Low Energy Effective Action of Topological Insulator

Recently, it has been argued that the low energy effective action for so-called *Topological Insulators* is described by the axionic electrodynamics, where the Maxwell Lagrangian is modified by a term proportional to $\theta \mathbf{E} \cdot \mathbf{B}$. The electric and magnetic fields are given by $\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t$ and $\mathbf{B} = \nabla \times \mathbf{A}$. The resulting action for the electromagnetism in the cgs unit is

$$S_0 = \int d^3x dt \Big[\frac{1}{8\pi} \big(\epsilon \mathbf{E}^2 - \frac{1}{\mu} \mathbf{B}^2 \big) + \frac{\theta}{2\pi} \frac{\alpha}{2\pi} \mathbf{E} \cdot \mathbf{B} \Big] \tag{0.1}$$

where ϵ and μ are permittivity and permeability, respectively. The parameter $\alpha = e^2/\hbar c \approx 1/137$ in the cgs unit is the fine structure constant, and θ is the phenomenological axionic parameter. As ϵ jumps at dielectric material, θ is supposed to jump to some value inside a topological insulator.
(1) Find the axionic Maxwell equation when ϵ, μ, θ depend on space and time. Note that when θ is constant, the θ contribution disappear.
(2) Assume that $\epsilon = \epsilon_0$ and $\mu = \mu_0$ take the vacuum value, so that $c = 1/\sqrt{\epsilon_0 \mu_0}$, and the axionic parameter θ jumps from θ_1 ($z < 0$) to θ_2 ($z > 0$) along the boundary plane $z = 0$ in three dimensional space. Set up a boundary condition for the electric and magnetic fields.
(3) Find the electromagnetic field made by a point particle of electric charge e at position $z = z_0 > 0$ by the image method.
(4) Find the rotation of plane polarization of an incident photon from $z < 0$.

Ref: X. Qi, T. Hughes, and S.-C. Zhang, Phys. Rev. B78, 195424 (2008), 0802.3537; X.-L. Qi, R. Li, J. Zang, and S.-C. Zhang, Science 323, 1184 (2009)