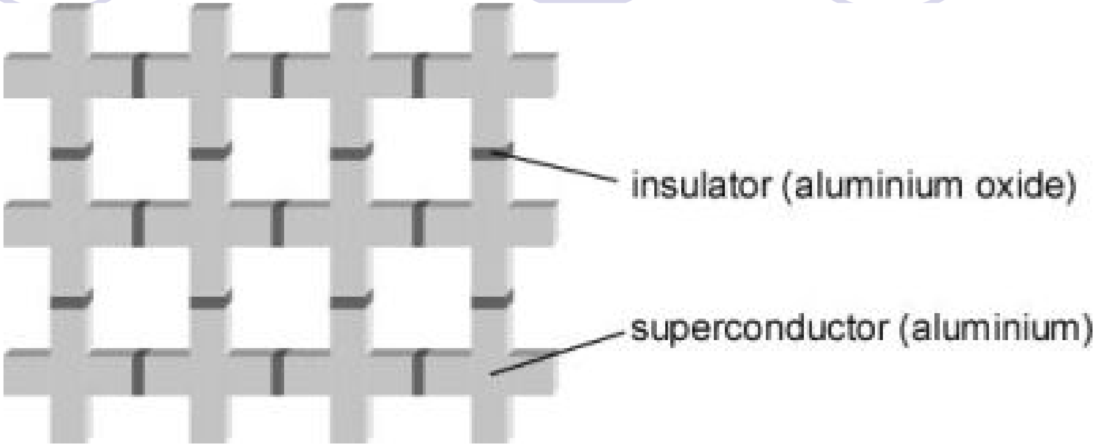
# Zero-temperature criticality of *2D Gauge Glass*

- 2D Randomly Frustrated XY Model
- Gauge-glass Ground State – Universal Features
- Density of States and Spin-Glass Stiffness
- Comparison with Monte Carlo Results
- Conclusions

in collaboration with Peiqing Tong (NJNU) and Shaolong Wan (USTC)

# Josephson-junction array



Josephson coupling

$$E = -J \sum_{\langle ij \rangle} \cos\left(\varphi_i - \varphi_j - A_{ij}\right)$$

frustration introduced by transverse magnetic field

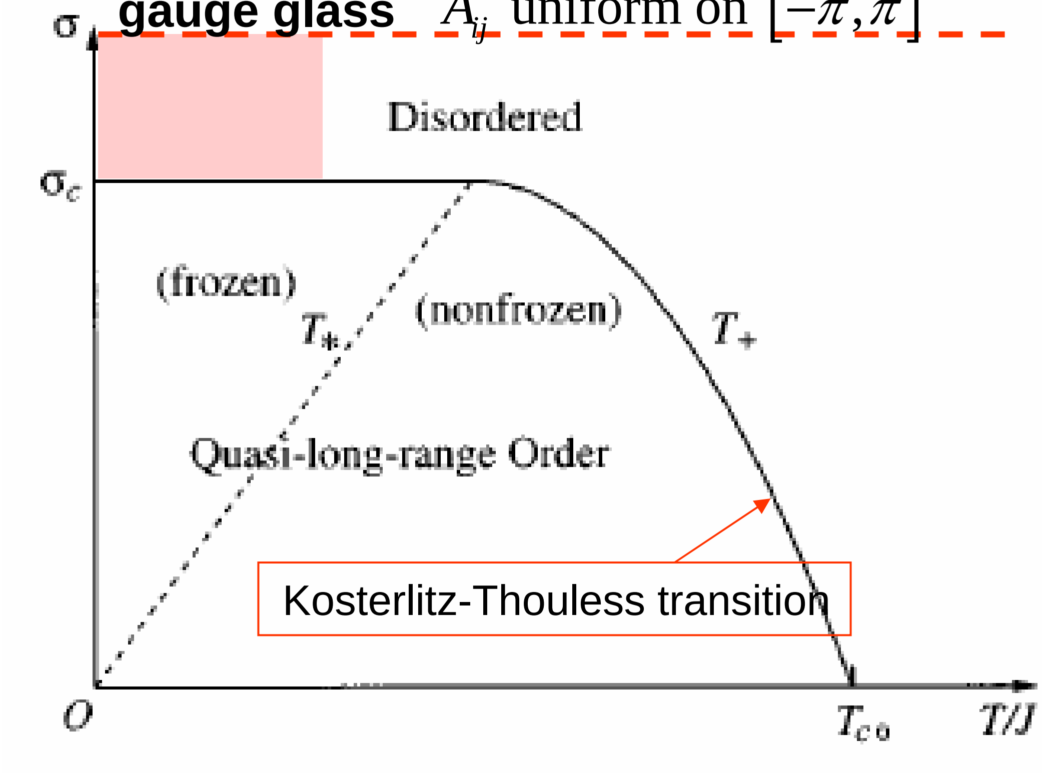
I-V characteristics highly sensitive to the applied field at low temperatures (superconducting, normal, or even insulating)

Interesting equilibrium and dynamical properties

# Phase diagram under random frustration

Cha & Fertig, PRL **74**, 4867 (95); T. Nattermann et al, J. Physique I **5**, 565 (95); LHT, PRB **54**, 3350 (96)

**gauge glass** $A_{ij}$ uniform on $[-\pi,\pi]$



Kosterlitz-Thouless transition

$$E = -J \sum_{\langle ij \rangle} \cos\left(\varphi_i - \varphi_j - A_{ij}\right)$$

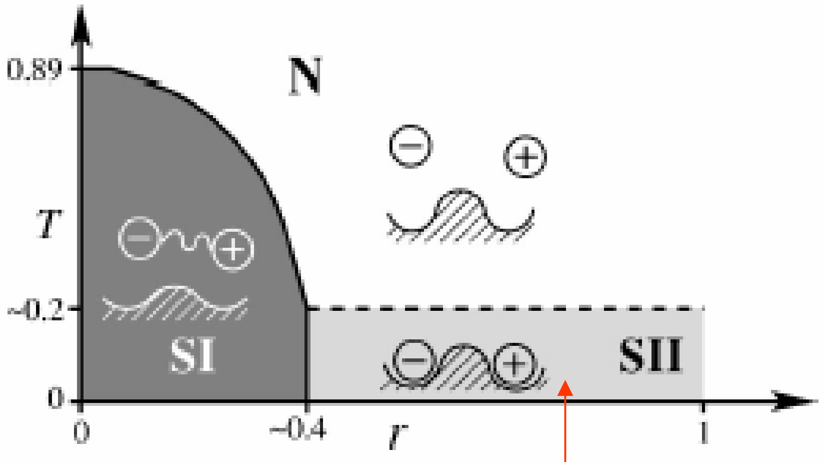
$$\left\langle A_{ij} \right\rangle = 0, \quad \left\langle A_{ij}^2 \right\rangle = \sigma$$

