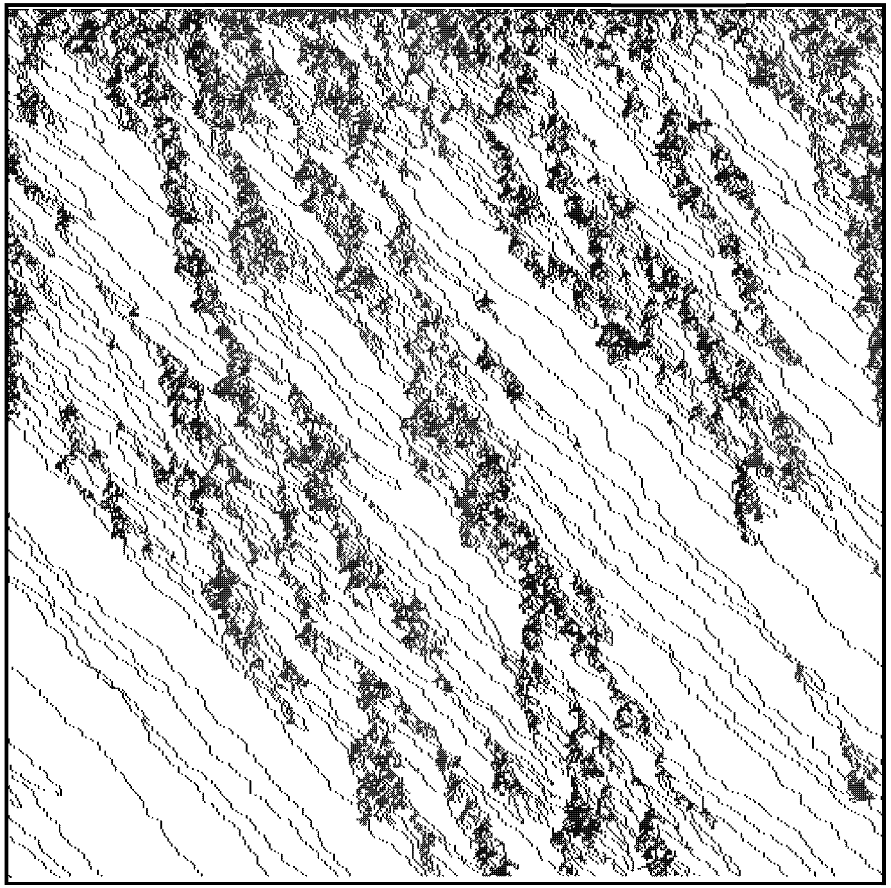
Driven Pair Contact Process with Diffusion



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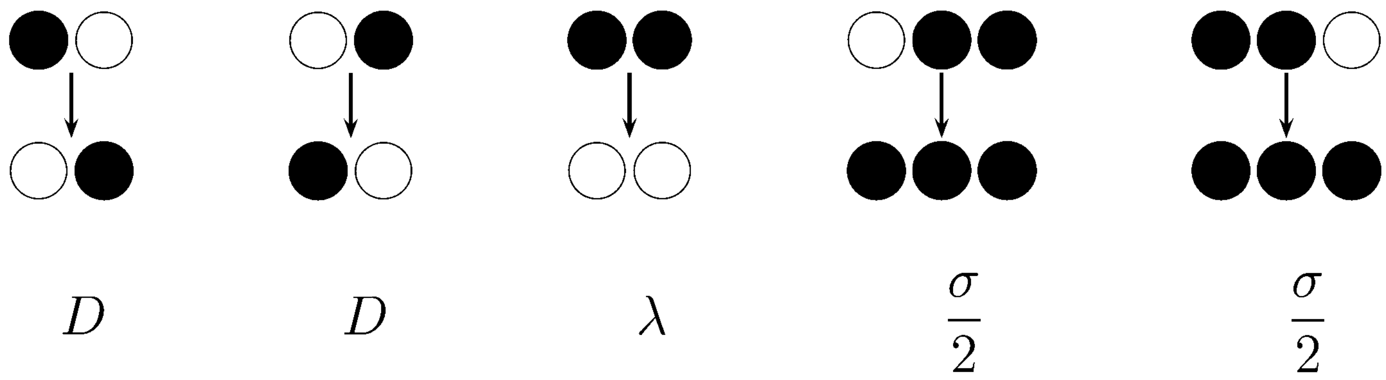
1 Motivation

1.1 PCPD and cyclically coupled (CC) model

- dynamic rules of PCPD



- CC
  - there are two species A and B.
  - A : annihilation random walks (solitary particles in PCPD)
  - B : contact process (pair in PCPD)
  - coupling : $2A \to B$ and $B \to A$.
- Question : CC is generic or for convenience?

