Seiberg Duality

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- $N=1$ SUSY, Nonabelian Gauge Theory + Matter Fields
- High Energy (short distance, weak coupling)

  Low Energy (long distance, strong coupling)
- Nonperturbative effect

  Supersymmetry

  new degrees of freedom
- Reference

  Modern Supersymmetry J. Terning

$\boxed{d=4 \ \mathcal{N}=1, \quad \text{SUSY}}$ $\qquad + - - -$

1. $\sigma^\mu{}_{\alpha\dot\alpha} = (1, \vec\sigma)$

$\bar\sigma^{\mu\,\dot\alpha\alpha} = (1, -\vec\sigma)$

$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}$

$\{\gamma^\mu \gamma^\nu\} = 2\eta^{\mu\nu}$

$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$P_L = \frac{1-\gamma^5}{2}, \quad P_R = \frac{1+\gamma^5}{2}$

$SO(1,3) = \underbrace{SU(2)_L \times SU(2)_R}_{SO(2,1)_L \otimes SO(2,1)_R}$

2. α : left haded

$\dot\alpha$: right handed

$\psi_\alpha(x), \quad \psi^+_{\dot\alpha}(x)$

$\psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta \qquad \epsilon_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$\psi^+_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\psi^{+\dot\beta} \qquad \epsilon^{\dot\alpha\dot\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$(\epsilon_\alpha)^+ = \epsilon$

$\epsilon\cdot\psi = \epsilon^\alpha\psi_\alpha = +\epsilon_{\alpha\beta}\epsilon^\alpha\psi^\beta = -\epsilon_{\alpha\beta}\psi^\beta\epsilon^\alpha$

$= \epsilon_{\beta\alpha}\psi^\beta\epsilon^\alpha = \psi\cdot\epsilon \qquad \leftarrow$ lorentz inv

$\epsilon^+\cdot\psi^+ = \epsilon^+_{\dot\alpha}\cdot\psi^{+\dot\alpha} = \psi^+\epsilon^+ \qquad \bar\psi = \psi^+\gamma^0$

3. $\Psi = \begin{pmatrix} \chi_\alpha \\ \psi^{+\dot\alpha} \end{pmatrix}, \quad \Psi^c = \begin{pmatrix} \psi_\alpha \\ \chi^{+\dot\alpha} \end{pmatrix} = -i\gamma^0\gamma^2(\bar\Psi)^T$

$\bar\Psi_1\Psi_2 = \chi_1^+\psi_2^+ + \psi_1\chi_2$

4. $\Psi_m = \Psi_m^c = \begin{pmatrix} \chi_\alpha \\ \chi^{+\dot\alpha} \end{pmatrix} \qquad$ Majorana

5. Gauge ~~Matter~~ Lagrangian $A_\mu = A_\mu^a T^a$, $\lambda = \lambda^a T^a$, D

$$\mathcal{L} = \frac{1}{\cancel{4}2 e^2} \mathrm{Tr}\left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \lambda^{\dagger\cancel{*}} \bar{\sigma}^\mu D_\mu \lambda + \frac{1}{2} D^2 \right) + \xi D$$

$\uparrow$ FI term

$$\begin{cases} F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu] \\ D_\mu \lambda = \partial_\mu \lambda - i [A_\mu, \lambda] \\ D \end{cases}$$

degrees of freedom

on-shell $A_\mu : 2$, $\lambda : 2$, $D : 0$

off-shell $A_\mu : 3$ $\lambda : 4$ $D : 1$

$$\begin{cases} \delta A_\mu = \frac{-1}{\sqrt{2}} \left(\epsilon^\dagger \bar{\sigma}_\mu \lambda + \lambda^\dagger \bar{\sigma}_\mu \epsilon \right) \\ \delta \lambda = -\frac{i}{2\sqrt{2}} (\sigma^\mu \bar{\sigma}^\nu \epsilon) F_{\mu\nu} + \frac{1}{\sqrt{2}} \epsilon D \\ \delta \lambda^\dagger = \frac{i}{2\sqrt{2}} \epsilon^\dagger \bar{\sigma}^\nu \sigma^\mu F_{\mu\nu} + \frac{1}{\sqrt{2}} \epsilon^\dagger D \end{cases}$$

$$\delta D = \frac{-i}{\sqrt{2}} \left(\epsilon^\dagger \bar{\sigma}^\mu D_\mu \lambda^* - D_\mu \lambda^\dagger \bar{\sigma}^\mu \epsilon \right)$$

6. matter $\phi_j \quad \psi_j$

$$\left.\begin{aligned} D_\mu \phi_f &= \partial_\mu \phi_f - i A_\mu \phi_f \\ D_\mu \psi_f &= \partial_\mu \psi_f - i A_\mu \psi_f \end{aligned}\right\} \quad F_f$$

$$\begin{aligned} \delta \phi_f &= \epsilon \psi_f \\ \delta \psi_f &= -i (\sigma^\mu \epsilon^\dagger)_\alpha D_\mu \phi_f + \epsilon_\alpha F_f \\ \delta F_f &= -i \epsilon^\dagger \bar{\sigma}^\mu \underset{D_\mu \psi_f}{\cancel{D_\mu \psi_f}} \mp \sqrt{2}\, T (\epsilon^\dagger \lambda^\dagger) \phi_f \end{aligned}$$

$$\boxed{\begin{aligned} \mathcal{L}_{\text{matter}} = {} & D^\mu \bar{\phi}_f D_\mu \phi_f + i \psi_f^\dagger \bar{\sigma}^\mu D_\mu \psi_f \\ & + \bar{F}_f F_f + \sqrt{2} g (\phi^\dagger \lambda \psi + \cancel{\lambda^\dagger \psi^\dagger \phi} \\ & \qquad + \psi^\dagger \lambda^\dagger \phi) \\ & - \cancel{\phi^\dagger T} - \phi^\dagger D \phi + \cancel{h.c.} \\ & + \tfrac{1}{2} W^j F_j - \tfrac{1}{2} W^{jk} \psi_j \psi_k \end{aligned}}$$

$\xi = -$

$$W(\phi_j), \quad W^j = \frac{\partial W}{\partial \phi_j}, \quad W^{jk} = \frac{\partial^2 W}{\partial \phi_j \partial \phi_k}$$

$$V(\phi, \phi^*) = \bar{F}^i F_i + \tfrac{1}{2} D^s D^s = W_i^* W^i + \tfrac{1}{2} \Big(\sum_j \bar{\phi}_j T^s \phi_j\right)^2$$

susy $\boxed{V(\phi, \phi^*) \geq 0}$

SUSY GAUGE Theory

quark chiral field

1. $\left(\begin{array}{ll} Q_i = (q_i, \psi_i, F_i) & i = 1 \cdots f \\ \bar{Q}_i = (\bar{q}_i, \bar{\psi}_i, \bar{F}_i) & i = \cdots f' \end{array} \right.$

anti-quark chiral field

Conjugate

$Q_i^* = (q_i^*, \psi_i^*, F_i^*)$

bare coupling ~~bare~~ f

2. Superfield $\qquad \tau_0 = \frac{\theta_0}{2\pi} + \frac{i 4\pi}{e_0^2}$

$$\mathcal{L}_{SQCD} = \frac{1}{8\pi} \mathrm{Im} \left(\tau_0 \int d^2\theta \, W^\alpha W_\alpha \right)$$

$$+ \int d^4\theta \left(Q_f^\dagger e^{-2V} Q_f + \tilde{Q}_f e^{2V} \tilde{Q}_f^\dagger \right)$$

$$+ \int d^2\theta \, W(Q_f, \tilde{Q}_f)$$

V: vector field

W_α : chiral field strength

$\left. \begin{array}{l} Q_f \\ \tilde{Q}_f \end{array} \right|$ chiral field.

- $Q_f \to e^{-i\Lambda} Q_f$ $\qquad \Lambda$: chiral $= \Lambda^a T^a$

$\tilde{Q}_f \to e^{-i\tilde{\Lambda}} Q_f$ $\qquad \tilde{T}^a = -T^{a*} = -(T^a)^T$

$e^{-2V} Q_f = e^{-2V^a T^a} Q_f$, $\quad e^{-2\tilde{V}} \tilde{Q}_f = e^{-2\tilde{T}^a V^a} \tilde{Q}_f$

3. Symmetry $\quad SU(N_c)$ gauge

	$SU(N)$	$SU(N_F)_L$	$SU(N_F)$	$U(1)_B$	$U(1)_A$	$U(1)_R$
Q_f	N	N_F	1	1	1	n
$\tilde{Q}_f$	$\bar{N}$	1	$\bar{N}_F$	-1	1	n

- $U(1)_A$: $\quad Q_f(x,\theta) \to e^{i\alpha} Q_f(x,\theta)$

$\quad \tilde{Q}_f(x,\theta) \to e^{i\alpha}\tilde{Q}_f(x,\theta)$

- $U(1)_R \quad \theta_\alpha \to e^{-i\beta}\theta_\alpha, \quad \bar{\theta}^{\dot\alpha} \to e^{i\beta}\bar{\theta}^{\dot\alpha}$

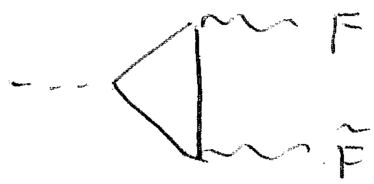
$$\begin{cases} Q(y,\theta) \to e^{in\beta} Q_f(y, e^{-i\beta}\theta) \\ V \to V(x, e^{-i\beta}\theta; e^{i\beta}\bar\theta) \\ W^a_\alpha(y,\theta) \to e^{i\beta} W^a_\alpha(y, e^{-i\beta}\theta) \end{cases}$$

$$-\left[\begin{array}{l} \int d^2\theta\, W^{a\alpha} W^a_\alpha = W^{a\alpha}W^a_\alpha\big|_{\theta^2} \quad = \text{inv} \\ \int d^4\theta\, Q_f^\dagger Q_f = \text{inv} \end{array}\right.$$

$U(1)_R$: $$\begin{cases} A_\mu,\ \lambda_\alpha,\ D \to A_\mu,\ e^{i\beta}\lambda_\alpha,\ D \\ q_f,\ \psi_f,\ F_f \to e^{i\beta} q_f,\ e^{i(n-1)}\psi_f,\ e^{i(n-2)}F_f \\ \epsilon_\alpha \to e^{i\beta}\epsilon_\alpha \ : \text{susy param.} \\ Q^\alpha \to e^{i\beta} Q^\alpha \quad \text{or} \quad [R, Q] = -Q \end{cases}$$

Anomalous Current

1) j_5^μ chiral current $\bar{\psi}\gamma_\mu\gamma_5\psi$



$$\partial_\mu j_5^\mu \simeq F\tilde{F}$$

$$j_5^\mu = A\,\bar{\lambda}\gamma_\mu\gamma_5\lambda + B\,\bar{\psi}_f\gamma_\mu\gamma_5\psi_f + C\,\bar{\tilde{\psi}}_f\gamma_\mu\gamma_5\tilde{\psi}_f$$

