Standard Model Phenomenology

The First Part

- Prelude
- Poincaré Group
- The Dirac Equation
- From Lagrangian to Cross Section

The Second Part

- The Standard Model
- SM Higgs Phenomenology

8th Summer Institute of Theoretical Physics @ KIAS

Prelude

- Goal : When you hear a rumor,
  - understand the physics immediately for yourself.
  - write a paper within a week.
- Baselines
  - 4–dimensional spacetime

$$x^\mu = (t, x, y, z), \quad x_\mu = (t, -x, -y, -z)$$
$$p^\mu = (E, p_x, p_y, p_z), \quad p_\mu = (E, -p_x, -p_y, -p_z)$$
$$\text{metric}: \; g_{\mu\nu} = g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$$
$$x^2 \equiv x^\mu x_\mu = g_{\mu\nu} x^\mu x^\nu = t^2 - x^2 - y^2 - z^2$$

  - Consider only the physics to be confirmed at colliders. (Gravitation is not considered in this lecture.)
- Framework : Renormalizable relativistic local field theory.
- Key concepts
  - Symmetry
  - Mass
- Basic strategy : Perturbation theory

Poincaré Group

The Poincaré group is the fundamental spacetime symmetry group of translations and Lorentz transformations; any physical object that lives in the Minkowski space of the four dimensional spacetime must belong to some representations of the Poincaré group.

The Lorentz transformation to the spacetime coordinate x^μ

$$x'^\mu = \Lambda^\mu{}_\nu \, x^\nu$$

leaving the length of the four vector x^μ invariant as

$$x'^2 = g_{\mu\nu} \, x'^\mu x'^\nu = x^2$$

should satisfy the relation:

$$g_{\mu\nu} \, \Lambda^\mu{}_\rho \Lambda^\nu{}_\tau = g_{\rho\tau}$$

The six (anti-symmetric) generators $M_{\rho\sigma}$ of the Lorentz group defined by

$$\Lambda^\mu{}_\nu = \left[\exp\left(-\frac{i}{2} \omega^{\rho\sigma} M_{\rho\sigma} \right) \right]^\mu{}_\nu$$

satisfy the Lie algebra

$$[M_{\mu\nu}, \, M_{\rho\sigma}] = i \, [g_{\mu\sigma} M_{\nu\rho} - g_{\mu\rho} M_{\nu\sigma} + g_{\nu\rho} M_{\mu\sigma} - g_{\nu\sigma} M_{\mu\rho}]$$

which can be expressed as

$$\begin{aligned} [J_i, \;\; J_j\,] &= +i \, \epsilon_{ijk} \, J_k \\ [J_i, \;\; K_j] &= +i \, \epsilon_{ijk} \, K_k \\ [K_i, \; K_j] &= -i \, \epsilon_{ijk} \, J_k \end{aligned}$$

in terms of the generators of rotations and boosts

$$J_i \equiv \frac{1}{2} \epsilon_{ijk} M_{jk}, \qquad K_i \equiv M_{0i}$$

The mixed algebra of $\{J_i, K_i\}$ can be diagonalized

$$\left[N_i^-,\ N_j^+\right] = 0$$
$$\left[N_i^-,\ N_j^-\right] = +i\,\epsilon_{ijk}\,N_k^-$$
$$\left[N_i^+,\ N_j^+\right] = +i\,\epsilon_{ijk}\,N_k^+$$

by introducing two generators

$$\mathcal{L} : N_i^- \equiv \frac{1}{2}(J_i + i\,K_i)$$
$$\mathcal{R} : N_i^+ \equiv \frac{1}{2}(J_i - i\,K_i)$$

In terms of N_i^- and N_i^+, we can construct SU(2)$_L$×SU(2)$_R$ representations of the Lorentz group

$$(n, m) \quad : \quad \begin{cases} N^{-2}|(n,m)\rangle = n\,(n+1)\,|(n,m)\rangle \\ N^{+2}|(n,m)\rangle = m(m+1)|(n,m)\rangle \end{cases}$$

The two SU(2) generators are related by Parity P

$$P \ : \ \begin{cases} J_i \ \to +J_i \\ K_i \ \to -K_i \end{cases} \quad \Rightarrow \quad P : \quad N_i^- \ \leftrightarrow \ N_i^+$$

Spinors

Now we introduce 2–component spinors

$$\left(\frac{1}{2}, 0\right) \ : \quad \psi_L(x) = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \{\eta_\alpha\} \quad \Rightarrow \quad \psi_L'(x') = \Lambda_L \psi_L(x)$$
$$\left(0, \frac{1}{2}\right) \ : \quad \psi_R(x) = \begin{pmatrix} \bar{\chi}^1 \\ \bar{\chi}^2 \end{pmatrix} = \{\bar{\chi}^{\dot{a}}\} \quad \Rightarrow \quad \psi_R'(x') = \Lambda_R \psi_R(x)$$

The generators in the spinor representation are expressed as

$$J_i = \frac{1}{2}\sigma^i, \qquad K_i = -\frac{i}{2}\sigma^i$$

with the Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \sigma^i\sigma^j = \delta_{ij} + i\,\epsilon_{ijk}\sigma^k$$

For the rotation of $\vec{\omega}$ and the boost of $\vec{\kappa}$, the generators are given by

$$\Lambda_L = \text{exp}\left\{-i(\vec{J}\cdot\vec{\omega} + \vec{K}\cdot\vec{\kappa})\right\} = \text{exp}\left\{-i\frac{\vec{\sigma}}{2}\cdot(\vec{\omega} - i\vec{\kappa})\right\}$$

$$\Lambda_R = \text{exp}\left\{-i(\vec{J}\cdot\vec{\omega} - \vec{K}\cdot\vec{\kappa})\right\} = \text{exp}\left\{-i\frac{\vec{\sigma}}{2}\cdot(\vec{\omega} + i\vec{\kappa})\right\}$$

The magnitude $\omega = |\vec{\omega}|$ is the rotation angle while the magnitude of the boost vector $\vec{\kappa}$ [which is parallel to the velocity vector $\vec{\beta}$] is called the rapidity related to the speed β as

$$\text{e}^{\kappa} = \gamma(1+\beta) = \sqrt{\frac{1+\beta}{1-\beta}}; \quad \cosh\kappa = \gamma, \ \sinh\kappa = \gamma\beta$$

Vectors

Polarization vectors of a vector boson in the frame where its momentum is chosen as 3–axis can be obtained by boosting the polarization vectors in the rest frame along the 3-axis:

$$p^\mu = (m,0,0,0) \quad \overset{\text{boost}}{\longrightarrow} \quad (E,0,0,p) \ \text{ with } \ E = \gamma m,\, p = \gamma\beta m$$

$$
\begin{array}{llll}
\epsilon^\mu(p,1) = (0,1,0,0) & & (0,1,0,0) & : \text{transverse} \\
\epsilon^\mu(p,2) = (0,0,1,0) & & (0,0,1,0) & : \text{transverse} \\
\epsilon^\mu(p,3) = (0,0,0,1) & \xrightarrow{\text{boost}} & (p/m,0,0,E/m) & : \text{longitudinal}
\end{array}
$$

These vectors satisfy

$$
p_\mu \epsilon^\mu(p,a) = 0, \quad \epsilon_\mu(p,a)\,\epsilon^\mu(p,b) = -\delta_{ab}\,; \quad a,b = 1,2,3
$$

and the helicity spin–1 states are determined from the above polarization vectors by

$$
\epsilon^\mu(p,\pm) = \frac{1}{\sqrt{2}}\left[\mp\,\epsilon^\mu(p,1) - i\,\epsilon^\mu(p,2)\right]
$$
$$
\epsilon^\mu(p,0) = \epsilon^\mu(p,3)
$$

The contraction of the helicity polarization vectors and γ^μ is

$$
\not\epsilon_\pm(p,+) = \pm\sqrt{2}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}
$$

$$
\not\epsilon_\pm(p,-) = \mp\sqrt{2}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}
$$

$$
\not\epsilon_\pm(p,\ 0) = \begin{pmatrix} \frac{1}{m}\,(p \mp E) & 0 \\ 0 & \frac{1}{m}\,(p \pm E) \end{pmatrix}
$$

The Dirac Equation

For a free particle, the only operator which can appear in the wave equation is the 4–momentum operator $p_\mu = i\,\partial_\mu$ or in the spinor notation

$$
p_\mu\ \sigma^\mu_{+\alpha\dot{\alpha}} \ = \begin{pmatrix} p_0 - p_z & -p_x + ip_y \\ -p_x - ip_y & p_0 + p_z \end{pmatrix}
$$

$$p_\mu \, \sigma_-^{\mu \, \dot\alpha\alpha} = \begin{pmatrix} p_0 + p_z & p_x - i p_y \\ p_x + i p_y & p_0 - p_z \end{pmatrix}$$

One simple wave equation can be a linear differential relation between the components of spinors, expressed by the operators $p_\mu \sigma^\mu_{+\alpha\dot\alpha}$ and $p_\mu \sigma^\mu_{-\dot\alpha\alpha}$

$$p_\mu \, \sigma^\mu_{+\alpha\dot\alpha} \bar\chi^{\dot\alpha} = m \, \eta_\alpha \qquad p_\mu \, \sigma_-^{\mu \, \dot\alpha\alpha} \, \eta_\alpha = m \bar\chi^{\dot\alpha}$$

where

$$\sigma^\mu_+ = (1, \ \ \vec\sigma) \qquad \sigma^\mu_- = (1, -\vec\sigma)$$

The need to use the mass in the wave equation implies the simultaneous consideration of two spinors (η_α and $\bar\chi^{\dot\alpha}$); with only one of these, it would not be possible to construct a relativistically invariant equation containing a dimensional parameter. The above relativistic wave equation is called the Dirac equation having been first derived by Dirac in 1928.

The spinor form of the Dirac equation is the most natural one, in the sense that its relativistic invariance is immediately apparent. In applications of the equation, however, other forms of the wave equation may be more convenient. We denote a four–component Dirac spinor by the symbol ψ. In the spinor representation, it is a bispinor

$$\psi = \begin{pmatrix} \eta_\alpha \\ \bar\chi^{\dot\alpha} \end{pmatrix}$$

The Dirac equation is put in terms of the 4–component spinor in the form

$$p_\mu \, \gamma^\mu \psi \equiv \not{p} \psi = m \, \psi$$

The spinor form of the wave equation with the components of the above bispinor corresponds to the 4×4 matrices γ^μ :

$$\text{Weyl/Chiral} \ : \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_+ \\ \sigma^\mu_- & 0 \end{pmatrix}$$

We introduce an additional gamma matrix γ_5 and two chiral projection operators :

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad P_\lambda = \frac{1+\lambda\gamma_5}{2} \qquad \lambda = \pm = \mathsf{R/L}$$

In the general case, the matrices γ^μ need to satisfy only the conditions ensuring that $p^2 = m^2$. To find these conditions, we multiply the Dirac equation by $\not{p}$

$$\not{p}\,\not{p}\psi = \frac{1}{2}p_\mu p_\nu\left(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu\right)\psi = m\,\not{p}\psi = m^2\psi$$

and we must therefore have

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\,g^{\mu\nu}$$

Free–particle Solutions

Let us solve the Dirac equation. We take into account a plane wave solution

$$\psi(x) = u(p)\,\mathsf{e}^{-ip\cdot x} \quad p^2 = m^2 \oplus p^0 > 0 \quad \Rightarrow\ (\gamma^\mu p_\mu - m)u(p) = 0$$

It is easiest to analyze this equation in the rest frame, where $p_0^\mu = (m, \vec{0})$; the solution for a general p can then be found by boosting the rest–frame solution

$$(m\gamma^0 - m)u(p_0) = m\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} u(p_0) = 0 \quad \Rightarrow\ u(p_0) = \sqrt{m}\begin{pmatrix} \xi \\ \xi \end{pmatrix}$$

for any 2–component spinor ξ normalized to be $\xi^\dagger\xi = 1$. Now that we have the general form of $u(p)$ in the rest frame, we can obtain $u(p)$ in any other frame by boosting. Consider a boost along the 3–direction. Then with the rapidity κ the boosted spinor is given by

$$\begin{aligned} u(p) &= \exp\left[-\frac{1}{2}\kappa\begin{pmatrix}\sigma^3 & 0\\ 0 & -\sigma^3\end{pmatrix}\right]\sqrt{m}\begin{pmatrix}\xi\\ \xi\end{pmatrix} \\ &= \left[\cosh\frac{\kappa}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \sinh\frac{\kappa}{2}\begin{pmatrix}\sigma^3 & 0\\ 0 & -\sigma^3\end{pmatrix}\right]\sqrt{m}\begin{pmatrix}\xi\\ \xi\end{pmatrix} \\ &= \begin{bmatrix}\left[\sqrt{E+p^3}\left(\frac{1-\sigma^3}{2}\right)+\sqrt{E-p^3}\left(\frac{1+\sigma^3}{2}\right)\right]\xi \\ \left[\sqrt{E+p^3}\left(\frac{1+\sigma^3}{2}\right)+\sqrt{E-p^3}\left(\frac{1-\sigma^3}{2}\right)\right]\xi\end{bmatrix} \end{aligned}$$

The last line can be simplified to give

$$u(p) = \begin{pmatrix}\sqrt{p\cdot\sigma_+}\,\xi \\ \sqrt{p\cdot\sigma_-}\,\xi\end{pmatrix}$$

where it is understood that in taking the square root of a matrix we take the positive root of each eigenvalue. This expression for $u(p)$ is not only more compact, but is also valid for any arbitrary direction of $\vec{p}$.

