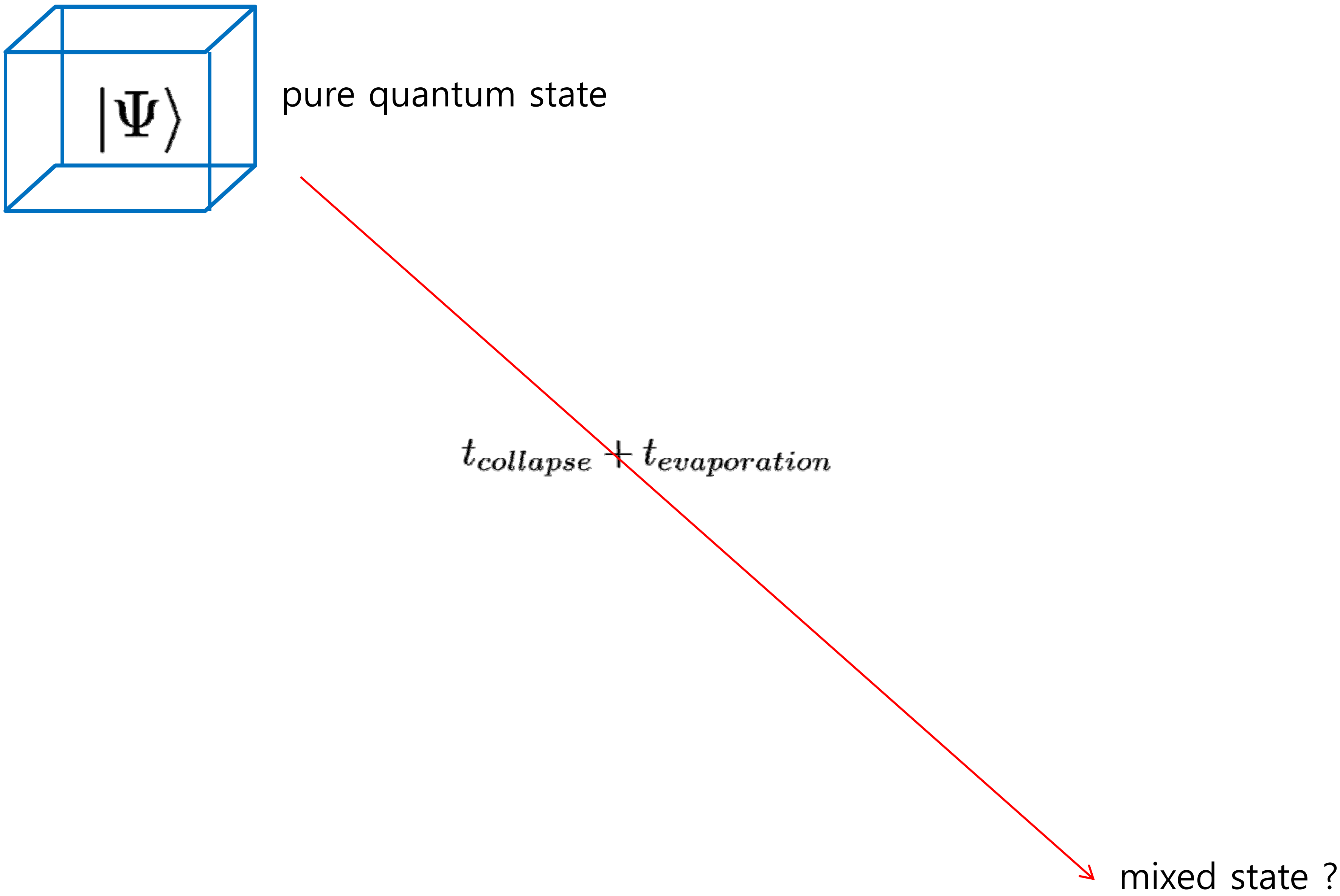
$L^2T^4G_N/h^3c^7 \sim 1$
$\log S$
fluid/gas at a
fixed temperature T
in a box of size L
black hole
of size L
$\log L$

do we lose quantum information ?


$|\Psi\rangle$
gravitational collapse
$T_H = \frac{hc^3}{16\pi^2 G_N M}$
Hawking radiation
Hawking radiation
nothing but radiation ?



$|\Psi\rangle$
gravitational collapse
$T_H = \frac{hc^3}{16\pi^2 G_N M}$
Hawking radiation
Hawking radiation
$t_{evaporation} \sim (M l_p)^2 \frac{l_p}{c}$
nothing but radiation ?


$|\Psi\rangle$
pure quantum state
$t_{collapse} + t_{evaporation}$
mixed state ?

do we lose quantum information ?

Hawking said a long time ago:

Yes we do. Therefore, we must abandon quantum unitarity principle
when we try to quantize gravity.

do we lose quantum information
when black holes are created and evaporated away ?

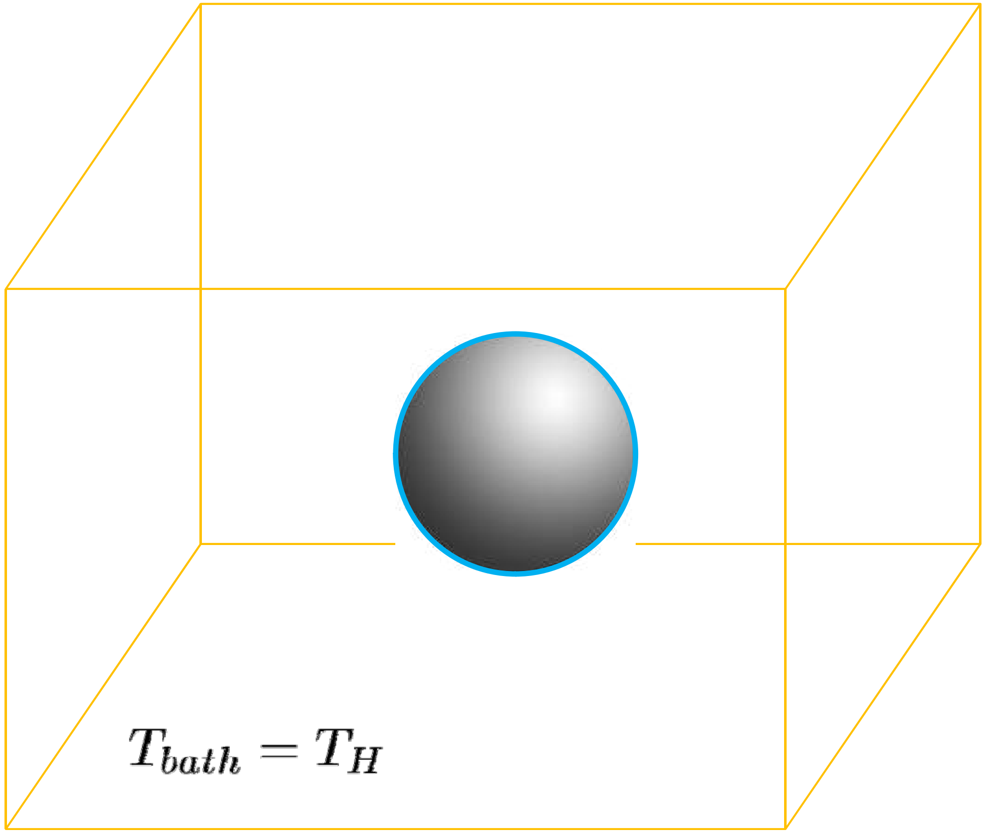
Most string theorists now believe:

Absolutely not.