1.2 Biased diffusion and critical behavior

1.2.1 field theory for single species model

- Example $A + A \to \emptyset$

$$\mathcal{L} = \bar{a}(\partial_t - D\nabla^2 + \boldsymbol{v} \cdot \nabla)a + \lambda_1 \bar{a} a^2 + \lambda_2 \bar{a}^2 a^2$$

Galilean transformation gauges away the $\boldsymbol{v} \cdot \nabla$ term. Bias has *no* role.

1.2.2 two species model with relative bias

- Example $A + B \to \emptyset$: Kang and Redner (1984)
  - No bias : $t^{-1/4}$ density decay in one dimension
  - relative bias : $t^{-1/2}$ density decay in one dimension

  Bias is a relevant perturbation.
- cf) $A + A \to B$, $B \to \emptyset$: bias has no role.
- how about PCPD? $\Rightarrow$ main question of this talk.

2 Driven branching annihilating random walks

2.1 Model definition

$$A\emptyset \xrightarrow{p} \emptyset A, \qquad AA \xrightarrow{p} \emptyset\emptyset, \qquad A \xrightarrow{1-p} \begin{cases} 2A & \text{DBAW1} \\ 3A & \text{DBAW2} \end{cases}$$

- NOTE
  - ☞ fully biased diffusion
  - ☞ branching is also biased for convenience
- algorithm

  ① at time t, a particle is selected randomly.

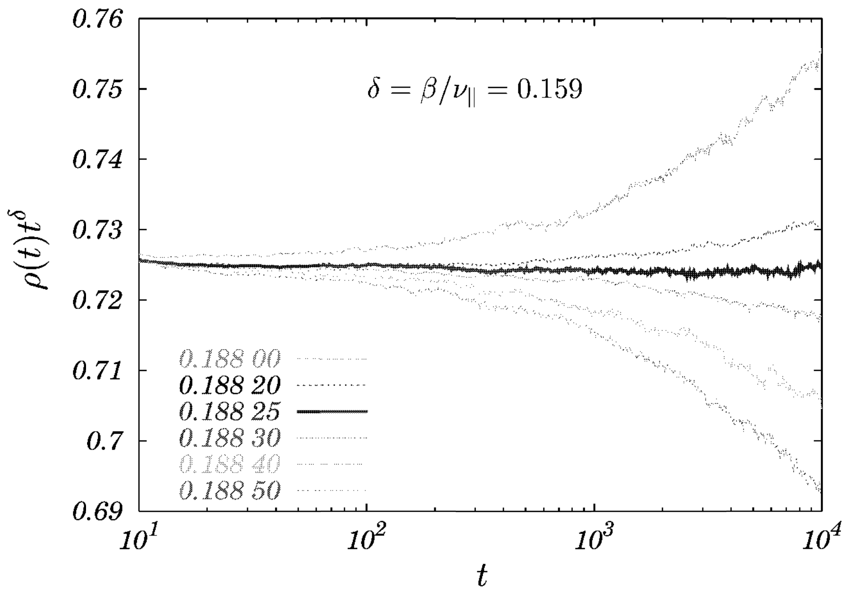
  ② with probability p, the chosen particle tries to hop to the right.

  ③ if a particle meets another particle, both are removed immediately.

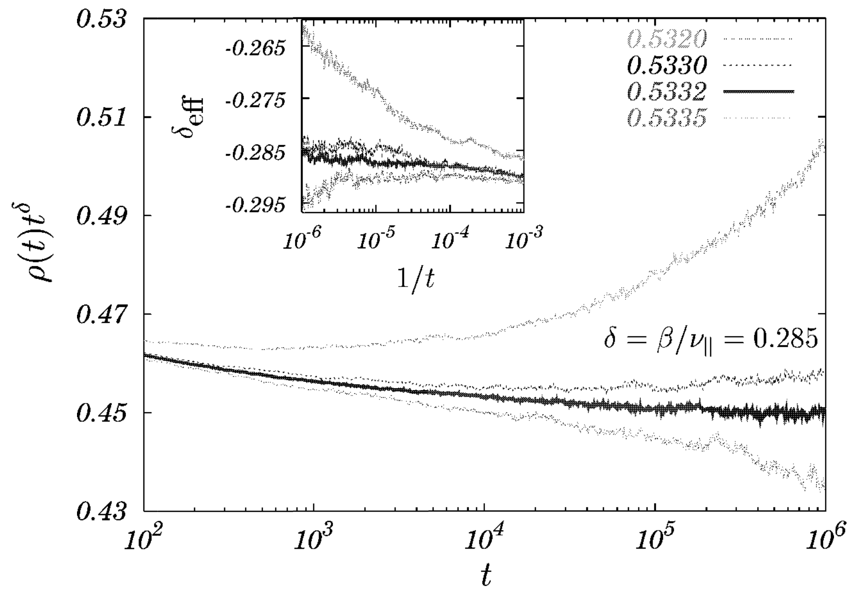
  ④ with probability $1-p$, branching event occurs in case the target sites are empty. If one of the target sites is occupied, nothing happens.