- ➢ No long-range phase ordering at sufficiently strong disorder
- ➢ Nature of the classical ground state at strong disorder (shaded region) not fully characterized

*A subject of controversy in the past ten years*

# Recent simulation studies on gauge glass

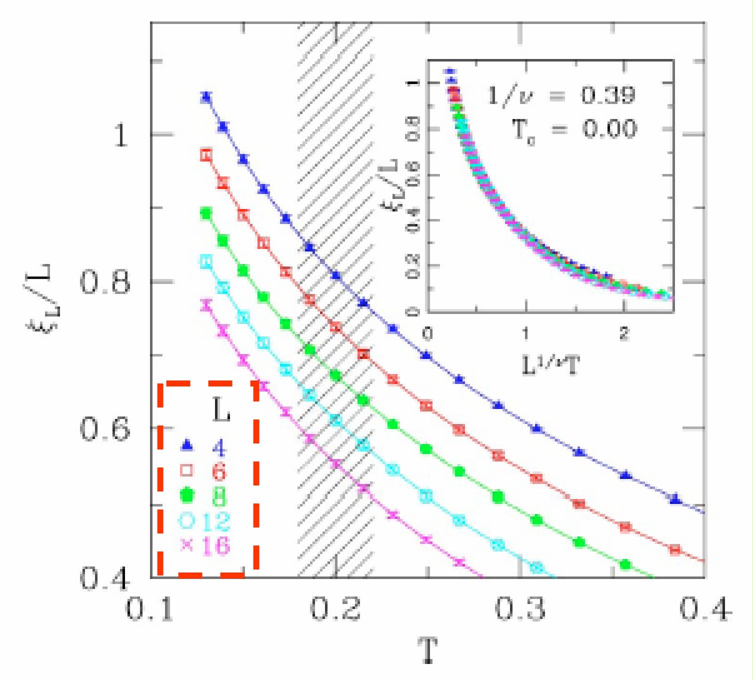
P. Holme et al., PRB **67**, 104510 (2003)



disordered superconductor at sufficiently low temperatures

Evidence: helicity modulus appears to approach finite value at sufficiently low temperatures

Refuted by H. Katzgraber, PRB **67**, 180402 (2003)



spin-glass susceptibility

$\xi \sim T^{-\nu} \sim T^{-2.5}$

# How can one do better?

- Better numerics

Multicanonical Monte Carlo sampling

- Analytical approach

better intuition about the spin-glass order in the ground state

likely a very challenging task

# The 2D Coulomb gas in a random-dipole field

decomposition of the phase field

$$\nabla \varphi_{\text{lattice}} = \nabla \varphi_{sw} + \sum_i m_i \hat{z} \times \frac{\mathbf{r}-\mathbf{r}_i}{\left|\mathbf{r}-\mathbf{r}_i\right|^2}$$

spin-wave and vortices decouple
in the continuum description

$$E = -J \sum_{\langle ij \rangle} \cos\left(\varphi_i - \varphi_j - A_{ij}\right) = E_{sw} + E_v$$

$$E_v = -\pi J \sum_{i \neq j} m_i m_j \ln r_{ij} + \sum_i \left( m_i^2 E_C + m_i V(\mathbf{r}_i) \right)$$

Coulomb potential

random dipole field

**Possible phases for a charge neutral system**

- **dielectric**

bound vortex-antivortex pairs

- **plasma**

free vortices/antivortices

- **Coulomb glass** ??

unpaired but localized vortices and antivortices

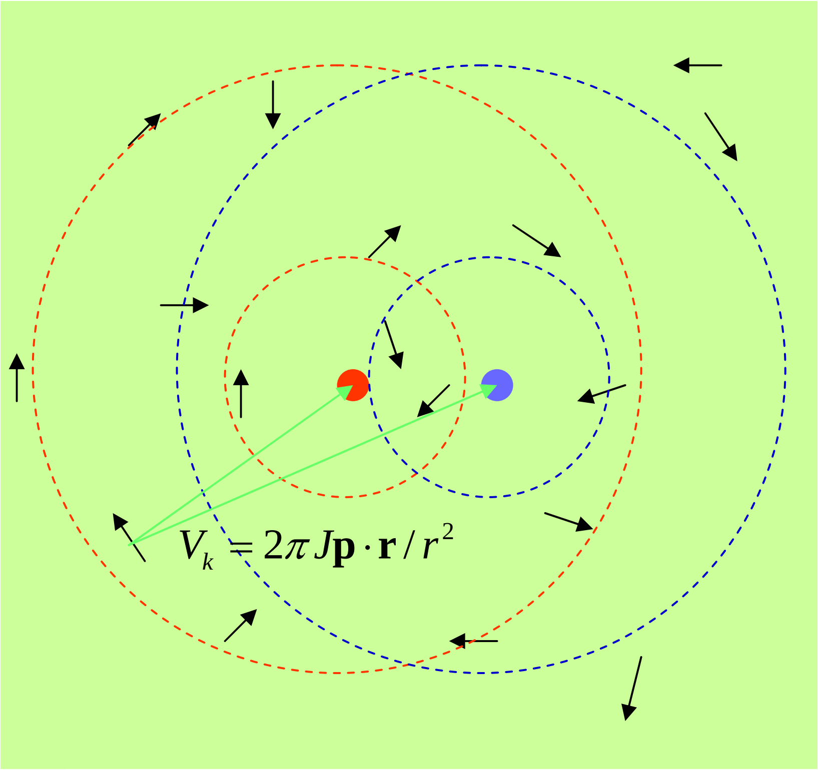
# A solvable case: the single-vortex problem

LHT, PRB **54**, 3350 (96), S. Scheidl, PRB 55, 457 (97).

