Driven Diffusive Systems

an Overview and some Recent Developments

R. K. P. Zia

Physics Department, Virginia Tech,
Blacksburg, Virginia, USA

B. Schmittmann
I. Geogiev

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Outline

- Overview (devoted to students and newcomers)
  - What are DDS? Why study them?
    What's the context?
  - Driven Ising Lattice Gas (the "standard" model) and Variations
  - Novel properties: many surprises…

    some understood, much to be understood

Outline

- Overview
- Some recent developments
    - Mysteries in a two-species, two-lane system
    - Solution to a class of mass transport models
        - a simple test on the chipping kernel…

          …to check if stationary distribution is factorizable or not
        - (if yes) explicit construction of distribution
- Conclusions

Take-home message:

Many-body systems, with very simple constituents and rules-of-evolution (especially “non-equilibrium” rules),

often display a rich variety of complex and surprising behavior.

How can they be predicted??

What are DDS? Why study them?

- Interacting many-particle systems
  (conservation laws, diffusion)…
- Driven far from thermal equilibrium
  (by some external force field, or some other energy reservoir)
- Motivated by the physics of
  super ionic conductors (Katz, Lebowitz, and Spohn, 1983,84)

What are DDS? Why study them?

- Interacting many-particle systems
  (conservation laws, diffusion)…
- Driven far from thermal equilibrium
  (by some external force field, or some other energy reservoir)
- Fundamental issue:

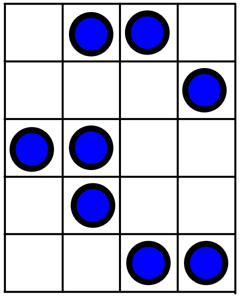
Can systems in *non-equilibrium* steady states be understood in the Boltzmann-Gibbs framework?

If not, what's the new game in town?

Ising Lattice Gas

- Take a well-known equilibrium system…

e.g., Ising lattice gas (2-d, Onsager)



C : { $n(x,y)$ } with $n = 0,1$

$$\mathbf{H}(\mathbf{C}) = -J \sum_{\boldsymbol{x},\boldsymbol{a}} n(\boldsymbol{x})\, n(\boldsymbol{x}+\boldsymbol{a})$$

+ periodic boundary condtions (PBC)

Ising Lattice Gas

- Take a well-known equilibrium system,
- evolving with a simple dynamics…

…going from **C** to **C** ′ with rates $R(\mathbf{C} \to \mathbf{C}')$ that obey **detailed balance:**

$$R(\mathbf{C} \to \mathbf{C}') \,/\, R(\mathbf{C}' \to \mathbf{C}) = exp\left[\{\mathbf{H}(\mathbf{C}') - \mathbf{H}(\mathbf{C})\}/kT\right]$$

e.g., simulators’ favorite (Metropolis, Kawasaki):

- ✓ pick particle-hole pair at random
- ✓ compute energy difference if they are exchanged
- ✓ if $\Delta\mathbf{H} \leq 0$, go (R=1)
- ✓ if $\Delta\mathbf{H} > 0$, go with probability $exp[-\Delta\mathbf{H}/kT]$

Ising Lattice Gas

- Take a well-known equilibrium system,
- evolving with a simple dynamics…

…going from $\mathbf{C}$ to $\mathbf{C}'$ with rates $R(\mathbf{C} \to \mathbf{C}')$ that obey **detailed balance:**

$$R(\mathbf{C} \to \mathbf{C}') \,/\, R(\mathbf{C}' \to \mathbf{C}) = exp\left[\{\mathbf{H}(\mathbf{C}') - \mathbf{H}(\mathbf{C})\}/kT\right]$$

…so that, in long times, the system is described by
the Boltzmann distribution:

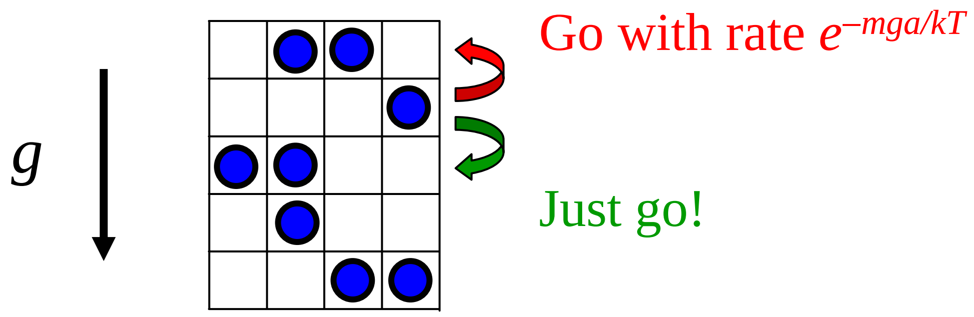
$$P^*(\mathbf{C}) \propto exp\left[-\mathbf{H}(\mathbf{C}) / kT\right]$$

Driven Ising Lattice Gas

- Take a well-known equilibrium system
  (Ising model in lattice gas langauge),
- and drive it far from thermal equilibrium
  (by some additional external force, so particles suffer *biased* diffusion.)

e.g., effects of gravity (at the *local* level!)

- Can't have PBC !!
- Get to equilibrium!

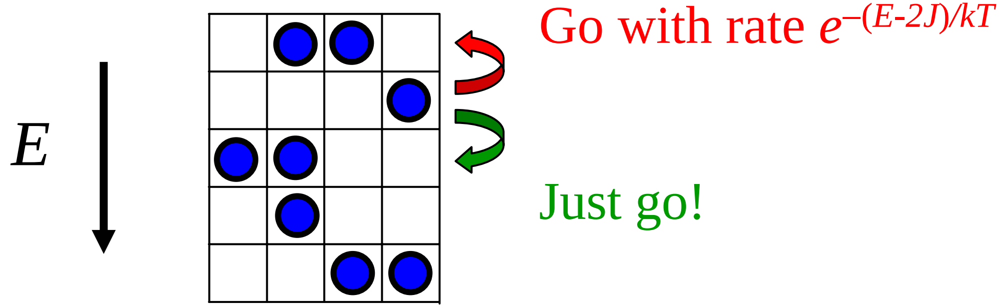


Driven Ising Lattice Gas

- Take a well-known equilibrium system
  (Ising model in lattice gas langauge),
- and drive it far from thermal equilibrium
  (by some additional external force, so particles suffer *biased* diffusion.)

PBC possible with “electric” field, E
(non-potential, rely on $\partial_t B$)

unit “charge” and a
with $E > 2J$



Driven Ising Lattice Gas

How does this differ from the equilibrium case?

- Condition of detailed balance violated.
- System goes into non-equilibrium steady state:
  non-trivial particle current *and*
  energy through-flux.

In most cases, this is not easy to see!

In this case, it has to do with the PBC.

Driven Ising Lattice Gas

How does this differ from the equilibrium case?