The amplitude of the plane wave contains one arbitrary two–component quantity ξ. Thus, for a given momentum, there are two different independent states, corresponding to the two possible values of the spin component. But, in the relativistic theory the orbital angular momentum $\vec{l}$ and the spin $\vec{s}$ of a moving particle are not separately conserved. Only the total angular momentum $\vec{j} = \vec{l} + \vec{s}$ is conserved. The component of the spin in any fixed direction is therefore also not conserved. However, the component of the spin in the direction of the momentum is conserved; since $\vec{l} = \vec{r}\times\vec{p}$ the product $\hat{p}\cdot\vec{s}$ is equal to the conserved product $\hat{p}\cdot\vec{j}$. This quantity is called the helicity. Helicity states correspond to plane waves in which $\xi = \xi_\lambda$ is an eigenvalue of the operator $\hat{p} \cdot \hat{\sigma}$:

$$\frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}\, \xi_\lambda(p) = \lambda\, \xi_\lambda(p) \qquad \lambda = \pm$$

If we write

$$\frac{\vec{p}}{|\vec{p}|} = (n_x, n_y, n_z) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

then the normalized helicity eigenstates can be expressed as

$$\xi_+(p) = \frac{1}{\sqrt{2(1+n_z)}} \begin{bmatrix} 1+n_z \\ n_x + i n_y \end{bmatrix} = \begin{bmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\, \mathrm{e}^{i\phi} \end{bmatrix}$$

$$\xi_-(p) = \frac{1}{\sqrt{2(1+n_z)}} \begin{bmatrix} -n_x + i n_y \\ 1+n_z \end{bmatrix} = \begin{bmatrix} -\sin\frac{\theta}{2}\, \mathrm{e}^{-i\phi} \\ \cos\frac{\theta}{2} \end{bmatrix}$$

apart from arbitrary phases. In the helicity basis we can express the Dirac spinor as

$$u(p,\lambda) = \begin{bmatrix} (E-\lambda p)^{1/2}\, \xi_\lambda(p) \\ (E+\lambda p)^{1/2}\, \xi_\lambda(p) \end{bmatrix} \equiv \begin{pmatrix} u_-(p,\lambda) \\ u_+(p,\lambda) \end{pmatrix}$$

The subscripts are so chosen as to satisfy

$$P_+ u(p,\lambda) = \frac{1+\gamma_5}{2} u(p,\lambda) = \begin{pmatrix} 0 \\ u_+(p,\lambda) \end{pmatrix}$$

$$P_-(p,\lambda) = \frac{1-\gamma_5}{2} u(p,\lambda) = \begin{pmatrix} u_-(p,\lambda) \\ 0 \end{pmatrix}$$

In addition to the positive–frequency plane wave solution $u(p)$, there exists a negative–frequency plane wave solution

$$\psi(x) = v(p)\, \mathrm{e}^{+ip\cdot x}$$

satisfying the same Dirac equation. The easiest way to find this solution is to consider charge conjugation represented by a unitary matrix C:

$$C\gamma_\mu^T C^\dagger = -\gamma_\mu \qquad CC^\dagger = 1 \qquad \Rightarrow \qquad C = -i\gamma^2\gamma^0$$
$$v(p,\lambda) = C\bar{u}^T(p,\lambda) = \left(-i\gamma^2\gamma^0\right)\gamma^0 u^*(p,\lambda) = \begin{pmatrix} 0 & -i\sigma^2 \\ i\sigma^2 & 0 \end{pmatrix} u^*(p,\lambda)$$

which can be derived through the following procedure

$$\begin{aligned}
&\left(i\gamma^\mu\partial_\mu - m\right)\psi = 0\\
&\psi^\dagger\left(-i\overleftarrow{\partial}_\mu\gamma^{\mu\dagger} - m\right) = 0\\
&\psi^\dagger\gamma^0\left(-i\overleftarrow{\partial}_\mu\gamma^0\gamma^{\mu\dagger}\gamma^0 - m\right) = 0\\
&\bar{\psi}\left(-i\overleftarrow{\partial}_\mu\gamma^\mu - m\right) = 0\\
&\left(-i\gamma^{\mu\,T}\partial_\mu - m\right)\bar{\psi}^T = 0\\
&\left(-iC\gamma^{\mu\,T}C^\dagger\partial_\mu - m\right)C\bar{\psi}^T = 0
\end{aligned}$$

Noting that $i\,\sigma^2\,\xi_\lambda^* = -\lambda\xi_{-\lambda}$ $[\lambda = \pm]$ we can obtain the negative–frequency solution in the helicity basis as

$$v(p,\lambda) = \lambda\left[\begin{array}{c} -(E+\lambda p)^{1/2}\,\xi_{-\lambda}(p) \\ (E-\lambda p)^{1/2}\,\xi_{-\lambda}(p)\end{array}\right] \equiv \begin{pmatrix} v_-(p,\lambda) \\ v_+(p,\lambda)\end{pmatrix}$$

It is an interesting exercise to check that the helicity spinors $u(p,\lambda)$ and $v(p,\lambda)$ satisfy the following relations

$$\bar{u}(p,\lambda)u(p,\lambda') = +2m\delta_{\lambda\lambda'} \qquad \sum_\lambda u(p,\lambda)\bar{u}(p,\lambda) = \not{p} + m$$
$$\bar{v}(p,\lambda)v(p,\lambda') = -2m\delta_{\lambda\lambda'} \qquad \sum_\lambda v(p,\lambda)\bar{v}(p,\lambda) = \not{p} - m$$

We have seen that the necessity of two spinors (η,χ) to describe a particle with spin $1/2$ is due to the mass of the particle. This necessity disappears if the mass is zero. The wave equation which describes such a particle can be derived from a single spinor, say the undotted spinor

$$p_\mu \, \sigma_-^{\mu\,\dot{\alpha}\alpha} \eta_\alpha = 0 \quad \Rightarrow \quad (E + \vec{p} \cdot \vec{\sigma})\, \eta = 0$$

The energy and momentum of a particle with $m = 0$ are related by $E = |\vec{p}|$ so that we have

$$(\hat{p} \cdot \vec{\sigma})\eta(p) = -\eta(p) \; : \quad \text{helicity } \lambda = -1/2 = -$$

On the other hand the dotted spinor $\bar{\chi}$ satisfies

$$(\hat{p} \cdot \vec{\sigma})\bar{\chi}(p) = -\bar{\chi}(p) \; : \quad \text{helicity } \lambda = +1/2 = +$$

Consequently, states of the massless particle with a definite momentum are necessarily helicity states, for which the spin component in the direction of motion has a definite value. If the particle spin is opposite to the momentum (helicity $-1/2$), the antiparticle spin is along the momentum (helicity $+1/2$). The neutrinos in the Standard Model were supposed to be such particles possessing these properties.

2–component Spinor Technique

For the contraction of a four–vector a^μ and γ^μ we write

$$\not{a} = \begin{pmatrix} 0 & \not{a}_+ \\ \not{a}_- & 0 \end{pmatrix} \qquad \not{a}_\pm = a_\mu \sigma_\pm^\mu$$

For the Pauli–adjoint of the four–component spinors we have

$$\bar{u}(p,\lambda) = u^\dagger(p,\lambda)\gamma^0 = \left(u_+^\dagger(p,\lambda), u_-^\dagger(p,\lambda)\right)$$

$$\bar{v}(p,\lambda) = v^\dagger(p,\lambda)\gamma^0 = \left(v_+^\dagger(p,\lambda), v_-^\dagger(p,\lambda)\right)$$

Hence strings with even and odd numbers of γ–matrices are expressed, respectively,

$$\begin{aligned}
\bar{u}(\bar{p},\bar{\lambda})P_{\pm}u(p,\lambda) &= u^{\dagger}_{\mp}(\bar{p},\bar{\lambda})u_{\pm}(p,\lambda) \\
\bar{u}(\bar{p},\bar{\lambda})\not{a}P_{\pm}u(p,\lambda) &= u^{\dagger}_{\pm}(\bar{p},\bar{\lambda})\not{a}_{\pm}u_{\pm}(p,\lambda) \\
\bar{u}(\bar{p},\bar{\lambda})\not{a}\not{b}P_{\pm}u(p,\lambda) &= u^{\dagger}_{\mp}(\bar{p},\bar{\lambda})\not{a}_{\mp}\not{b}_{\pm}u_{\pm}(p,\lambda)
\end{aligned}$$

and the similar relations hold for strings with v's and with u and v.

An Example

As a first example, let us calculate the helicity amplitude for the process

$$e^{-}(k,\sigma)+e^{+}(\bar{k},\bar{\sigma})\to\mu^{-}(p,\lambda)+\mu^{+}(\bar{p},\bar{\lambda})$$

in the lowest order. We choose the e^{-} momentum direction as the positive z-axis and assume that the muon pair is produced on the x-z plane with the μ^{-} scattering angle θ:

$$\begin{aligned}
&k=\frac{\sqrt{s}}{2}(1,0,0,+1) \qquad \bar{k}=\frac{\sqrt{s}}{2}(1,0,0,-1) \\
&p=\frac{\sqrt{s}}{2}(1,\beta\sin\theta,0,\beta\cos\theta) \quad \bar{p}=\frac{\sqrt{s}}{2}(1,-\beta\sin\theta,0,-\beta\cos\theta) \\
&q=k+\bar{k}=p+\bar{p} \qquad s=q^2 \qquad \beta=\sqrt{1-4m^2/s}
\end{aligned}$$

where m is the muon mass and the electron mass is neglected. In this coordinate system the electron and muon 2-component spinors are given by

$$u(k,+)_a=\delta_{a+}s^{1/4}\begin{pmatrix}+1\\0\end{pmatrix} \quad u(k,-)_a=\delta_{a-}s^{1/4}\begin{pmatrix}0\\+1\end{pmatrix}$$

$$v(\bar{k},-)_a = \delta_{a+} s^{1/4} \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad v(\bar{k},+)_a = \delta_{a-} s^{1/4} \begin{pmatrix} +1 \\ 0 \end{pmatrix}$$

$$u(p,+)_b = \omega_b \begin{pmatrix} c_h \\ s_h \end{pmatrix} \qquad \omega_b = (E + b\,p)^{1/2}$$

$$u(p,-)_b = \omega_{-b} \begin{pmatrix} -s_h \\ c_h \end{pmatrix}$$

$$v(\bar{p},-)_b = -b\omega_b \begin{pmatrix} s_h \\ -c_h \end{pmatrix}$$

$$v(\bar{p},+)_b = b\omega_{-b} \begin{pmatrix} c_h \\ s_h \end{pmatrix} \qquad c_h = \cos\frac{\theta}{2} \quad s_h = \sin\frac{\theta}{2}$$

The scattering amplitude due to the γ and Z exchanges is written as

$$\mathcal{M}(\sigma\bar{\sigma} : \lambda\bar{\lambda}) = \frac{e^2}{s} Q_{ab} \left[\bar{v}(\bar{k},\bar{\sigma})\gamma_\mu P_a u(k,\sigma)\right] \left[\bar{u}(p,\lambda)\gamma^\mu P_b v(\bar{p},\bar{\lambda})\right]$$

$$Q_{ab} = 1 + \frac{s}{s - m_Z^2 + i m_Z \Gamma_Z} a_a^e a_b^\mu$$

$$a_L^f = \frac{T_3^f - Q_f \sin^2\theta_W}{\cos\theta_W \sin\theta_W} \qquad a_R^f = -\frac{Q_f \sin^2\theta_W}{\cos\theta_W \sin\theta_W}$$

It is quite straightforward to evaluate the electron current (although you need to do a little exercise to get familiar with the technique)

$$\begin{aligned} j_+^\mu(\sigma\bar{\sigma}) &= \bar{v}(\bar{k},\bar{\sigma})\gamma^\mu P_+ u(k,\sigma) = v(\bar{k},\bar{\sigma})_+^\dagger \sigma_+^\mu u(k,\sigma)_+ \\ &= \delta_{\sigma+}\,\delta_{\bar{\sigma}-}\, s^{1/2}(0,-1)\,[1,\,\vec{\sigma}] \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \delta_{\sigma+}\,\delta_{\bar{\sigma}-}\sqrt{s}\,(0,-1,-i,0) \\ &= \delta_{\sigma+}\,\delta_{\bar{\sigma}-}\sqrt{2s}\,\epsilon^\mu(q,+) \end{aligned}$$

$$j_-^\mu(\sigma\bar{\sigma}) = \bar{v}(\bar{k},\bar{\sigma})\gamma^\mu P_- u(k,\sigma) = v(\bar{k},\bar{\sigma})_-^\dagger \sigma_-^\mu u(k,\sigma)_-$$

$$
\begin{aligned}
&= \delta_{\sigma-}\, \delta_{\bar\sigma+} s^{1/2}(1,0)\,[1,-\vec\sigma]\begin{pmatrix}0\\1\end{pmatrix}\\
&= \delta_{\sigma-}\, \delta_{\bar\sigma+}\sqrt{s}\,(0,-1,+i,0)\\
&= -\delta_{\sigma-}\, \delta_{\bar\sigma+}\sqrt{2s}\,\epsilon^{\mu}(q,-)
\end{aligned}
$$