2.2 Numerics

- fully occupied initial condition
- at criticality, $\rho(t) \sim t^{-\delta}$ or $\rho(t)t^{\delta}$ is constant.



DBAW1 ($\delta = 0.159(1)$)



DBAW2 ($\delta = 0.285(1)$)

☞ DP class : $\delta = 0.1595$, DI or PC class : $\delta = 0.285$. These results are consistent with the field theory (Galilean transformation).

3 PCPD and DPCPD

3.1 Dynamic Rules

$$A\emptyset \xrightarrow{D_R} \emptyset A, \quad \emptyset A \xrightarrow{D_L} A\emptyset, \quad AA \xrightarrow{p} \emptyset\emptyset, \quad \left.\begin{array}{c} AA\emptyset \\ \emptyset AA \end{array}\right\} \xrightarrow{(1-p)/2} AAA$$

✔ $D_R = D_L = 0$: PCP ✔ $D_R = D_L \neq 0$: PCPD

✔ $D_R \neq D_L$: DPCPD ✔ In what follows, we set $D_R + D_L = 1$

➼ algorithm

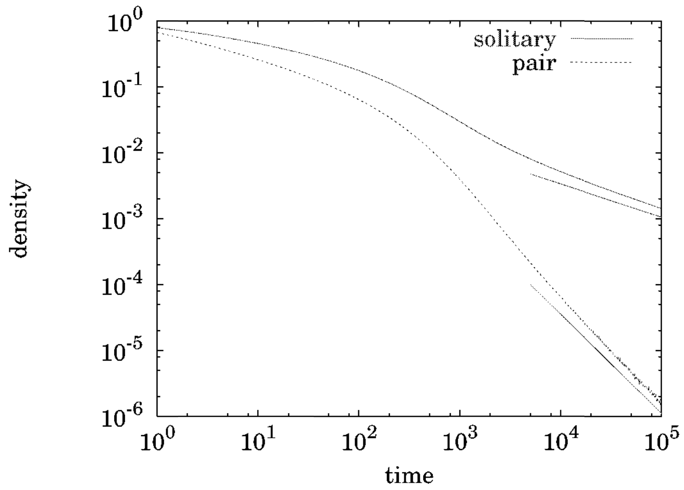
① A particle is selected at random.

② The chosen particle hops to the right (left) with prob. D_R (D_L).

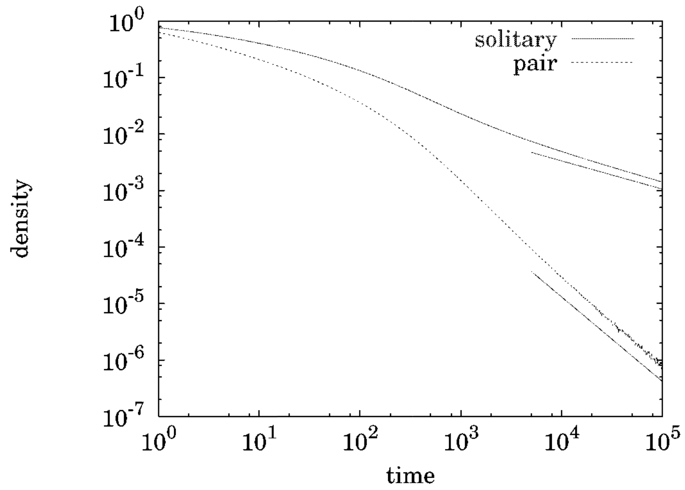
③ If the target site is occupied, with prob. p, two particles are annihilated, or with prob. $1 - p$, the chosen particle come back to its original sites and branches a new particle at one of the nearest neighbors of the pair.

