X

Charges associated with these are

$$Q_L = \int d^3x\, j^0_L = \bar{\psi}_L \gamma^0 \psi_L = \psi_L^\dagger \psi_L = N_L$$

$$Q_R = \int d^3x\, j^0_R = \bar{\psi}_R \gamma^0 \psi_R = \psi_R^\dagger \psi_R = N_R$$

Now recall

$$K_\mu = 4g^2 \epsilon_{\mu\nu\alpha\beta} \operatorname{Tr}\left\{ A_\nu \partial_\alpha A_\beta + \frac{2ig}{3} A_\nu A_\alpha A_\beta \right\}$$

$$\partial_\mu K^\mu = 2g^2 \operatorname{Tr} F\tilde{F}$$

$$\int d^3x\, K^0 = 32\pi^2 \times [\text{winding } \#]$$

Define $\quad j^\mu_A = \frac{1}{32\pi^2} K^\mu$

and

$$J^\mu = j^\mu_5 - 2N_f\, j^\mu_A$$

$$= j^\mu_R - j^\mu_L - 2N_f\, j^\mu_A$$

$$\partial_\mu J^\mu = \frac{2N_f}{16\pi^2} g^2 \operatorname{tr} F\tilde{F} - \frac{2N_f}{16\pi^2} g^2 \operatorname{tr} F\tilde{F}$$
$$= 0$$

$$Q = \int d^3x\, J^0 = N_R - N_L \mp (\text{winding } \#) \times (2N_f)$$

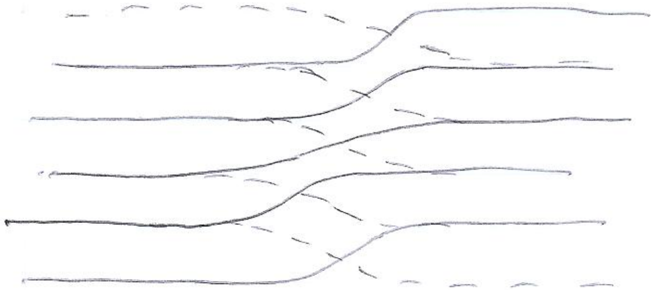
$J^\mu \to$ gauge-variant, but divergenceless

$A_0 = 0$ gauge, Q conserved

other gauges $\to$ flow through surface at $r \to \infty$

$\Rightarrow$ Shifting $|n\rangle$ vac (e.g., by tunneling), shifts chirality

Dirac sea picture:



$|n\rangle$ $|n+1\rangle$

New & old spectra equiv., just shifted

Adiabatic approx:

t_{init} : Vac $\rightarrow$ all neg E states filled
all pos E " empty

t_{final} : occupy 1 pos E R state
" 1 neg E L state

Effect is as if had term in effective $\mathcal{L}$ of form

$$\mathcal{L}_1 = \prod_j \bar{\psi}_R^j \psi_L^j$$

Under chiral transf.,

$$\psi_L^j \rightarrow U^L_{jk} \psi_L^k \qquad , U \in SU(N_f)$$

$$\psi_R^j \rightarrow U^R_{jk} \psi_R^k$$

$$\prod_j \psi_L^j \rightarrow \prod_j U^L_{jk} \psi_L^k = (\det U^L) \prod_k \psi_L^k$$

similar for $\prod_k \bar{\psi}_R^k$

$\Rightarrow$ Extra term in $\mathcal{L}_{eff}$ invariant under transf. with ~~$U^L = U^R$~~ $\det U^L = \det U^R = 1$

$\rightarrow$ invariant under chiral $SU(N_f) \times SU(N_f)$

$\rightarrow$ breaks chiral $U(1) \times U(1)$ symmetry

U(1) prob. of QCD

m_u, m_d small seems to $\Rightarrow$ approx $U(2) \times U(2)$

m_u, m_d, m_s " " " $\Rightarrow$ approx $U(3) \times U(3)$

spont. broken $\Rightarrow$ $m=0$ Goldstone bosons if exact, light pseudoscalars if approx

<u>But</u> $m_\pi \approx 139$ MeV
$m_\eta \approx 549$ MeV $\gg m_\pi$

OR

$m_\pi \approx 139$
$m_K \approx 495$
$m_\eta \approx 549$
$m_{\eta'} \approx 958$ $\gg \pi, K, \eta$