Note that the positron helicity is always opposite to the electron helicity. On the other hand, it is a little demanding to evaluate the muon current:

$$
\begin{aligned}
J^\mu_+(+,-) &= \bar u(p,+)\gamma^\mu P_+ v(\bar p,-) = -\omega_+^2(c_h,s_h)\,[1,\,\vec\sigma]\begin{pmatrix}s_h\\-c_h\end{pmatrix}\\
&= \frac{\sqrt{s}}{2}(1+\beta)\,(0,-\cos\theta,i,\sin\theta)\\
J^\mu_-(-,+) &= \bar u(p,-)\gamma^\mu P_- v(\bar p,+) = -\omega_+^2(-s_h,c_h)\,[1,\,-\vec\sigma]\begin{pmatrix}c_h\\-s_h\end{pmatrix}\\
&= \frac{\sqrt{s}}{2}(1+\beta)\,(0,-\cos\theta,-i,\sin\theta)\\
J^\mu_+(+,+) &= \bar u(p,+)\gamma^\mu P_+ v(\bar p,+) = \omega_+\omega_-(c_h,s_h)\,[1,\,\vec\sigma]\begin{pmatrix}c_h\\s_h\end{pmatrix}\\
&= m\,(1,\sin\theta,0,\cos\theta)\\
J^\mu_-(-,-) &= \bar u(p,-)\gamma^\mu P_- v(\bar p,-) = \omega_-\omega_+(-s_h,c_h)\,[1,\,-\vec\sigma]\begin{pmatrix}s_h\\-c_h\end{pmatrix}\\
&= m\,(-1,\sin\theta,0,\cos\theta)
\end{aligned}
$$

With these expressions we obtain the helicity amplitudes:

$$
\begin{aligned}
\mathcal{M}(\sigma\bar\sigma:\lambda\bar\lambda) &= \frac{e^2}{s}Q_{ab}\,j_a(\sigma,\bar\sigma)\cdot J_b(\lambda,\bar\lambda)\\
&\equiv e^2\,\delta_{\sigma,-\bar\sigma}\,\delta_{\sigma a}\,Q_{ab}\,\langle\sigma:\lambda\bar\lambda\rangle
\end{aligned}
$$

$$
\langle + : ++\rangle = \frac{m}{\sqrt{s}}\sin\theta
$$

$$\langle + : +- \rangle = \frac{1}{2}(1+\beta)(-1-\cos\theta)$$
$$\langle + : -+ \rangle = \frac{1}{2}(1+\beta)(+1-\cos\theta)$$
$$\langle + : -- \rangle = \frac{m}{\sqrt{s}}\sin\theta$$
$$\langle - : ++ \rangle = \frac{m}{\sqrt{s}}\sin\theta$$
$$\langle - : +- \rangle = \frac{1}{2}(1+\beta)(+1-\cos\theta)$$
$$\langle - : -+ \rangle = \frac{1}{2}(1+\beta)(-1-\cos\theta)$$
$$\langle - : -- \rangle = \frac{m}{\sqrt{s}}\sin\theta$$

At high energies the amplitude is greatly simplified because of chirality conservation

$$\mathcal{M}(\sigma, -\sigma : \lambda, -\lambda) = -e^2 Q_{\sigma\lambda}\,(\sigma\lambda + \cos\theta)$$

Gauge Interactions

- Obtained by the replacement

$$\partial_\mu \quad \Rightarrow \quad D_\mu = \partial_\mu + igT^a A^a_\mu$$

- Universality : there is only one coupling constant for each simple group. The gauge interaction of a particle is totally determined by knowing the representation of the particle.

- Conserves fermion chirality

$$\bar{\psi}\gamma_\mu(v - a\gamma_5)\,\psi Z^\mu = \left[(v+a)\bar{\psi}_L\gamma_\mu\,\psi_L + (v-a)\bar{\psi}_R\gamma_\mu\,\psi_R\right] Z^\mu$$

List of Possible Renormalizable Interactions

Spin	0	1/2	1
1	Gauge $\|D_\mu\varphi\|^2$	Gauge $\bar\psi \not{D}\psi$	Gauge $F^{\mu\nu}F_{\mu\nu}$
1/2	Yukawa $\bar\psi\psi\varphi$	No	No
0	Scalar φ^3, φ^4	No	No

Breaking of fermion chirality is entirely due to a mass term or a Yukawa interaction [apart from anomalies].

- Non–renormalizable effective interaction of gauge bosons can be constructed from D_μ and $F_{\mu\nu}$ such as

$$\bar\psi\sigma_{\mu\nu}\psi F^{\mu\nu} \qquad \bar\psi\gamma_\mu D_\nu\psi F^{\mu\nu}$$

Prescription of Model Building

- Fix the gauge group
  - Gauge bosons are determined
  - Parameters : gauge couplings

- Fix the representations of fermions and scalars
  - Gauge interactions of matter particles are fixed (no new parameters)
  - The total fermion representation must be anomaly–free
- Give global symmetries if needed
- Write down all possible mass terms and interactions compatible with the symmetries
  - Parameters
    * scalar potential parameters ($\varphi^2, \varphi^3, \varphi^4$)
    * fermion masses and Yukawa couplings

From Lagrangian to Cross Section

- Particle states in the Hilbert space
  - The vacuum

$$\langle 0|0\rangle = 1 \qquad |0\rangle \ : \text{dim} = 0$$

  - One–particle state

$$\langle p|p'\rangle = 2p^0(2\pi)^3\delta^3(\vec{p}-\vec{p'}) \qquad |p\rangle \ : \text{dim} = -1$$

$$1 = |0\rangle\langle 0| + \sum_p |p\rangle\langle p| + \cdots \qquad \sum_p = \frac{d^3p}{(2\pi)^3 2p^0}$$

- Quantum fields

$$\varphi(x) = \sum_p \left[a(p)\,\mathrm{e}^{-ip\cdot x} + a^\dagger(p)\,\mathrm{e}^{ip\cdot x}\right]$$

$$\psi(x) = \sum_p \sum_{\lambda=\pm} \left[a_\lambda(p) u(p,\lambda)\,\mathrm{e}^{-ip\cdot x} + b_\lambda^\dagger(p) v(p,\lambda)\,\mathrm{e}^{ip\cdot x}\right]$$

$$A^\mu = \sum_p \sum_{\lambda=\pm,0} \left[a_\lambda(p)\,\epsilon^\mu(p,\lambda)\,\mathrm{e}^{-ip\cdot x} + a_\lambda^\dagger(p)\,\epsilon^{\mu*}(p,\lambda)\,\mathrm{e}^{ip\cdot x}\right]$$

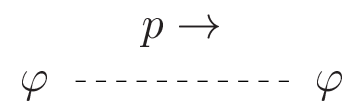
- S matrix

$$S_{fi} \equiv \langle f\ \text{out}|i\ \text{in}\rangle = 1_{fi} + i\,(2\pi)^4\,\delta^4(P_f - P_i)\,T_{fi}$$

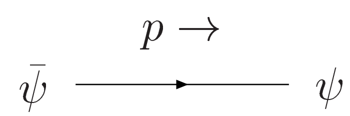
$$\text{Unitarity} \ : SS^\dagger = S^\dagger S = 1 \quad \Rightarrow \quad T^\dagger T = -i\left(T - T^\dagger\right)$$

Lagrangian to Feynman Rule

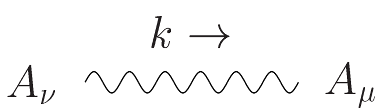
- Free parts (or kinetic terms) ⇒ Propagators; Interactions ⇒ Vertices



$$i\,\frac{1}{p^2-m^2+i\varepsilon}$$



$$i\,\frac{(\not{p}+m)}{p^2-m^2+i\varepsilon}$$



$$i\,\frac{-g_{\mu\nu}+\frac{k_\mu k_\nu}{m^2}}{k^2-m^2+i\varepsilon}$$

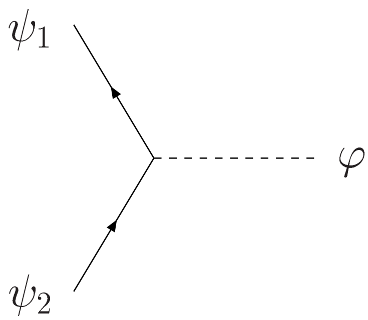
Massless : $$i\,\frac{-g_{\mu\nu}+(1-\xi)\frac{k_\mu k_\nu}{k^2}}{k^2+i\varepsilon}$$

$\xi = 1$: Feynman gauge

$\xi = 0$: Landau gauge

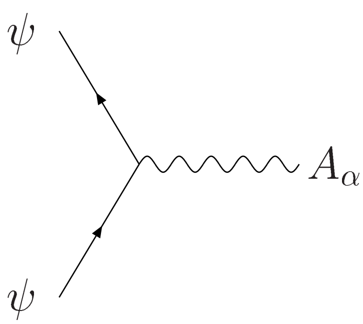
- Vertices : from $i\,\mathcal{L}_{\text{int}}$
  - No derivatives

$$\boxed{\mathcal{L} = i\,f\bar{\psi}_1\gamma_5\psi_2\varphi}$$



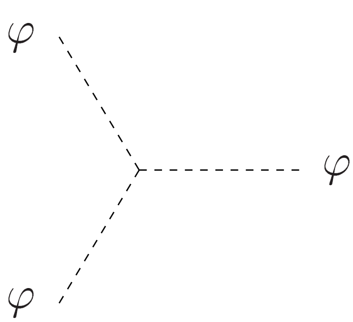
$$i \cdot i\,f\gamma_5 = -f\,\gamma_5$$

$$\boxed{\mathcal{L} = -e\bar{\psi}\gamma_\mu\psi\,A^\mu}$$



$$i \cdot (-e)\gamma^\mu g_{\mu\alpha} = -i\,e\,\gamma_\alpha$$

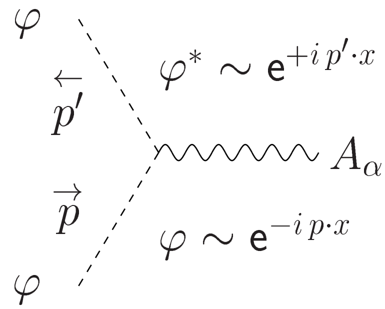
$$\boxed{\mathcal{L} = \mu\varphi^3 = 6\,\mu\left(\tfrac{1}{3!}\varphi^3\right)}$$



$$i \cdot 6\,\mu = 6\,i\mu$$

– With derivatives

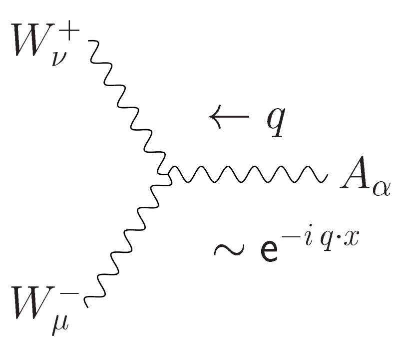
$$\boxed{\mathcal{L} = -ie(\varphi^* \partial_\mu \varphi - \partial_\mu \varphi^* \, \varphi) A^\mu}$$



$$i \cdot (-ie) \cdot [(-ip_\alpha) - (ip'_\alpha)]$$

$$= -i\,e\,(p + p')_\alpha$$

$$\boxed{\mathcal{L} = -i\,e\kappa\,W^+_\mu W_\nu\,(\partial^\mu\, g^{\nu\alpha} - \partial^\nu g^{\mu\alpha})\,A_\alpha}$$

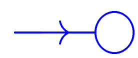
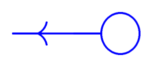
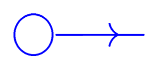
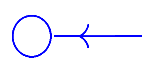


$$i \cdot (-i\,e\,\kappa)\,[(-iq_\mu) g_{\nu\alpha} - (-iq_\nu) g_{\mu\alpha}]$$

$$= -i\,e\,\kappa\,(q_\mu\, g_{\nu\alpha} - q_\nu\, g_{\mu\alpha})$$

Feynman Rules to Scattering Amplitude

A Feynman graph is a sum of all possible graphs for a given process using the vertices and propagators of the model.

- Vertex $\rightarrow$ vertex factor
- Internal line $\rightarrow$ Propagator
- Loop $\rightarrow \int \frac{d^4k}{(2\pi)^4}$
- External line $\rightarrow$ wave function
  - scalar : 1
  - fermion :
    - initial
      - particle $u(p)$ 
      - antiparticle $\bar{v}(p)$ 
    - final
      - particle $\bar{u}(p)$ 
      - antiparticle $v(p)$ 
  - vector :
    - initial $\epsilon_\mu(p)$ final $\epsilon^*_\mu(p)$
- Closed fermion loop $\rightarrow$ factor (-1)
- A graph with an exchanged fermion pair $\rightarrow$ factor (-1)

Following the above prescription one can obtain $iT_{fi} \equiv i\mathcal{M}$ where $\mathcal{M}$ is called the scattering or transition amplitude.

Scattering Amplitude to Cross Section/Width

- Decay rate (in the rest frame)

$$d\Gamma(p \to k_1 + \cdots + k_n) = \frac{1}{2M} \overline{\sum} |\mathcal{M}|^2 \, d\Phi_n$$

- Scattering cross section

$$d\sigma(p_1 + p_2 \to k_1 + \cdots + k_n) = \frac{1}{2\sqrt{(p_1 \cdot p_2)^2 - M_1^2 M_2^2}} \overline{\sum} |\mathcal{M}|^2 \, d\Phi_n$$

where $\bar{\Sigma}$ denotes the average for the initial states and the sum for the final states.