$B = C$

2)
$$\left(\begin{array}{l} \mathrm{Tr}\left(T^a_{\text{fund}}\, T^b_{\text{fund}}\right) = \frac{1}{2}\delta^{ab} \\ \mathrm{Tr}\left(T^a_{\text{adj}}\, T^b_{\text{adj}}\right) = N\delta^{ab} \end{array}\right.$$

$$NA + \frac{2N_f}{2}B = 0 \qquad B = -\frac{N}{N_f}A \quad , \quad A = 1$$

3) Anomaly Free $U(1)_{AF}$

$$\boxed{\lambda : 1 . \qquad \psi_f \ \tilde{\psi}_f : -\frac{N_c}{N_f}}$$

$W_\alpha = \lambda_\alpha + .$

$$\left[\begin{array}{l} W_\alpha(x,\theta) \to e^{i\alpha} W_\alpha(x, e^{-i\alpha}\theta_\alpha) \\ Q_f(x,\theta) \to e^{\cancel{i\alpha} + i(1-\frac{N_c}{N_f})\alpha} Q_f(x, e^{-i\alpha}\theta_\alpha) \end{array}\right.$$

4. 1-loop β-function

$$\beta_e = \mu \frac{de}{d\mu} = -\frac{e^3}{16\pi^2}\left(\frac{11}{3}T(Ad) - \frac{2}{3}\underset{\substack{\uparrow \\ \text{chiral}}}{T(F)} - \frac{1}{3}\underset{\substack{\uparrow \\ \text{complex scal}}}{T(S)}\right)$$

$$\equiv -\frac{e^3}{16\pi^2} b$$

<u>SUSY</u> • vector $\quad T(Ad) = N, \quad T(F) = N\frac{1}{2}$

$$b_{vect} = \left(\frac{11}{3} - \frac{2}{3}\right)\cdot N = 3N$$

• $Q_f, \tilde{Q}_f$

$$\left.\begin{array}{l} T(F) = N_F \cdot \frac{1}{2} \cdot 2 = N_F \\ T(S) = N_F \cdot \frac{1}{2} \cdot 2 = N_F \end{array}\right\} b_f = -N_f$$

$$b_{sQCD} = 3N - N_F$$

– <u>N=2 SUSY</u>

A_μ		$-N$
$\lambda\ \psi$	adjoint	$2N$
ϕ		$-N$

$\left.\begin{array}{l}\psi\ \tilde{\psi} \\ q\ \tilde{q}\end{array}\right\}$ hyper fundamental

$$b_{vec}\left(\frac{11}{3} - \frac{4}{3} - \frac{1}{3}\right)N = \frac{6}{3}N = 2N$$

$$b_{hyp} = -N_f$$

$$b_{N=2} = 2N - N_f = 0 \text{ if } N_f = 2N$$

$SU(2)$ $\quad N_f = 4$

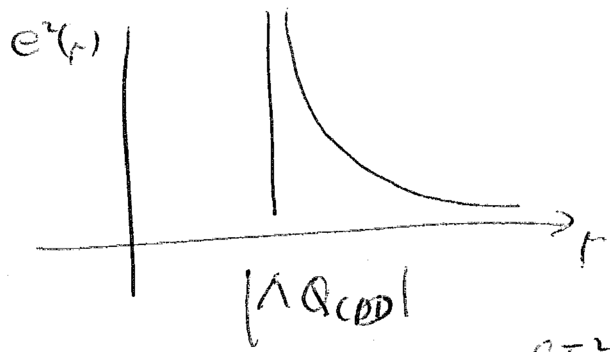
– $N=4$ SUSY $\quad A_\mu,\ \lambda\ \psi,\ \phi \quad \Big| \quad \left.\begin{array}{l}\psi\ \tilde{\psi} \\ q\ \tilde{q}\end{array}\right\}$ adjoint hyper $\quad \frac{N}{\cancel{2}} \cdot 2 \cdot \cancel{2}$ $\quad b = 2N$

$$b = b_V - 2N = 0$$

4'

$$\mu \frac{d}{d\mu}\left(\frac{1}{e^2(\mu)}\right) = \frac{b}{8\pi^2} \qquad b = 3N - N_f$$

$$\frac{1}{e^2(\mu)} = \frac{b}{8\pi^2} \ln \frac{\mu}{|\Lambda_{QCD}|}$$



$$|\Lambda_{QCD}| = \mu\, e^{-\frac{8\pi^2}{e^2(\mu) b}} \qquad \text{indep of } \mu$$

$$\sim \mu\, e^{+\frac{2\pi i}{b}\underbrace{\left[\frac{4\pi i}{e^2(\mu)} + \frac{\theta(\mu)}{2\pi}\right]}_{\tau_{eff}(\mu)}}$$

$$\simeq \mu\, e^{\frac{2\pi i}{b}\cdot \tau_{eff}(\mu)}$$

$b > 0$ asymptotically free

$$\frac{1}{e^2(\mu)} - \frac{1}{e^2(\mu_0)} = \frac{b}{8\pi^2} \ln \mu/\mu_0$$

$$\frac{1}{e^2(\mu)} = \frac{b}{8\pi^2} \ln\left(\frac{\mu}{\underbrace{\mu_0 e^{-\frac{8\pi^2}{b\, e_0^2}}}_{=|\Lambda|}}\right) \quad \rightarrow \quad e^2(\mu) = \frac{1}{\ln\left|\frac{\mu}{\Lambda}\right|^{\frac{8\pi^2}{b}}}$$

$$\boxed{\frac{e^2(\mu)}{4\pi} = \frac{1}{\ln\left|\frac{\mu}{\Lambda}\right|^{\frac{2\pi}{b}}}}$$

$$\boxed{\text{weak coupling} \quad e_0^2 \to 0 \quad |\Lambda| \to 0}$$

5 Classical Moduli space (Vacuum)

- $D^a = q_f^+ T^a q_f - \tilde{q}_f T^a \tilde{q}_f^+ = 0$

$$\to \mathrm{Tr}\left(T^a (q q^+ - \tilde{q}^+ \tilde{q})\right) = 0$$

$$\mathcal{U} = \frac{1}{2} D^a D^a$$

- $D = q_f q_f^+ - \tilde{q}_f^+ \tilde{q}_f \quad \overset{\swarrow \text{FI for } U(1)}{\xi} \ : N \times N$

$$D = 0 \Rightarrow \sum_f q_f q_f^+ = \sum_f \tilde{q}_f^+ \tilde{q}_f + \alpha \overset{\swarrow \text{identity}}{I} \ : N \times N$$

- Diagonalize $q_f q_f^+$ $\quad N_c \times N_c \to \tilde{q}_f^+ \tilde{q}_f$ diagonal

- $q_f = \begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_F \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\qquad \tilde{q}_f = \begin{pmatrix} \tilde{v}_1 & & \\ & \ddots & \\ & & \tilde{v}_F \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{pmatrix}$

$N_F \leq N_c$ $\qquad N_F \leq N_c$

rank N_F $\qquad SU(N_c) \to SU(N_c - N_F)$

- $N_F \geq N$

$$q_f = \begin{pmatrix} v_1 & & & 0 & 0 \\ & \ddots & & 0 & 0 \\ & & v_{N_c} & 0 & 0 \end{pmatrix} \qquad \tilde{q}_f^T = \begin{pmatrix} \tilde{v}_1 & & & 0 & 0 \\ & \ddots & & \vdots & \vdots \\ & & \tilde{v}_{N_c} & 0 & 0 \end{pmatrix}$$

$|v_i|^2 = |\tilde{v}_i|^2 + \rho$. indep of i

$SU(N_F) \to$ broken completely

- meson chiral field $\qquad Q_f,\ \tilde{Q}_f$

$$M_{fg} = Q_f \tilde{Q}_f Q_g \qquad N_F \times N_F$$

- baryon chiral field $\qquad N_F \geq N_c$

$$B_{f_1 \cdots f_{N_c}} = \underset{\substack{\text{color} \\ N \cdot N_c}}{\mathrm{Det}}\ Q_{f_1} Q_{f_2} \cdots Q_{f_{N_c}}$$

$$\tilde{B}_{f_1 \cdots f_{N_c}} = \underset{N_c}{\mathrm{Det}}\ \tilde{Q}_{f_1} \tilde{Q}_{f_2} \cdots \tilde{Q}_{f_{N_c}}$$

$\boxed{\text{Nonperturbative Effects}}$

1. Anomaly

$$\partial_\mu j^\mu_A = \frac{T(r)}{16\pi^2} F^{a\mu\nu} \tilde{F}^a_{\mu\nu}$$

2. Instanton

$$\begin{cases} D^\mu F^a_{\mu\nu} = 0 \\ F_{\mu\nu} = \tilde{F}_{\mu\nu} \\ A_\mu = 2 \cdot \frac{\quad}{r^2} \rightarrow i\partial_\mu U U^\dagger \end{cases}$$

- $\pi_3(SU(2)) = \mathbb{Z}$

~~$2N_c$ gaugino zero mode~~

$$\frac{1}{32\pi^2}\int d^4x\, F^a_{\mu\nu} \tilde{F}^a_{\mu\nu} = n \quad \text{integer}$$

$$S_E = \frac{1}{4e^2}\int d^4x (F^a_{\mu\nu})^2 = \frac{8\pi^2}{e^2} = 2\pi \cdot \frac{4\pi}{e^2} \cdot n$$

- $n_\psi - n_{\bar\psi} = 2T(r) \cdot n$

- $\mathcal{L}_{CP} = -\frac{\theta}{32\pi^2} F^{a\mu\nu} \tilde{F}^a_{\mu\nu}$

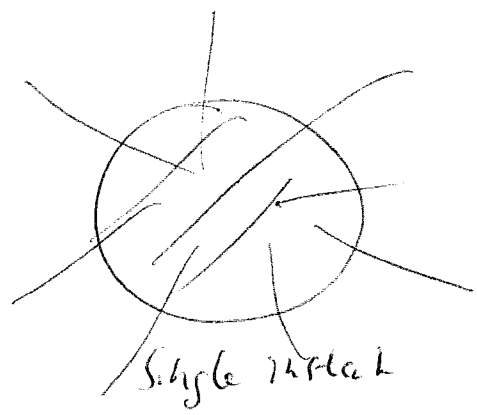
$$\boxed{-S_E = \int d^4x\, i\mathcal{L}_{CP} - S_E^{YM} = i\theta - \frac{8\pi^2}{e^2} = 2\pi i \cdot \tau}$$