Now understand, bec. instanton effective term $\sim (\bar\psi\psi)^{N_F}$ breaks chiral U(1)

Note: effects are <u>not</u> exponentially suppressed

B - violation in $SU(2) \times U(1)$

① Effective instanton

→ With $\langle \phi \rangle = v$, S_{Euc} decreases with instanton size λ

⇒ classically, only $\lambda \to 0$ singular solution

<u>But</u>, expect loop effects to stabilize scale at $\lambda \sim m_W^{-1}$

⇒ fixed instanton size, no integral over λ

② Consider (almost) massless fermions coupled to $SU(2)$ field strength:

$$\underbrace{(\nu, e)_L, \ (u, d)_L}_{\text{in 3 colors}}$$

⇒ Envision processes with $\Delta L = \pm 2$, $\Delta B = \pm 2$

→ B-L conserved, but B+L broken

e.g., $p + n \to e^+ + \bar{\nu}$

Rate: proportional to $\left(e^{-S_{inst}}\right)^2 = e^{-\frac{2(8\pi^2)}{g^2}} = e^{-\frac{16\pi^2}{g^2}}$

evaluate g^2 at $\sim m_W$; $g = \frac{e}{\sin\theta_W}$

$$\frac{16\pi^2}{g^2} = 4\pi \left(\frac{4\pi}{e^2}\right) \sin^2\theta_W \approx \cancel{200}\ 400$$

$$\Rightarrow \text{rate} \sim \cancel{10^{-400}}\ \cancel{\sim e^{-174}}\ e^{-400} \sim 10^{-176}$$

Note: obs. univ. has $\sim 10^{78}$ protons, ~~t_{univ}~~

$t_{univ} \sim 10^{10}$ yrs $\sim 10^{40} \times (10^{-23}$ sec$)$

⇒ Negligible effect at $T = 0$

But: $T \neq 0$ → fluctuate over barrier

→ Sphalerons

→ Implic for Baryogenesis

Effects of $\theta F\tilde{F}$ term

$\epsilon_{\mu\nu\alpha\beta}$ $\Rightarrow$ violates P, T
$\Rightarrow$ by CPT, must violate CP

Could this be explanation for observed CP violation?

<u>NO</u>

Consider: a) observed ~~CP~~ in K-system

$$\text{e.g.,} \quad \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \sim 10^{-3}$$

b) neutron elec. dipole moment
[Violates T because $\langle \vec{E} \cdot \vec{d} \rangle \sim \langle \vec{E} \cdot \vec{S} \rangle$
(cf., mag. dipole moment)]

$< 0.29 \times 10^{-25}$ e-cm
Natural scale might be $\sim 10^{-13}$ e-cm
$\Rightarrow$ suppression of $\sim 10^{-12} - 10^{-13}$

$\Rightarrow$ need $\theta \lesssim 10^{-9}$

This would be too small to explain ~~CP~~ in K-system
Latter is fit by CKM

<u>But</u> have problem: Why is θ so small?
Strong CP-problem

Now include effect of fermion mass terms

Usually write

$$m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

Suppose m complex:

$$m\bar{\psi}_L\psi_R + m^*\bar{\psi}_R\psi_L$$

Could make m real by $\psi \to e^{i\beta\gamma_5}\psi$, with approp. β

Noether-type args, using fact that this is ~~not~~ anomalous

$$\Rightarrow \Delta\mathcal{L} = 2i\beta m\bar{\psi}\gamma^5\psi + \frac{2\beta}{16\pi^2}\,\mathrm{tr}\, F\tilde{F}$$

$$\Rightarrow \quad m \to m(1+i\beta), \quad \theta \to \theta + \beta$$

$\Rightarrow$ Change in phase of $m \Leftrightarrow$ change of θ

Generalize to many fields: $[\ \bar{\psi}_L M \psi_R\ ]$

$$\bar{\theta} = \theta - \arg\det M$$

is the invariant quantity, relevant number for strong CP violation

If masses from Higgs (of arbitrary phase), even less likely to get small θ

Solutions ?.

1) Finely-tuned θ

2) One quark massless [m_u?], so phase undetermined, can rotate ~~away~~ away θ

3) Add extra scalar, extra sym. (Peccei - Quinn)
$\Rightarrow \theta_{eff}$ dynamically $\rightarrow 0$

But then have pseudo-Goldstone boson - AXION