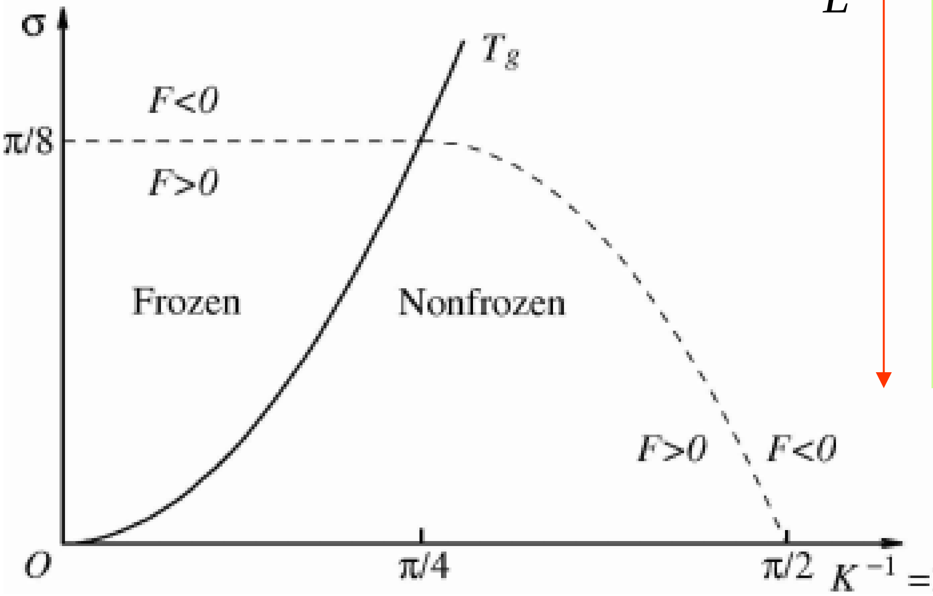
$$Z = L^{-\pi K} \sum_i \exp(-V_i / T)$$

$$\langle V_i \rangle = 0, \quad \left\langle \left( V_i - V_j \right)^2 \right\rangle = 2\pi\sigma J^2 \ln r_{ij}$$

maps exactly to REM/DP on Cayley tree in the limit *L* → ∞.



$$V_k = 2\pi J \mathbf{p} \cdot \mathbf{r} / r^2$$

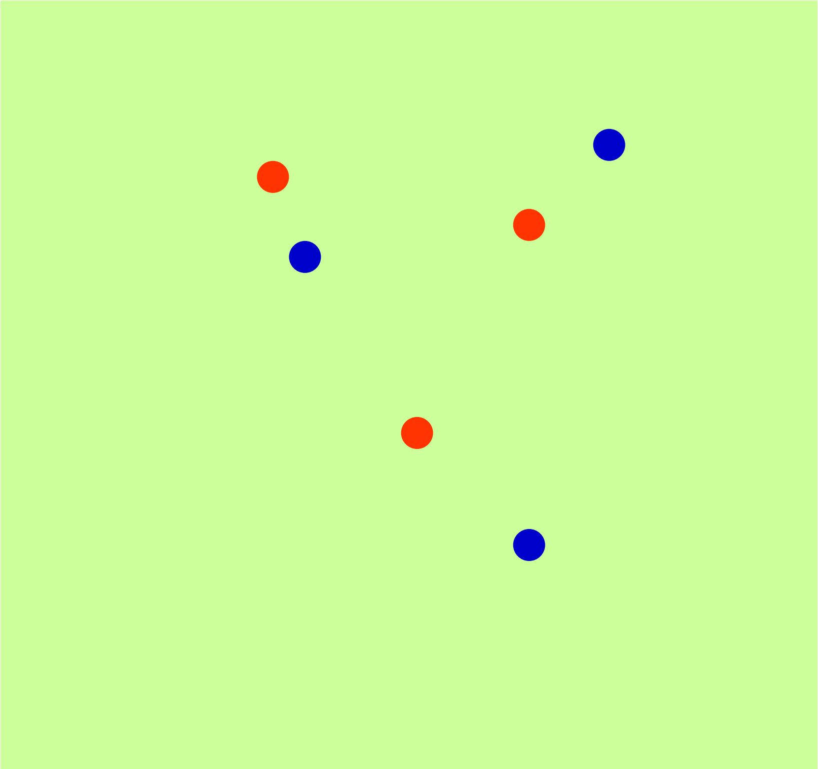


$\sigma \geq \pi / 8:$ ground state unstable against single vortex excitation

# Ground state at strong frustration: *numerical experiment*

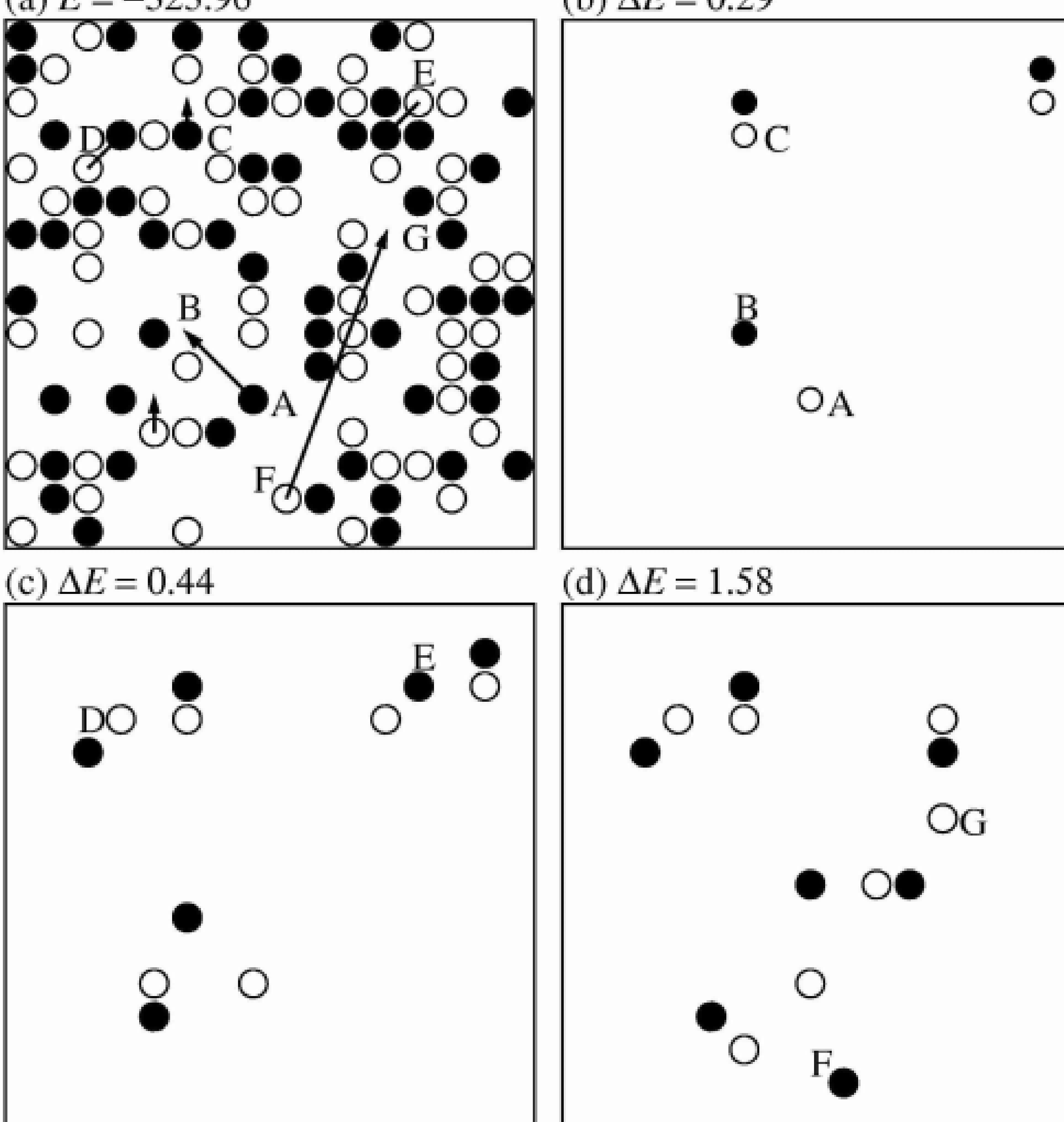
**Build up the ground state using a greedy algorithm**:

- filling in low potential energy sites one by one, while taking into account interactions as the process continues
- very good low energy states can be constructed this way



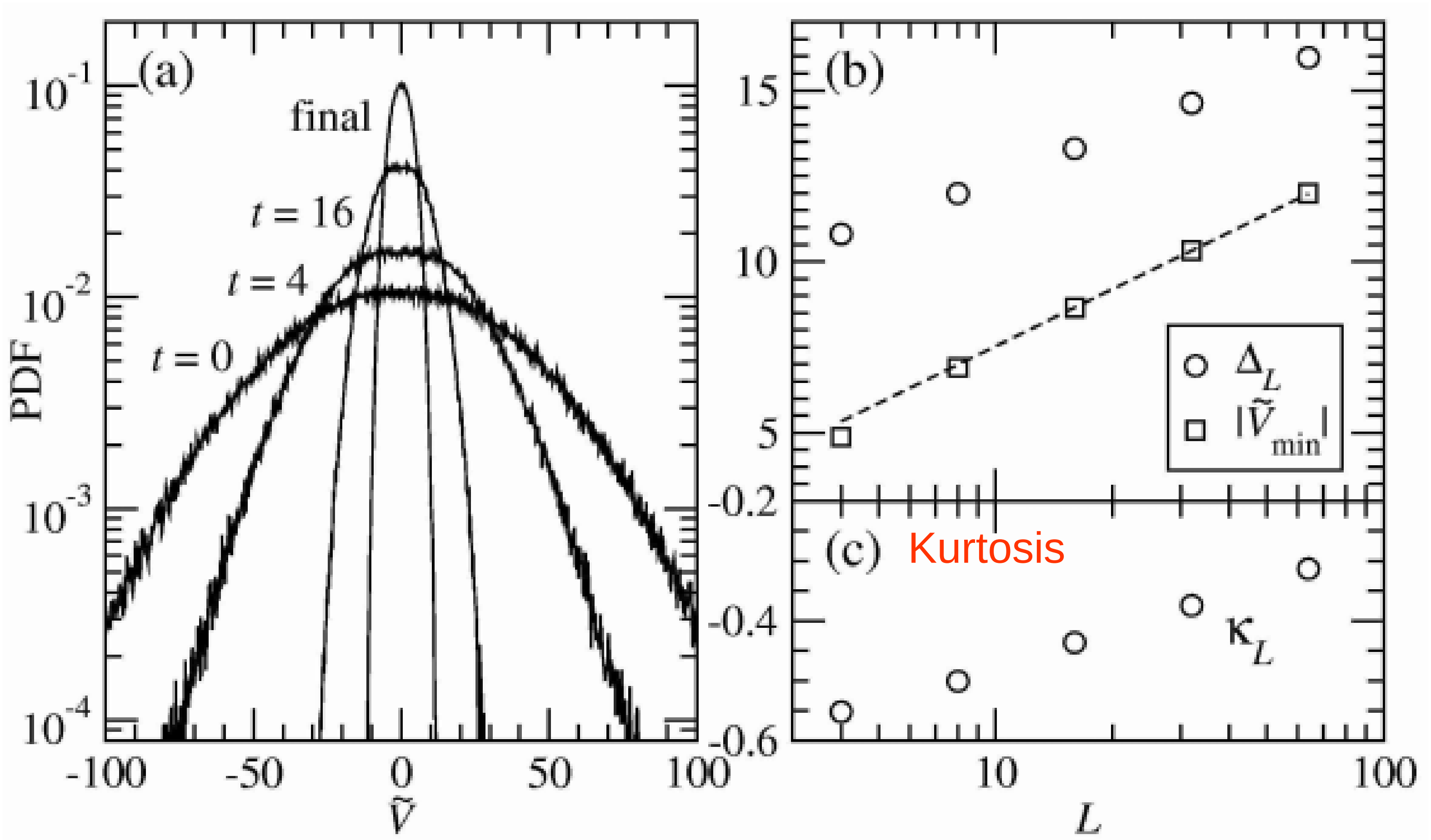
# Minimal energy states in 1000 trials

- Most ground state vortices are strongly localized
- Weak correlation exists for vortices near the ``Fermi energy''



# Evolution of the effective disorder potential in the filling process

$\tilde{V}_i(t)$ = potential at site $i$ due to disorder and existing vortices after $t$ iterations