Single Instanton effect $e^{-S_E} = e^{2\pi i \tau}$

- Instanton

λ : $2N_c$ zero modes

ψ_f : single zeromode for each $\psi_f, \tilde{\psi}_f$



single instanton

$$\boxed{\lambda^{2N_c}\ \psi_1 \cdots \psi_{N_f}\ \tilde{\psi}_1 \cdots \tilde{\psi}_{N_f} \times e^{+2\pi i\tau}}$$

vertex approach

$$(\lambda\lambda)^{N_c} \det\psi \det\tilde{\psi}\, e^{2\pi i\tau}$$

3. Anomaly in path integral

$$S_F = \int d^4x\, i\bar{\psi}\,\bar{\sigma}^\mu D_\mu \psi$$

$$\psi \to e^{i\alpha(x)}\psi$$

$$S_F \to S_F - \int d^4x\, \alpha(x)\,\partial_\mu \underbrace{(\bar{\psi}\bar{\sigma}^\mu\psi)}_{j^\mu_A}$$

$$Z[A_\mu] = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\, e^{-S_E}$$

indep of axial rotation

$$\mathcal{D}\psi\mathcal{D}\bar{\psi} \to \mathcal{D}\psi\mathcal{D}\bar{\psi}\, e^{-i\int d^4x\, \alpha(x) A(x)}$$

$$A(x) = \frac{T(r)}{16\pi^2} F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}$$

$$\Rightarrow \quad \partial_\mu j^\mu_A(x) = \frac{T(r)}{16\pi^2} F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}$$

4. θ-term $\qquad \mathcal{L}_E = -\frac{\theta}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu}$

$$e^{-S_E} = + i \int d^4x \; \frac{\theta(x)}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu}$$

$U(1)_{AF}$: $\lambda \to e^{i\alpha}\lambda, \quad \psi_f, \tilde{\psi}_f \to e^{i\beta} \psi_f, \tilde{\psi}_f$

$$\theta \to \theta - 2N_c\,\alpha - 2N_f\,\beta$$

For A-F $U(1)_{AR}$ $\quad \beta = \frac{-N_c}{N_f} \quad \Rightarrow \theta \to \theta$

5

Holo~~nomy~~morphic

i. non-renormalization theorem

- Wilsonian Effective Action

$$\int_{|p|\geq\mu} \mathcal{D}\phi(p)\, e^{iS} = e^{iW_{eff}(\phi)}\Big|_{|p|\leq\mu}$$

high energy degrees of freedom integrated

- Regard ~~holom~~ "coupling constants" holomorphic ← background field.

- $W_{tree} = \frac{m}{2}\phi^2 + \frac{\lambda}{3}\phi^3$

ϕ: chiral , λ: zero

- $[W]_R = 2$, $[\lambda]_R = \cancel{0}1$ $[\phi] = \frac{2}{3}$

- $\Phi \to e^{i\alpha}\Phi$: $U(1)$

$U(1) \times U(1)_R$ $\quad$ m, λ: Spurion {background}

	$U(1)$	$U(1)_R$
ϕ	1	1
m	-2	0
λ	-3	-1

← invariant under $U(1)\times U(1)_R$

— $W_{eff} = m\phi^2 h\left(\frac{\lambda\phi}{m}\right) = \sum_n a_n \lambda^n m^{1-n}\phi^{2+n}$

{ perturbative $n \geq 0$; massless limit $1-n\geq 0$ } $\quad n = 0, 1$

$W_{eff} = \frac{m}{2}\phi^2 + \frac{\lambda}{3}\phi^3 = W_{tree}$ ← not renormalized

2. Wave fn Renormalisation & ~~Physical~~

Running mass & coupling

$$\mathcal{L}_{Kin} = Z\, \partial_\mu \phi^+ \partial^\mu \phi + i Z\, \psi^+ \bar{\sigma}^\mu \partial_\mu \psi$$

$$\left(Z = Z(m, \lambda, m^+, \lambda^+, \mu, \Lambda) \right.$$ cut off

$\mu > m$

$$Z = 1 + c\lambda\lambda^+ \ln\left(\frac{\Lambda^2}{\mu^2}\right)$$, c : a number

$\mu < m$

$$Z = 1 + c\lambda\lambda^+ \ln\frac{\Lambda^2}{|m|^2}$$

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running m, λ

$$m(\mu) = \frac{m}{Z}, \quad \lambda(\mu) = \frac{\lambda}{Z}$$

3. Coupled Fields

$$W = \frac{1}{2} M \phi_H^2 + \frac{\lambda}{2} \phi_H \phi^2$$

	$U(1)_A$	$U(1)_B$	$U(1)_R$
ϕ_H	1	0	1
ϕ	0	1	$\frac{1}{2}$
M	-2	0	0
λ	-1	-2	0

$U(1)_A$, $U(1)_B$: spurion M, $\lambda \neq 0$

Integrate ϕ_H down to $\mu < M$

- $W_{eff} \sim \lambda^p M^k \phi^j$
- $W_{eff} = \lambda^2 M^{-1} \phi^4 = -\dfrac{\lambda^2 \phi^4}{8M}$

$\Rightarrow$ perturbative result (no higher order correction)

- Algebra eq

$$\frac{\partial W}{\partial \phi_H} = 0 \qquad M \phi_H + \frac{\lambda}{2} \phi^2 = 0, \quad \phi_H = -\frac{\lambda \phi^2}{2M}$$

4.

$$W = \frac{1}{2} M \phi_H^2 + \frac{\lambda}{2} \phi_H \phi^2 + \frac{y}{6} \phi_H^3$$

$$\frac{\partial W}{\partial \phi_H} \qquad M\phi_H + \lambda \phi^2 + \frac{y}{2} \phi_H^2 = 0$$

$$\phi_H = \frac{-M \pm \sqrt{M^2 - 2\lambda y \phi^2}}{y}$$

$$= -\frac{M}{y}\left(1 \pm \sqrt{1 - \frac{\lambda y \phi^2}{M^2}}\right)$$

$y \to 0 \qquad \phi_H = -\frac{\lambda \phi}{2M} \quad$ (prev)

$\infty \quad$ (odd)

$$W_{eff} = \frac{M^3}{3y^2}\left(1 - \frac{3\lambda y \phi^2}{2M^2} \pm \left(1 - \frac{\lambda y \phi^2}{M^2}\right)\sqrt{1 - \frac{\lambda y \phi^2}{M^2}}\right)$$

Singular point of W_{eff} : ϕ_H massless

ϕ_H mass $\qquad \frac{\partial^2 W}{\partial \phi_H^2} = M + y\phi_H = \mp M\sqrt{1 - \frac{\lambda y \phi^2}{M^2}}$

y : U(1)A U(1)B U(1)R
$-3 \quad 0 \quad -1$

$$W_{eff} = \frac{M^3}{y^2} f\left(\frac{\lambda y \phi^2}{M^2}\right)$$

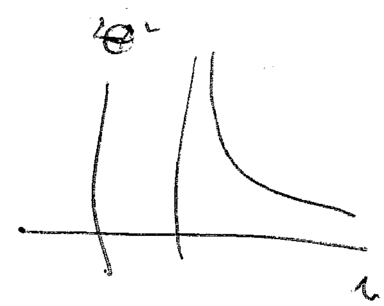
$$\boxed{\text{SYM} \quad (N_F = 0)}$$

1. $$W_\alpha^a = -i\lambda_\alpha^a(y) + \theta_\alpha D^a - (\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu}^a(y)$$
$$- (\theta\theta)\sigma^\mu D_\mu \lambda^{a\dagger}(y) + \cdots$$

$$\tau = \frac{\theta_{YM}}{2\pi} + i\frac{4\pi}{e^2(\mu)}$$

2. $$\mathcal{L} = \frac{1}{8\pi} \mathrm{Im}\left(\tau\, \cancel{\mathrm{tr}}\, W^{\alpha a} W_\alpha^a\right)\Big|_{\theta^2}$$
$$= -\frac{F_{\mu\nu}^a F^{a\mu\nu}}{4e^2} - \frac{\theta}{32\pi^2} F_{\mu\nu}^a \tilde{F}^{a\mu\nu}$$
$$\tilde{F}^{a\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}^a$$

3. $$\cancel{\mathrm{He}}\quad \mu\frac{d}{d\mu}\left(\frac{1}{e^2(\mu)}\right) = \frac{b}{8\pi^2}$$



$$\underset{\text{1-loop}}{\frac{4\pi}{e^2(\mu)}} = \frac{b}{2\pi}\ln\frac{\mu}{|\Lambda_{QCD}|}$$

$$\underset{\text{1 loop}}{\tau_{\cancel{QCD}}} = \frac{\theta}{2\pi} + i\frac{4\pi}{e^2(\mu)}$$ ← holomorphic coupling
$$= \frac{1}{2\pi i}\ln\left[\left(\frac{|\Lambda|}{\mu}\right)^b e^{i\theta}\right]$$

$$\boxed{\Lambda \equiv |\Lambda|\, e^{\frac{i\theta_{YM}}{b}}}$$ ← holomorphic scale

4. $$\tau_{1\text{-loop}} = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right)$$

- $$\theta_{YM} F\tilde{F} = \theta \partial^\mu W_\mu$$

no perturbation correction

$$\frac{\theta}{32\pi^2} \int d^4x\, F^a_{\mu\nu} \tilde{F}^{a\mu\nu} = n\, \theta_{YM}$$

periodic $\theta_{YM} \to \theta_{YM} + 2\pi$ $\quad e^{iS} \to e^{iS}$

5. Instanton

$$F_{\mu\nu} \pm \tilde{F}_{\mu\nu} = 0 \qquad \int d^4x\, F^2 = \int d^4x\, F\tilde{F} = 32\pi^2 n$$

$$\boxed{\begin{aligned} e^{-S_E} &= e^{-\frac{8\pi^2}{e^2} + i\theta} = e^{2\pi i \tau(\mu)} \\ &= \left(\frac{\Lambda}{\mu}\right)^b \qquad \text{holomorphic} \end{aligned}}$$

Anti instanton $\underline{n=-1}$ $\quad e^{-\frac{8\pi^2}{e^2} - i\theta} = e^{-2\pi i \bar{\tau}} = \left(\frac{\bar{\Lambda}}{\mu}\right)^b$

6. ~~perturbat~~ Effective Coupling

$$\int_{p>\mu} \mathcal{D}A\, \mathcal{D}\lambda\; e^{-\int d^4x \left[\left(\frac{1}{8\pi} \operatorname{Im}(\tau W^{\alpha a} W^a_\alpha) + h.c.\right)\right]}$$

$$= e^{-\int d^4x \left[\frac{1}{8\pi} \operatorname{Im}\left(\tau_{eff}(\mu) W^{a\alpha} W^a_\alpha\right) + h.c.\right]}$$

- $\tau_{eff}(\mu) = \overset{\text{1-loop}}{\frac{b}{2\pi i}\ln\left(\frac{\Lambda}{\mu}\right)} \left(\begin{array}{l} + \text{higher order} \\ + \text{nonperturbative th} \end{array}\right)$

$$+ \underset{\text{holomorphic}}{f\left(\frac{\Lambda}{\mu}\right)}$$

$\theta_{YM} \to \theta_{YM} + 2\pi \quad \xrightarrow{\text{sym}} \quad \Lambda \to \Lambda e^{\frac{2\pi i}{b}}$

- Under $\Lambda \to \Lambda e^{\frac{2\pi i}{b}}$ $\quad \boxed{\tau_{eff}(\mu) \to \tau_{eff} + 1}$.

