KIAS-SNU Physics Winter Camp (January 5-18, 2005)

Mini Research Topic:
Suppressed backscattering in integer quantum Hall effect

When electrons in a two-dimensional electron gas (2DEG) are subject to a high magnetic field, transport properties may show nontrivial behaviors. In particular, when the electron density n_e is close to an integer multiple of eB/hc, the longitudinal resistance drops to zero while the Hall resistance has the quantized value $h/\nu e^2$, where $\nu = n_e/(eB/hc)$. This effect is the so-called integer Hall effect.

This mini research topic aims to understand why the longitudinal resistance becomes zero or more specifically why the backscattering is suppressed in the integer quantum Hall effect. The steps provided below provide a pathway leading to the understanding of the suppressed backscattering. Though the description in the suggested steps is self-contained, you may still find useful the following references for integer quantum Hall effect:

- *Introduction to Solid State Physics* by C. Kittel, Chap. 19.
- *The Quantum Hall Effect* by R. E. Prange and S. M. Girvin (Spinger-Verlag, New York, 1990).

[Step A] Electron motion in a 2DEG with magnetic field perpendicular to the 2DEG can be described by the following Schrödinger equation,

$$\left[E_s + \frac{1}{2m}\left(\frac{\hbar}{i}\nabla + \frac{e}{c}\mathbf{A}\right)^2 + U(\mathbf{r})\right]\psi(\mathbf{r}) = E\psi(\mathbf{r}). \tag{1}$$

Without loss of generality, we may take $E_s = 0$. And for simplicity, let us assume $U(\mathbf{r}) = 0$. The Schrödinger equation then describes electron motion in an impurity-free infinite system. Let us also make the following gauge choice, $\mathbf{A} = (A_x, A_y) = (-By, 0)$. Eigenfunctions of the system can be written as

$$\psi(x, y) \propto \exp(ikx)\chi(y).$$

Find the equation that $\chi(y)$ should satisfy.
(If you get the result right, you should find that the equation for $\chi(y)$ is nothing but a Schrödinger equation for a harmonic oscillator whose potential energy minimum location, say y_0, depends on k, that is, $y_0 = y_0(k)$.)

[Step B] Solve the harmonic oscillator equation for $\chi(y)$ and thus find the eigenenergies $E_n(k)$ and eigenfunctions $\psi_n(k)$ of the Schrödinger equation (1). Here $n = 0, 1, 2, 3, \ldots$ is the quantum number for the harmonic oscillator. Show

that the eigenstates are extended in the x-direction while they are strongly localized in the y-direction within the typical length scale $l_B = \sqrt{\hbar c/eB}$, which can be smaller than other relevant length scales in the large B limit. Show also that despite the wavefuntions being extended in the x-direction, *they do NOT carry electric current.*
(If you get the result correct, you should find the famous Landau levels.)

[Step C] Now we restore the potential $U(\mathbf{r})$. Let us assume that $U(\mathbf{r})$ describes the confining potential in the y-direction, i.e., $U(\mathbf{r}) = U(y)$. For sufficiently strong magnetic field, we may assume that the typical length scale over which $U(y)$ varies is much larger than l_B. In this large magnetic field limit, show that the eigenfunctions are essentially unaffected by $U(y)$ while the new eigenvalues $E_n^U(k)$ are given by $E_n(k) + U(y_0(k))$. Since the wavefunction for k is strongly localized near $y = y_0(k)$, we may say with good accuracy that an electron at $y = y_0(k)$ has the energies $E_n^U(k)$ and thus we may define a relation between the energies and y. Draw this relation for the following $U(y)$,

$$U(y) = \begin{cases} 0 & \text{for } |y| < L_y/2 \\ \frac{1}{2}m\omega_0^2(|y| - L_y/2)^2 & \text{for } |y| > L_y/2 \end{cases}$$

where L_y, $\sqrt{\hbar/m\omega_0} \gg l_B$. Here the region with $|y| < L_y/2$ may be regarded as the bulk part and the regions with $|y| > L_y/2$ as the edge parts of the system.

[Step D] Show then that the current in the bulk part is zero and only two edge parts ($y > L_y/2$ and $y < -L_y/2$) carry current. Show also that in the upper (lower) edge part, there are only left-(right-)moving electrons (for $B > 0$).

[Step E] We then attach two voltage contacts 1 and 2 to the upper edge and one voltage contact 3 to the lower edge. Show that the contacts 1 and 2 will couple only to the left-moving electrons while the contact 3 will only to the right-moving electrons.

[Step F] Let us assume that the chemical potential is chosen in such a way that in the bulk part, electrons with $n \leq m$ are occupied (or $m + 1$ Landau levels are occupied). Then how many transport channels appear in the upper and lower edge parts? Calculate the longitudinal resistance $R \equiv (V_1 - V_2)/I$ and the Hall resistance $R_H \equiv (V_2 - V_3)/I$. Show that R is equal to the ideal 4-terminal resistance value and R_H to the ideal 2-terminal resistance value.

[Step G]: *Suppressed backscattering in Integer Quantum Hall effect*
Up to now, it has been assumed that the system is free of impurities. Here let us consider effects of impurities. The leading contribution to scattering amplitudes (first-order Born approximation) comes from the overlap integral $\langle k|U|k'\rangle$, where $|k\rangle$ and $|k'\rangle$ represent current-carrying states in the upper and lower edges, respectively. Show that the overlap integral is exponentially small. Show that thus back-scattering by impurities is strongly suppressed in the high magnetic field limit. This explains why the quantized resistance values in [Step

F] can be indeed measured in integer quantum Hall experiments even in the presence of impurities.