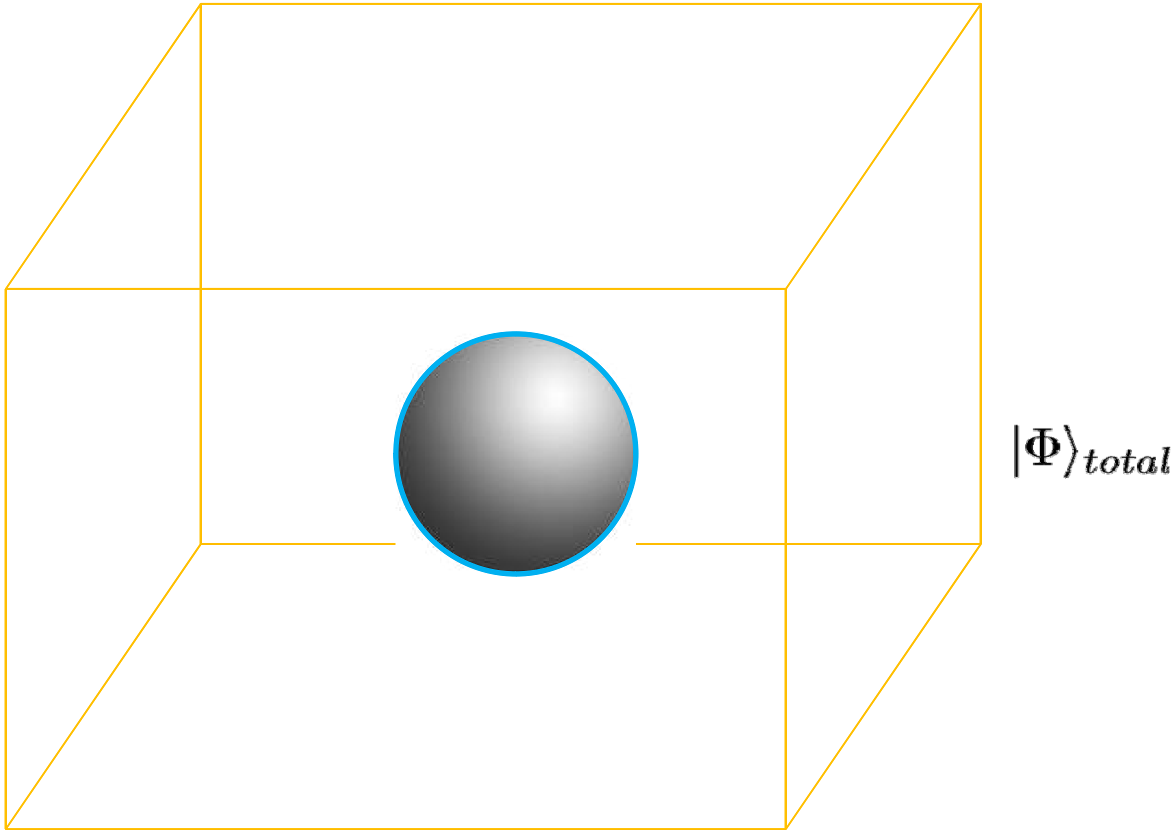
the entropy of (at least some) Black Holes can be shown to come from quantum microstates which are entirely described as quantum wavefunctions.

It is only because we have no effective probe for these internal states that we find finite entropy and thermal behavior.

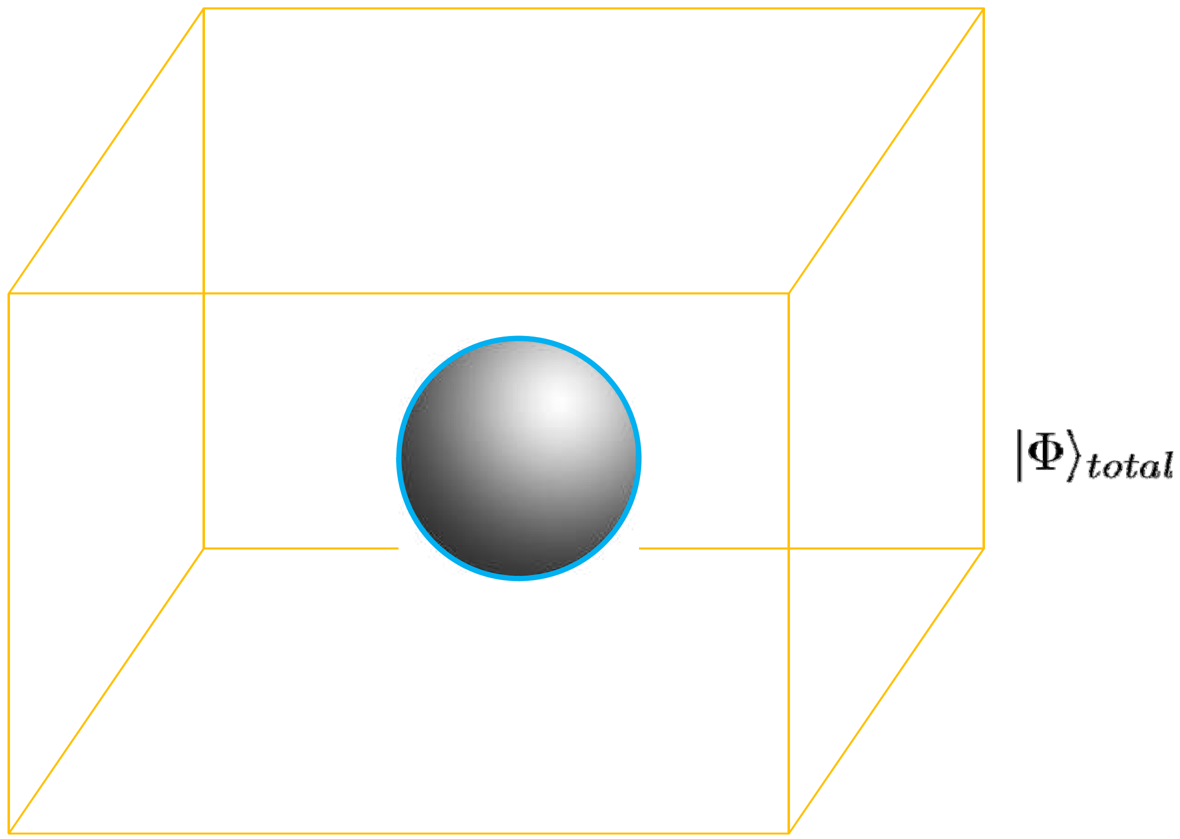
BH entropy as entanglement entropy



Black hole contributes entropy, proportional to its area,
to this equilibrium state. How ?