- Final–state phase space

$$d\Phi_n \;=\; (2\pi)^4 \, \delta^4 \left(p - \sum_{i=1}^{n} k_i \right) \prod_{i=1}^{n} \frac{d^3 k_i}{(2\pi)^3 2k_i^0}$$

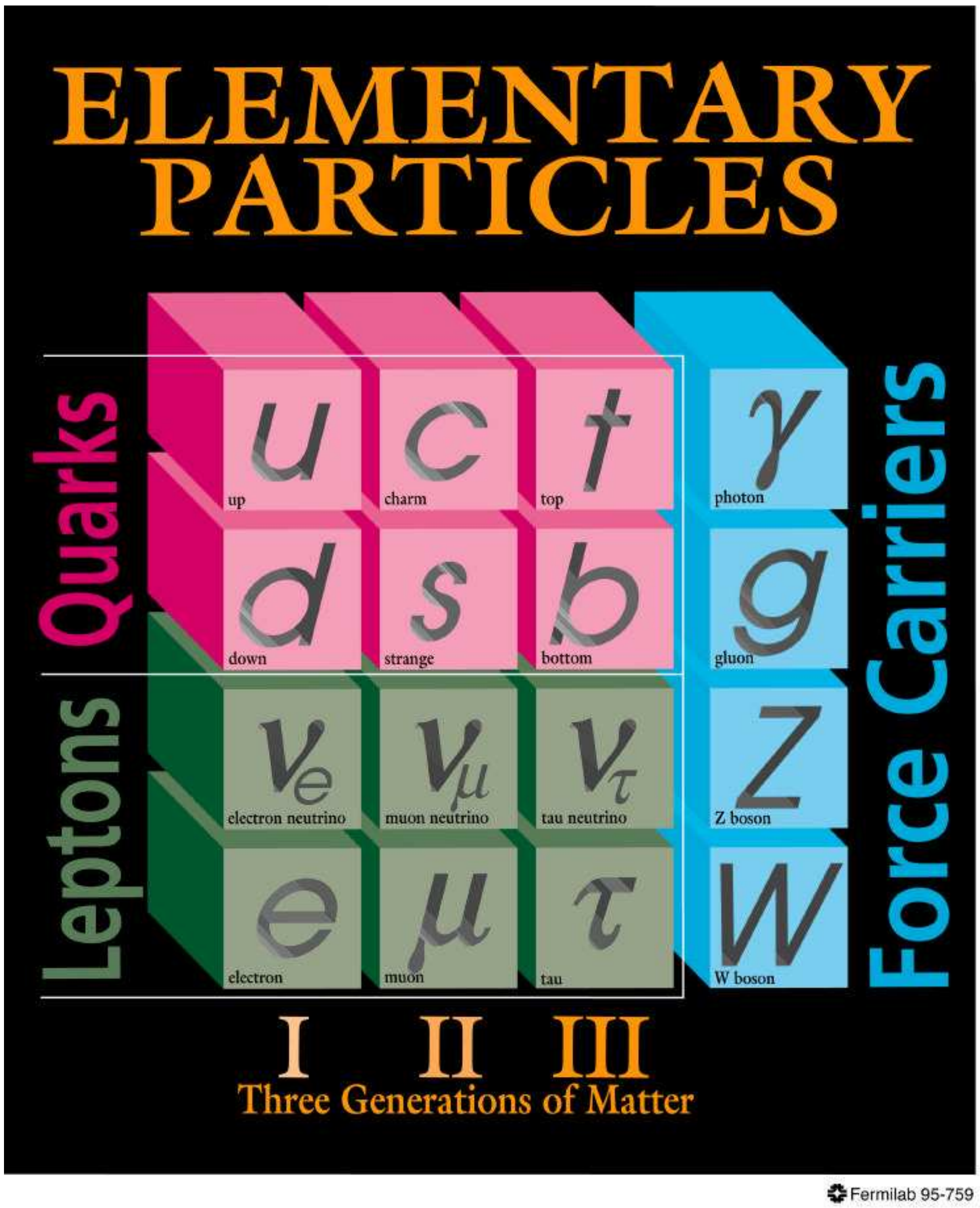
The evaluation of the phase space integrals is facilitated by the identity

$$\frac{d^3 k_i}{2k_i^0} = d^4 p_i \, \delta(k_i^2 - m_i^2) \, \theta(k_i^0)$$

You are recommended to derive the 2–body phase space

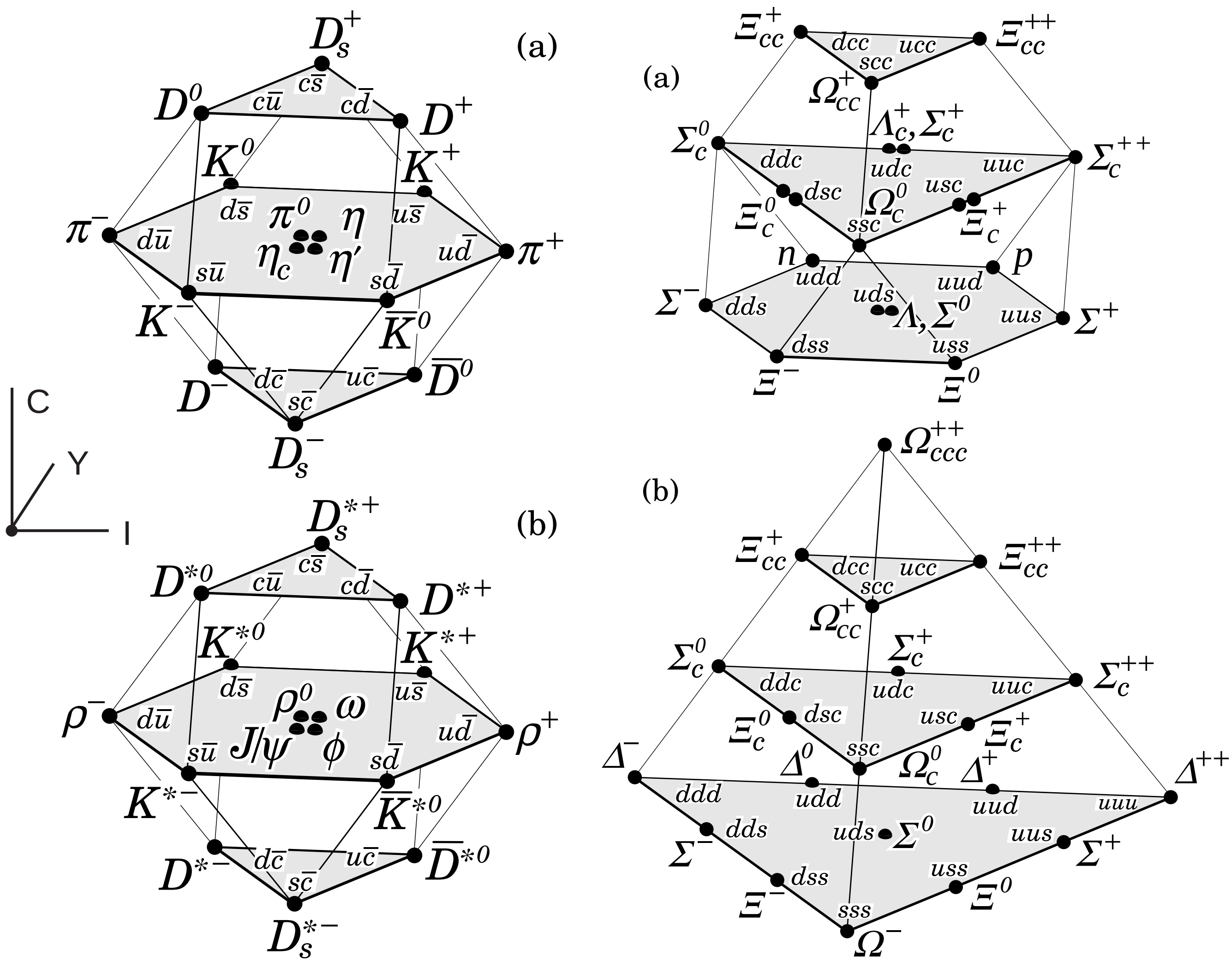
$$d\Phi_2 = \frac{\bar{\beta}_f}{32\pi^2} \sin\theta_1 d\theta_1 d\phi_1 \qquad \bar{\beta}_f^2 = \frac{s^2 - 2(m_1^2 + m_2^2)s + (m_1^2 - m_2^2)^2}{s^2}$$

Standard Model

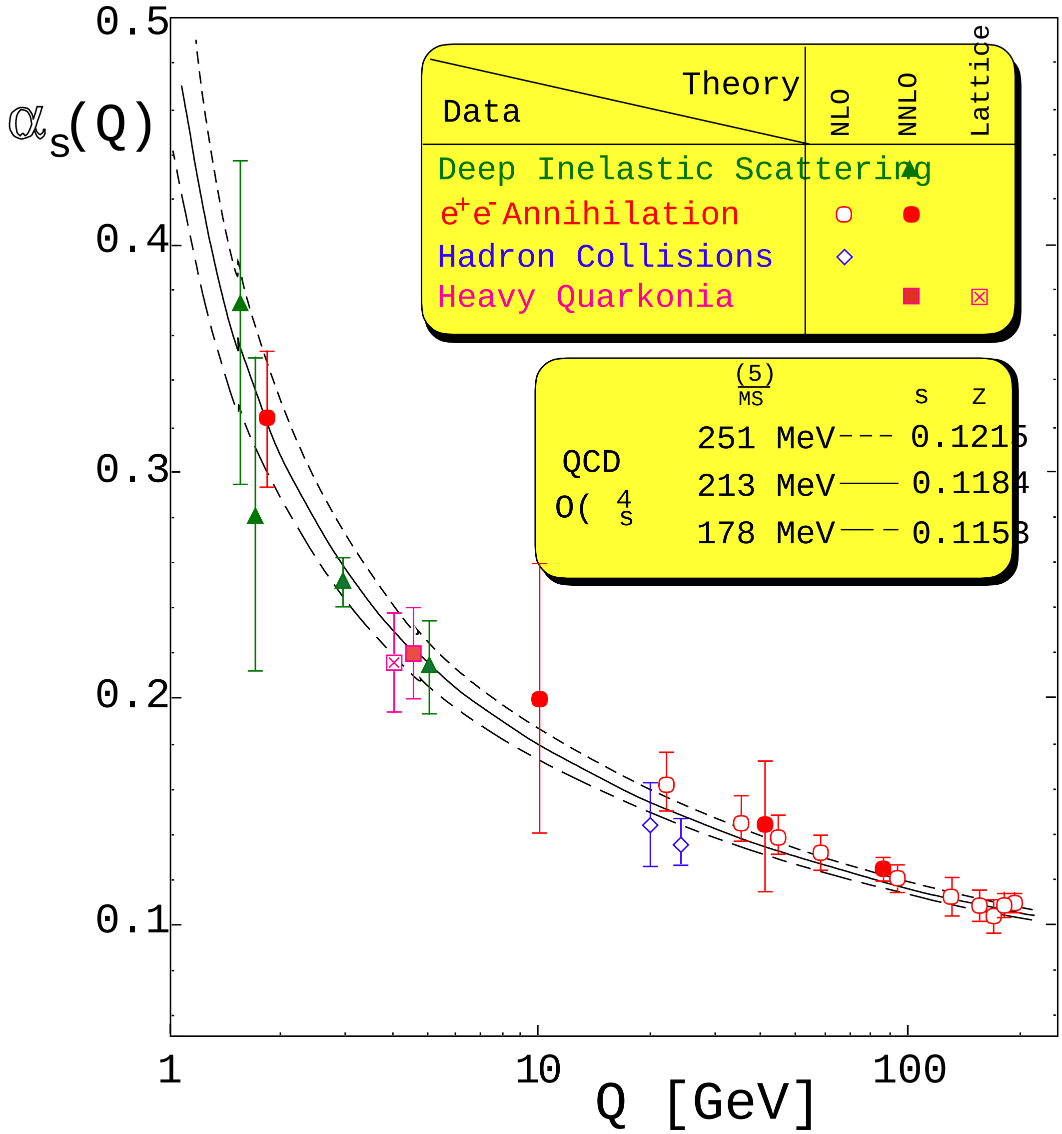


Quantum Chromodynamics

Hadrons – mesons and baryons – are "bound states" of quarks and gluons.