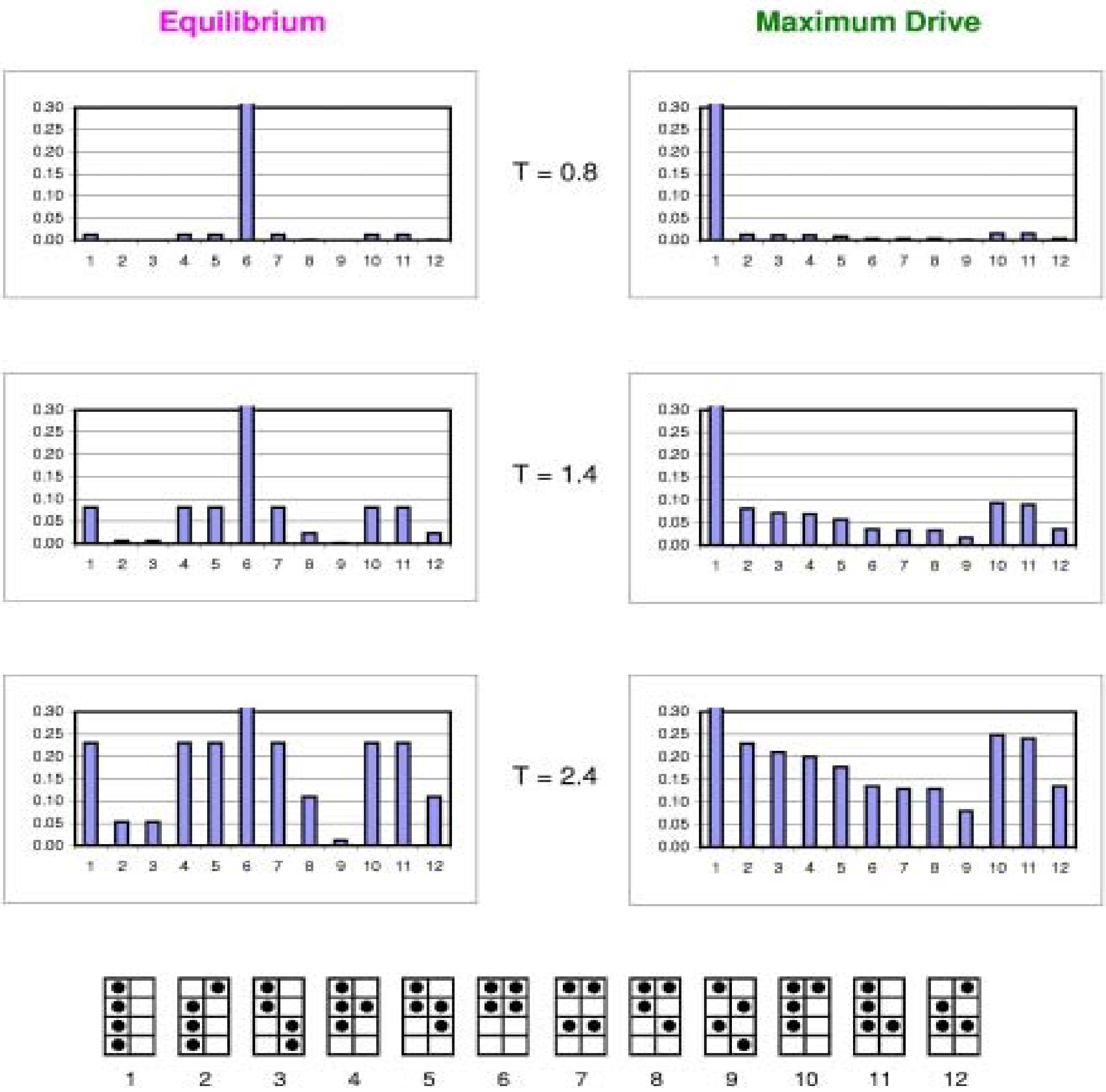
- ❖ Condition of detailed balance violated.
- ❖ System goes into non-equilibrium steady state
- ❖ Stationary distribution, $P^*(\mathbf{C})$, exists…

...but very different from Boltzmann.

A simple, exactly solvable, example: 2×4

Overview

Relative Probabilities of Configurations



Driven Ising Lattice Gas

How does this differ from the equilibrium case?

- ❖ Condition of detailed balance violated.
- ❖ System goes into non-equilibrium steady state
- ❖ Stationary distribution, $\boldsymbol{P}^*(\mathbf{C})$, exists…
  ...but very different from Boltzmann.
- ❖ Usual fluctuation-dissipation theorem violated.

Even simpler example: 2×3 ($E=\infty$)

- "specific heat" $-\partial_\beta\langle U\rangle$ has a *peak* at $\ell n 9/(8J)$
- energy fluctuations $\langle\Delta U^2\rangle$ *monotonic* in β

Driven Ising Lattice Gas

How does this differ from the equilibrium case?

- Condition of detailed balance violated.
- System goes into non-equilibrium steady state
- Stationary distribution, $P^*(\mathbf{C})$, exists…

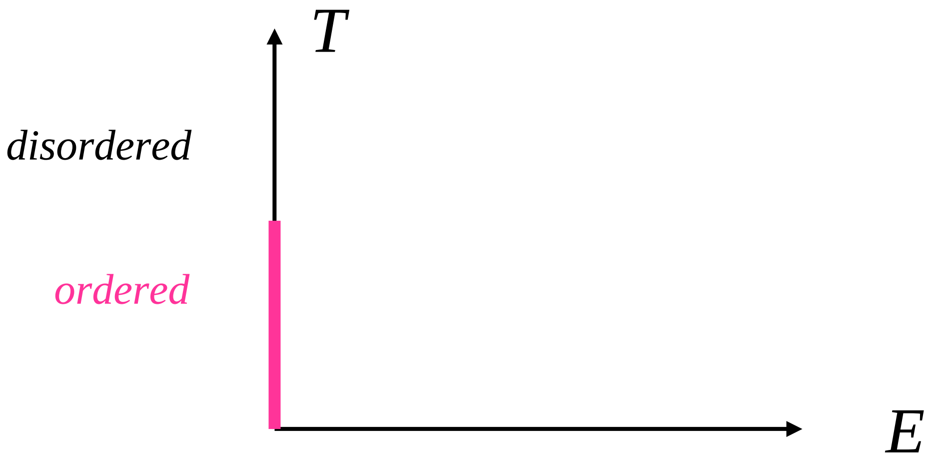
  ...but very different from Boltzmann.
- Usual fluctuation-dissipation theorem violated.
- The many *surprises* they bring!!

Driven Ising Lattice Gas

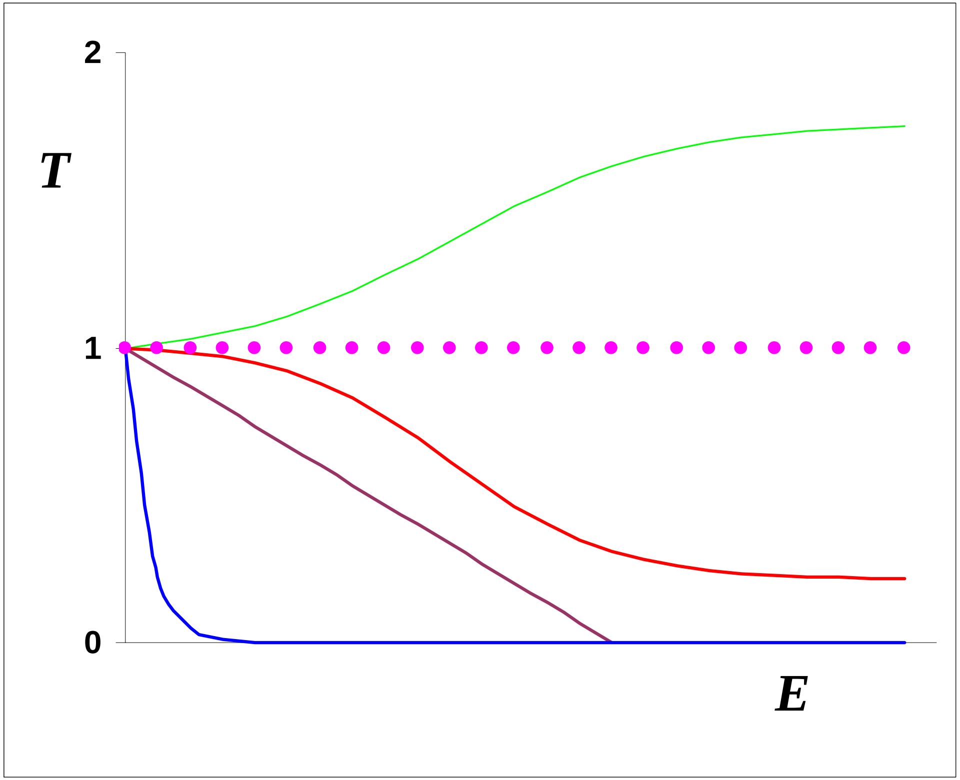
The surprises they bring!!