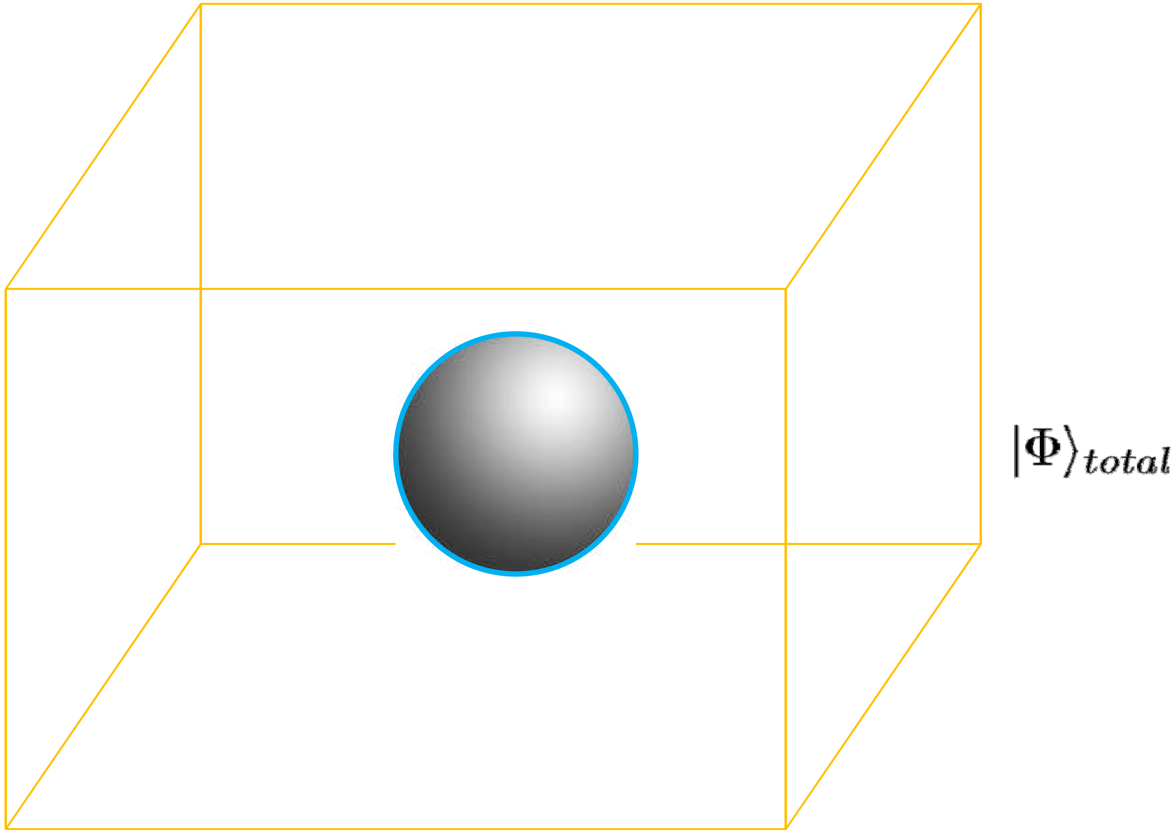


Imagine that the total state is in some pure quantum state
instead of a thermal ensemble,
with perfect correlations between black hole states and outside states,
in which case all entropy, if any, must come from the black hole.

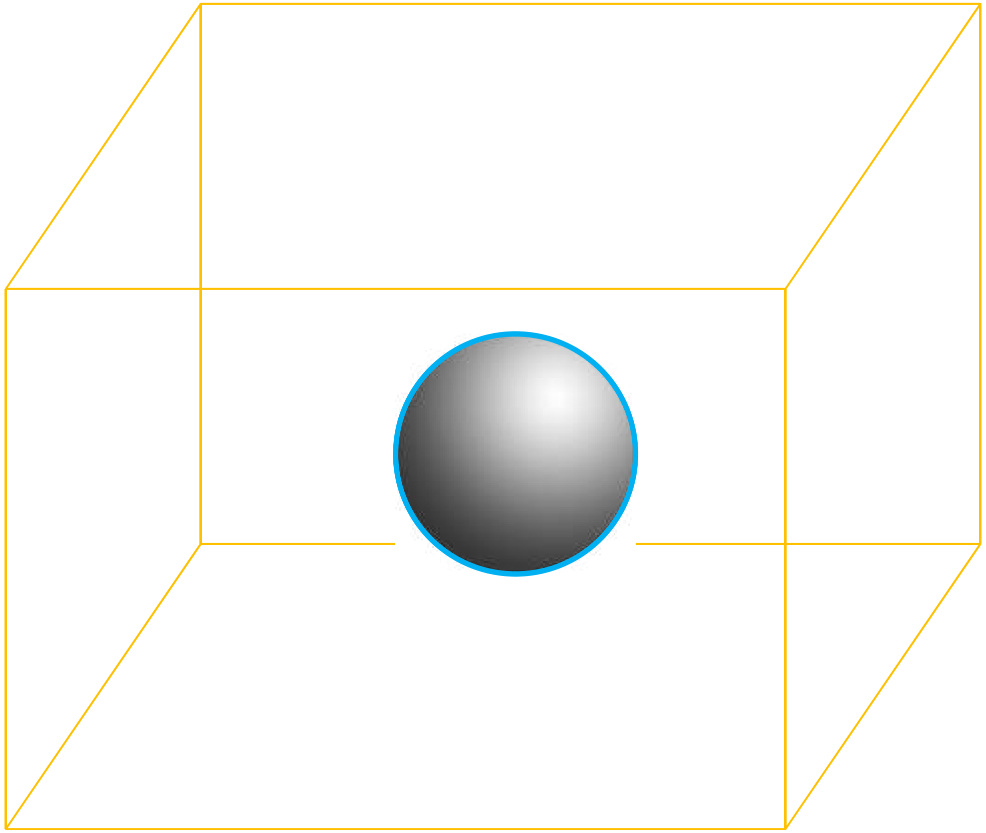


$$|\Phi\rangle_{total} = \sum_{n=1}^{N_{BH}} \frac{1}{\sqrt{N_{BH}}} |n\rangle_{black\ hole} \otimes |\psi_n\rangle_{outside}$$