- Asymptotic freedom : the effective coupling vanishes at high energies



– Interactions of quarks and gluons visible at high energies
– Perturbation applicable, but parameters connecting quarks/gluons to hadrons need to be known
  - ∗ structure functions (parton distributions)
  - ∗ decay constants ($f_\pi, \cdots$)
  - ∗ wave functions (quarkonia)

⋮

- Color confinement : neither free quarks nor free gluons
  - Crucial to separate short– and long–distance physics
- QCD Lagrangian
  - Gauge group : SU(3)

    gluons 8

    quarks $(3_L + 3_R) \times n_f$

    n_f : #(flavors)

  - Parameters : masses generated by Higgs mechanism

    gauge coupling $\alpha_s = \frac{g_s^2}{4\pi}$

    quark masses $m_u, m_d, m_s, m_c, m_b, m_t$

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} + \sum_q \bar{q}(i\not{D} - m_q) q$$

Gauged Higgs System

- SU(2)$_L$ doublet : φ with hypercharge $Y = 1/2$

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} i(\varphi_1 - i\varphi_2) \\ \varphi_0 - i\varphi_3 \end{pmatrix}$$

$$\mathrm{SU}(2) \ : \ \varphi \to \exp(iT_a\theta^a)\,\varphi$$

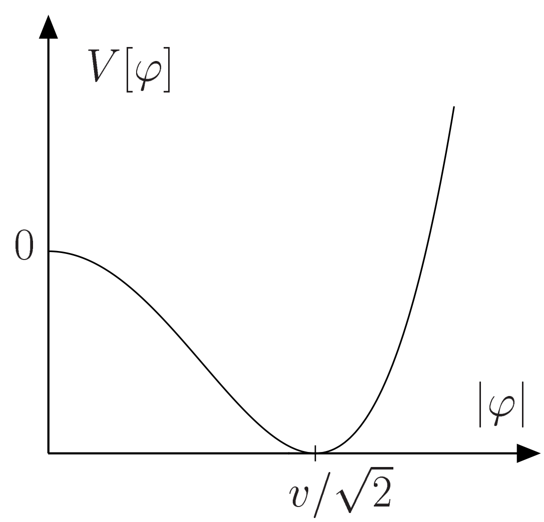
$$\mathrm{U}(1) \ : \ \varphi \to \exp(iY\theta)\,\varphi$$

- Potential : The gauge invariant and renormalizable Lagrangian

$$\mathcal{L}_{Higgs} = (D^\mu\varphi)^\dagger D_\mu\varphi - V(\varphi)$$

for the Higgs doublet is given by the most general potential

$$V(\varphi) = \mu^2\varphi^\dagger\varphi + \lambda(\varphi^\dagger\varphi)^2$$



– O(4) $\equiv$ SU(2)$_L\times$SU(2)$_R$: $\varphi^\dagger\varphi = \frac{1}{2}\left(\varphi_0^2 + \varphi_1^2 + \varphi_2^2 + \varphi_3^2\right) \equiv \frac{1}{2}\eta$

Spontaneous Breaking of SU(2)$_L$×U(1)$_Y$

- $\mu^2 < 0$ and $\lambda > 0$ (vacuum stability) :

$$\frac{\partial V}{\partial \eta} = \frac{1}{2}\mu^2 + \frac{1}{2}\lambda\eta = 0 \;\Rightarrow\; \eta = -\frac{\mu^2}{\lambda} \equiv v^2$$

$$\varphi_0 = v, \quad \varphi_a = 0 \quad (a = 1, 2, 3)$$

with losing the generality leading to the vacuum expectation value (vev) of the Higgs doublet

$$\langle\varphi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

- Unbroken gauge symmetry leaving $\langle\varphi\rangle$ invariant

$$Q = T_3 + Y = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow SU(2)_L \times U(1)_Y \to U(1)_{EM}$$

- The potential itself has the custodial SU(2)$_D$ symmetry.

$$SU(2)_L \times SU(2)_R \to SU(2)_D$$

This group is the symmetry of the Higgs potential only; in the full theory SU(2)$_R$ and SU(2)$_D$ are explicitly broken by the U(1)$_Y$ gauge interactions.

Gauge Boson Masses

- $SU(2)_L \times U(1)_Y$ covariant derivative

$$D_\mu \varphi = \left(\partial_\mu + ig\frac{\tau_a}{2} W_\mu^a + ig'\frac{1}{2} B_\mu \right) \varphi$$

- Gauge boson mass terms come from the Higgs kinetic term :

$$\mathcal{L} = (\mathcal{D}^\mu \langle\varphi\rangle)^\dagger \mathcal{D}_\mu \langle\varphi\rangle \Leftarrow \langle\varphi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{v^2}{8} \left\{ g^2 \left[(W_\mu^1)^2 + (W_\mu^2)^2 \right] + (gW_\mu^3 - g'B_\mu)^2 \right\}$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp iW_\mu^2 \right)$$

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(gW_\mu^3 - g'B_\mu \right)$$

$$= \frac{1}{4} g^2 v^2 W_\mu^\dagger W^\mu + \frac{1}{8}(g^2 + g'^2) v^2 Z^\mu Z_\mu \equiv m_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} m_Z^2 Z^\mu Z_\mu$$

$$m_W = \frac{1}{2} g v \qquad v \approx 246 \text{ GeV}$$

$$m_Z = \frac{1}{2} \sqrt{g^2 + g'^2}\, v > m_W$$

– The state orthogonal to Z_μ

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g'W_\mu^3 + gB_\mu \right)$$

is massless so that it is nothing but the photon field.

– The weak mixing angle θ_W is defined as

$$\tan\theta_W = \frac{g'}{g} \Rightarrow \begin{cases} Z_\mu = W_\mu^3 \cos\theta_W - B_\mu \sin\theta_W \\ A_\mu = W_\mu^3 \sin\theta_W + B_\mu \cos\theta_W \end{cases}$$

- Among the four scalar fields $\varphi_i (i = 0, 1, 2, 3)$ the three fields $(\varphi_1, \varphi_2, \varphi_3)$ become the longitudinal components of the massive gauge bosons $(W^\pm, Z)$ while $\varphi_0 = (v + H)/\sqrt{2}$ remains as a physical field - this is called the Higgs boson.

- The so–called ρ parameter

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{g_Z^2/m_Z^2}{g^2/m_W^2} = \frac{\text{NC Fermi coupling}}{\text{CC Fermi coupling}}$$

– SM Higgs : $\rho = 1$

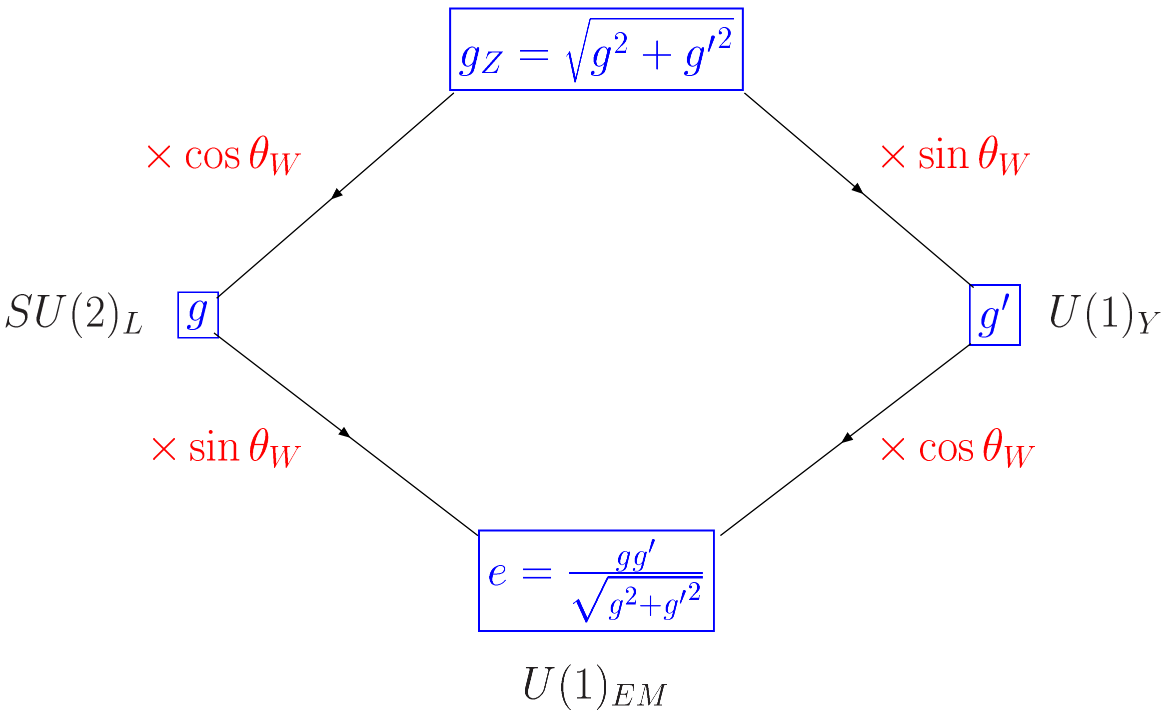
– General Higgs :

$$\begin{aligned} m_W^2 &= \frac{1}{2}\langle I_3|(I^+I^- + I^-I^+)|I_3\rangle g^2 \langle\varphi\rangle^2 \\ &= \left[I(I+1) - I_3^2\right] g^2 \langle\varphi\rangle^2 \qquad\qquad I^\pm = I^1 \pm iI^2 \\ m_Z^2 &= 2I_3^2\,(g^2 + g'^2)\langle\varphi\rangle^2 \end{aligned}$$

$$\Rightarrow \rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{I(I+1) - I_3^2}{2I_3^2}$$

$$\begin{aligned} (I, I_3) &= \left(\frac{1}{2}, \pm\frac{1}{2}\right) \to \rho = 1 \qquad (1,\ \pm 1) \to \rho = \frac{1}{2} \\ & \quad (1,\ 0) \to \rho = \infty \qquad \left(\frac{3}{2}, \pm\frac{3}{2}\right) \to \rho = \frac{1}{3} \quad \cdots \end{aligned}$$

$$\boxed{\tan\theta_W = \frac{g'}{g}} \qquad \sin / \cos\theta_W = \frac{g'/g}{\sqrt{g^2+g'^2}}$$



SU(2)×U(1) Gauge Interactions

- Covariant derivative in terms of physical gauge bosons

$$D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}}\left(I^+ W_\mu + I^- W_\mu^\dagger\right) + igI_3 W_\mu^3 + ig' Y B_\mu$$

$$\mathrm{Y} = \mathrm{Q} - \mathrm{I}_3 \qquad \{\mathrm{W}^3, \mathrm{B}\} \to \{\mathrm{Z}, \mathrm{A}\}$$

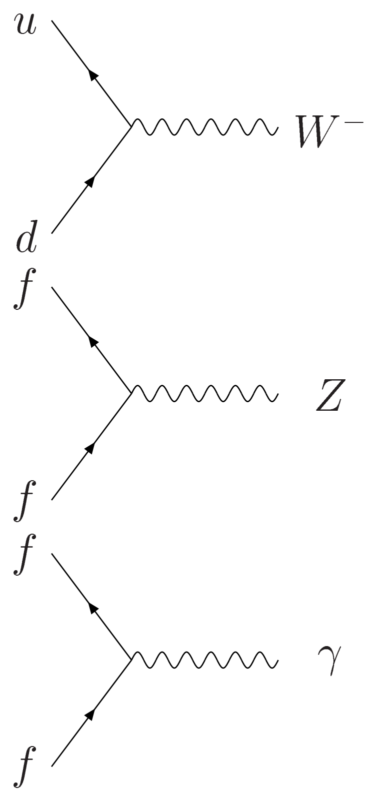
$$= \partial_\mu + \frac{ig}{\sqrt{2}}\left(I^+ W_\mu + I^- W_\mu^\dagger\right) + ig_Z\left(I_3 - Q\sin^2\theta_W\right) Z_\mu + ieQA_\mu$$

  - $W^\pm$ couples with pure SU(2) gauge coupling
  - Z couples to a linear combination of SU(2) and EM charge
  - γ couples to the electric charge $Q \Rightarrow$ QED

Particle Names	I	$Y = \langle Q \rangle$	$SU(3)_C$
$l_L = \begin{bmatrix} \nu \\ e \end{bmatrix}_L$	$\frac{1}{2}$	$-\frac{1}{2}$	1
e_R	0	-1	1
$q_L = \begin{bmatrix} u \\ d \end{bmatrix}_L$	$\frac{1}{2}$	$\frac{1}{6}$	3
u_R	0	$\frac{2}{3}$	3
d_R	0	$-\frac{1}{3}$	3
$\varphi = \begin{bmatrix} \varphi^+ \\ \varphi^0 \end{bmatrix}$	$\frac{1}{2}$	$\frac{1}{2}$	1
$\tilde{\varphi} = \begin{bmatrix} -\varphi^{0*} \\ \varphi^- \end{bmatrix}$	$\frac{1}{2}$	$-\frac{1}{2}$	1

- Fermion gauge interactions

$$\mathcal{L} = \sum_{\text{fermions}} \bar{\psi}_L\, i\, \not{D} \psi_L + \bar{\psi}_R\, i\, \not{D} \psi_R$$



$$-\frac{i}{\sqrt{2}}\, g\gamma_\mu P_L$$

$$-ig_Z\gamma_\mu\left(I_3 P_L - Q\sin^2\theta_W\right)$$

$$-ieQ\gamma_\mu$$

- Yang–Mills Interactions

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

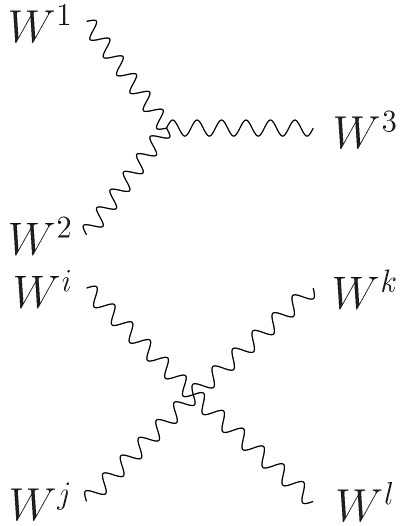
$$[\mathsf{T^a}, \mathsf{T^b}] = \mathsf{i\, f^{abc} T^c} \qquad \mathsf{Tr}(\mathsf{T^a T^b}) = \frac{1}{2}\delta^{\mathsf{ab}}$$

