Project for “Topological Matter”

From the lecture, you have learned about the topological insulators both in quantum Hall and spin quantum Hall states. You have also been introduced to that various topological excitations existing in both the single and bilayer quantum Hall systems. These excitations generically carry a topological charge, whose density is given by the following formulae

$$\rho(\vec{r}) = \frac{1}{8\pi}\varepsilon^{\alpha\beta}\vec{m}\cdot\partial_\alpha\vec{m}\times\partial_\beta\vec{m}$$

where $\vec{m}(\vec{r})$ stands for the spin texture and $\alpha,\beta = x,y$.
For the meron, the spin texture $\vec{m}(\vec{r})$ can be written by

$$m_x = \frac{r\cos\theta}{\sqrt{\lambda^2 + r^2}}, \quad m_y = \frac{r\sin\theta}{\sqrt{\lambda^2 + r^2}}, \quad m_z = \frac{\lambda}{\sqrt{\lambda^2 + r^2}}$$

(1) Plot the meron spin texture and obtain the charge density profile of the meron. Explicitly show that the total charge is quantized to be 1/2 independent of λ.

(2) Propose a spin texture for the anti-meron excitation with total charge -1/2.

(3) In the paper of Ref[1], the authors have shown that the HgTe quantum wells undergo a quantum phase transition at certain sample thickness from the typical insulator to the quantum spin Hall insulator. They also mention that the meron of a sort have a role in the phase transition. Explain this topological quantum phase transition based on the meron picture following Ref[1].

Reference

About Quantum Spin Hall Effect in HgTe Quantum Wells

1. B. Bernevig, T. Hughes, S.C. Zhang, Science 314, 1757 (2006).
   <Pdf file available upon asking>

About Solitons in general;

2. R. Rajaraman, “Solitons and Instantons”.

About skyrmions and merons in quantum Hall system

3. K. Moon et al., Phys. Rev. B 51, 5138 (1995).