3.2 Numerics

3.2.1 in absorbing phase



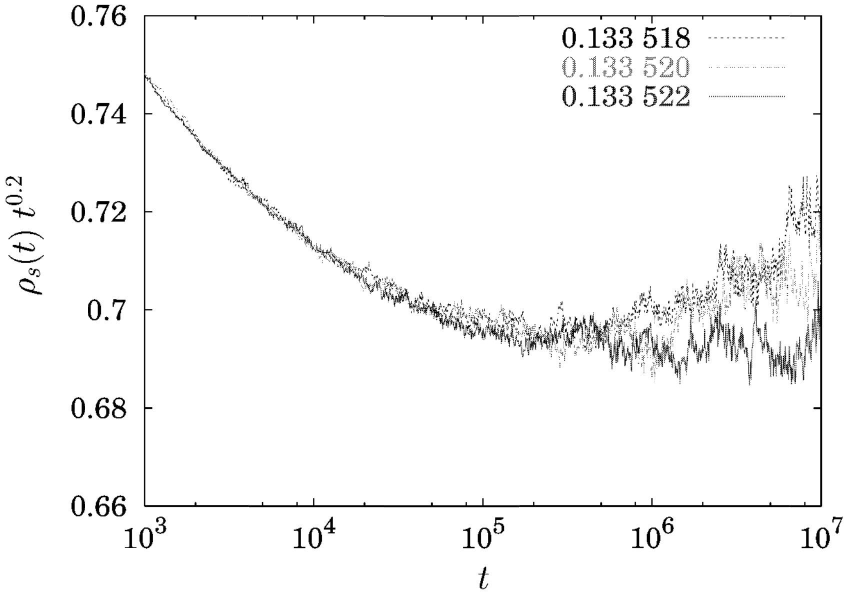
PCPD at $p = 0.16$



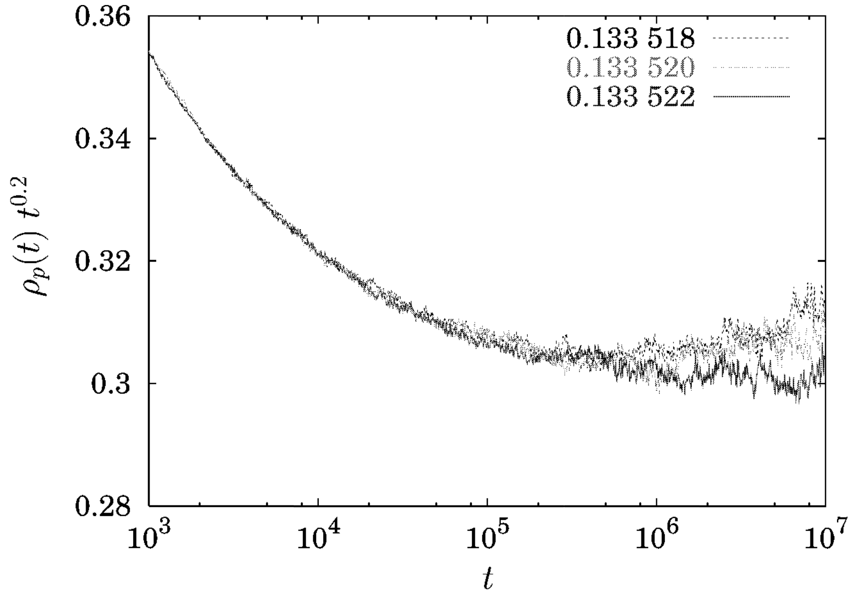
DPCPD at $p = 0.18$

- ☞ exponential decay followed by power law decay for both cases
- ☞ $\rho_s(t) \sim t^{-0.5}$ and $\rho_p(t) \sim t^{-1.5}$
- ☞ PCPD and DPCPD share the annihilation fixed point $(A + A \to \emptyset)$.