$$N_{BH} = e^{S_{BH}}$$

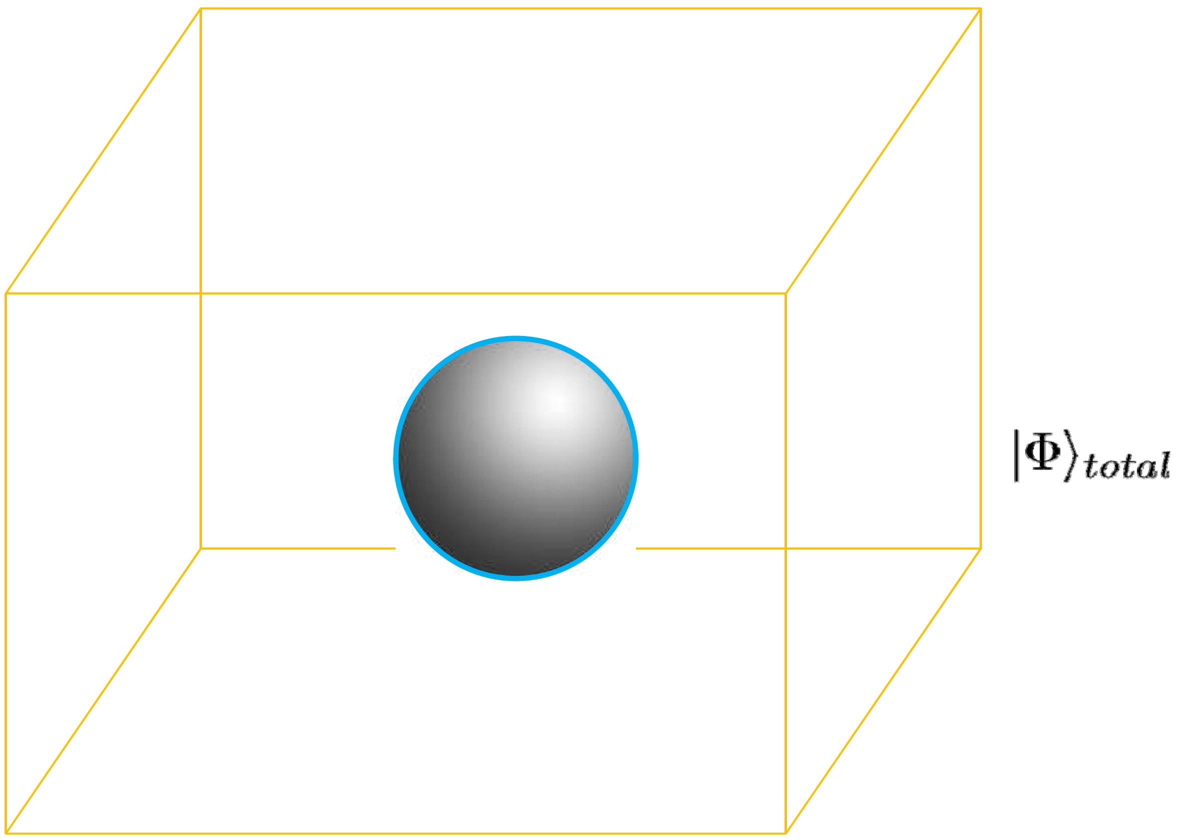


Since we are completely ignorant about inside of BH horizon,
for physical process involving degrees of freedom outside,
we can trace over internal Hilbert space first
and use the resulting (outside) density matrix

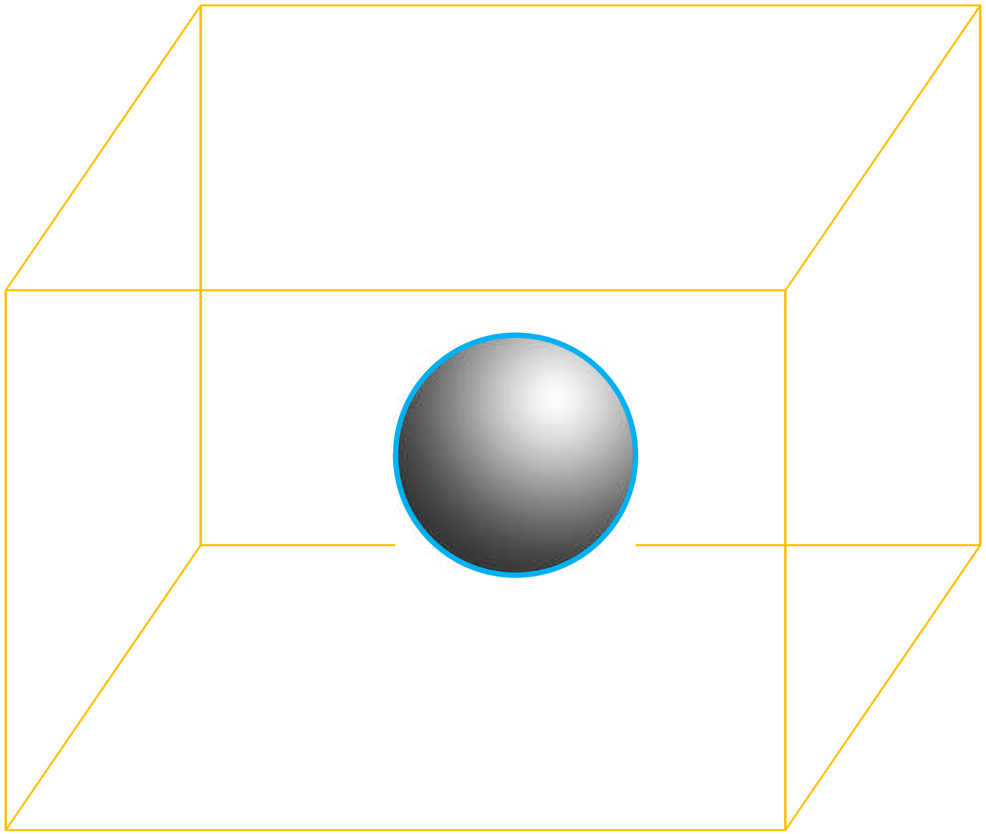


$$\rho_{outside} = \mathrm{Tr}_{BH} |\Phi\rangle\langle\Phi| = \frac{1}{N_{BH}} \sum_{n=1}^{N_{BH}} |\psi_n\rangle\langle\psi_n|$$

$$N_{BH} = e^{S_{BH}}$$



for large enough box, all of these outside states can be independent of one another



$$\rho_{outside} = \mathrm{Tr}_{BH}|\Phi\rangle\langle\Phi| = \frac{1}{N_{BH}}\sum_{n=1}^{N_{BH}}|\psi_n\rangle\langle\psi_n|$$