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} \\ &= -\frac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 - g f^{abc} \partial_\mu A^a_\nu A^{b\,\mu} A^{c\,\nu} \\ &\quad -\frac{1}{4} g^2 f^{abe} f^{cde} A^a_\mu A^b_\nu A^{c\mu} A^{d\nu} \end{aligned}$$

  – SU(2) : $f^{abc} = \epsilon^{abc}$ $[a = 1, 2, 3]$
  – Gauge boson self couplings*

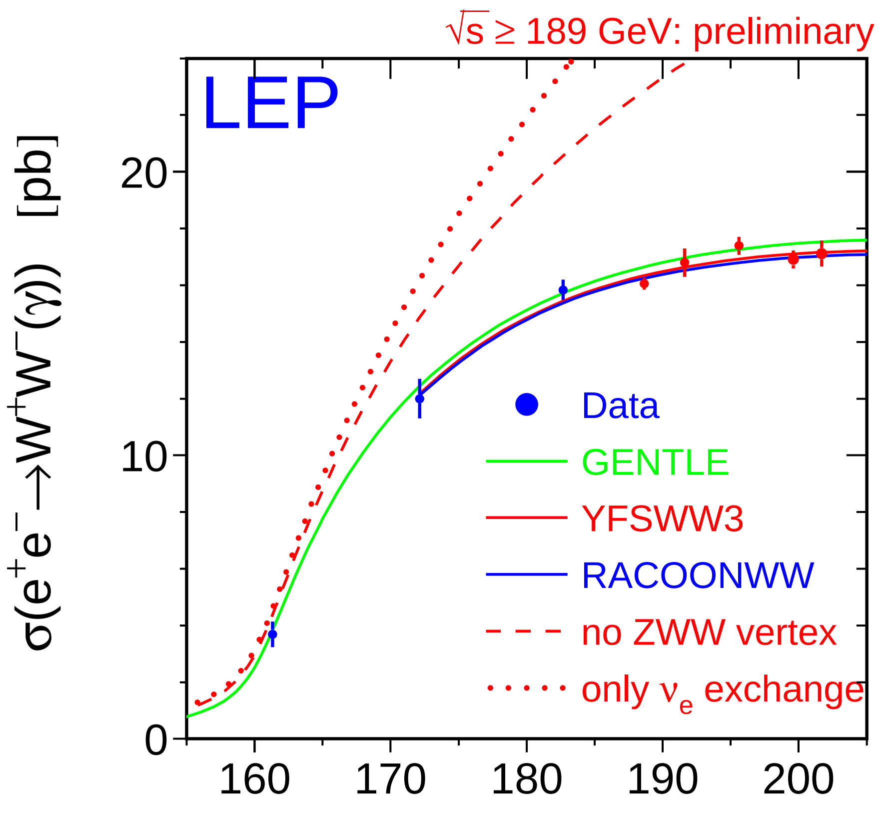
---

*Higgs–gauge/Higgs–fermion interactions later



$W^+W^-[Z/\gamma]$

$W^+W^-[W^+W^-/ZZ/Z\gamma/\gamma\gamma]$



Fermion Masses

- Only Higgs doublets can give known fermion masses while any nontrivial Higgs representation can give gauge boson masses
- Fermions cannot have SU(2)×U(1) invariant mass term, i.e. quarks and leptons are massless before symmetry breaking
- A single doublet φ can generate masses of all quarks, leptons and $W^{\pm}, Z$
- Yukawa interactions [only one generation is considered] :

$$
\begin{aligned}
-\mathcal{L} &= f\,(\bar{q}_L \bar{d})\,\varphi - h\,(\bar{q}_L \bar{u})\,\tilde{\varphi} + \text{h.c.} \\
&\qquad \langle\varphi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \langle\tilde{\varphi}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} -v \\ 0 \end{pmatrix} \\
&= \frac{1}{\sqrt{2}} f v(\bar{d}d) + \frac{1}{\sqrt{2}} h v(\bar{u}u) \equiv \mathrm{m_d \bar{d} d} + \mathrm{m_u \bar{u} u}
\end{aligned}
$$

$$\Rightarrow \mathrm{m_d} = \frac{1}{\sqrt{2}}\mathrm{fv} \quad \mathrm{m_u} = \frac{1}{\sqrt{2}}\mathrm{hv}$$

Minimal Model based on SU(2)×U(1)×SU(3)

- The only dimensionful parameter is v; all other masses are secondary

$$
\begin{aligned}
m_W &= \tfrac{1}{2} g v & m_Z &= \tfrac{1}{2} g_Z v \\
m_u &= \tfrac{1}{\sqrt{2}} h_u v & m_d &= \tfrac{1}{\sqrt{2}} f_d v \\
m_l &= \tfrac{1}{\sqrt{2}} f_l v & m_H &= \sqrt{2}\lambda^{1/2} v
\end{aligned}
$$

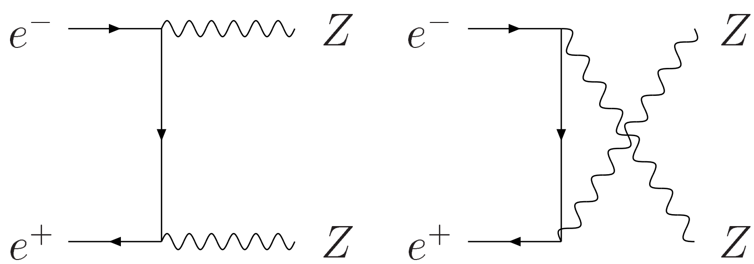
- Neutrinos were exactly massless within the SM ⇒ Not any more
- Baryon and lepton numbers are automatically conserved (no renormalizable B/L–violating interactions) ⇒ Probably not any more

Higgs Phenomenology

Minimal Higgs Boson

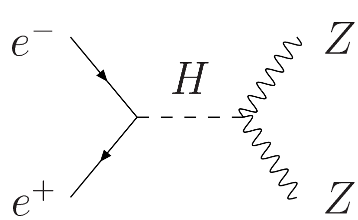
Something needed in $J = 0$ sector

- Assume SU(2)×U(1) without Higgs and with $W^{\pm}, Z$ masses put by hand
  - $e^+e^- \to ZZ$

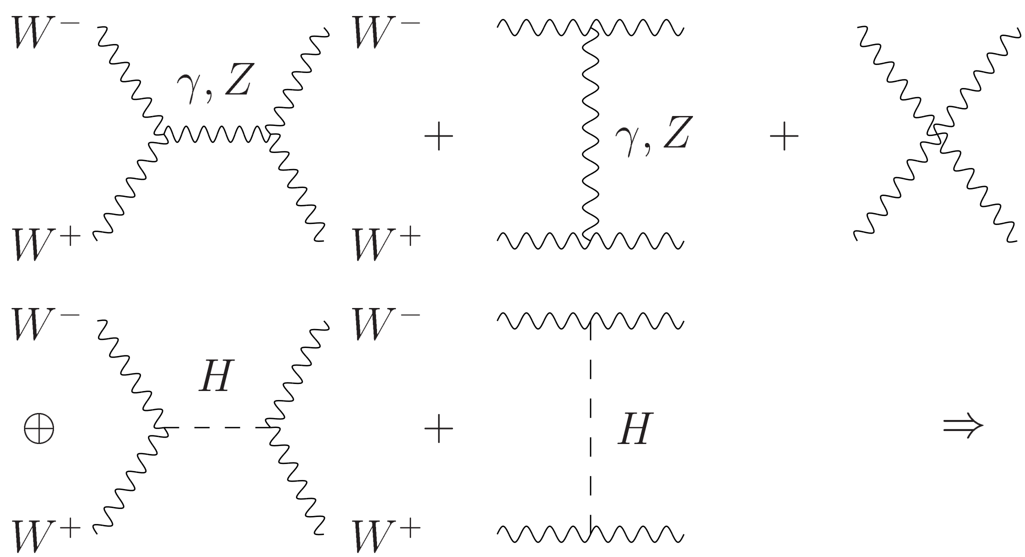


$$\frac{d\sigma}{d\cos\theta} \sim \frac{\pi\alpha}{16 s_W^2 c_W^2} \frac{m_e^2}{m_Z^4}$$

This comes from $J = 0$ partial wave and eventually violates unitarity at high energies; so something is needed to cure the $J = 0$ part. Adding the standard Higgs cancels the ill behavior entirely. [Higgs–fermion coupling must be proportional to the fermion mass !]



– $W^+W^- \rightarrow W^+W^-$



$\mathcal{M} \sim E^2$

$\frac{d\sigma}{d\cos\theta} \sim s$

$\Rightarrow$ $\mathcal{M} \sim$ constant

$\frac{d\sigma}{d\cos\theta} \sim 1/s$

- SU(2) doublet with 4 components - 1 physical $H \oplus$ 3 unphysical $(W^{\pm}, Z)$
- Higgs : vibration of the vacuum

$$J = 0,\ P = +,\ C = +,\ Q = 0,\ \cdots$$

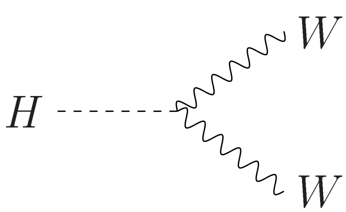
Higgs Couplings

Higgs couplings can be derived by replacing v by $v + H$

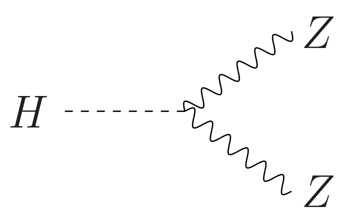
$$m \Rightarrow m \left(1 + \frac{g}{2m_W} H\right)$$

- With gauge bosons

$$\begin{aligned}
\mathcal{L} &= m_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} m_Z^2 Z^\mu Z_\mu \\
&\to m_W^2 \left(1 + \frac{g}{2m_W} H\right) W_\mu^\dagger W^\mu + \frac{1}{2} m_Z^2 \left(1 + \frac{g_Z}{2m_Z} H\right) Z^\mu Z_\mu \\
&= (\text{masses}) + g m_W H W_\mu^\dagger W^\mu + \frac{1}{2} g_Z m_Z H Z_\mu Z^\mu \\
&\qquad + \frac{1}{4} g^2 H^2 W_\mu^\dagger W_\mu + \frac{1}{8} g_Z^2 H^2 Z_\mu Z^\mu
\end{aligned}$$



$ig m_W g_{\alpha\beta}$

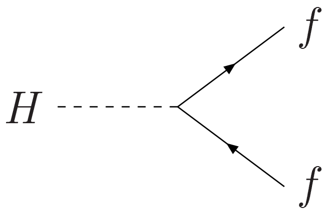


$ig_Z m_Z g_{\alpha\beta}$

No $HZ\gamma$, $H\gamma\gamma$, Hgg at tree level

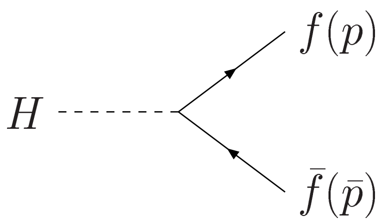
- With fermions

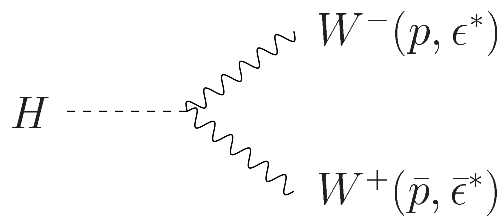
$$\begin{aligned}
\mathcal{L} &= -m_f \bar{f} f \\
&\to -m_f \left(1 + \frac{g}{2m_W} H\right) \bar{f} f = (\text{mass}) - \frac{g m_f}{2m_W} H \bar{f} f
\end{aligned}$$



$-i\,\frac{gm_f}{2m_W}$ flavor–conserving scalar coupling (Q^{++})

Higgs Decays





$$\mathcal{M} = -\frac{gm_f}{2m_W}\,\bar{u}(p)v(\bar{p})$$

$$\mathcal{M} = gm_W\,\epsilon^* \cdot \bar{\epsilon}^*$$

Partial Decay Widths

$$\Gamma(H \to f\bar{f}) = \frac{\alpha m_f^2 m_H}{8m_W^2 \sin^2\theta_W} N_C^f \beta_f^3$$

$$= \frac{G_F m_f^2 m_H}{4\sqrt{2}\pi} N_C^f \beta_f^3 \qquad \beta_f = \sqrt{1 - \frac{4m_f^2}{m_H^2}}$$

$$\Gamma(\mathrm{H} \rightarrow \mathrm{W}_\mathrm{T}^+\mathrm{W}_\mathrm{T}^-) = \frac{\alpha \mathrm{m}_\mathrm{W}^2 \beta_\mathrm{W}}{2\mathrm{m}_\mathrm{H} \sin^2\theta_\mathrm{W}} \qquad \beta_\mathrm{W} = \sqrt{1 - \frac{4\mathrm{m}_\mathrm{W}^2}{\mathrm{m}_\mathrm{H}^2}}$$

$$\Gamma(\mathrm{H} \rightarrow \mathrm{W}_\mathrm{L}^+\mathrm{W}_\mathrm{L}^-) = \frac{\alpha \mathrm{m}_\mathrm{H}^3 \beta_\mathrm{W}}{16\mathrm{m}_\mathrm{W}^2 \sin^2\theta_\mathrm{W}} \left(1 - \frac{2\mathrm{m}_\mathrm{W}^2}{\mathrm{m}_\mathrm{H}^2}\right)$$

$$\Gamma(H \rightarrow W^+W^-) = \frac{\alpha m_H^3 \beta_W}{16 m_W^2 \sin^2\theta_W} \left(1 - \frac{4m_W^2}{m_H^2} + \frac{12 m_W^4}{m_H^4}\right)$$
$$= \frac{G_F m_H^3 \beta_W}{8\sqrt{2}\pi} \left(1 - \frac{4m_W^2}{m_H^2} + \frac{12 m_W^4}{m_H^4}\right)$$

$$\Gamma(H \rightarrow ZZ) = \frac{1}{2} \cdot \frac{G_F m_H^3 \beta_Z}{8\sqrt{2}\pi} \left(1 - \frac{4m_Z^2}{m_H^2} + \frac{12 m_Z^4}{m_H^4}\right)$$