- breakdown of well founded intuition

for example, consider phase diagram:



What's your bet?

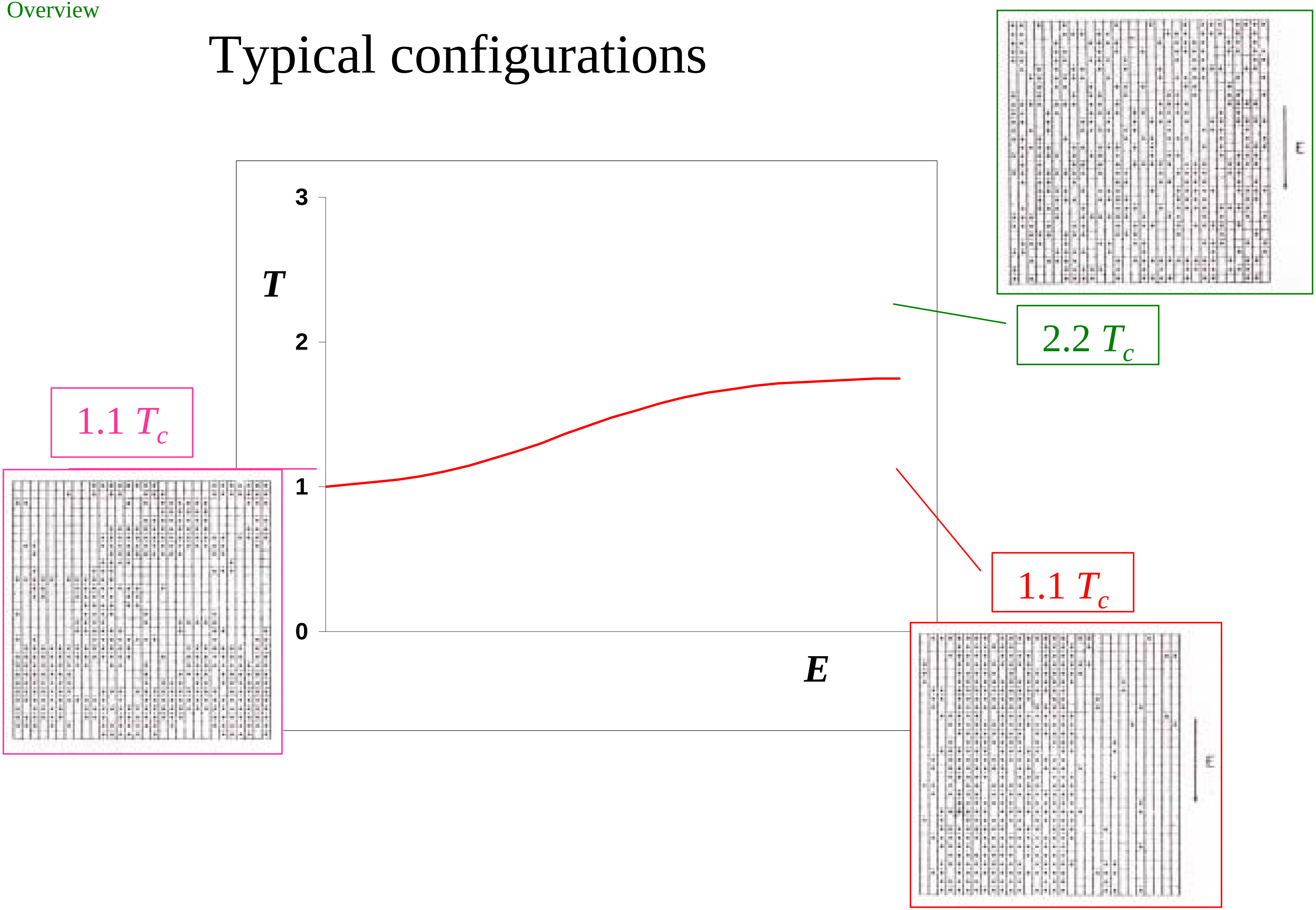


T_c goes up!!

My first guess…

Guesses based on energy-entropy intuition.

Typical configurations



Driven Ising Lattice Gas

The surprises they bring!!

- breakdown of well founded intuition
- negative responses…

… "Getting more by pushing less!"

American Journal of Physics **70,** 384 (2002)

…an entirely different example:

1-d Ising with two T's

M. Droz, Z. Racz, P. Tartaglia, *Phys. Rev.* **A41**, 6621 (1990)

Driven Ising Lattice Gas

The surprises they bring!!

- breakdown of well founded intuition
- negative responses
- generic long range correlations: r^{-d} (*all* T not near T_c)

Driven Ising Lattice Gas

The surprises they bring!!

- breakdown of well founded intuition
- negative responses
- generic long range correlations: r^{-d} (*all T* not near T_c)
- new universality classes, e.g.,

$d_c = 5\ [3]$ for uniformly [randomly] driven case

Driven Ising Lattice Gas

The surprises they bring!!

- breakdown of well founded intuition
- negative responses
- generic long range correlations: r^{-d} (*all* T not near T_c)
- new universality classes
- anomalous interfacial properties, e.g.,

$G(q) \sim q^{-0.67}\,[1/(|q|+c)]$ for uniformly [randomly] driven case

$\Rightarrow$ interfacial widths do not diverge with L !

Driven Ising Lattice Gas

The surprises they bring!!

- breakdown of well founded intuition
- negative responses
- generic long range correlations: r^{-d} (*all* T not near T_c)
- new universality classes
- anomalous interfacial properties
- new ordered states if PBC $\rightarrow$ SPBC, OBC

$\vdots$

Driven Ising Lattice Gas

The surprises they bring!!

- breakdown of well founded intuition
- …need new intuition/paradigm…

One way forward is

to study many other, similar systems

Other Driven Systems

- Various drives:
  - AC or random *E* field (more accessible experimentally)
  - Two (or more) temperatures (as in cooking)
  - Open boundaries (as in real wires)
  - Mixture of Glauber/Kawasaki dynamics (e.g., bio-motors)

    ⋮

Other Driven Systems

- Various drives
- Multi-species:
  - Two species (e.g., for ionic conductors)

    Baseline Study: driven in opposite directions, with "no" interactions

    "American football, Barber poles, and Clouds"

    ⋮
  - Pink model (with 10 or more species) for bio-membranes

    ⋮

Other Driven Systems

- Various drives
- Multi-species
- Anisotropic interactions and jump rates
  - Layered compounds
  - Lamella amphiphilic structures.