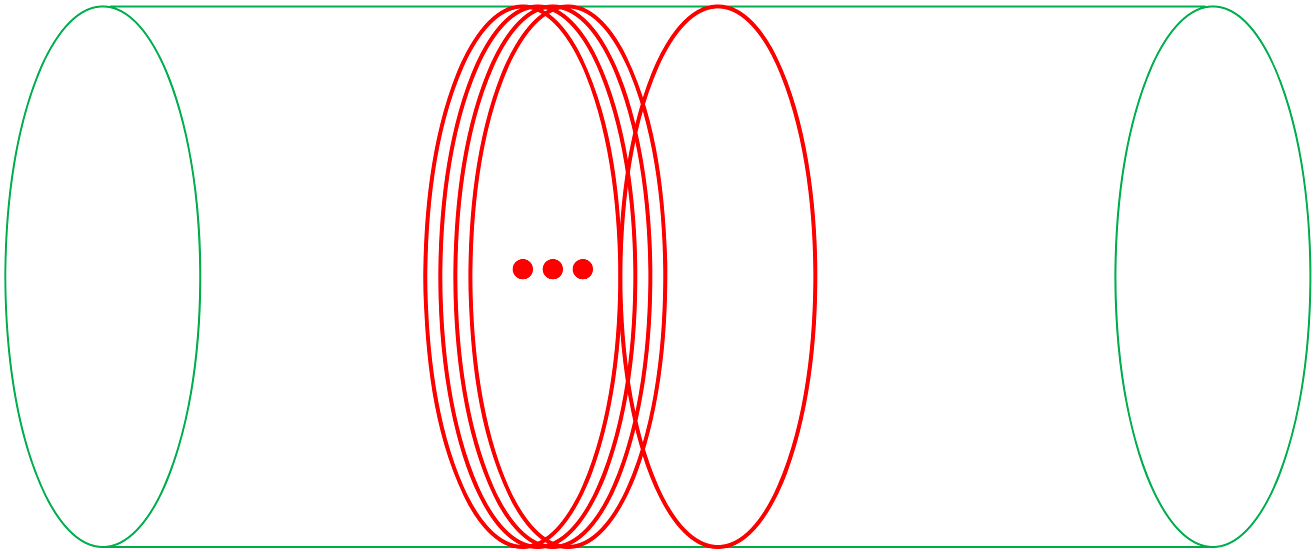
$$S = -\mathrm{Tr}\ \rho_{outside}\log\rho_{outside} = N_{BH}\times\frac{1}{N_{BH}}\log N_{BH} = \log N_{BH} = S_{BH}$$

Therefore, if one can show that black holes are equipped with such a large number of internal quantum states,
it becomes more likely that evolution of black hole is also governed by the same old quantum principle.

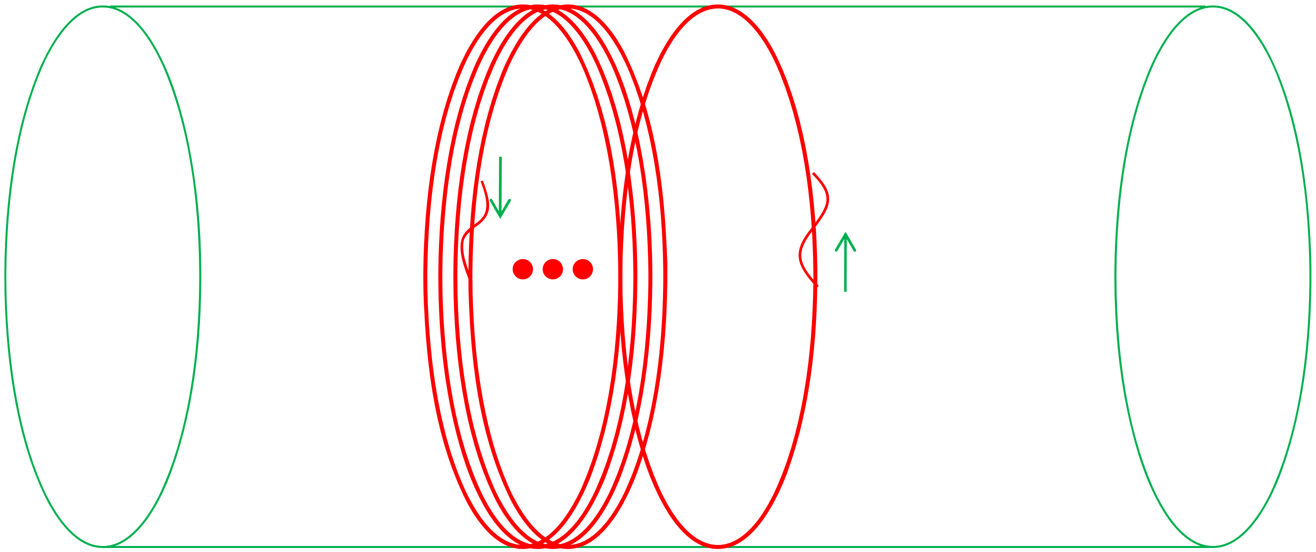
$N_{BH} = e^{S_{BH}}$ black hole microstates?

$N_{BH} = e^{S_{BH}}$ black hole microstates from string theory ?

Or, at least, degeneracy that grows exponentially with the energy ?