- ~~low energy~~ weak coupling limit $\Lambda \to 0$
- higher order in perturbation $\quad [\ln\Lambda]^n \to \left(\ln\Lambda + \frac{2\pi i}{b}\right)^n$
- nonperturbative instant. $\quad \left(\frac{\Lambda}{\mu}\right)^b \to \left(\frac{\Lambda}{\mu}\right)^b$

$$\Rightarrow \boxed{\tau_{eff}(\Lambda, \mu) = \frac{b}{2\pi i}\ln\left(\frac{\Lambda}{\mu}\right) + \sum_{n=1}^{\infty} a_n \left(\frac{\Lambda}{\mu}\right)^{bn}}$$

$U(1)_R$

7. Chiral Symmetry Breaking

- $U(1)_A = U(1)_R$ broken by anomaly

- Instanton
$$(\lambda^2)^{N_c}\, e^{2\pi i \tau} \sim (\bar{\Lambda}^3)^{N_c}\, e^{i\theta - \frac{8\pi^2}{e^2}}$$

- $\left.\begin{array}{l}\lambda \to e^{i\alpha}\lambda \\ \bar{\lambda} \to e^{-i\alpha}\bar{\lambda}\end{array}\right\}$ is equivalent to $\theta \to \theta \mp 2N_c\alpha$

- $\alpha = \frac{\pi}{N_c} = \frac{2\pi}{2N_c} \Rightarrow \theta \to \theta - 2\pi$ periodic

- Thus Instanton vertex is inv under
$$\bar{\lambda}^{\dot{\alpha}}_{\alpha} \to e^{-i\frac{2\pi}{2N_c}}\bar{\lambda}^{\dot{\alpha}}_{\alpha}$$
$b = 3N_c$

or $U(1)_R \longrightarrow Z_{2N_c}$
explicitly broken

- $$\langle (\bar{\lambda}^2)^{N_c} \rangle = \langle 0 | (\lambda^2)^N (\bar{\lambda}^2)^{N_c} e^{2\pi i \tau} \rangle \sim e^{2\pi i \tau}$$
$$\sim (\Lambda^3)^{N_c}$$

- Spurion Symmetry: $\lambda \to e^{i\alpha}\lambda,\ \theta \to \theta - N\alpha$

- ~~this~~

8. W_{eff}

- regard $\tau(\mu)$ as a chiral field (spurion)

- no massless degree of freedom (confinement)

- massive color singlet . $\underline{gg, \lambda\lambda, g\lambda}$

- R-charge restored if $\left(\begin{array}{l}\lambda \to e^{i\alpha}\lambda \\ \tau \to \tau + \frac{2N_c}{2\pi}\alpha\end{array}\right.$ $\quad \theta \to \theta + 2N_c\alpha$

$[W_{\text{eff}}] = 2$ $\qquad \int d^4x\, d^2\theta\, W$

$\dim [W_{\text{eff}}] = \text{mass}^3$

$b = 3N_c$

- $$\boxed{\begin{aligned} W_{\text{eff}} &= c\,\mu_0^3\, e^{\frac{2\pi i \tau(\mu_0)}{N_c}} \\ &= c\cdot \Lambda_{QCD}^3 \end{aligned}}$$

$$\Lambda_{QCD} = |\Lambda|\, e^{\frac{i\theta}{b}} = \mu_0\, e^{-\frac{1}{3N_c}\cdot\frac{8\pi^2}{g^2} + \frac{i\theta}{3N_c}}$$

- $$\boxed{W_{\text{eff}} = N_c\, \Lambda_{QCD}^3}$$ $\quad N_F = 0$ $\qquad c = N_c$ later argument

unbroken Z_{2N_c} : $\theta \to \theta + 2\pi$

$\alpha = \frac{2\pi}{2N_c}$, $\quad \Lambda \to \Lambda\, e^{\frac{2\pi i}{3N_c}}$ $\qquad \Lambda^3 \to \Lambda^3\, e^{\frac{2\pi i}{N_c}}$

~~W_{eff}: not invariant und $\alpha = \frac{2\pi}{2N_c}\cdot k \quad k = 0, 1, \ldots N_c$~~

$(\tau + \theta^2 F_\tau)(\lambda\bar\lambda + \cdots)$

$\frac{1}{16\pi i} \tau W^a_\alpha W^a_\alpha + h.c.$

9. gluino condensation

- $$\langle \lambda^a \lambda^a \rangle = 16\pi i \frac{\partial}{\partial F_\tau} \ln Z = 16\pi i \frac{\partial}{\partial F_\tau} \left(W_{eff} \big|_{\theta^2} \right)$$

$$= 16\pi i \frac{\partial}{\partial \tau} W_{eff} = 16\pi i \cdot \Lambda^3 \cdot N_c \cdot \frac{2\pi i}{N_c}$$

$$= -32\pi^2 \cdot \Lambda^3$$

- Unbroken Z_{2N_c}

$$\begin{cases} \lambda \to \lambda e^{i\frac{2\pi}{2N_c}} \\ \lambda^a\lambda^a \to \lambda^a\lambda^a \cdot e^{i\frac{2\pi}{N_c}} \end{cases}$$

$$\Rightarrow \boxed{\langle \lambda^a \lambda^a \rangle = -32\pi^2 \Lambda^3 e^{i\frac{2\pi k}{N_c}}}$$

$\langle \cancel{f} \rangle \qquad k = 0, 1, \ldots N_c - 1$

$Z_{2k} \to Z_2$ spontaneously broken

$\boxed{N_c \text{ degenerated susy vacuums}}$

10 Veneziano - Yankielowicz Pot

$$S = -\langle 0| \frac{\mathrm{tr}\,\lambda\lambda}{16\pi^2} |0\rangle = \langle 0| \frac{\lambda^a \lambda^a}{32\pi^2} |0\rangle$$

$$= \Lambda^3 e^{i\frac{2\pi k}{N_c}} \quad \leftarrow \text{plan} \quad \Lambda^{3N_c} \sim e^{2\pi i \tau}$$

$$\mathcal{L}_{eff} = 2\pi i \tau(\mu) S - N_c S \ln \frac{S}{\mu^3} + N_c S$$

$$= -S \left[\ln \left(\frac{S}{\Lambda^3} \right)^{N_c} - N_c \right]$$

$$\frac{\partial \mathcal{L}}{\partial S} = \ln \left(\frac{S}{\Lambda^3} \right)^{N_c} = 0 \Rightarrow S = \Lambda^3 e^{\frac{2\pi i k}{N_c}}$$

$$\mathcal{L}_{eff} = N_c \Lambda^3 e^{\frac{2\pi i k}{N_c}}$$

$\boxed{\text{SQCD } 0 < N_f < N_c}$ Affleck-Dine-Seiberg

1. $\langle \phi_f \rangle = \langle \tilde{\phi}_f^\dagger \rangle = \begin{pmatrix} v_1 & 0 & 0 \\ & \ddots & \\ & & v_{N_f} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

- $SU(N_c) \to SU(N_c - N_f)$

broken generators $(N^2-1) - ((N_c - N_f)^2 - 1)$

$= 2N_c N_f - N_f^2$ → eaten to massive vector multiplet

1	$\frac{1}{2}$	0
2	4	2

- remaining scalar $2N_f N_c - (2N_c N_f - N_f^2)$

$= N_f^2$: massless

- $M_{f\bar{g}} = \phi_f \tilde{\phi}_g$ chiral, N_f^2 massless boson

singlet under $SU(N_c)$

$U(1)_A : 2$

$U(1)_B : 0$

$U(1)_{AF} = (2 - \frac{2N_c}{N_f})$

$= -2\left(\frac{N_c - N_f}{N_f}\right)$

2. Low energy effective ~~potential~~ Lagrangian

$b = 3N - N_f$

	$U(1)_A$	$U(1)_{AF}$
$W_\alpha W^\alpha$	0	2
Λ^b	$2N_f$	0
$\det M$	$2N_f$	$2(N_f - N_c)$
[illegible]	0	2

→ unbroken $SU(N_c - N_f)$ confining

$$W_{eff} \sim (\Lambda^b)^l \overbrace{(\mathrm{tr} W_\alpha W^\alpha)^m}^{\text{mass}} (\det M)^n$$

$l \geq 0$

- locality $\quad m \geq 0 \quad$ • pert. lim $\quad l \geq 0$

- $U(1)_A \quad l + n = 0 \qquad 2(N_f - N_c) \neq 0$

$\underline{m=1} \rightarrow n = l = 0$

$m=0 \rightarrow n + l = 0, \quad n = -l = \dfrac{-1}{N_c - N_f} < 0$

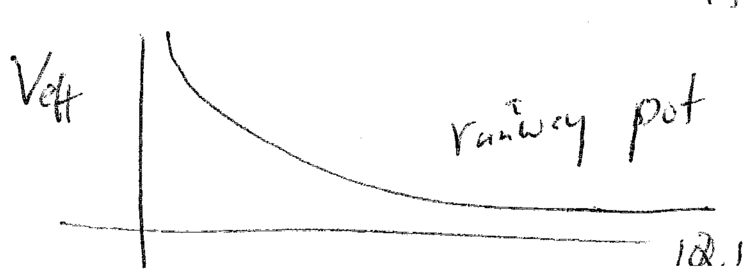
$$\boxed{W_{eff} = C_{N_c, N_f} \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}}}$$

Affleck - Dine - Seiberg

3. $\quad \theta \rightarrow \theta + 2\pi i \qquad (N_c - N_f)$

$W_{eff} \rightarrow W_{eff} \cdot e^{\frac{2\pi i}{N_c - N_f}} \qquad$ degeneracy in gluon condensate

$$\mathcal{U} = |F_f|^2 + |\tilde{F}_f|^2 = \left|\frac{\partial W}{\partial \phi_f}\right|^2 + \left|\frac{\partial W}{\partial \tilde{\phi}_f}\right|^2$$



4. Massive flavor

$SU(N_f)_L \times SU(N_f)_R \times U(1)_{AF} \times U(1)_A$

- $W_{\text{superpot}} = m_{f\bar{g}} \tilde{Q}_f Q_g$
$= \mathrm{tr}(mM)$

$M \to LMR^{-1}, \quad m \to RmL^{-1}$ $\quad \mathrm{tr}\, mM$: $U(1)_{AF} = 0$

$U(1)_A: \quad M = +2 \quad m = -2, \quad \cancel{W_{eff} = 2}$

$U(1)_{AF} \quad M: 2(1 - \frac{N_c}{N_f}), \quad m: \cancel{\ } \frac{N_c}{N_f})$
$= \frac{2(N_c - N_f)}{N_f}$