- In the limit of $m_H \gg m_W, m_Z$

$$\Gamma(H \rightarrow ZZ) = \frac{1}{2}\Gamma(H \rightarrow WW)$$

- $H \rightarrow \gamma\gamma, Z\gamma, gg$: only via fermion and/or W loops leading to small branching ratios in general

- Equivalence theorem : $\Gamma(H \rightarrow V_L V_L) \gg \Gamma(H \rightarrow V_T V_T)$

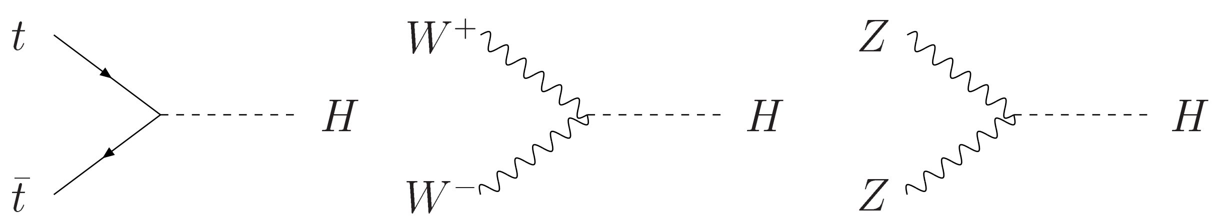
$$\Gamma(H \rightarrow f\bar{f}) \sim \alpha_W m_H \times \left(\frac{m_f}{m_W}\right)^2 \sim (\text{Yukawa})^2 m_H$$

$$\Gamma(H \rightarrow VV) \sim \alpha_W m_H \times \left(\frac{m_H}{m_V}\right)^2 \sim \lambda m_H \not\sim (\text{gauge})^2 m_H$$

in the limit of $m_H \gg m_V$.

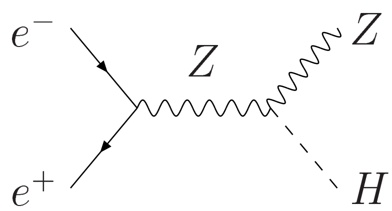
Higgs Boson Production

- 1st and 2nd generation quarks/leptons couple extremely weakly to H
- "Large" coupling required, i.e. production via heavy particles



- Main production modes

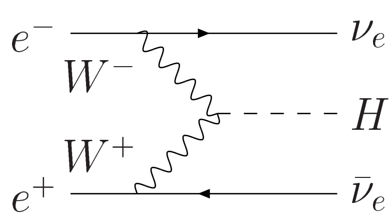
(a) Higgs–strahlung :



$\sigma \sim \frac{G_F^2 M_Z^4}{96\pi s} \kappa_S \beta$

$\sqrt{s}$ lower part

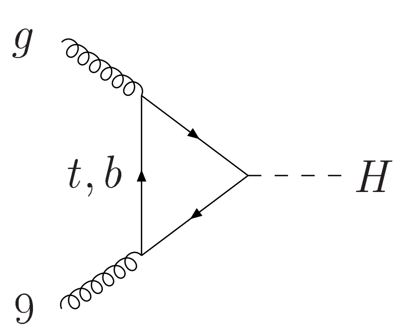
(b) W fusion :



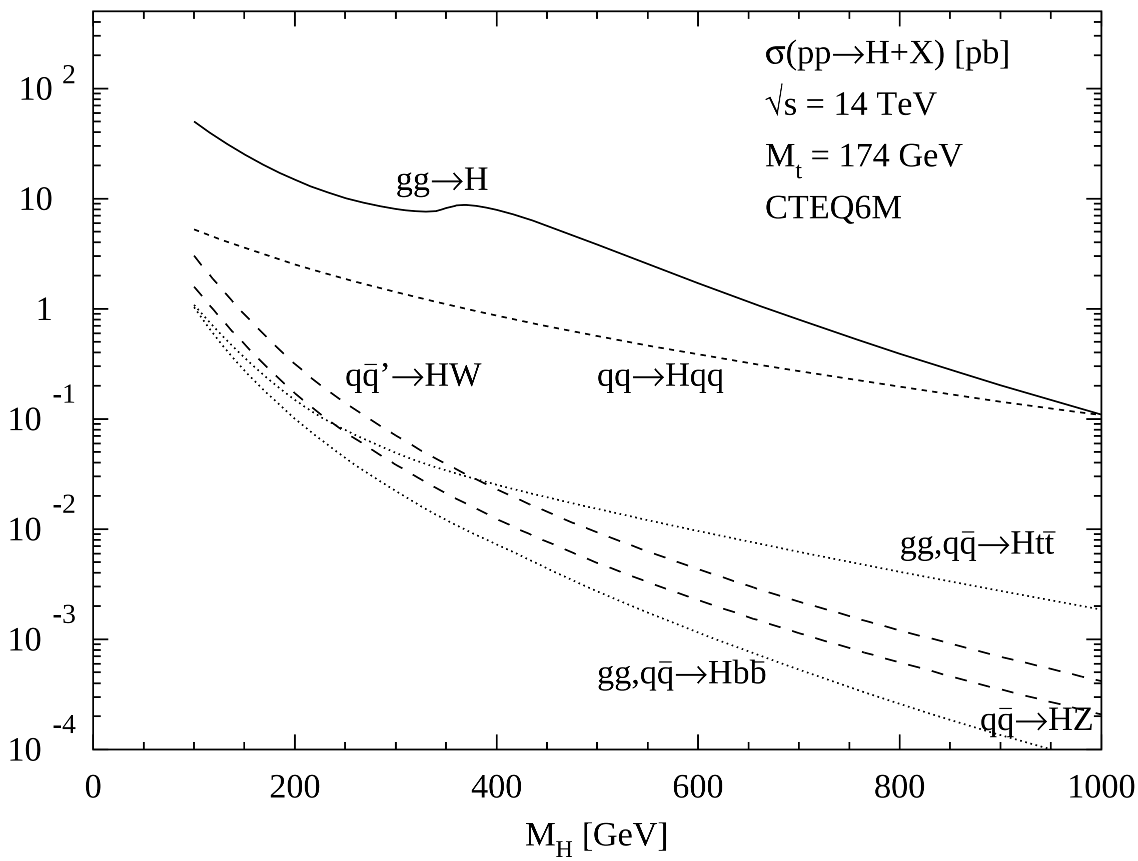
$\sigma \sim \frac{G_F^3 M_W^4}{4\sqrt{2}\pi^3} \log \frac{s}{M_H^2}$

$\sqrt{s}$ upper part

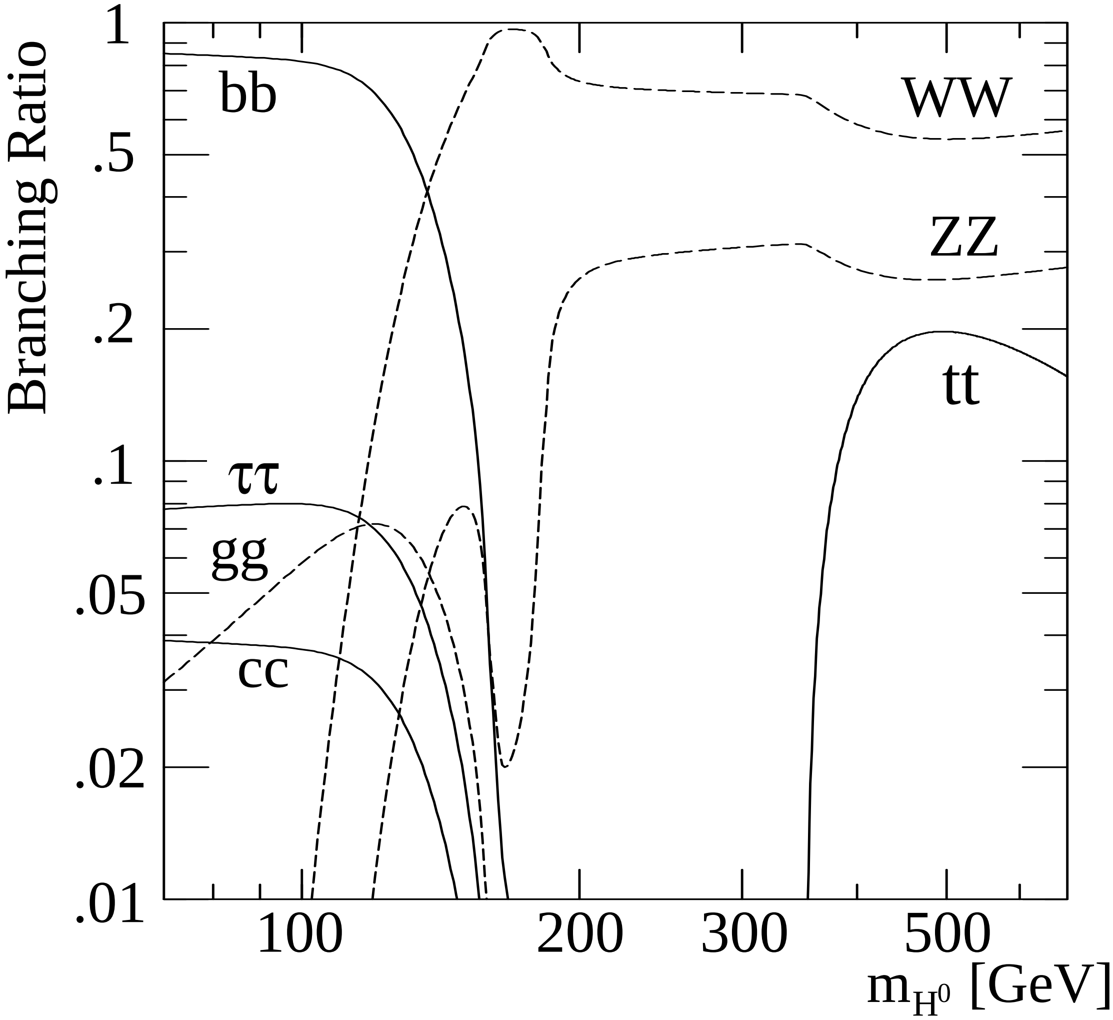
(c) Gluon fusion :