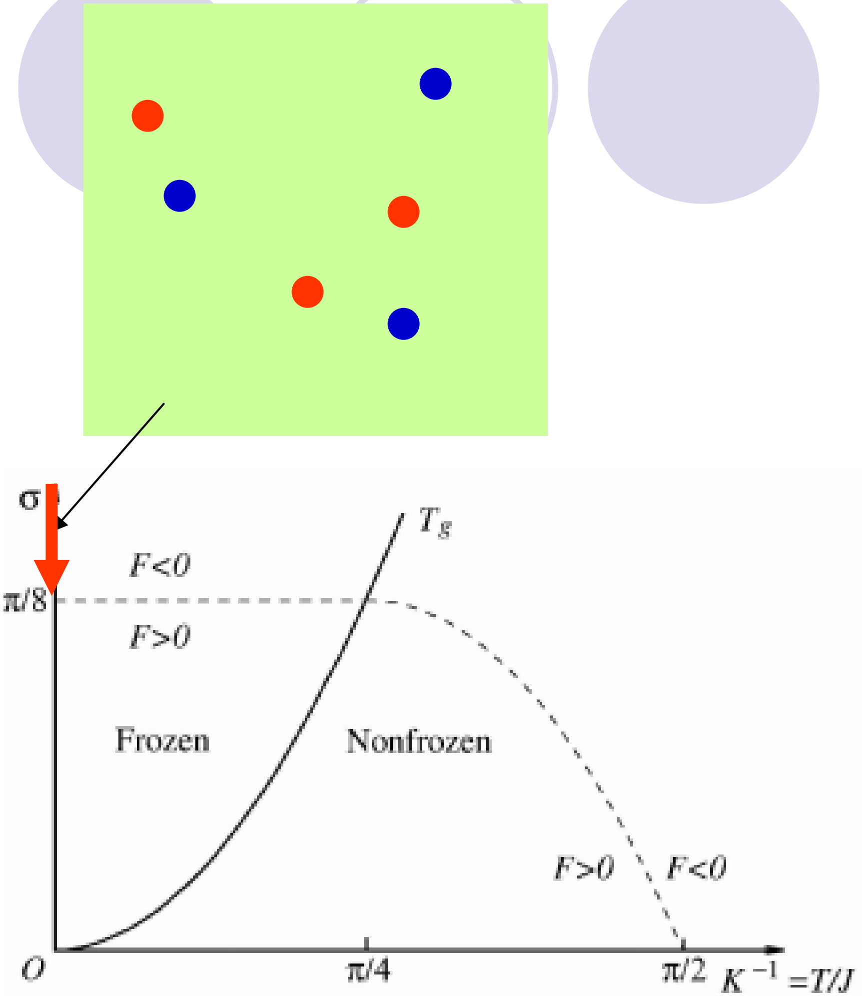
Kurtosis

# Ground state is critical!

- **Criticality**: Energy can not be lowered by inserting/removal of vortex-antivortex pairs

- **No Coulomb gap**: On the other hand, pair-pair interaction decays as $r^{-2}$, not nearly enough to produce a gap at the Fermi energy.

- **Finite DOS** for vortex-antivortex pairs



# The Stiffness Exponent

N. Akino and J. M. Kosterlitz, PRB **66**, 054536 (2002)

effective spin-glass stiffness $J_{\text{eff}} = aL^{-\theta}, \quad \theta = 0.36 - 0.45$

Dimensional consideration:

Density of pair states: $f_p(0) = cJ^{-1}$

Dielectric screening: $dJ^{-1}/dl = 2\pi^2 f_p(0) = 2\pi^2 cJ^{-1}$

$\Rightarrow$ $J = J_B L^{-2\pi^2 c}$

Finite temperature: $J \square T \quad \rightarrow \quad \xi \square T^{-\nu}, \quad \nu = 1/2\pi^2 c$

# Flat-histogram MC

(Torrie & Valleau, 77, B. A. Berg, 92)

Given an energy function

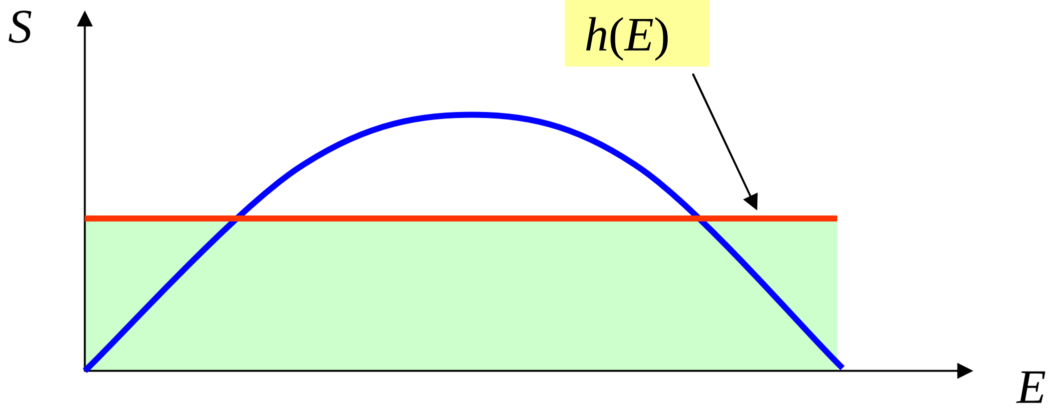
$$E(\varphi_1,\varphi_2,\ldots,\varphi_N)$$

Sample the phase space according to the multicanonical weight

$$P(\{\varphi\}) = \exp\left(-S(E\{\varphi\})\right)$$

energy histogram of an MCMC run

$$h(E) \propto \rho(E)\exp\left(-S\right) = \mathbf{1}$$



samples the whole (or any specified) energy range with equal frequency, including the lowest energy state.