- Low energy effective action

$$W_{eff} = \left(\frac{\Lambda_{N_c,N_F}^{3N_c - N_F}}{\det M} \right)^{\frac{1}{N_c - N_f}} \cdot f\left(\underbrace{t = (\mathrm{tr}(mM) \Upsilon^{-1})}_{U(1)_{AF} = 0} \right)$$

$\equiv \Upsilon \quad \boxed{\cancel{U(1)_A = 2}}$
$\cancel{U(1)_{AF} = 2(N_c - N_f)}$

massless limit $\quad f(0) = C_{N_f, N_c} \quad W_{eff} =$

weak coupling $\quad f_t = W_{eff} = \mathrm{tr}(mM)$
$\Lambda \to 0$

$\Rightarrow \quad f(t) = C_{N_f, N_c} + t$

$$\boxed{W_{eff} = C_{N_f, N_c} \left(\frac{\Lambda_{N_c,N_F}^{3N_c - N_F}}{\det M} \right)^{\frac{1}{N_c - N_F}} + \mathrm{tr}\, mM}$$

5. Recursion Relations (I) between C_{N_F, N_c}

(1) $\delta \det M = (\det M)\, \mathrm{tr}\, M^{-1} \delta M$

make one flavor to be very massive, which would decouple $\quad m_{f,g} = \underline{m}\, \delta_{f,N_F}\, \delta_{g,N_F}$

$$\left(\begin{array}{ccc|c} & & & 0 \\ & & & 0 \\ & & & 0 \\ \hline 0 & 0 & 0 & m \end{array} \right)$$

$$\frac{\partial W_{eff}}{\partial M_{N_F N_F}} = C_{N_F, N_c} \left(\frac{\Lambda^{3N_c - N_F}}{\det M} \right)^{\frac{1}{N_c - N_F}} \cdot \left[\frac{-(N_c - N_F)^{-1}}{M_{N_F N_F}} \right] + m_{N_F N_F} = 0$$

$$\frac{\partial W_{eff}}{\partial M_{N_F, f}} = C_{N_F, N_c} \left(\frac{\Lambda^{3N_c - N_F}}{\det M} \right)^{\frac{1}{N_c - N_F}} \cdot \left[\frac{-1}{N_c - N_F} \right] \cdot \frac{\mathrm{cof}(M_{N_F, f})}{\det M} = 0 \quad \because m_{N_F, f} = 0$$

$(M^{-1})_{N_F, f}$

$f \neq N_F$

$\Rightarrow M_{N_F, f} = 0 \quad f \neq N_F \qquad M_{N_F} = \begin{pmatrix} \tilde{M} & 0 \\ 0 & M_{N_F N_F} \end{pmatrix}$

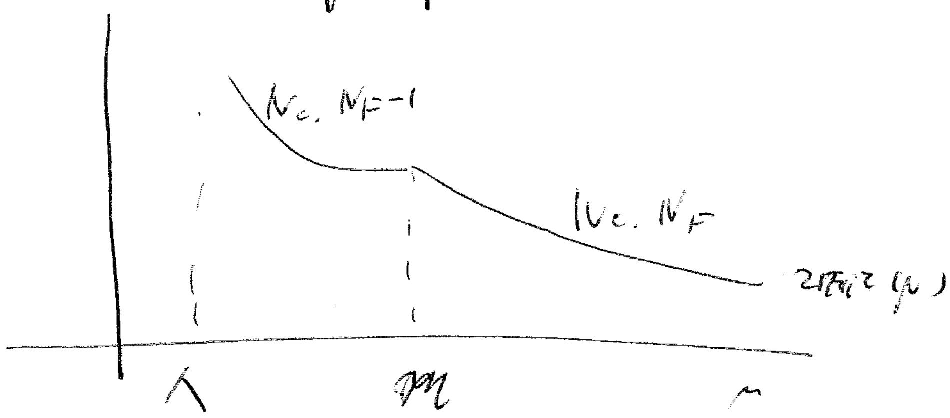
$$\frac{C_{N_F, N_c}}{N_c - N_F} \left(\frac{\Lambda^{3N_c - N_F}}{\det M} \right)^{\frac{1}{N_c - N_F}} = m_{N_F N_F} \cdot M_{N_F N_F}$$

$\downarrow$

$$W_{eff} = (N_c - N_F + 1)\, m M_{N_F N_F}$$

$$= (N_c - N_F + 1) \cdot \left[\frac{C_{N_F N_c}}{N_c - N_F} \right]^{\frac{N_c - N_F}{N_c - N_F + 1}} \cdot \left(\frac{\Lambda^{3N_c - N_F}}{\det \tilde{M}} \right)^{\frac{1}{N_c - N_F + 1}} \cdot m^{\frac{1}{N_c - N_F + 1}}$$

Continuation of coupling $\overline{DR}$ scheme



$$2\pi i \tau(m_{N_c,N_F}) = \left(\frac{\Lambda_{N_F N_c}}{m}\right)^{b = 3N-N_F} = \left(\frac{\Lambda_{N_c,N_F-1}}{m}\right)^{3N-N_F+1}$$

$$\Rightarrow \quad m\,\Lambda_{N_c,N_F}^{3N-N_F} = \Lambda_{N_c,N_F-1}^{3N-N_F+1}$$

$$\Rightarrow$$

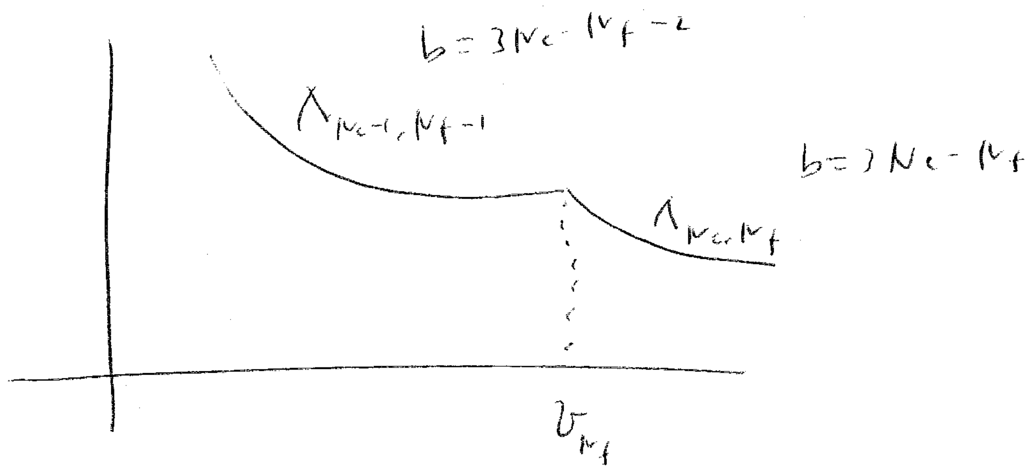
$$W_{eff} = (N-N_F+1)\cdot\left(\frac{C_{N_c N_F}}{N_c-N_F}\right)^{\frac{N_c-N_F}{N_c-N_F+1}}\cdot\left(\frac{\Lambda_{N_c,N_F-1}^{3N-N_F+1}}{\det\tilde{m}}\right)^{\frac{1}{N_c-N_F+1}}$$

$$\Rightarrow \quad \boxed{C_{N_c,N_F-1} = (N_c-N_F+1)\left(\frac{C_{N_c,N_F}}{N_c-N_F}\right)^{\frac{N_c-N_F}{N_c-N_F+1}}}$$

6. Recursion Relation II

$$\Phi_f = \tilde{\Phi}_f^{\dagger} = \begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_{N_F} \\ 0 & \cdots & 0 \end{pmatrix}$$

- large higgs expectation value
  $$v_{N_F} \gg v_f,\ \Lambda.$$
  $$SU(N_c) \to SU(N_c-1)$$
- massive vect boson $N_c^2-1-((N_c-1)^2-1) = 2N_c-1$
- N_F-1 flavor of $SU(N_c-1)$ gauge theory : low energy
  $$b = 3(N_c-1)-(N_F-1) = 3N_c-N_F-2$$



- $$2\pi i \tau = b' \ln\left(\frac{\Lambda_{N_c-1,N_f-1}}{v}\right) = b \ln\left(\frac{\Lambda_{N_c,N_f}}{v}\right)$$

$$v^2 \cdot \Lambda_{N_c-1,N_f-1}^{3N_c-N_f-2} = \Lambda_{N_c N_f}^{3N_c-N_f}$$

- $$W_{eff} = C_{N_f,N_c}\left(\frac{\Lambda_{N_c N_f}^{3N_c-N_f}}{v_{N_f}^2 \det M_{N_f-1}}\right)^{\frac{1}{N_c-N_f}}$$

$$= \underbrace{C_{N_f,N_c}}_{= C_{N_f-1,N_c-1}}\left(\frac{\Lambda_{N_c-1,N_f-1}^{3N_c-N_f-2}}{\det M}\right)^{\frac{1}{N_c-N_f}}$$

$$\Rightarrow \boxed{C_{N_f,N_c} = C_{N_c-N_f}}$$

- $$\Rightarrow C_{N_c-N_f+1} = (N_c-N_f+1)\left(\frac{C_{N_c-N_f}}{N_c-N_f}\right)^{\frac{N_c-N_f}{N_c-N_f+1}}$$

$$\Rightarrow \boxed{C_{N_c-N_f} = (N_c-N_f)\, C^{\frac{1}{N_c-N_f}}}$$

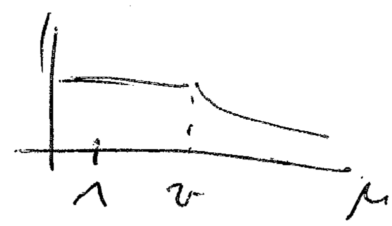
7

weak coupling calculation

- $N_f = N_c - 1$
- instanton $\Lambda^b = \Lambda^{3N_c - (N_c-1)} = \Lambda^{2N_c+1}$

$v \gg \Lambda$ also

$\phi_f = \tilde{\phi}_f^+ = \begin{pmatrix} v & & \\ & \ddots & \\ & & v \\ 0 & & 0 \end{pmatrix}$



weak coupling

single instanton correction

$\Rightarrow \quad c = 1$

$C_{N_c - N_F} = N_c - N_F$

8. $N_F = 0$ case

$$W_{exact} = (N_c - N_F)\left(\frac{\Lambda^{3N-N_F}}{\det M}\right)^{\frac{1}{N_c - N_F}} + \operatorname{tr} mM)$$

- start with $m = \begin{pmatrix} m_1 & & 0 \\ & \ddots & \\ 0 & & m \end{pmatrix} = m\, I_{N_F \times N_F}$

- δM $\quad M_{ij} = (m^{-1})_{ij}\left(\frac{\Lambda^{3N-N_F}}{\det M}\right)^{\frac{1}{N-N_F}}$