    ⋮

Other Driven Systems

- Various drives
- Multi-species
- Anisotropic interactions and jump rates
- Quenched impurities

⋮

Other Driven Systems

- Various drives
- Multi-species
- Anisotropic interactions and jump rates
- Quenched impurities

    ⋮

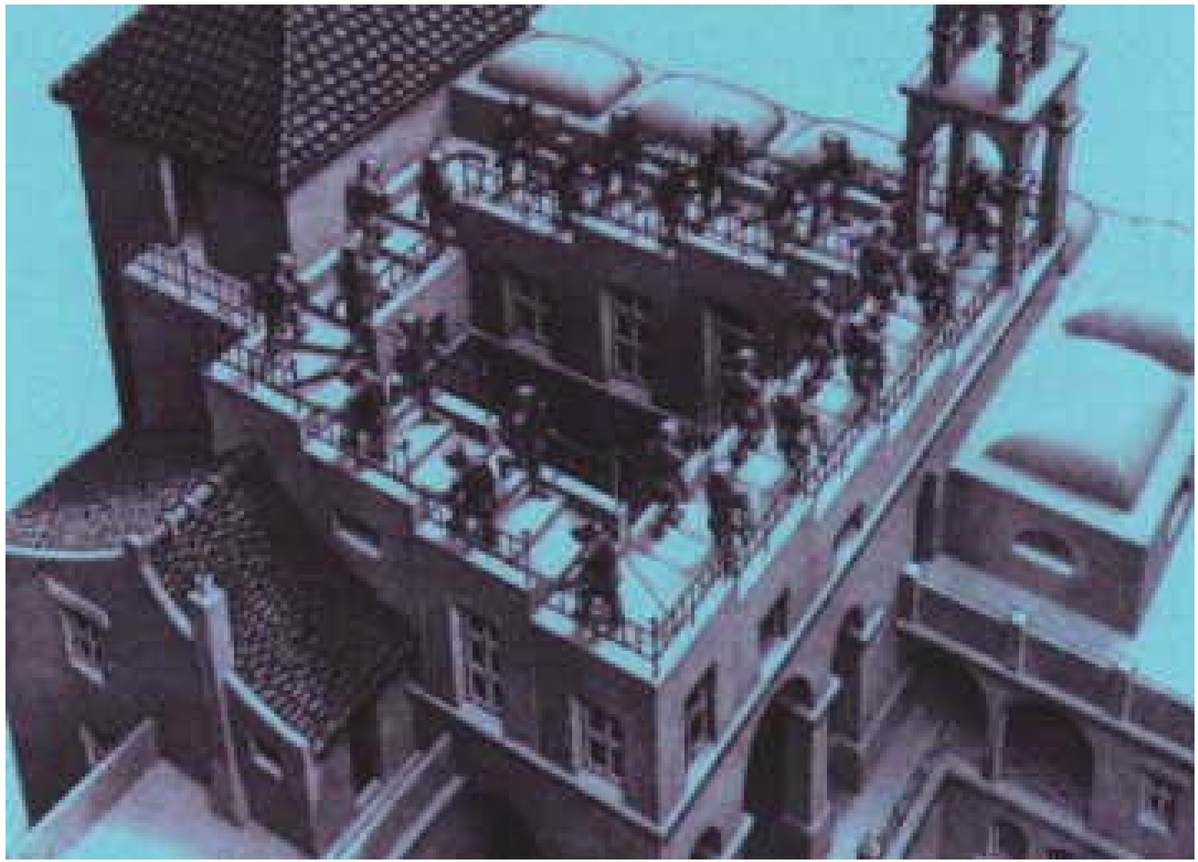
- Exactly solvable models in 1-d

…provided lots of insight for a specialized class of DDS

Domb and Lebowitz
series on

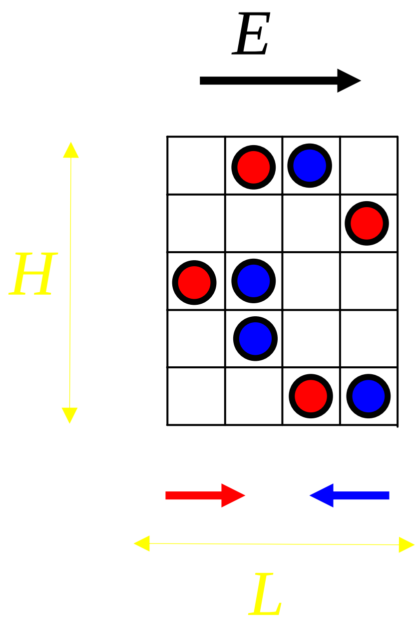
Phase Transitions
and
Critical Phenomena

Vol. 17



Recent Developments

Biased diffusion of two species in Quasi-one-dimensional lattices



all particle-hole exchanges with rate: 1

…except jumps against E : $exp(-E)$

all "charge" exchanges with rate: γ

…except jumps against E : γe^{-E}

e.g., $E=\infty$, $\gamma = 0.1$, half-filled, $N=N$

Recent Developments

Biased diffusion of two species in Quasi-one-dimensional lattices

- MC on 2-d shows a jamming transition, but…
- *No transitions* in 1-d (exact results and MC)
- Early data on *two lanes* and a conjecture
- Fast coarsening
- Lane preference

Snapshots of $H = 1,2$ systems

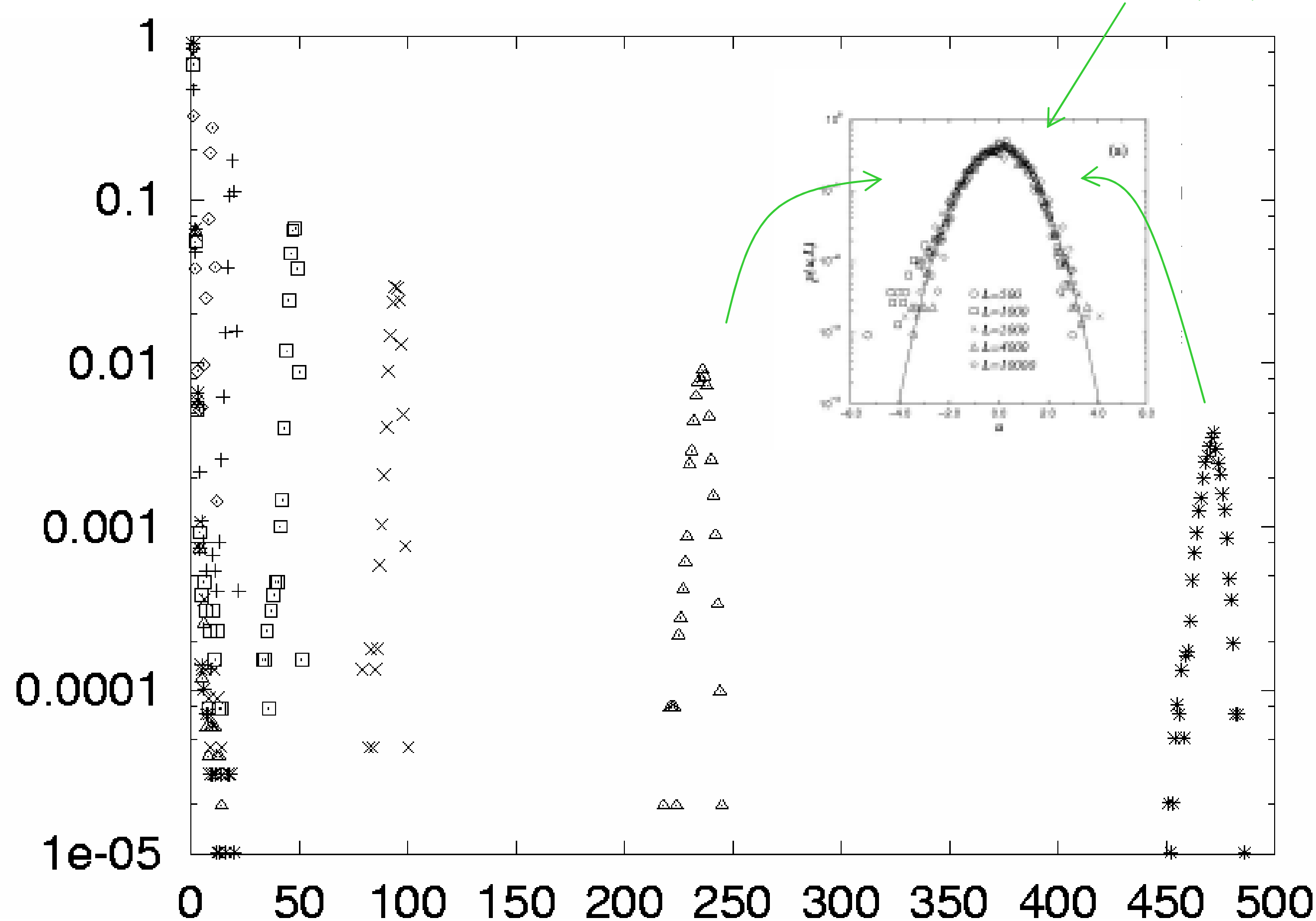
(both in steady state)