# Iterative determination of MCMC weight

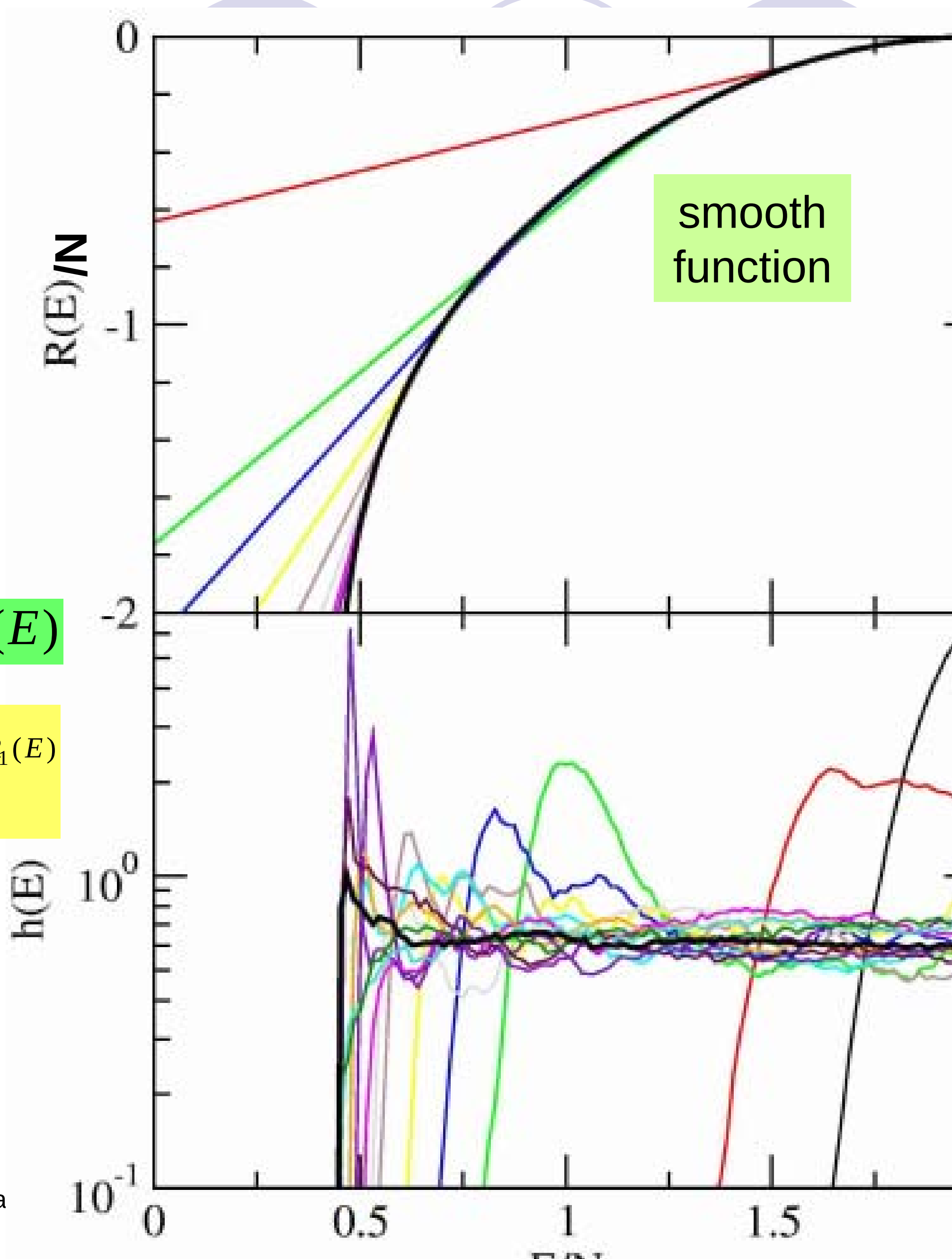
**Berg's iterative scheme**

- start with a trial $P_0(E) = e^{-R_0(E)}$
- MC sampling $\Rightarrow h_0(E)$
- Obtain an estimate of DOS $\rho_0(E) = h_0(E) / P_0(E)$
- Construct an improved $P_1(E) = \dfrac{1}{\rho_0(E)} = e^{-R_1(E)}$
- MC sampling $\Rightarrow h_1(E)$