$\det M = (\det m)^{-1}\left(\frac{\Lambda^{3N-N_F}}{\det M}\right)^{\frac{N_F}{N_c-N_F}}$

$\Rightarrow$ ~~$\det M =$~~ $\boxed{M_{ij} = (m^{-1})_{ij}\left(\frac{\Lambda^{3N-N_F}}{\det m}\right)^{\frac{1}{N_c}}}$

- weak coupling $\boxed{OK}$
- massless $\det m \to 0$

8. $N_F = 0$ case

$$W_{exact} = (N_c - N_F)\left(\frac{\Lambda^{3N_c - N_F}}{\det M}\right)^{\frac{1}{N_c - N_F}} + \operatorname{tr} m M$$

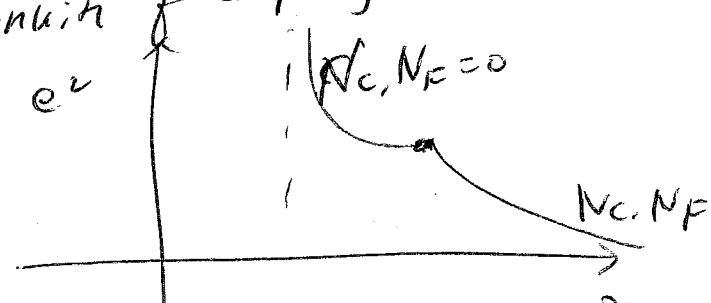
δM

$$M_{ij} = (m^{-1})_{ij}\left(\frac{\Lambda^{3N-N_F}}{\det M}\right)^{\frac{1}{N-F_N}}$$

$$\Rightarrow W_{exact} = N_c\left(\frac{\Lambda^{3N_c - N_F}}{\det M}\right)^{\frac{1}{N_c - N_F}}$$

$$= N_c \cdot \left(\det m \, \Lambda^{3N_c - N_F}\right)^{\frac{1}{N_F}}$$

- Continuity of coupling constant



$$e^{2\pi i \tau} = \left(\frac{\Lambda_{N_c, N_F = 0}}{m}\right)^{b = 3N_c} = \left(\frac{\Lambda_{N_c, N_F}}{m}\right)^{b = 3N_c - N_F}$$

$$\Rightarrow W_{exact}^{N_c, N_F = 0} = N_c \left(\Lambda_{N_c, N_F = 0}\right)^3$$

~~exact~~ as claimed before.

$N_F < 3N_c$

$\boxed{\text{SQCD} \quad N_F \geq N_c}$

1 Phases of gauge theories (effective pot) <u>IR limit</u>

Coulomb $V(R) = \frac{1}{R}$: conformal

Free electric $V(R) = \frac{1}{R\ln(R\Lambda)}$ } dual

Free magnetic $V(R) = \frac{\ln(R\Lambda)}{R}$

Higgs $V(R) = $ constant } dual

confining $V(R) = \sigma R$

2.

	$SU(N_c)$	$SU(N_F)_L$	$SU(N_F)_R$	$U(1)_B$	$U(1)_{AF}$	$U(1)_A$
Q_f	N	N_F	0	1	$\frac{1}{N_F}(N_c - N_F)$	1
$\tilde{Q}_f$	$\bar{N}$	$\bar{N}_F$	$\bar{N}_F$	-1		1
W_α^a	N^2-1	0	0	0	1	0

3.

$$\phi_f = \begin{pmatrix} v_1 & & & 0 & 0 \\ & \ddots & & 0 & \vdots \\ & & v_{N_c} & 0 & \vdots \end{pmatrix} \qquad \tilde{\phi}_f = \begin{pmatrix} \tilde{v}_1 & & & 0 & \\ & \ddots & & 0 & \\ & & \tilde{v}_{N_c} & 0 & \end{pmatrix}$$

$\downarrow N_c \qquad \xrightarrow{\quad} N_F$

$|v_i|^2 - |\tilde{v}_i|^2 = \alpha$ = indep of i

- $SU(N_c) \to$ completely broken
- massless chiral bosons $2N_F N_c - (N^2-1)$ n

light degrees of freedom · $M_{f\bar{g}} = Q_f \tilde{Q}_{\bar{g}}$ $\to N_F^2$

$2 \cdot \frac{N_F!}{N_c!(N_F-N_c)!} \left(\begin{array}{l} B_{f_1 \ldots f_{N_c}} = \epsilon_{n_1 \ldots n_{N_c}} Q_{f_1}^{n_1} \cdots Q_{f_{N_c}}^{n_{N_c}} \\ \tilde{B}_{f_1 \ldots f_{N_c}} \end{array} \right)$

$\exists$ constraints

- $B_{f_1 \cdots f_{N_c}} \tilde{B}_{\bar{s}_1 \cdots \bar{s}_{N_c}} = \epsilon_{f_1 \cdots f_{N_c}} M_{f_1 s_1} \cdots M_{f_N s_N}$

4. $$\langle M \rangle = \begin{array}{c|c} & \begin{pmatrix} v_1\tilde{v}_1 & & & 0 & \\ & \ddots & & & \\ 0 & & v_{N_c}\tilde{v}_{N_c} & & \\ & & & 0 & \\ & 0 & & & 0 \end{pmatrix} \\ N_F & \\ & N_F \end{array}$$ rank N_c

$$\langle B_{1\cdots N} \rangle = v_1 \cdots v_N, \quad \langle \tilde{B}_{1\cdots N} \rangle = \tilde{v}_1 \cdots \tilde{v}_N$$

5.

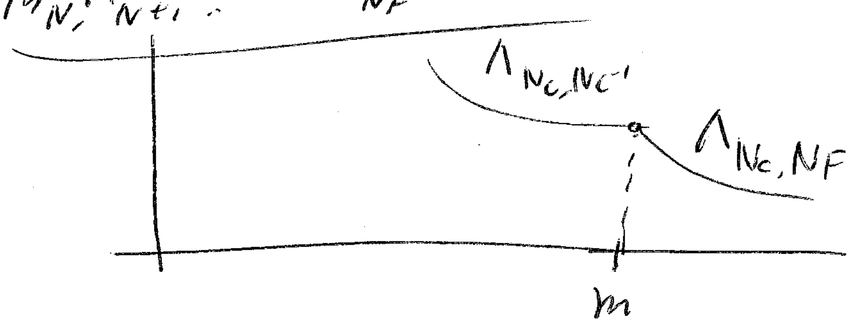
$$\boxed{M_{f\bar{g}} = (m^{-1})_{f\bar{g}} \left(\det m \, \Lambda^{3N_c - N_F} \right)^{\frac{1}{N_c}}}$$ [true for generic msu]

from ADS pot $\quad m^{\frac{N_F}{N_c}-1} = m^{\frac{N_F-N_c}{N_c}}$

- $\det m \neq 0 \quad N_f < N_c \quad m \to 0 \quad M \to \infty$ runaway soln

  $N_f > N_c \quad m \to 0 \quad M \to 0$

- weak coupling $\Lambda \to 0 \quad M \to 0$ fine if $3N_c - N_F > 0$
- $m_N, m_{N+1} \ldots m_{N_F} \gg m \gg \quad N_F - (N-1)$



$$e^{i2\pi\tau(m)} = \left(\frac{\Lambda_{N_c,N_F}}{m} \right)^{3N_c - N_F} = \left(\frac{\Lambda_{N_c,N_F-1}}{m} \right)^{3N_c-(N_c-1)}$$

- $\Rightarrow \det m \, \Lambda_{N,F}^{3N-N_F} = (\cancel{\det m}) \cdot \Lambda_{N_c,N_c-1}^{3N_c-(N_c-1)}$

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$$m = \begin{pmatrix} m_L & \\ & m_{N_c \cdots N_F} \end{pmatrix}$$

- $$M_{L\, f\bar{5}} = (m_L^{-1}) \left(\det m \det \tilde{m}\, \Lambda^{3N_c - N_F} \right)^{\frac{1}{N_c}}$$

$$= m_L^{-1} \cdot \left(\det m\, \Lambda_{N_c, N_c-1}^{3N_c - (N_c - 1)} \right)^{\frac{1}{N_c}} \qquad \underline{\underline{OK}}$$

6. $3N > N_F \geq N_c$

- vacuum degenaracy is not lifted
- quantum modificat of the relat betw M & B

7. $\frac{3N_c}{2} \leq N_F \leq 3N_c$

- $N_c \to \infty \qquad 3N_c - N_F \simeq 0 \to$

IR fixed point $\quad g_*^2 = \frac{8\pi^2}{3} \frac{N}{N^2-1} \epsilon$

- superconformal field theory
  ~~exact~~ NSVZ exact β fn
- $D(M) = 3 - \dfrac{3N_c}{N_F} \geq 1 \Rightarrow \quad \frac{3N}{2} \leq N_F \leq N_c$

dim

$\Rightarrow$ • non abelian Coulomb phase $(\langle M \rangle = 0$
- superconformal field thy.
- no particle interpretn

$\Rightarrow$ ~~[illegible]~~

$\frac{3}{2}N_c < N_F < 3N_c$

$\boxed{\text{IR fixed Point \& SCFT}}$

[38]

1. Banks Zaks

physical $\beta(g) = -\frac{g^3}{16\pi^2} \frac{(3N - F(1-\gamma))}{1 - N g^2/8\pi^2}$

Near $F \simeq 3N_c$, $N_c \to \infty$

$N_F = 3N_c - \epsilon N_c$

$$\gamma = -\frac{e^3}{8\pi^2} \frac{N^2-1}{N} + O(e^4)$$

$$16\pi^2 \beta(e) \simeq -g^3 \epsilon N + \frac{g^5}{8\pi^2}\left(3(N^2-1) + O(\epsilon)\right) + O(e^7)$$

$$e_*^2 = \frac{8\pi^2}{3} \cdot \frac{N}{N^2-1} \epsilon$$

2. $g = g_*$: superconformal

$\dim = \frac{3}{2} R_{sc}$ chiral superfield

$$\dim(M) = 2 + \gamma_* = \frac{3}{2} \cdot 2 \cdot \frac{N_F - N_c}{N_F}$$

$\frac{3N}{F} \leq 2$

$$= 3 - \frac{3N_c}{N_F}$$

$2 - \frac{3N}{F} \geq 0$

$\dim(M) \geq 1 \Rightarrow N_F \geq \frac{3N_c}{2}$

$\xleftarrow{}$ scalar

$\boxed{N_F = \frac{3}{2}N_c \text{ : Free}}$

'tHooft Anomaly Matching Condition

- Yang-Mills + Matter

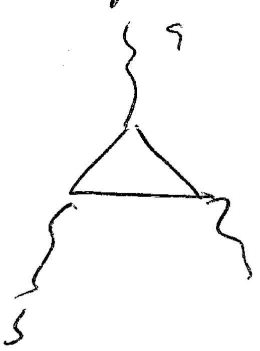
  gauge sym G_{color}

  global symmetry G ← three currents $J^a_\mu, J^b_\mu, J^c_\mu$

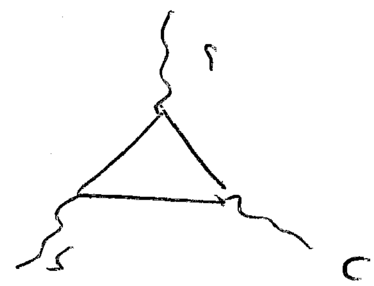
  $Tr(T^a\{T^b, T^c\})$

- $\boxed{\text{Thm}}$ Anomaly of three currents

$\sum$ elementary fermions UV



$=$ $\sum$ massless ferm IR



- proof. Add leptons or massless fermions which are neutral in G_{color}, and carry nontrivial G charges such that G-anomaly cancels totally in UV region
  - We gauge global G and ends up with consistent G-gauge theory.
  - Anomaly should cancel at low energy too. Massive fermions don't contribute to the anomaly at the low energy.