3.2.2 PCPD near criticality



density of particles

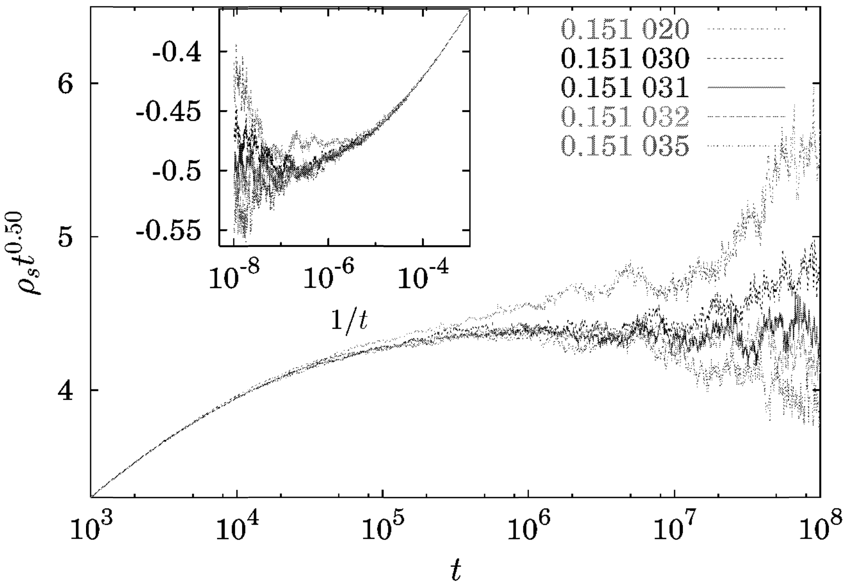


density of pairs

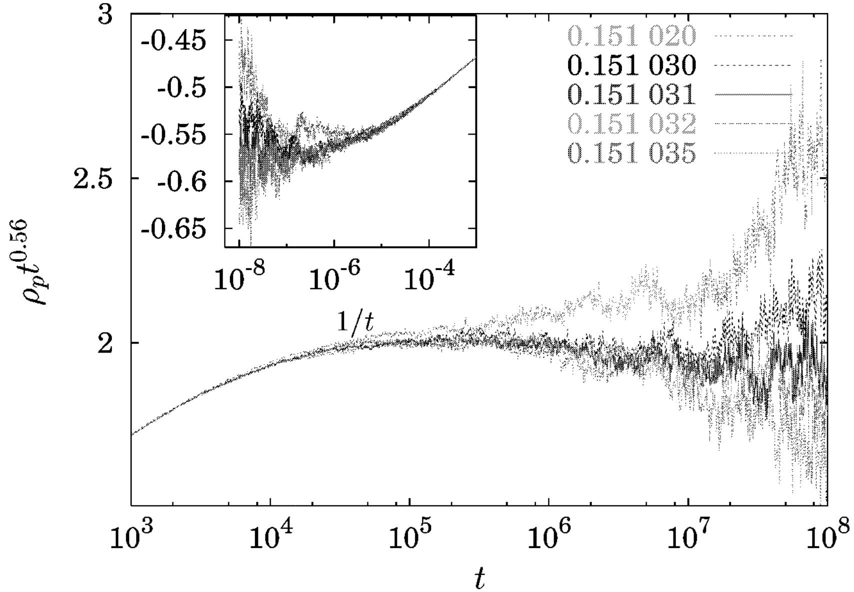
$$p_c = 0.133\ 522(2).$$

$\rho_s(t) \sim \rho_p(t) \sim t^{-0.2}$ Kockelkoren and Chaté (2002)

3.2.3 DPCPD near criticality



density of particles



density of pairs

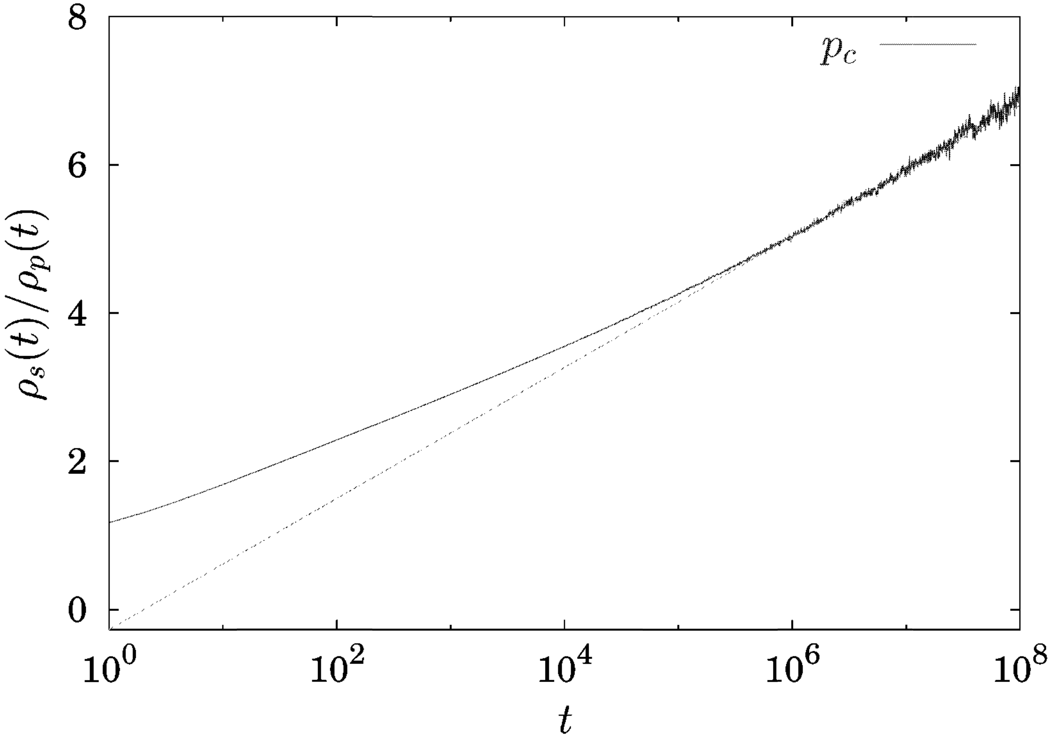
$$p_c = 0.151\ 031(1)$$

$\rho_s(t) \sim t^{-0.5}$ $\rho_p(t) \sim t^{-0.56}$

☞ 0.5 and 0.56 are clearly different from 0.2

☞ The exponent difference is the symptom of logarithmic corrections

- t vs $\rho_s(t)/\rho_p(t)$ semi-log plot

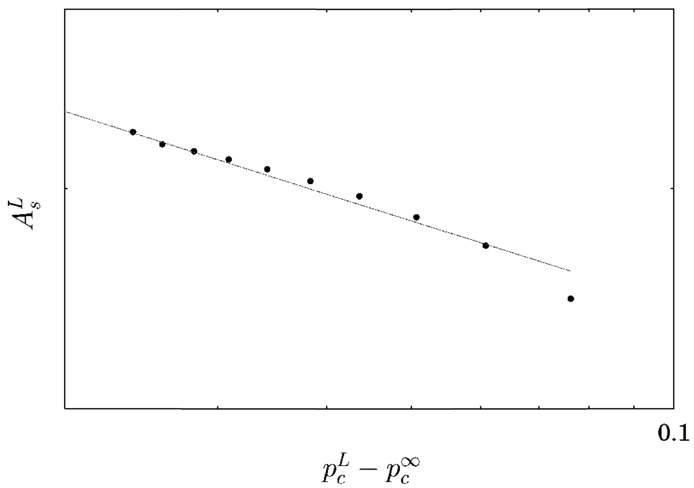


- This should be compared with the 2-D PCPD simulation.
- The upper critical dimension of DPCPD is 1 (?).
- For PCPD, bias plays a drastic role.