$$E = \infty \,,\ \gamma = 0.1 \,,\ L = 1000$$

Residence distribution for 2×*L* cases

L = 2K, 4K, 10K

$p(\ell)$



ℓ

Coarse grained
“length” of cluster

Conjecture

- The two lane system, just like the $1 \times L$ case, is homogeneous in the *thermodynamic limit*.
- The "jam" will not scale with L for "large enough L, " with cross-over length beyond those in MC (may be as large as 10^{10} or even 10^{70}).
- …based on MC + solution of similar model

N. Rajewsky, T. Sasamoto, and E.R. Speer, Physica A**279**, 123 (2000).
T. Sasamoto and D. Zagier, J. Phys. **A34**, 5033 (2001).

- …and criterion associated with asymptotic properties of currents of finite clusters

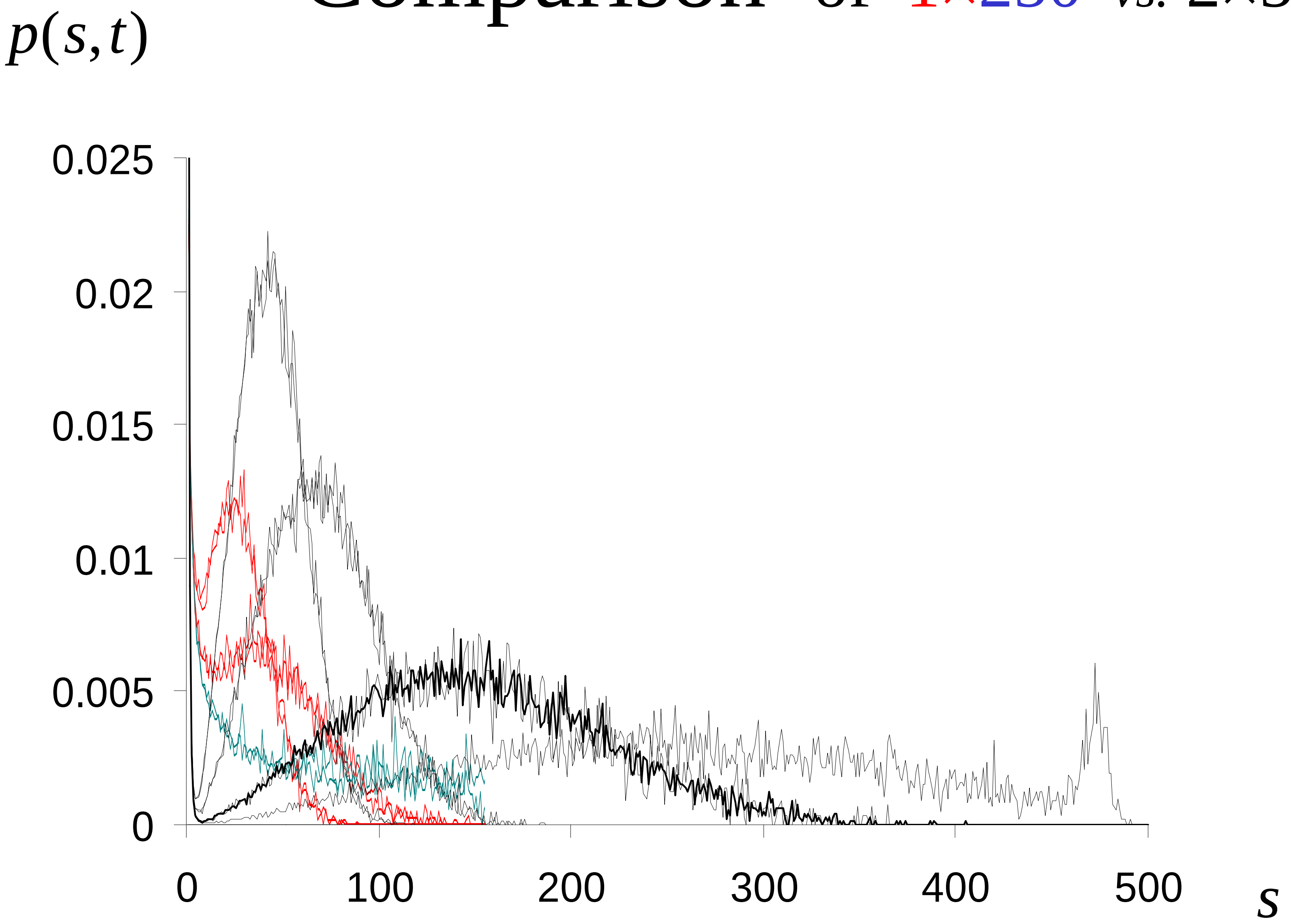
Y. Kafri, E. Levine, **D. Mukamel, G.M. Schütz**, and J. Török, Phys. Rev. Lett. **89**, 035702 (2002).

Regardless of the issues of $L \to \infty$,
we may ask:

How do the $1{\times}L$ and $2{\times}L$ systems evolve toward these very different steady states (for *presently accessible* L's) ?

Investigate the
t-dependent residence distribution
$p(\ell,t)$!

Comparison of 1×250 *vs.* 2×500



Fast coarsening in $2\times L$

- Study residence distribution $p(\ell,t)$
- Average cluster size grows faster than typical:

$$\bar{\ell}(t) \equiv \sum \ell\, p(\ell,t) \quad \sim t^{2/3}$$

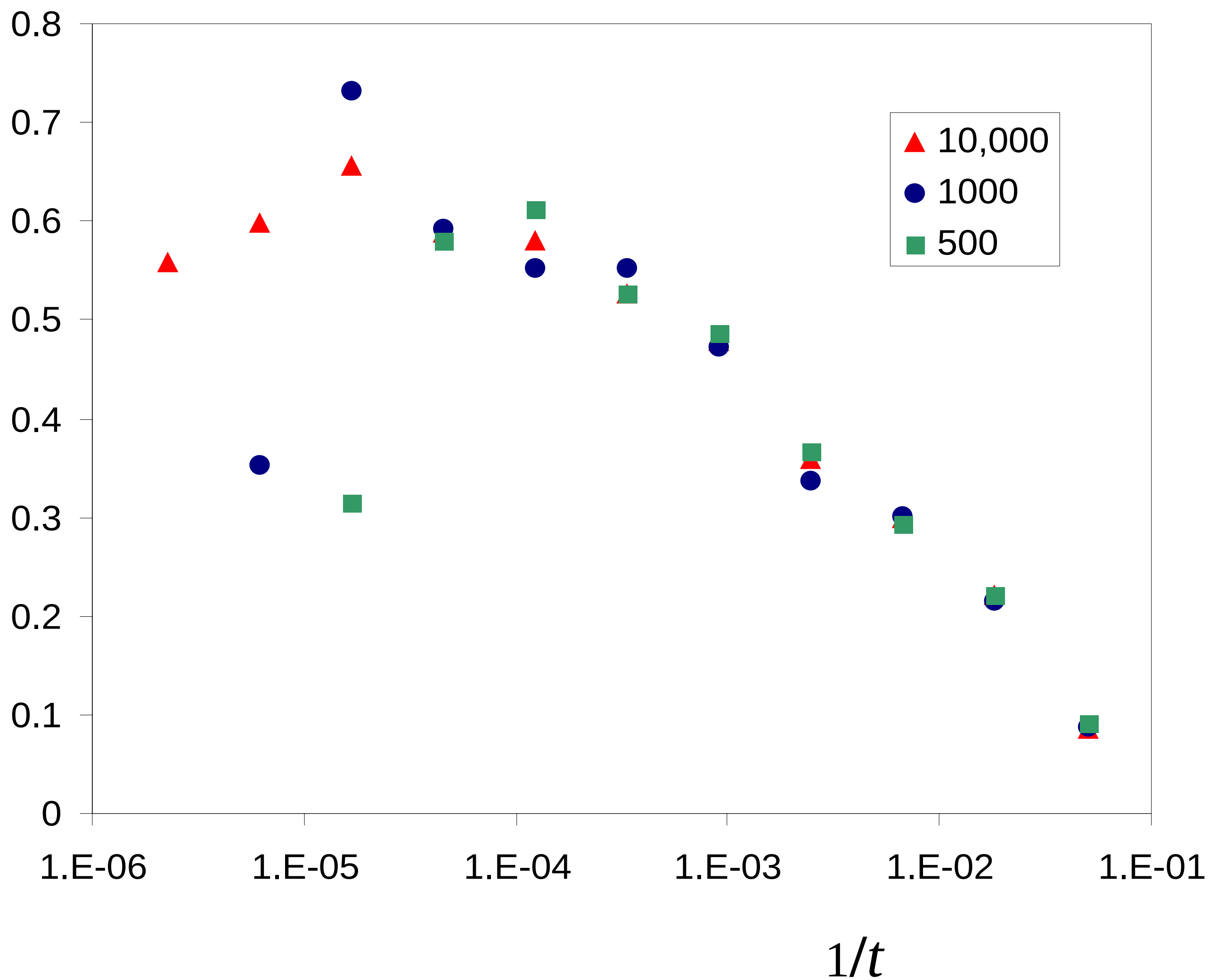
- Dynamic scaling qualitatively present :

$$\bar{\ell}(t)p(\ell,t) \cong f(x) \qquad x \equiv \ell / \bar{\ell}(t)$$

- Features are L independent (during growth regime).

J.T. Mettetal, B. Schmittmann, and R.K.P. Zia, Europhys. Lett. **58**, 653 (200
B. Schmittmann, *et.al.*, in *Computer Simulation Studies in Condensed Matter Physics X*
eds. D.P. Landau, S.P. Lewis, and H. B. Schüttler (Springer, *in pres*

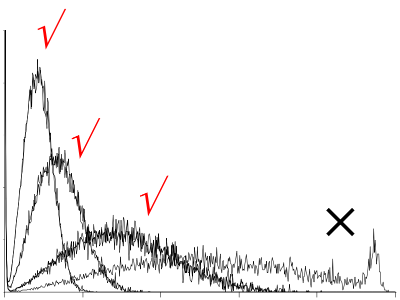
"local exponent" (slope in log-log plot) : $d(\ln \bar{\ell})/d(\ln t)$