8. – Dual Theory

- $SU(N_F - N_c)$ gauge theory
  - dual gauge particle & gaugino
  - dual quarks & squarks
  - dual mesino's

$$\tilde{N}_c$$

	$SU(N_F - N_c)_c$	$SU(N_F)_L$	$SU(N_F)_R$	$U(1)_B$	$U(1)_{AF}$
q	N	$\bar{N}_F$	1	$\frac{N_c}{N_F - N_c}$	$\frac{N}{N_F}$
$\tilde{q}$	$\bar{N}$	1	N_F	$-\frac{N_c}{N_F - N_c}$	$\frac{N}{N_F}$
$\tilde{M}$ ($N_F \times N_F$)	1	N_F	$\bar{N}_F$	0	$2\left(\frac{N_F - N_c}{N_F}\right)$

- 't Hooft Anomaly Matching = Dual Theory

$A(N) = 1$

$A(Adj) = 0$

$SU(N_F)^3_L \qquad -(N_F - N_c) + N_F = N_c$

$U(1)_B\, SU(N_F)^2 \qquad \frac{N_c}{N_F - N_c} \cdot \frac{1}{2} \cdot (N_F - N_c) = \frac{N_c}{2}$

$U(1)_R\, SU(N_F)^2 \qquad \frac{(N_c - 1)}{N_F} \cdot \frac{1}{2} (N_F - N_c) + 2\left(\frac{N_F - 2N_c}{N_F}\right) \cdot \frac{1}{2} \cdot N_F = -\frac{N_c^2}{2F}$

$U(1)_B^2 \qquad = 0$

$U(1)_B\, U(1)_R^2 \qquad 0$

$\mathrm{Tr}\, U_B(1) = 0 \leftarrow$ gravitational anomaly

- $\mathrm{Tr}\, U(1)_R$ $\quad \frac{N-N_F}{N_F}\cdot 2(N_F-N)\cdot N_F + \frac{N_F-2N_c}{N_F}\cdot N_F^2$

$$+ (N_F-N_c)^2 - 1 = -N^2-1$$

- $U(1)_R$: $\left(\frac{N-N_F}{N_F}\right)^3 \cdot 2(N_F-N_c)N_F + \left(\frac{N_F-2N_c}{N_F}\right)^3 N_F^2$

$$+ (N_F-N_c)^2-1 = -\frac{2N^2}{N_F^2} + N_c^2 - 1$$

- $U(1)^2 U(1)_R$ $\quad \left(\frac{N}{F-N}\right)^2 \cdot \frac{N-N_F}{N_F} \cdot 2N_F(N_c-N_F) = -2N^2$

- Unique superpotential

$$W = \lambda \tilde{M}_{ij} q_i \tilde{q}_j$$

- Dual baryon

$$\left(\begin{array}{l} b^{i_1 \cdots i_{(N_F-N_c)}} \\ \tilde{b} \end{array}\right. = \epsilon_{n_1 \cdots n_{\tilde{N}_c = N_F - N_c}} q^{n_1}_{i_1} \cdots q^{n_{\tilde{N}_c}}_{i_{\tilde{N}_c}}$$

- moduli space

$$M_{ij} \longleftrightarrow \tilde{M}_{ij}$$

$$\begin{array}{l} B_{i_1 \cdots i_N} \\ \tilde{B} \end{array} \longleftrightarrow \epsilon_{i_1 \cdots i_N j_1 \cdots j_{N_F-N_c}} \begin{array}{l} b^{j_1 \cdots j_{N_F-N_c}} \\ \tilde{b} \end{array}$$

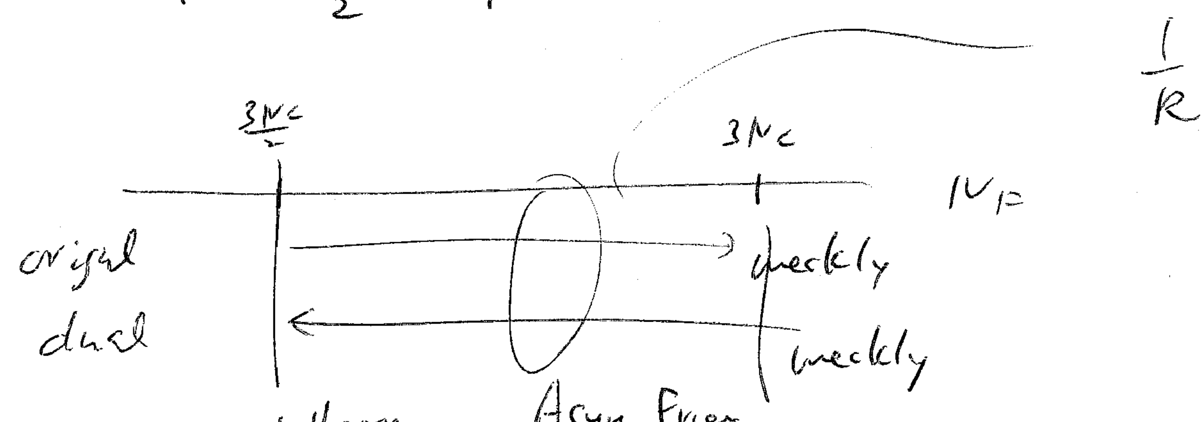
- 1-loop β-ftn

$$\beta(\tilde{g}) = -\tilde{g}^3 (3\tilde{N}_c - F)$$
$$= -\tilde{g}^3 (2N_F - 3N_c) \quad <0 \text{ if } \tfrac{3}{2}N_c < N_F$$
$$>0 \text{ if } \tfrac{3}{2}N_c > N_F$$

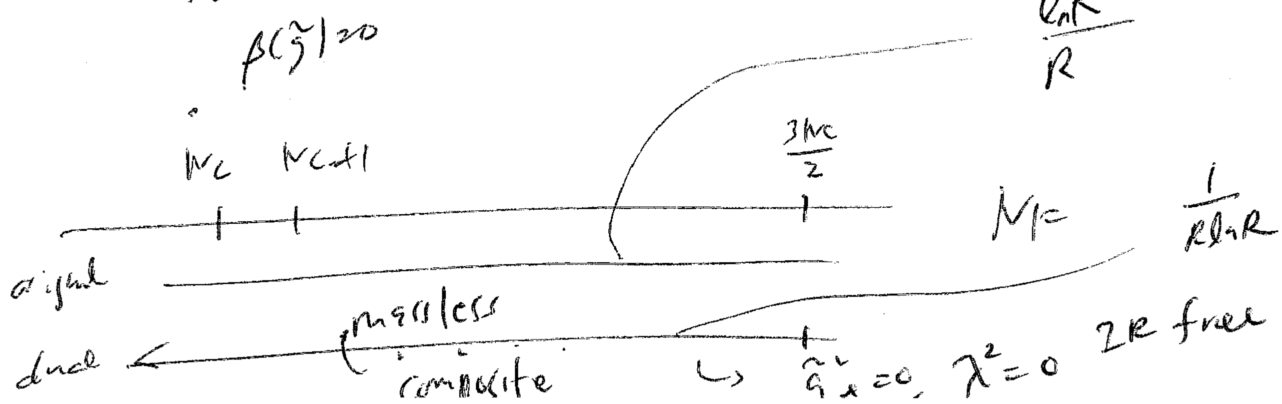
- Bank – Zak's approach for the conformal theory

$$\begin{cases} \tilde{g} \neq 0 \\ \lambda = 0 \end{cases}$$

- Interacting IR fixed point for $\frac{3N_c}{2} < N_F < 3N_c$



- dual theory $\beta(\tilde{g}) \geq 0$



9. Integrating out a flavor

$N_F \to N_F - 1$

$$\left(W_{\text{new}} = m\, Q_{N_F} \tilde{Q}_{N_F} \right.$$

dual $W_d = \lambda \tilde{M}_{f\tilde{f}} q_f \tilde{q}_{\tilde{f}} + \mu \tilde{M}_{N_F N_F}$

$$\lambda \tilde{M} = \frac{M}{\Lambda}$$

$$\frac{\partial W_d}{\partial M_{N_F N_F}} = 0 = \quad \lambda q_{N_F} \tilde{q}_{N_F} = -\mu$$

<u>dual</u> consistent

$S(N_F - N_c - 1)$, $SU(N_F - 1)$

10. quantum moduli space

$2 N_c N_F - (N_c^2 - 1)$ } preserved

if $N_F = N_c + 1$ $\to$ N_F^2

11. $N_F = N_c$, $N_F = N_c + 1$ $\to$ dual gauge theory $SU(0)$, $SU(1)$

is not there.

$$\boxed{N_F = N_c}$$

1. low energy $\quad M_{f\bar{f}},\ B = \epsilon_{i_1 \dots i_{N_c}} Q_1^{i_1} \cdots Q_{N_c}^{i_{N_c}}$, $\tilde{B}$

$\det M = B\tilde{B}$ $\quad$ singular manifold

2. Seiberg $\qquad b = 3N_c - N_F$

$$\det M - B\hat{B} = \Lambda^{2N_c} \doteq e^{2\pi i \tau}$$

$\uparrow$ instanton effect

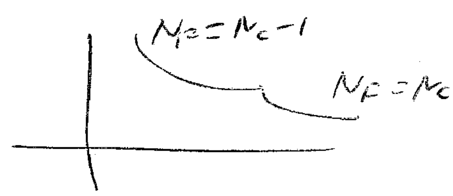
$M : N_F^2,\ B : \hat{B} : 2,\ \ 1$ constraint

$\Rightarrow \quad N_F^2 + 1$ dim complex vacuum moduli space

smooth

3. mass term $\qquad W = \mathrm{tr}\, m M \qquad m = \begin{array}{cc} & N_F \\ N_F & \left(\begin{array}{c|c} 0 & \\ \hline & m \end{array}\right) \end{array}$