……

Until convergence is reached

2D Ga


smooth function

# MCMC result on gauge glass

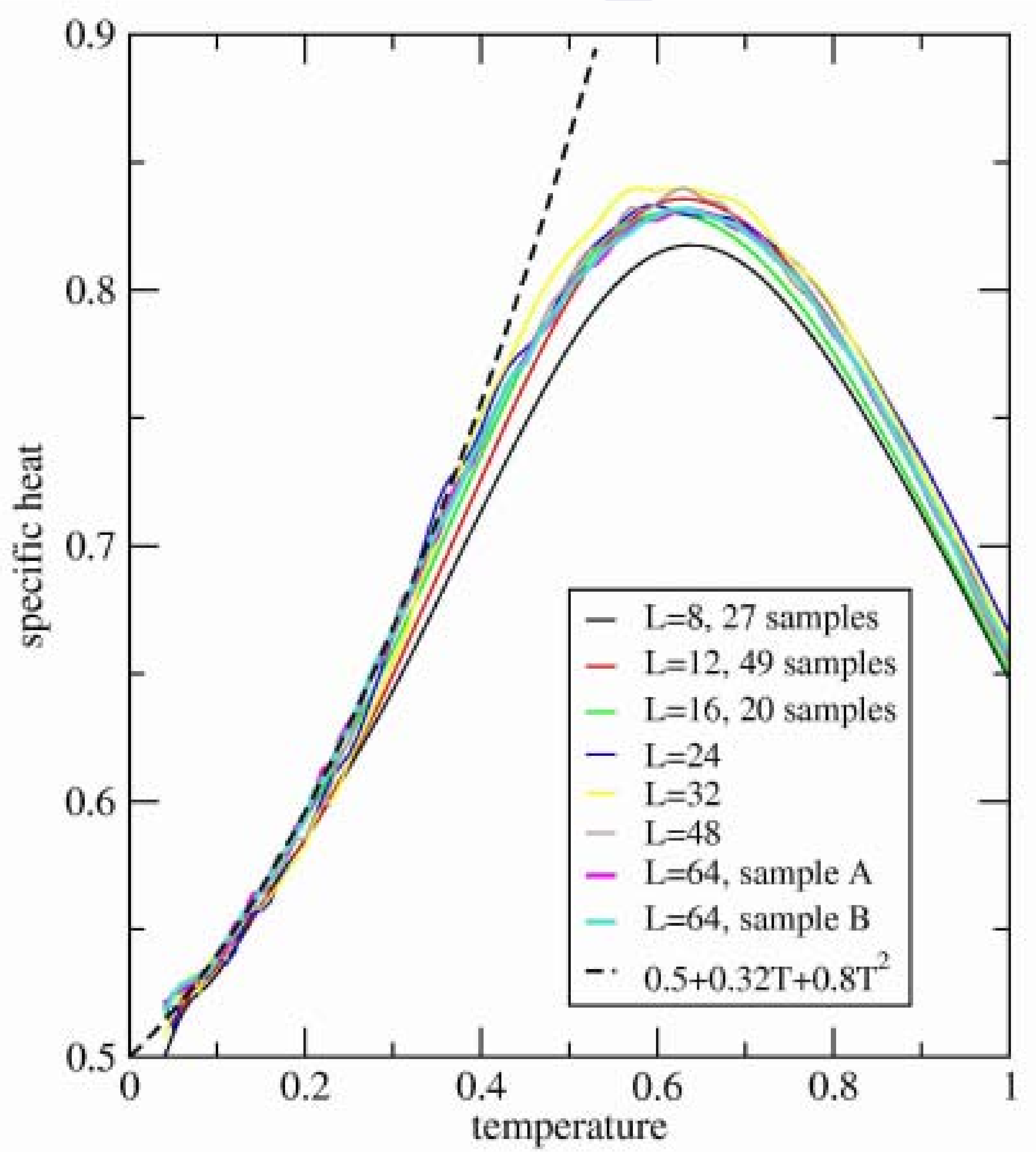
Specific heat

$$C_V \;\square\; 0.5 + \underbrace{0.32T + 0.8T^2}_{\text{vortices}}$$

classical
spin waves

Density of states for vortex /antivortex excitations

$$\rho(\varepsilon) \;\square\; \rho_0 + \rho_1 \varepsilon$$

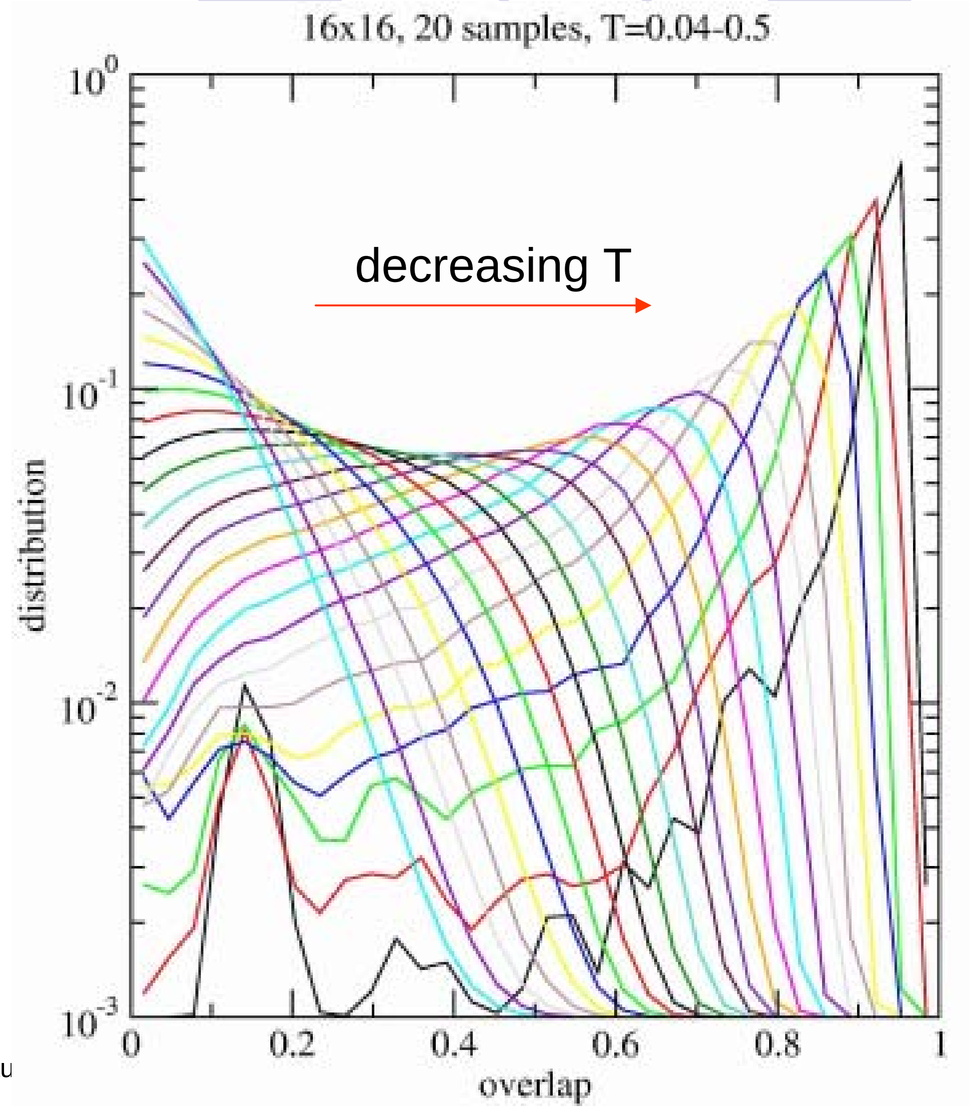


# Overlaps and spin-glass susceptibility

Overlap of two replicas α and β:

$$q_{\alpha\beta} = \frac{1}{N}\sum_j e^{i\left(\varphi_j^\alpha - \varphi_j^\beta\right)}$$

dominated by a unique ground state.