w times wound string



w times wound string with n unit of momenta

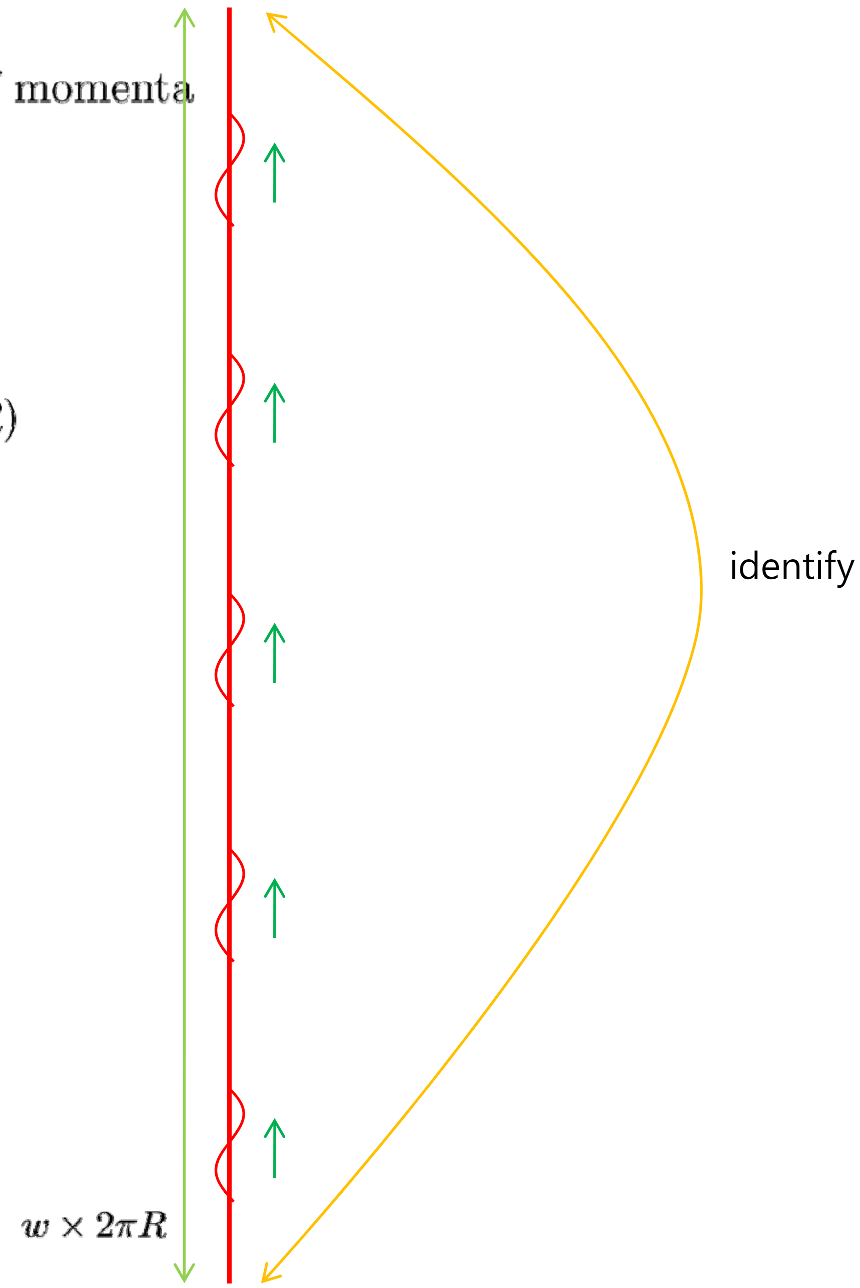
$$\text{energy} \sim \sqrt{w^2(2\pi R)^2\tau_{string}^2 + n^2/R^2}$$

w times wound string with n unit of momenta

total momenta $= n/R$

momentum quanta $= k/(w \times R)$

$k \in Z_+$



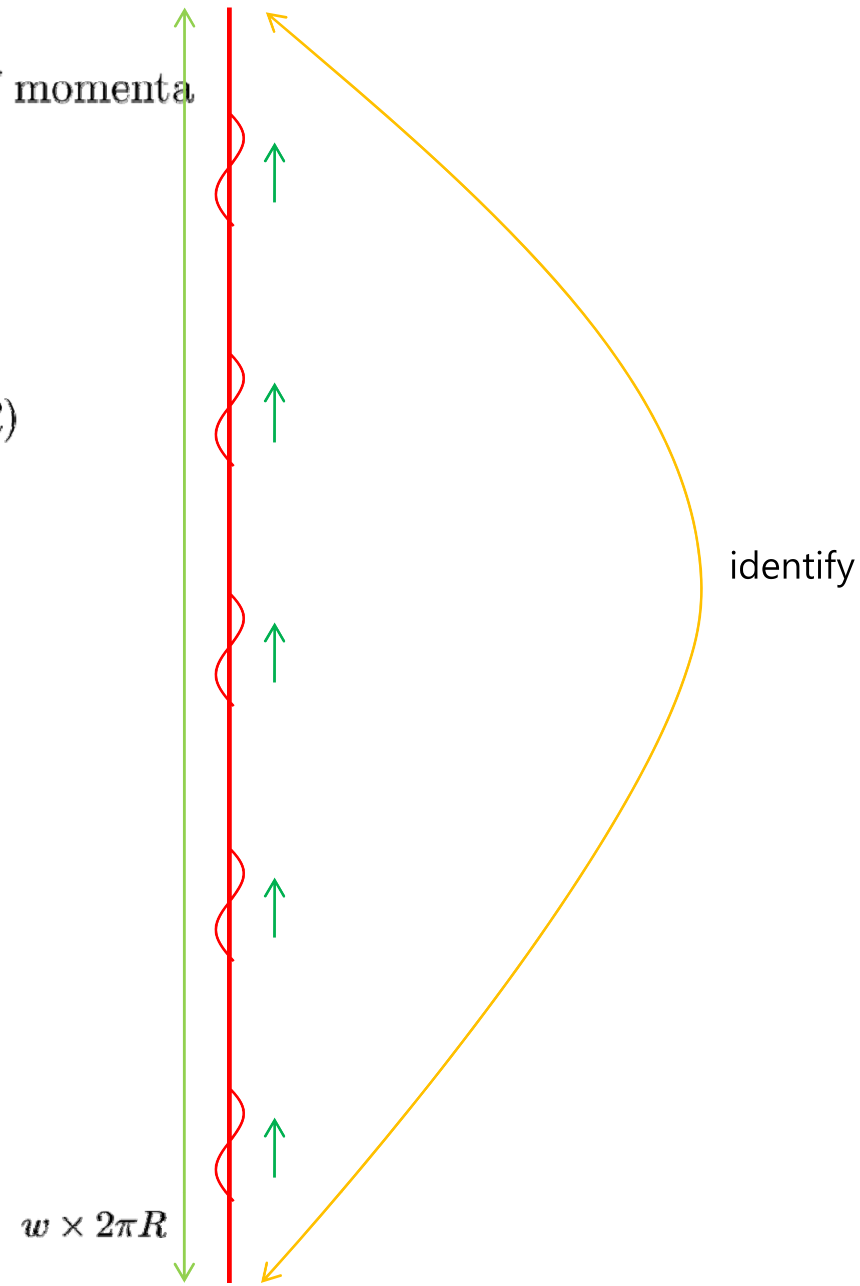
w times wound string with n unit of momenta

$$\text{total momenta} = n/R$$

$$\text{momentum quanta} = k/(w \times R)$$

$$k \in Z_+$$

degeneracy =
the number of possible
collection of integers $\{k_a\}$
such that
$n \times w = \sum_a k_a$