- $N_F' = N_c - 1$ $\quad$ for $\Lambda < \mu < m$
- continuity



$$\left(\frac{\Lambda_{N_c N_c}}{m}\right)^{2N_c} = \left(\frac{\Lambda_{N_c, N_c - 1}}{m}\right)^{2N_c + 1}$$

$$\Rightarrow \quad \Lambda_{N_c N_c}^{2N_c} = m^{-1} (\Lambda_{N_c, N_c - 1})^{2N_c + 1}$$

- Mlef $\qquad M_{ij} = (m^{-1}) \left(\det m\, \Lambda^{3N - N_F}\right)^{\frac{1}{N_c}}$

$\det M = \det m^{-1} \cdot \det m \cdot \Lambda^{2N_c} = \Lambda^{2N_c}$

$\left[\ N_F = N_c,\right.$

$\det m \neq 0 \rightarrow \langle B \rangle = 0 \leftarrow$ "integrate out baryonic particles"

$N_F = N_c$

1 no dual gauge theory

low energy M_{fj}, $B = \epsilon_{i \dots i_N} Q_1^{i_1} \dots Q_{N_c}^{i_{N_c}}$

$\tilde{B}$

$\det M = B\tilde{B}$ singular manifold

2. Seiberg

$$\det M - B\hat{B} = \Lambda^{2N_c} = e^{2\pi i \tau} \quad \leftarrow \text{instanton effect}$$

$\uparrow$ only possible correction in dim

N_F^2 $\quad$ 2

M: N_F^2 } parameters $-$ 1 constraint $=$
$B, \hat{B}$: 2

$N_F^2 + 1$ dim smooth vacuum moduli space

3. Relation to

- $M_{ij} = (m^{-1})(\det m \Lambda^{3N_c - N_F})^{\frac{1}{N_c}}$ $\rightarrow$ $B \cdot \hat{B} = 0$
- as $\det M \neq 0$, Baryons decouple

$\boxed{\det M = \Lambda^{2N_c}}$ for $N_F = N_c$ $\rightarrow$ consistent

4. Add mass term

$$W = X(\det M - B\tilde{B} - \Lambda^{2N}) + \operatorname{tr} mM$$

$\uparrow$ constraint

4: Mass $m = \left(\begin{array}{c|c} \tilde{m} & \begin{array}{c}0\\ \vdots\end{array} \\ \hline 0\ 0\ 0 & m \end{array}\right)$, $M = \left(\begin{array}{c|c} \tilde{M} & N \\ \hline P & Y \end{array}\right)_{N_c N_c}$

$$\left(\frac{\partial W}{\partial B} = 0 = \frac{\partial W}{\partial \tilde{B}} \Rightarrow B = \tilde{B} = 0 \quad \text{if } X \neq 0 \right.$$

$$\frac{\partial W}{\partial N_i} = \frac{X}{\det M} \operatorname{cof} N_i = 0. \qquad \frac{\partial W}{\partial P_i} = \frac{X}{\det m} \operatorname{cof} P_i = 0$$

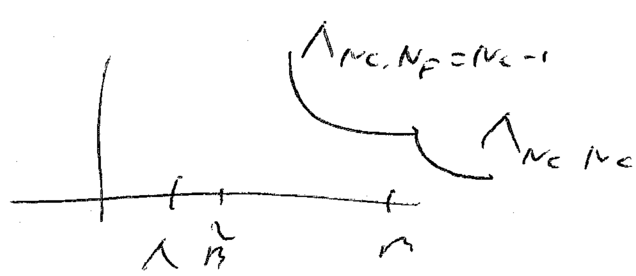
$$\Rightarrow \quad N_i = P_i = 0$$

$$\frac{\partial W}{\partial Y} = X \det \tilde{M} + m = 0 \quad \to X = -\frac{m}{\det \tilde{M}}$$

$$\frac{\partial W}{\partial X} = 0 \qquad Y \det \tilde{M} - \Lambda^{2N_c} = 0$$

- $W_{eff}\Big|_{sol} = mY = \dfrac{m \Lambda^{2N_c}}{\det \tilde{M}}$

- continuity of coupling



$$e^{2\pi i \tau} = \left(\frac{\Lambda_{N_c N_c}}{m}\right)^{2N_c} = \left(\frac{\Lambda_{N_c, N_c - 1}}{m}\right)^{2N_c + 1}$$

$$\Rightarrow m \Lambda_{N_c N_c}^{2N_c} = (\Lambda_{N_c, N_c-1})^{2N_c+1}$$

$$\Rightarrow \quad W_{eff} = \frac{(\Lambda_{N_c, N_c - 1})^{2N_c + 1}}{\det \tilde{M}} = W_{ADS}$$

5. Special moduli space points

- $M^{\tilde f}_{f\bar g} = \Lambda^2 \delta_{f\bar g}$, $B = \tilde B = 0$

unbroken global symmetry

	$SU(N_F)_V$	$U(1)_B$	$U(1)_{AF}$	f
$M - \mathrm{tr}M$	adj	0	0	$F1$
$\mathrm{tr}M$	1	0	0	-1
B	1	N	0	-1
$\tilde B$	1	$-N$	0	-1

~~d~~ tr M become massive

- Anomaly matching

	elem	comp
$U(1)_B^2 U(1)_{AF}$	$-2FN$	$-2N^2$
$U(1)_{AF}$	$-2FN + N^2 - 1 =$	$-(F^2-1) - 1 - 1$
$U(1)_{AF}^3$	..	..
$U(1)_B SU(N_F)_V^2$	$-2N + N =$	$-N$

massless fermion

$M - dM$, B, $\tilde B$: $N_f^2 + 1$

6. Special point E

$M = 0, \quad B = -\tilde{B} = \Lambda^{N_c}$

- global $SU(N_f)_L \times SU(N_f)_R \times U(1)_{AF}$

  $U(1)_B$ spontaneously broken

- $B + \bar{B}$ gets mass $\rightarrow$ $N_f^2 + 1$ massless

- massless

	$SU(N_F)_L$	$SU(N_F)_R$	$U(1)_{AF}$
M	N	$\bar{N}$	0
B	1	1	0
$\bar{B}$	1	1	0

- elements

  $\lambda : (1,1)_1^{N_c^2}, \quad \psi_f \, (N_F, 1)_{-1}^{N_c}, \quad \bar{\psi}_f \, (1, N_f)_{-1}$

- compact

  $\psi_M \, (N_f \bar{N}_F)_{-1}, \quad \underbrace{\psi_B \, (1,1)_{-1}, \quad \cancel{\psi_{\tilde{B}} \, (1,1)_{-1}}}$

- Anomaly Match

	e	c
$SU^3(N_F)$	N_c	N_F
$U(1)_{AF} SU(N_F)^2$	$-N \cdot \frac{1}{2}$	$-F \cdot \frac{1}{2}$
$U(1)_{AF}$	$-2 N_c N_F + N^2 - 1 =$	$- F^2 - 1$ ✓
$U(1)^3_{AF}$	"	"

$\boxed{N_F = N_c + 1}$

1. M_{fs} $\quad N_F^2$ $\quad \epsilon_{k_1 \cdots k_{N_c}} Q^{k_1}_{j_1} \cdots Q^{k_{N_c}}_{j_{N_c}}$

$B_i = \epsilon_{i i_1 \cdots i_{N_c}} \left(B_{j_1 \cdots i_{N_c}} \right.$ $\cancel{= Q^1_{i_1} \cdots} Q$

$\tilde{B}_i$

classical constraint

$B_i M_{ij} = 0, \quad M_{ij} \tilde{B}_j = 0$

$(\det M)(M^{-1})_{ij} - B_i \tilde{B}_j = 0$

$C_1 \quad M_{ij} B_i \tilde{B}_j = \det M = 0$

N_F^2 dim

Vacuum moduli space

$$\begin{array}{cc} & N_c \\ N_f & \left(\begin{array}{c|c} & \vdots \\ \hline \cdots & \vdots \end{array} \right) \end{array}$$

$B = 0, \; \tilde{B} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ N^c \end{pmatrix}$

2. $$W = \frac{1}{\Lambda^{2N_c-1}} \left(B_i M_{ij} \tilde{B}_j - \det M \right) + \operatorname{tr} m M$$

$$\frac{\partial W}{\partial M_{ij}} = 0 = \frac{B_i \tilde{B}_j - (M^{-1})_{ij} \det M}{\Lambda^{2N_c-1}} + m_{ij} = 0$$

$$\frac{\partial W}{\partial B_i} = 0 = \frac{\partial W}{\partial \tilde{B}_i} : \quad M_{ij} \tilde{B}_j = 0 = M_{ij} B_i$$

?

$$m = \left(\begin{array}{cccc|c} & & & & 0 \\ & & & & 0 \\ & & & & 0 \\ \hline 0 & 0 & 0 & 0 & m \end{array}\right)$$

$$M = \left(\begin{array}{c|c} \tilde{M} & N \\ \hline P & X \end{array}\right)$$

$$B_i = (U_i, \tilde{B}')$$

$$\hat{B}_i = (\tilde{U}_i, \hat{B}')$$

$$P_i: \quad \frac{\partial W}{\partial P_i} = \frac{1}{\Lambda^{2N-1}} (B' \tilde{U}_i - \mathrm{cof}\, P) = 0$$

$$N_i \quad \frac{\partial W}{\partial N_i} = \frac{1}{\Lambda^{2N-1}} (U \tilde{B}' - \mathrm{cof}\, N_i) = 0$$

$$U \quad \frac{\partial W}{\partial U_i} = \frac{1}{\Lambda^{2N-1}} \cdot N_i \tilde{B}_i = 0$$

$$\tilde{U} \quad \frac{\partial W}{\partial \tilde{U}_i} = \frac{1}{\Lambda^{2N-1}} P_i B_i = 0$$

$$X \quad \frac{\partial W}{\partial X} = \frac{1}{\Lambda^{2N-1}} (B' \tilde{B}' - \det \tilde{M}) + m = 0$$

$$\Rightarrow \quad P_i = N_i = U_i = \tilde{U}_i = 0$$

$$\boxed{\det \tilde{M} - B' \tilde{B}' = m \Lambda^{2N-1}_{N_c, N_f = N_c+1} = \Lambda^{2N}_{N_c, N_f = N_c}}$$

$$W_{eff} = \frac{X}{\Lambda^{2N-1}} \left(B' \tilde{B}' - \det M + \cancel{m \Lambda^{2N}} \; \Lambda^{2N}_{N_c N_c} \right)$$

4. At point $M_{ij} = B_i = \tilde{B}_i = 0$

only $M, B, \tilde{B}$ are massless

confining but no chiral symmetry breaking

"s-confinement"

Anomaly matching

	el		comp
$SU(N_F)_L^3$	N	$=$	$F-1$
$U(1)_B\, SU(N_F)^2$	$\frac{N}{2}$		$N\frac{1}{2}$
$U(1)_A\, SU(N_F)^2$	$-\frac{N^2}{2F}$		
& $U(1)_{AF}$			
& $U(1)_{AF}^3$			

well matched ex) complete the table