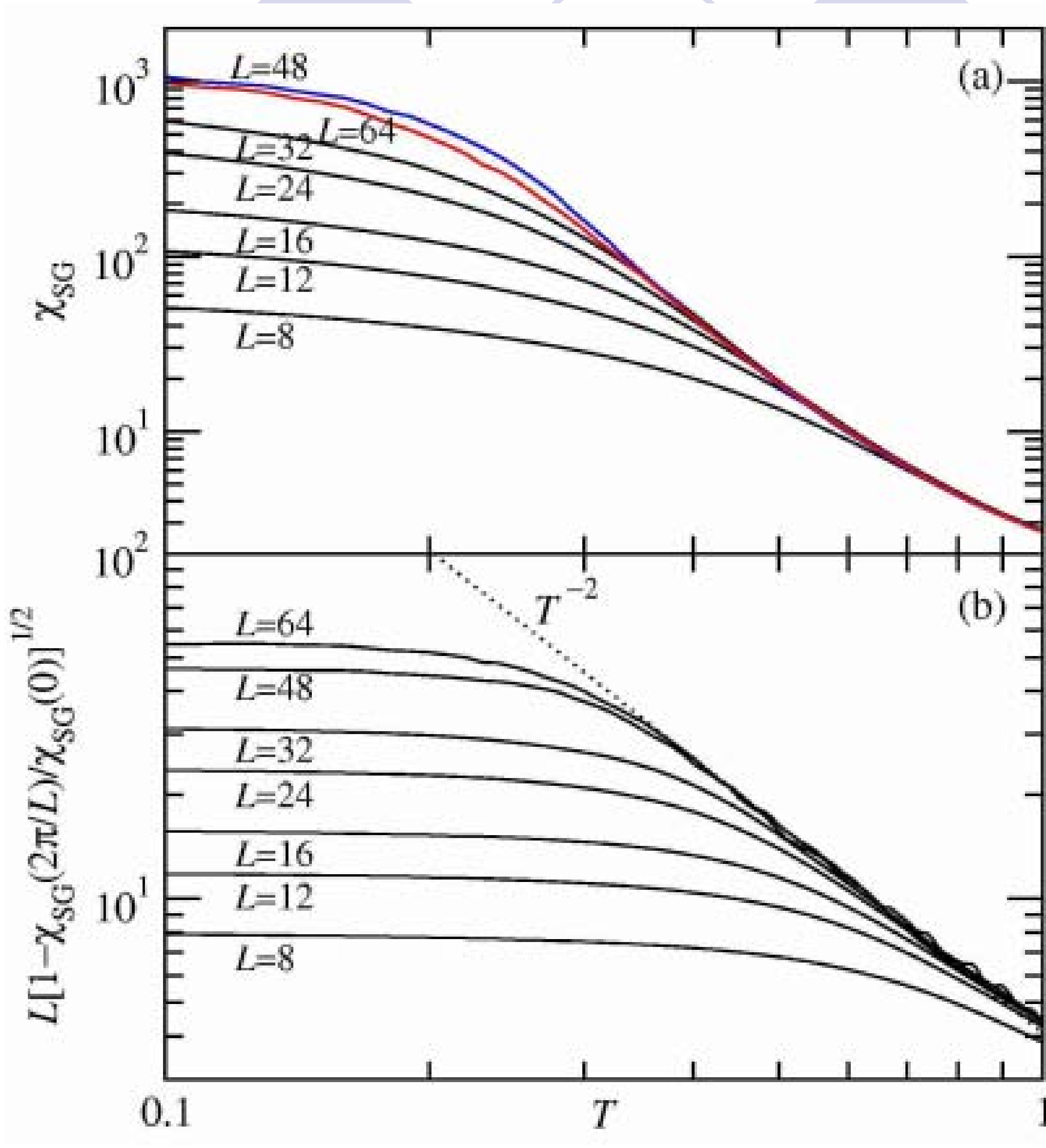
# Spin-glass susceptibility

$$\chi_{SG} = N\overline{\left\langle |q|^2 \right\rangle} = \sum_j \overline{\left|\left\langle e^{i(\varphi_j - \varphi_0)} \right\rangle\right|^2}$$

$$\sim \xi^2$$

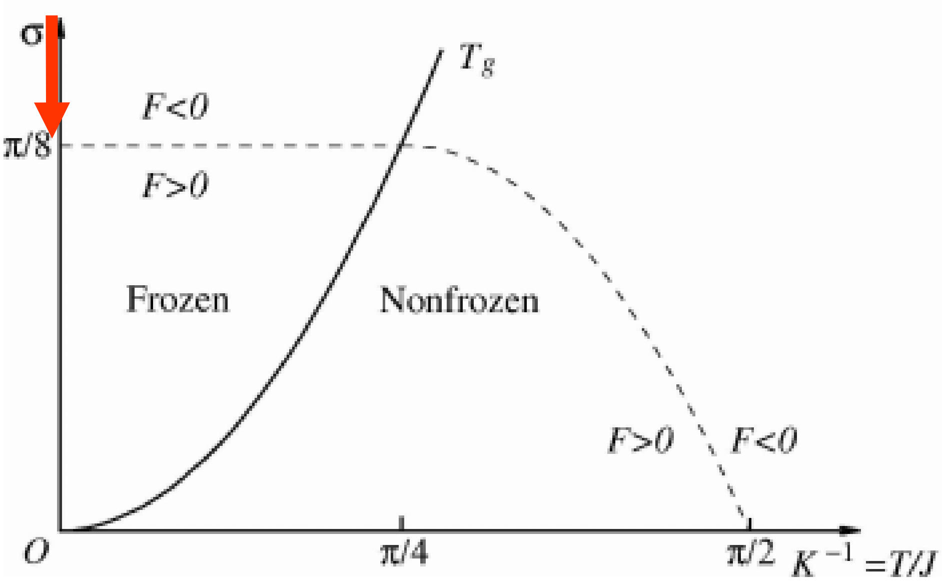
Data fits well to

$$\xi \sim T^{-2}$$



# Conclusions

- ➢ The ground state vortex configuration of the 2D Gauge Glass Model has a relatively simple structure, with only weak correlation effects
- ➢ There exists a finite density of pair states at the Fermi energy, giving rise to screening at zero temperature
- ➢ Power-law decay of spin-glass stiffness at zero temperature
- ➢ Full renormalization group treatment in progress



# Acknowledgements

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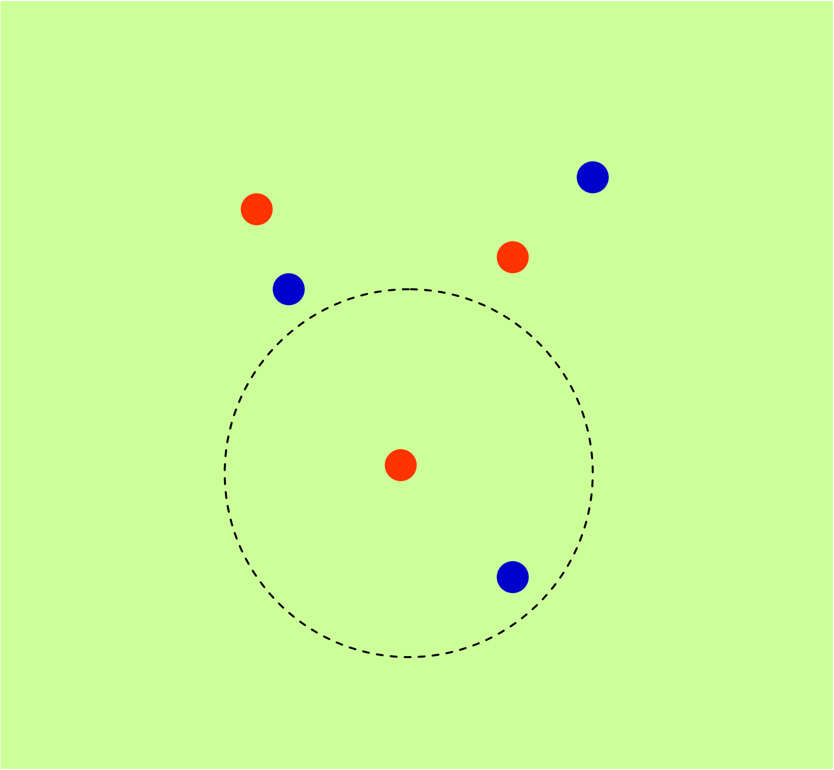
# Debye-Hückle for the density of states



Integrated density of states

$$N(\varepsilon) = \varepsilon^{1+\mu}$$

Screened interaction

$$u_{\text{eff}}(r) \sim r^{-\alpha}$$

Debye-Hückle

$$\nabla^2 u_{\text{eff}} = 2\pi(\rho_+ - \rho_-) = 4\pi N(u_{\text{eff}})$$

$$\mu = 0 \text{ (no gap),}$$

$$\alpha = \infty \text{ (exponential decay)}$$

is a solution.

# Major advantages and disadvantages

**Advantages**

✓ Sample entire energy range in a single run ⇒ thermodynamic averages for all temperatures

✓ Free energy/entropy calculated directly

✓ Possible to obtain estimates of energy barriers (such as surface tension at phase coexistence)

✓ Can significantly reduce critical slowing down/low temperature glassiness

**Disadvantages**

❖ Non-scalable: minimal equilibration time ~ $N^2$

❖ Efficiency of low-temperature sampling problem-dependent, usually does not eliminate slowing down completely though one can often do better

❖ Convergence of MCMC iteration with minimal sampling can be a big headache