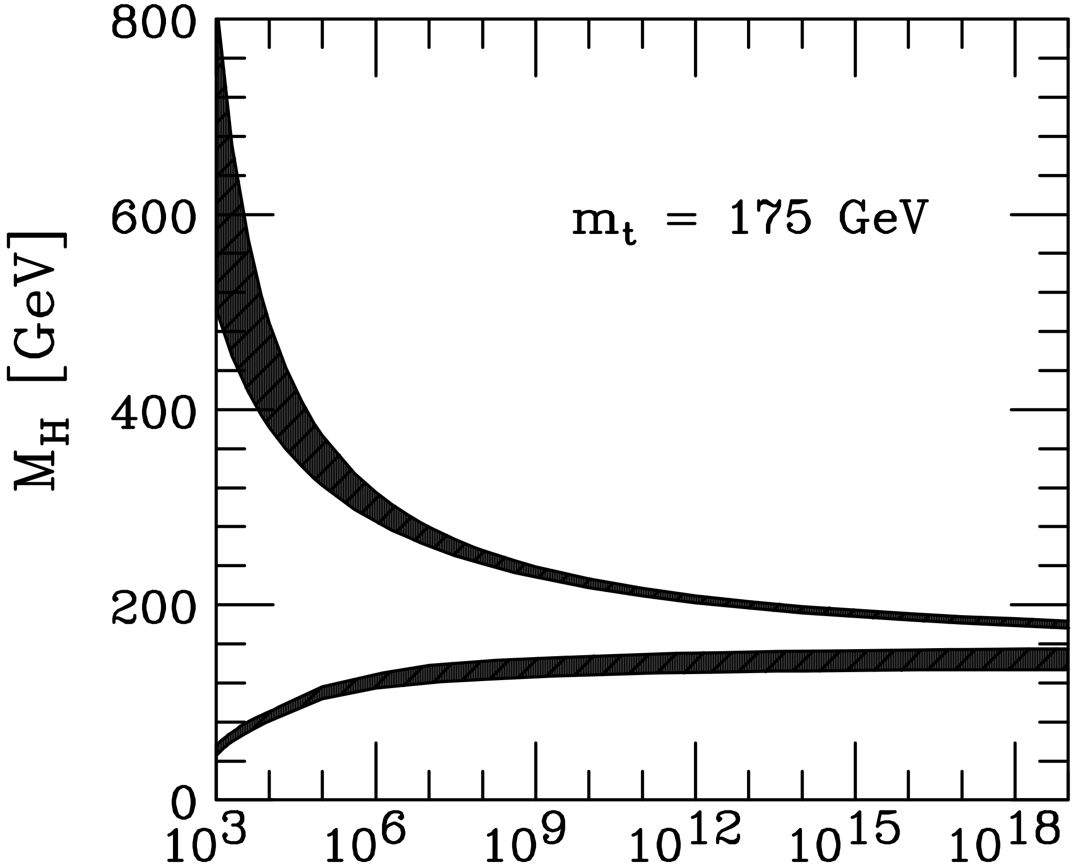


LHC

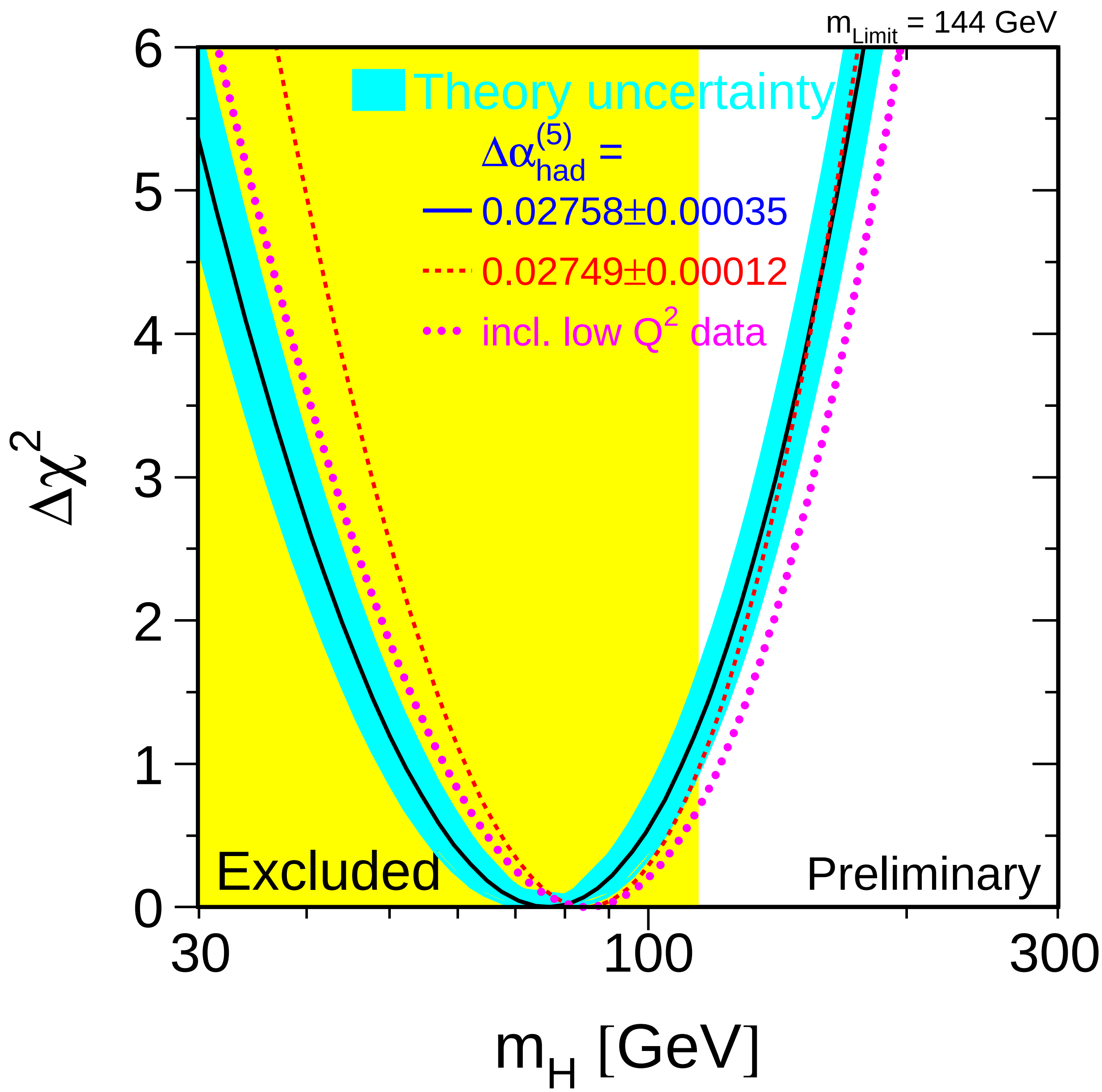


σ(pp→H+X) [pb]
√s = 14 TeV
M_t = 174 GeV
CTEQ6M
gg→H
qq̄'→HW
qq→Hqq
gg,qq̄→Htt̄
gg,qq̄→Hbb̄
qq̄→HZ
10^2
10
1
10^-1
10^-2
10^-3
10^-4
0
200
400
600
800
1000
M_H [GeV]

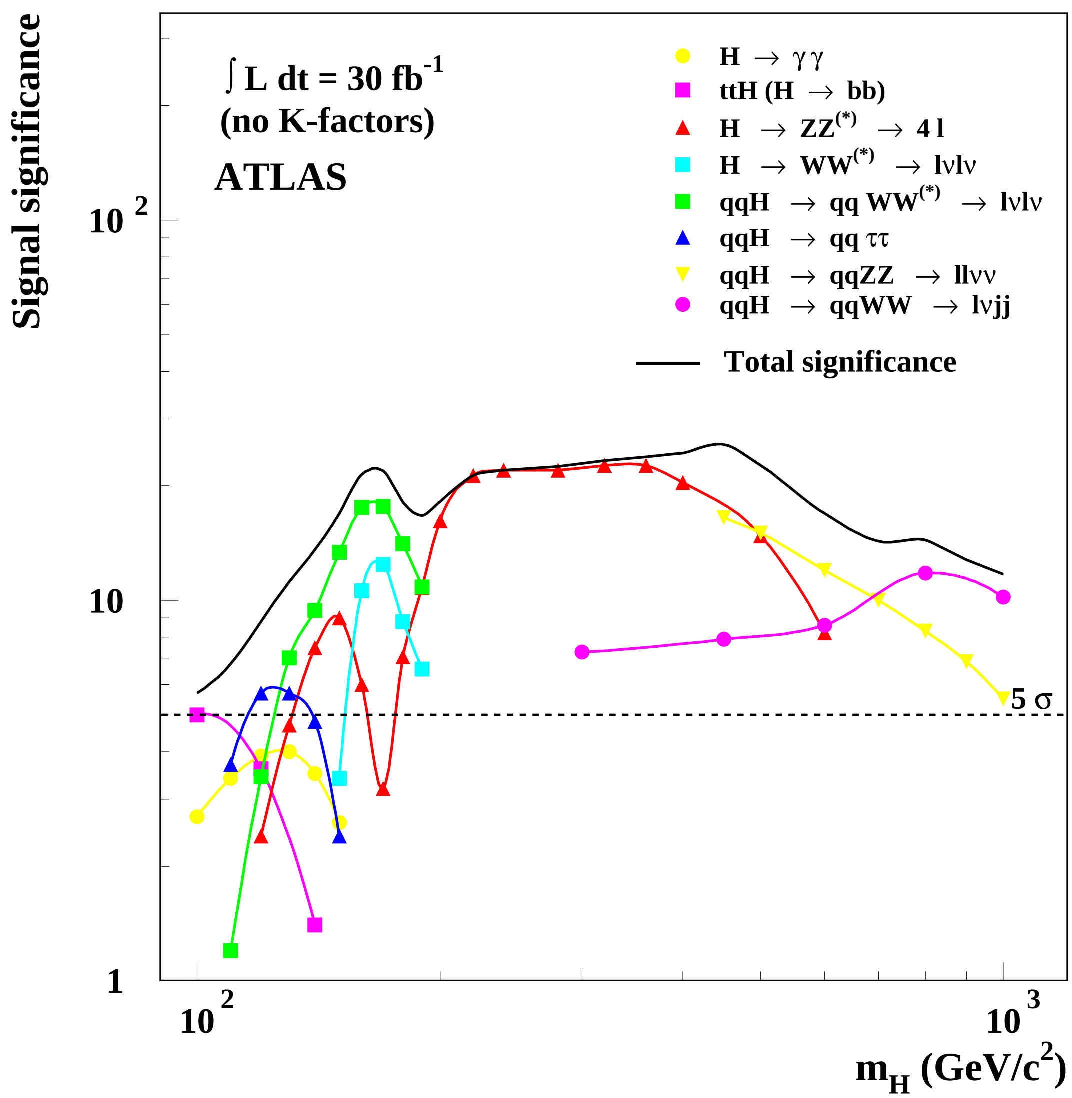

Branching Ratio
1
.5
.2
.1
.05
.02
.01
bb
ττ
gg
cc
WW
ZZ
tt
100
200
300
500
m_{H^0} [GeV]



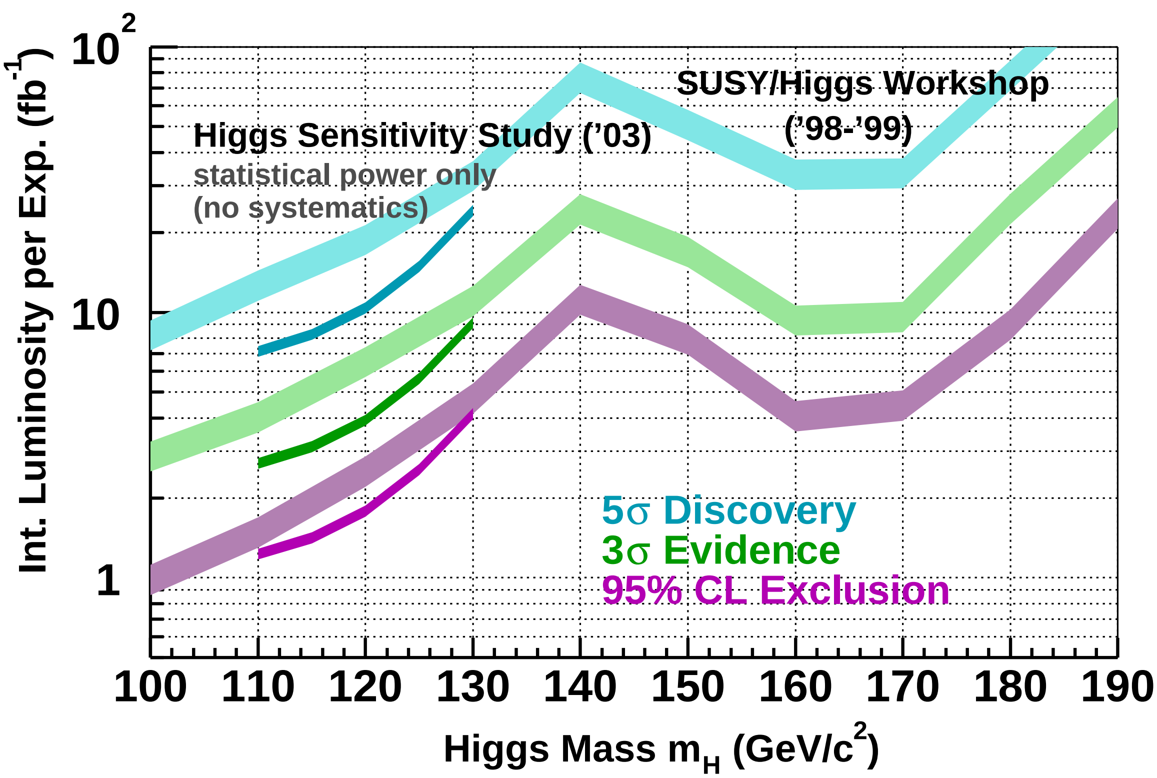
800
600
400
200
0
M_H [GeV]
m_t = 175 GeV
10^3
10^6
10^9
10^{12}
10^{15}
10^{18}
Λ [GeV]



m_Limit = 144 GeV
Theory uncertainty
Δα(5)had =
0.02758±0.00035
0.02749±0.00012
incl. low Q2 data
Excluded
Preliminary
Δχ2
mH [GeV]
0
1
2
3
4
5
6
30
100
300



Signal significance
∫L dt = 30 fb^-1
(no K-factors)
ATLAS
H → γγ
ttH (H → bb)
H → ZZ(*) → 4 l
H → WW(*) → lνlν
qqH → qq WW(*) → lνlν
qqH → qq ττ
qqH → qqZZ → llνν
qqH → qqWW → lνjj
Total significance
5 σ
10^2
10
1
10^2
10^3
m_H (GeV/c^2)



SUSY/Higgs Workshop
('98-'99)
Higgs Sensitivity Study ('03)
statistical power only
(no systematics)
5σ Discovery
3σ Evidence
95% CL Exclusion
Int. Luminosity per Exp. (fb^{-1})
Higgs Mass m_H (GeV/c^2)
10^2
10
1
100
110
120
130
140
150
160
170
180
190

Higgs Physics Menu

m_H

elementary or composite ?
electroweak physics
$t\bar{t}$ condensation ?

Γ_H

Higgs sector
Resonance shape (if wide)
How to measure ? (if narrow)

$J^{PC} = 0^{++}$

Branching Ratios

$H \to W^+W^-/ZZ \to$ custodial SU(2) symmetry
$H \to t\bar{t}, \tau^+\tau^-, \cdots \to$ Yukawa; minimal or not ?
$H \to \gamma\gamma, Z\gamma, gg \to$ one–loop structure