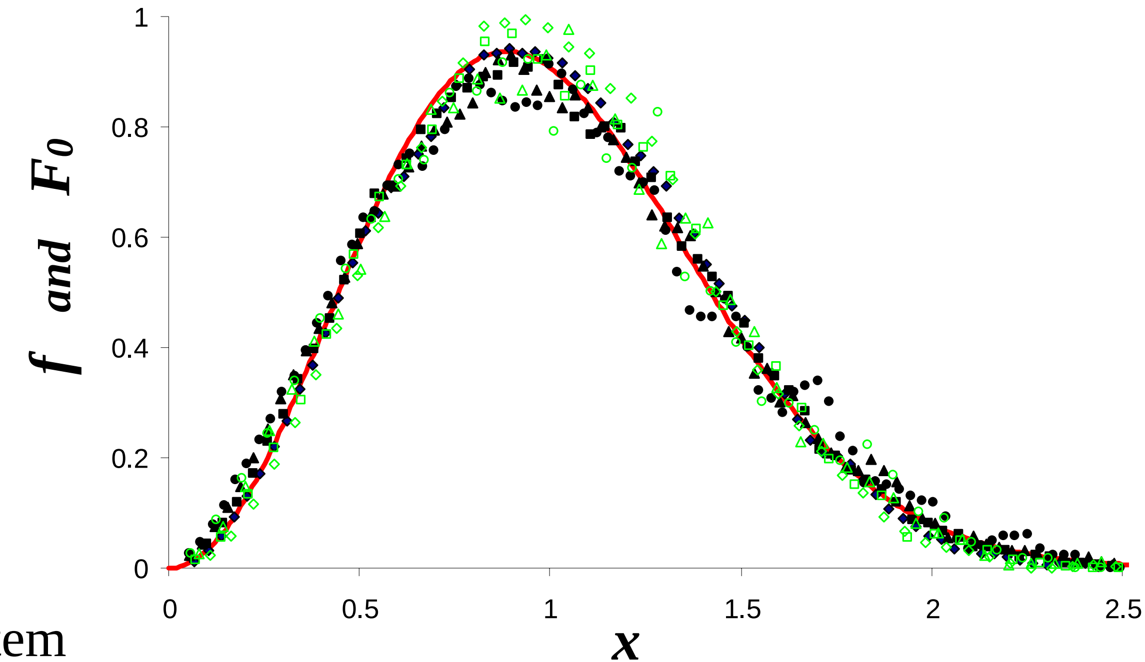


Dynamic scaling

$t \approx 1, 3, 8, 22$ KMCS

$L = 500$ $L = 2000$





F_0 from a “reference” system

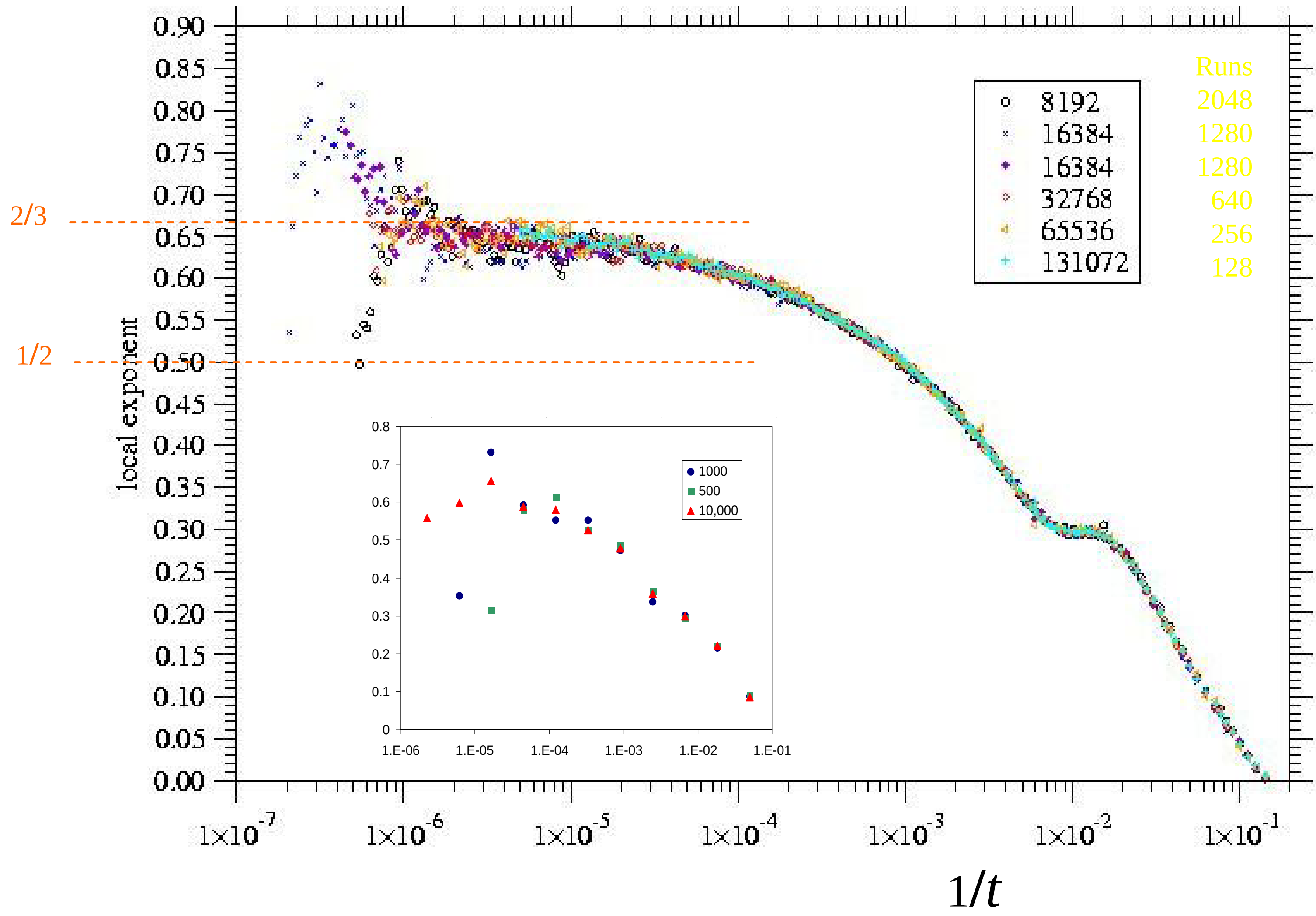
more recent surprises

(…work in progress, to be published soon)

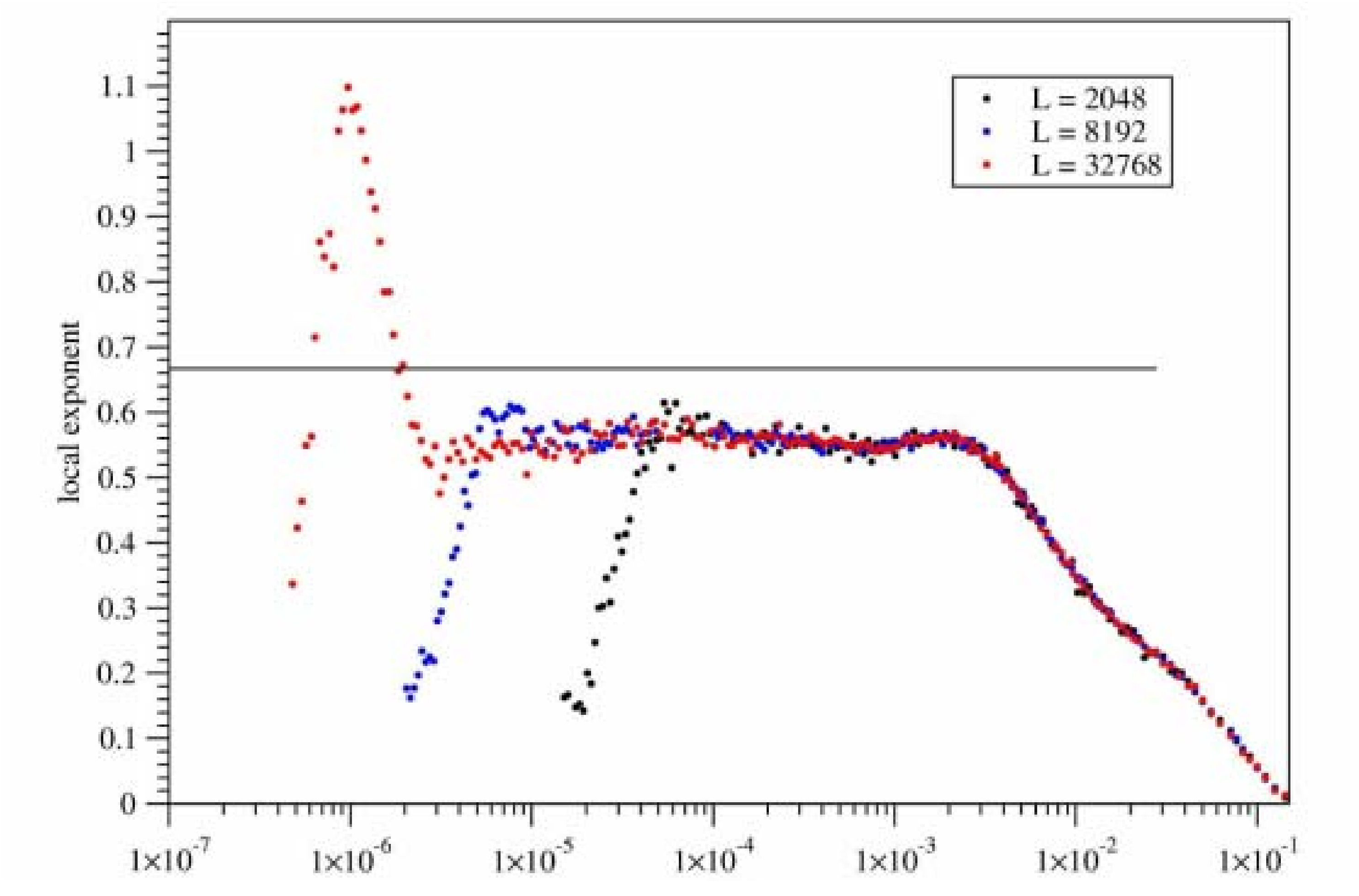
- Fast coarsening: (Ivan Georgiev)

  larger L; varying γ; multi-lanes ($H > 2$)