3.3 CAM analysis (up to $L = 13$)

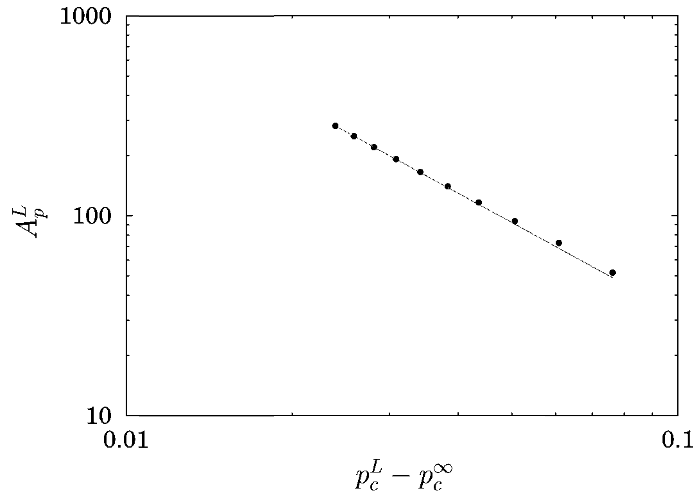
3.3.1 cluster approximation for PCPD

☞ From A_s^L



$\beta_s = 0.85$ but fitting is not good.

☞ From A_p^L



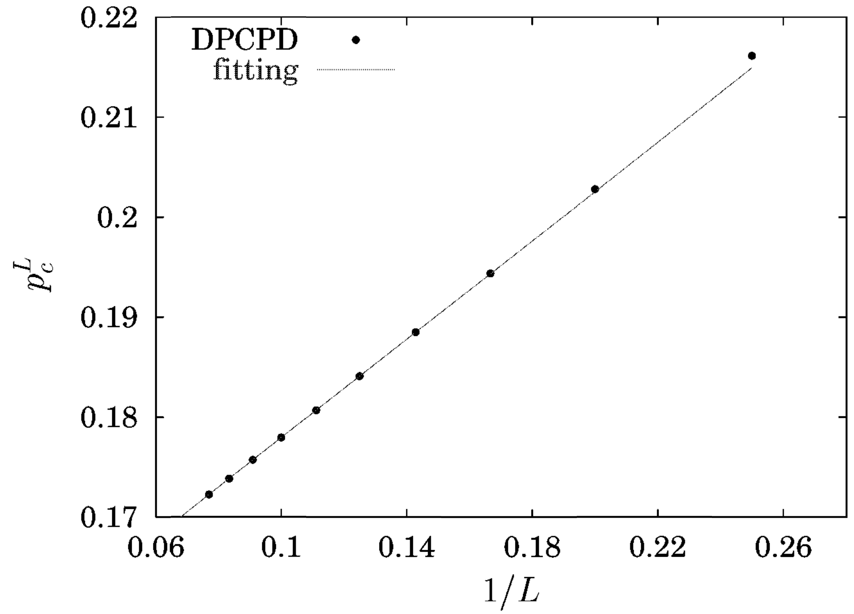
$\beta_p \simeq 0.5$

☞ Previous simulation results

$$\beta = 0.37(2) \; : \; \text{Kockelkoren and Chate (2000)}$$

3.3.2 cluster approximation for DPCPD

L	p_c^L	A_s^L	A_p^L
4	0.2161403513	4.253	44.36
5	0.2028009465	4.355	56.08
6	0.1943817410	4.404	66.47
7	0.1885031907	4.422	76.38
8	0.1841024774	4.424	85.76
9	0.1806898311	4.419	94.89
10	0.1779543360	4.409	103.7
11	0.1757117674	4.397	112.3
12	0.1738378803	4.383	120.7
13	0.1722478976	4.367	128.9
∞	0.151031(1)		