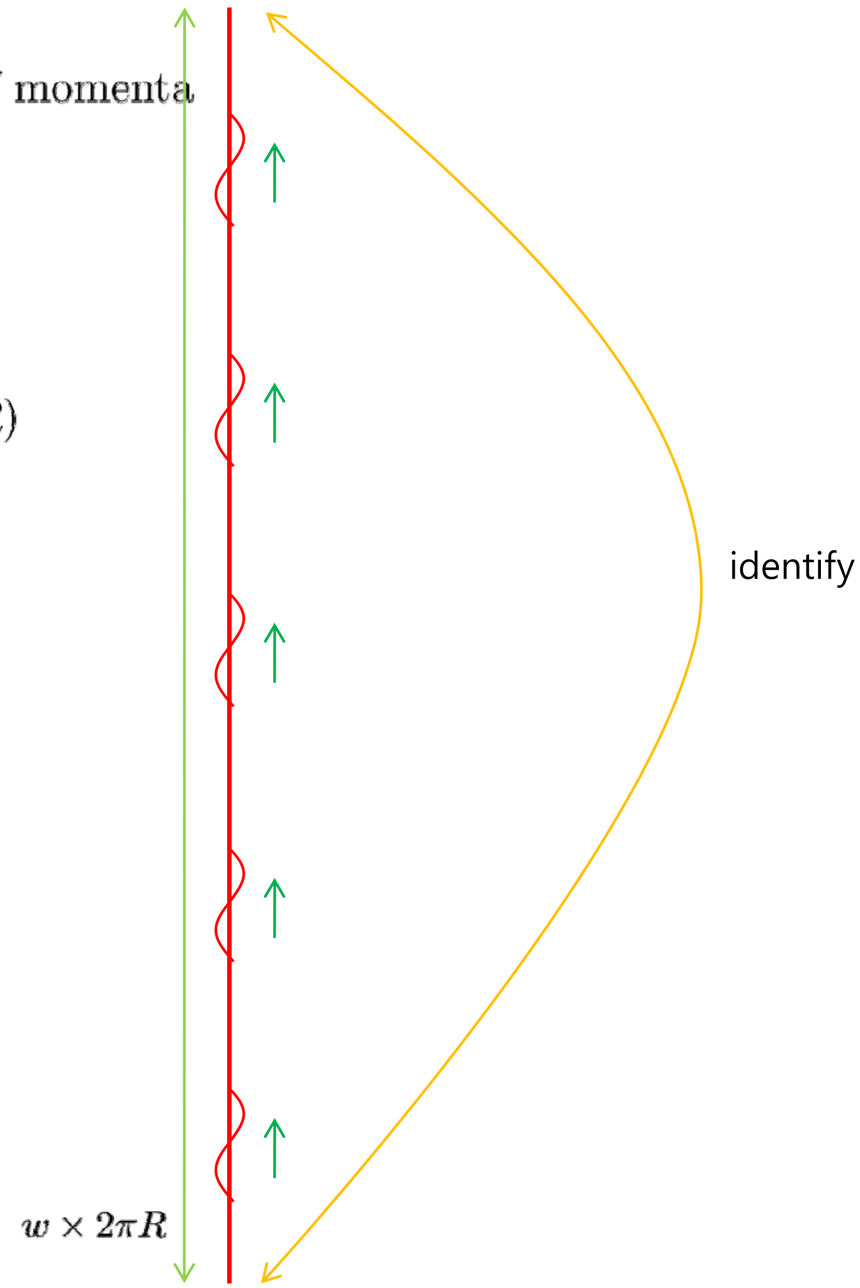
w times wound string with n unit of momenta

total momenta $= n/R$

momentum quanta $= k/(w \times R)$

$k \in Z_+$

degeneracy =
the number of
partitions of
nw



$p(N) \equiv$ # of partitions of N

$p(1) = 1$
$p(2) = 2$
$p(3) = 3$
$p(4) = 5$
$p(5) = 7$
$p(6) = 11$
$p(7) = 15$
$p(8) = 22$
$p(9) = 30$
$p(10) = 42$
$p(100) = 190{,}569{,}292$
$p(200) = 3{,}972{,}999{,}029{,}388$
$p(1000) = 24{,}061{,}467{,}864{,}032{,}622{,}473{,}692{,}149{,}727{,}991$

$p(N) \equiv$ # of partitions of N

$$\sum_N p(N) x^N = \prod_{m=1}^{\infty} \frac{1}{1-x^m}$$

$N \gg 1$ → $p(N) \sim \frac{1}{4\sqrt{3}\,N} \exp(\pi\sqrt{2N/3})$

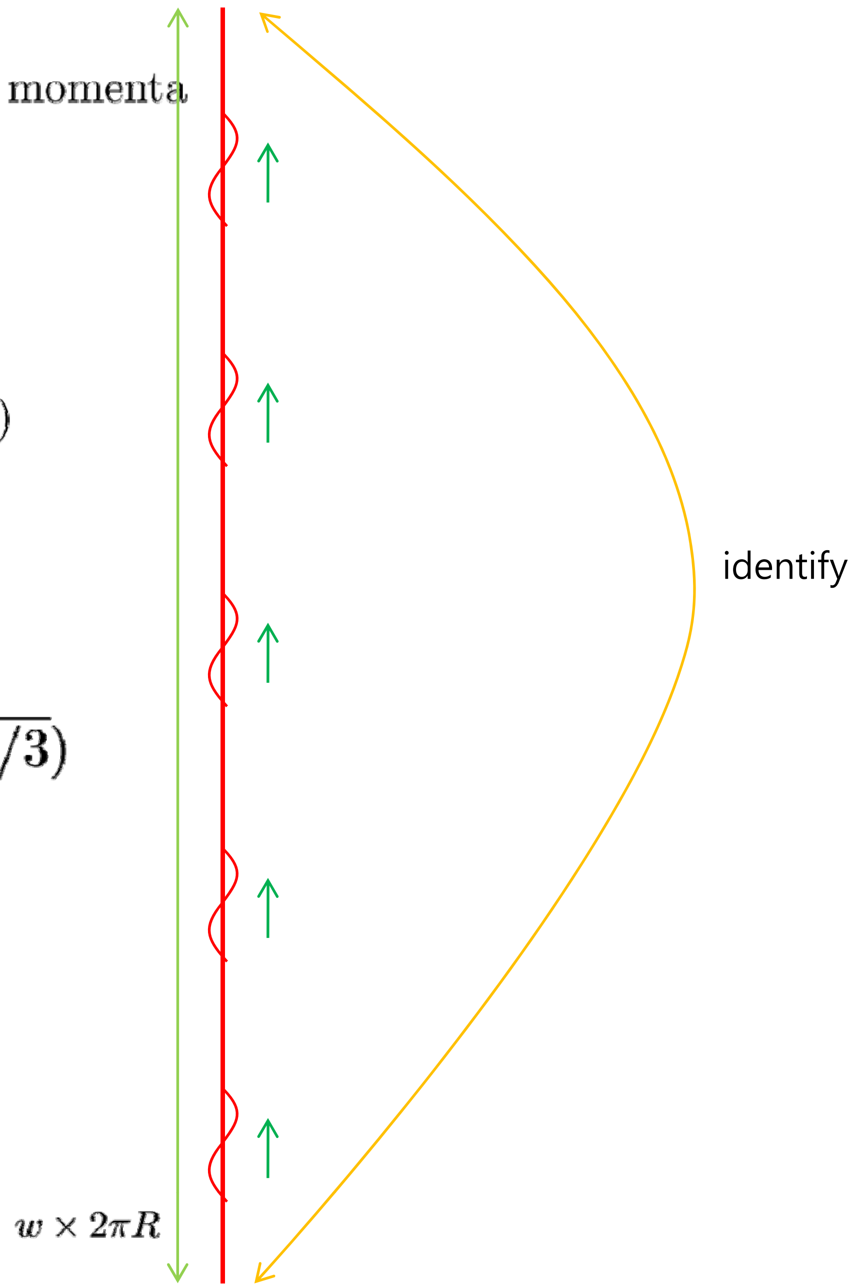
w times wound string with n unit of momenta

total momenta $= n/R$

momentum quanta $= k/(w \times R)$

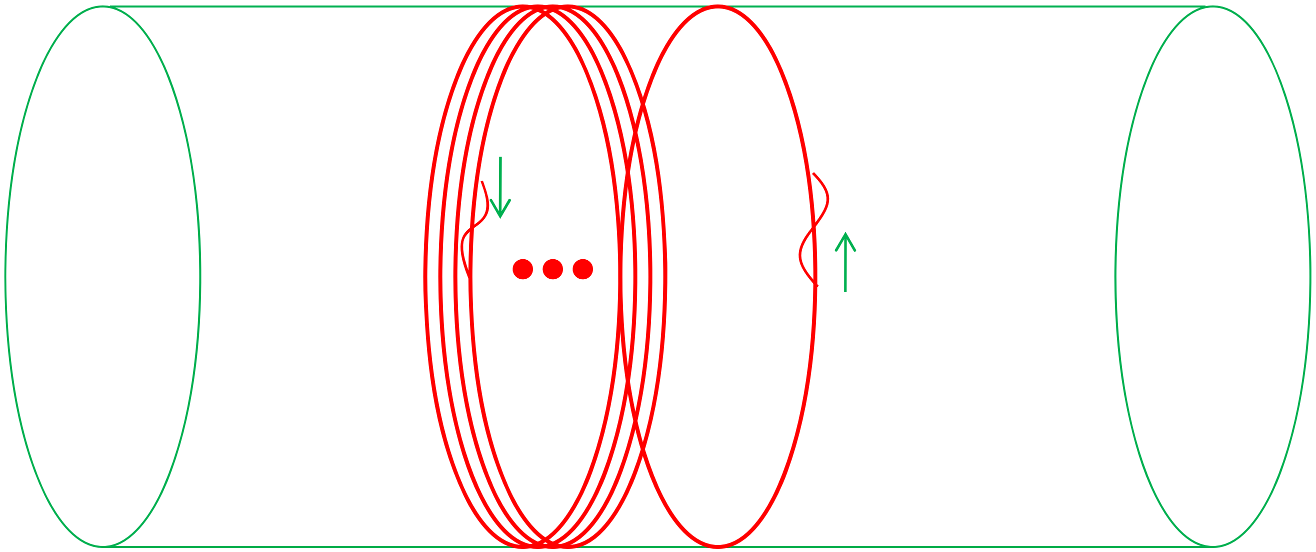
$k \in Z_+$

degeneracy $\sim \exp(\pi\sqrt{2nw/3})$



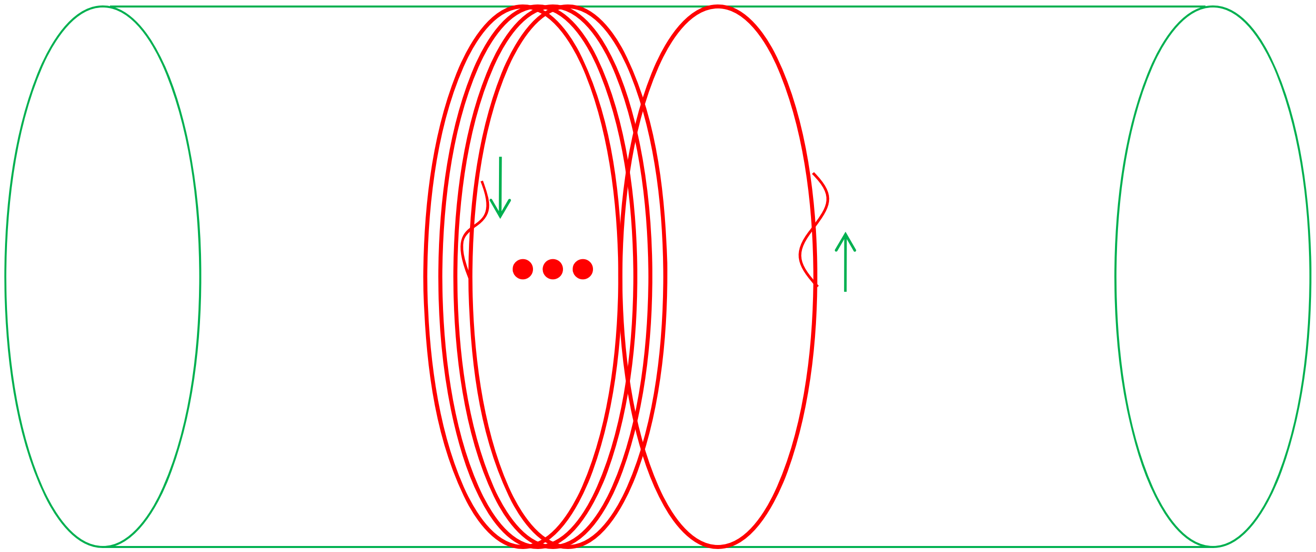
$$\text{energy} \sim \sqrt{w^2(2\pi R)^2\tau^2_{string} + n^2/R^2}$$

$$\log\,(\text{degeneracy}) \sim \sqrt{nw}$$



w times wound string with n unit of momenta

$$\text{degeneracy} \sim \exp(\# \cdot \text{energy}/\sqrt{\tau_{string}}) \;\; \text{if} \;\; n \sim 2\pi R^2 \tau_{string} w$$



w times wound string with n unit of momenta

In string theory, as we increase the gravitational coupling,
this example is known to become a special type of
supersymmetric black holes
whose horizon area (instead of length) is proportional to
the black hole mass

More generally, diverse supersymmetric black holes have known quantum counterpart in string theory (identified via masses and conserved charges), whereby the quantum degeneracy of the latter matches exactly that of the black hole entropy.

However, so far, this is done only for so-called BPS black holes, all of which has zero Hawking temperature, or its very energy excitations with very low Hawking temperature

Even so, it would not tell us exactly
how Hawking's unitarity violation argument is evaded.

The resolution would require the knowledge of
how radiations at different time are correlated with one another at
quantum level.

this puzzle, widely known as

the Black Hole information problem,

remains an unsolved problem.