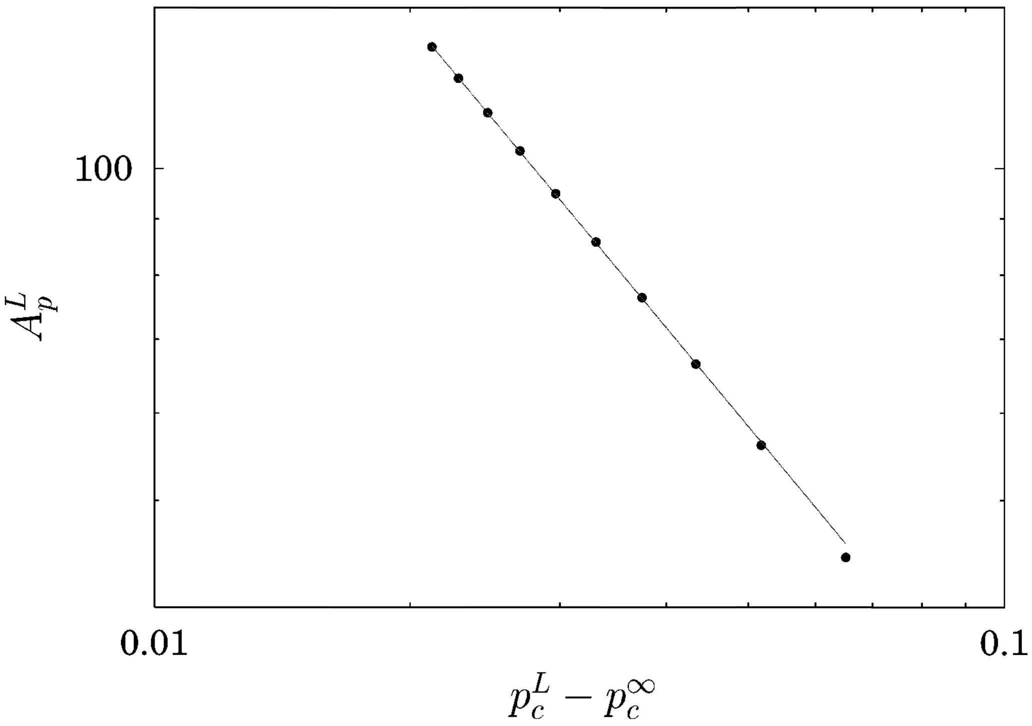
$$(p_c^L - p_c^\infty)^{\nu_\perp} = \frac{A}{L} + \frac{B}{L^2}$$

or

$$p_c^L = p_c^\infty + \left(\frac{A}{L} + \frac{B}{L^2}\right)^{1/\nu_\perp}$$

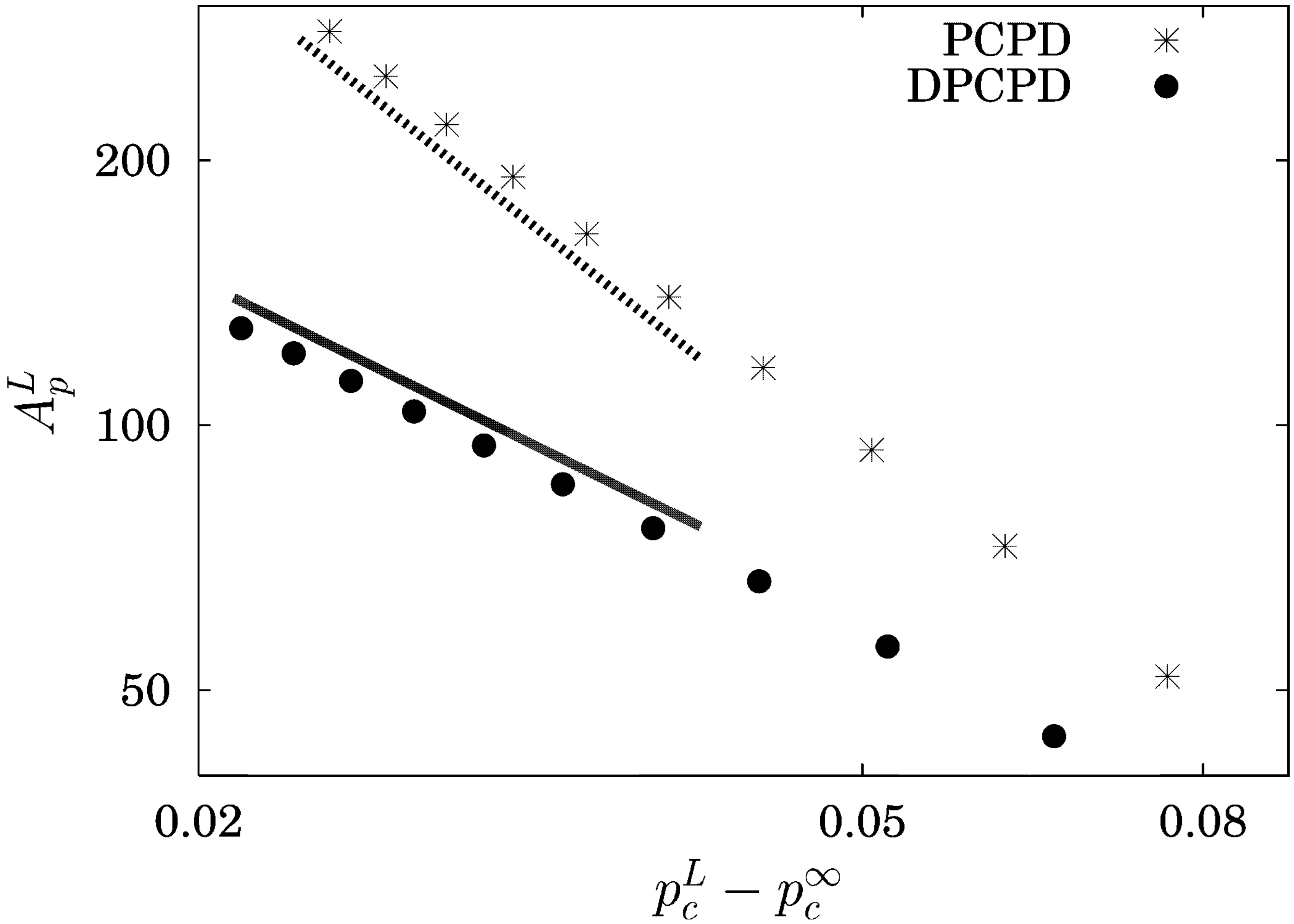
$p_c^\infty = 0.1515(5)$ and $\nu_\perp = 1.05(5)$.

- estimation of β
  - From A_s^L: $\beta_s^{\mathrm{MF}} - \beta_s \simeq 0$
  - From A_p^L: $\beta_p^{\mathrm{MF}} - \beta_p \simeq 0.93 \to \beta_p \simeq 1.07$



  - $\beta = 1$ with log correction : Mean-Field like

☞ comparison



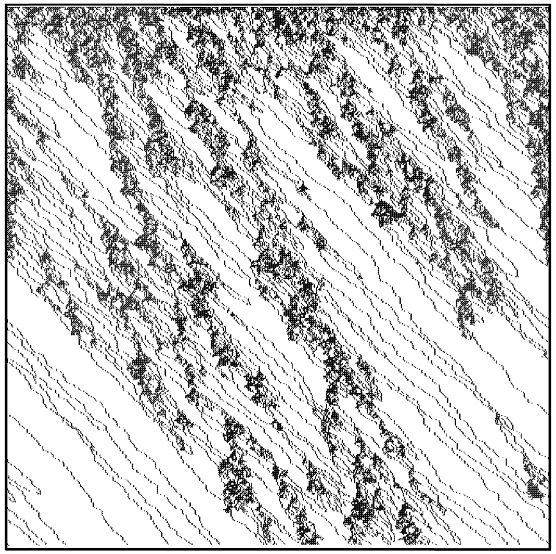
4 Discussion

- Is the Galilean transformation argument wrong? No!

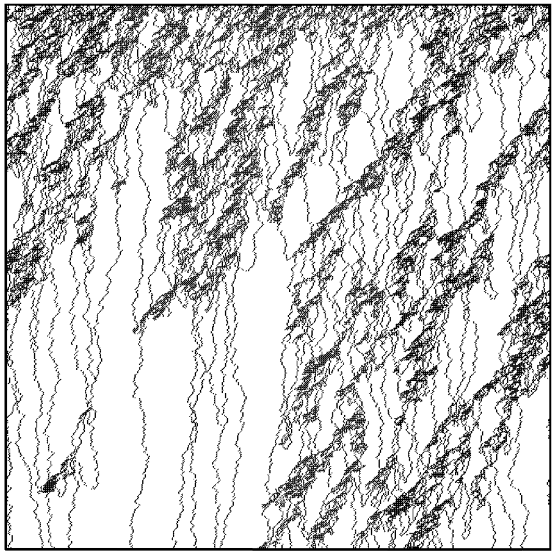
  Bosonic model (with soft constraint) with biased diffusion → PCPD!

  This model has peculiarity in that the biased diffusion *does not* generate the relative bias between solitary "excitations" and pair "excitations."

- space-time configuration for DPCPD



not transformed : (x, t)



transformed : $(x - vt, t)$

- DPCPD counter part of bosonic model
  - treat single particle occupied and multiple particles occupied sites differently. If a site is occupied by a single particle, this particle performs biased random walk. Otherwise, unbiased random walks.
  - We found that this model shares the critical behavior with DPCPD.
- Cyclically coupled model with relative bias : DPCPD behavior
- What about the field theory for PCPD?
  - simple-minded single species field theory is not suitable because the pair should be treated as an independent field with well defined propagator.
  - CC may be a candidate for field theory

- Universality issue : Our numerics exclude the possibility PCPD's belonging to the DP class because DP is described by Reggeon field theory which is basically single species model, but PCPD is generically two species model.
- Open question : Why does DPCPD show mean field like behavior in one dimension?

5 Conclusion

- PCPD is generically two species model
- PCPD does not belong to DP class.