"local exponent": $d(\ln \bar{\ell}) / d(\ln t)$ for $\gamma = 0.1$ 2×L

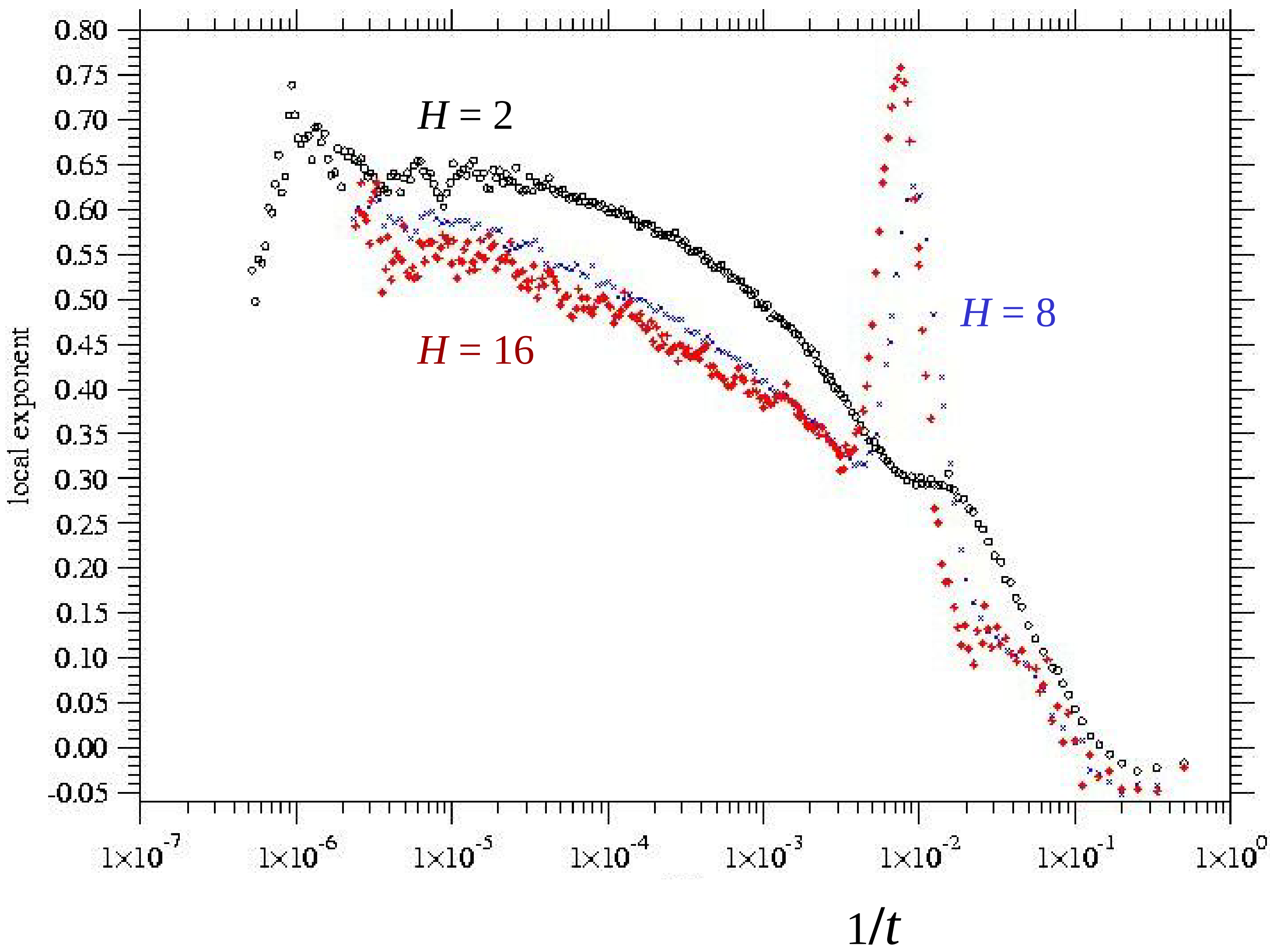


local exponent for $\gamma = 0.4$ $2\times L$



$1/t$

local exponent for γ = 0.1 *H*×8192



more recent surprises

(…work in progress)

- Fast coarsening: (Ivan Georgiev)

  larger L; varying γ; multi-lanes ($H > 2$)

- Effects of “lane preference” (Justin Krometis)

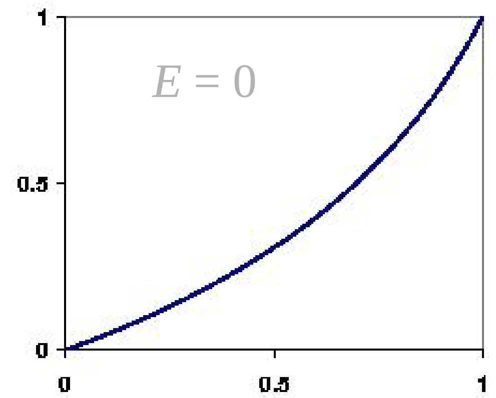
  probability of going to “preferred” lane > 0

Lane preference

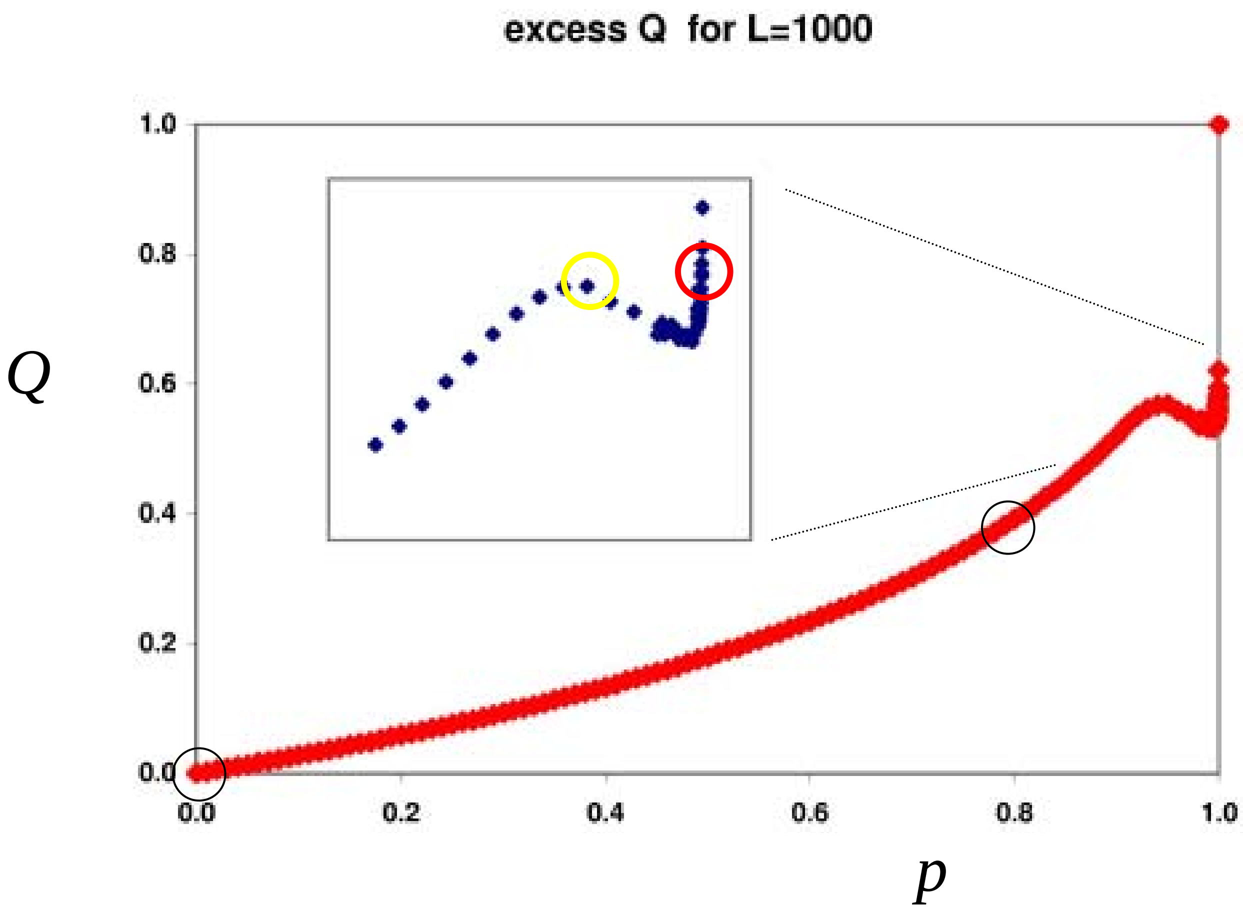
- "cars/trucks" (sometimes) tend to stay in "fast/slow" lane
- p probability for choosing "preferred" lane
- $p = 0, 1$ cases are clear:
  - jam (as before) *vs* free flow
  - equal mix (on the average) *vs* pure cars/trucks

    i.e., $Q \equiv$ "excess" $= 0$ *vs* $\equiv 1$
- *expect* $Q(p)$ to be

  monotonically increasing, e.g.,…



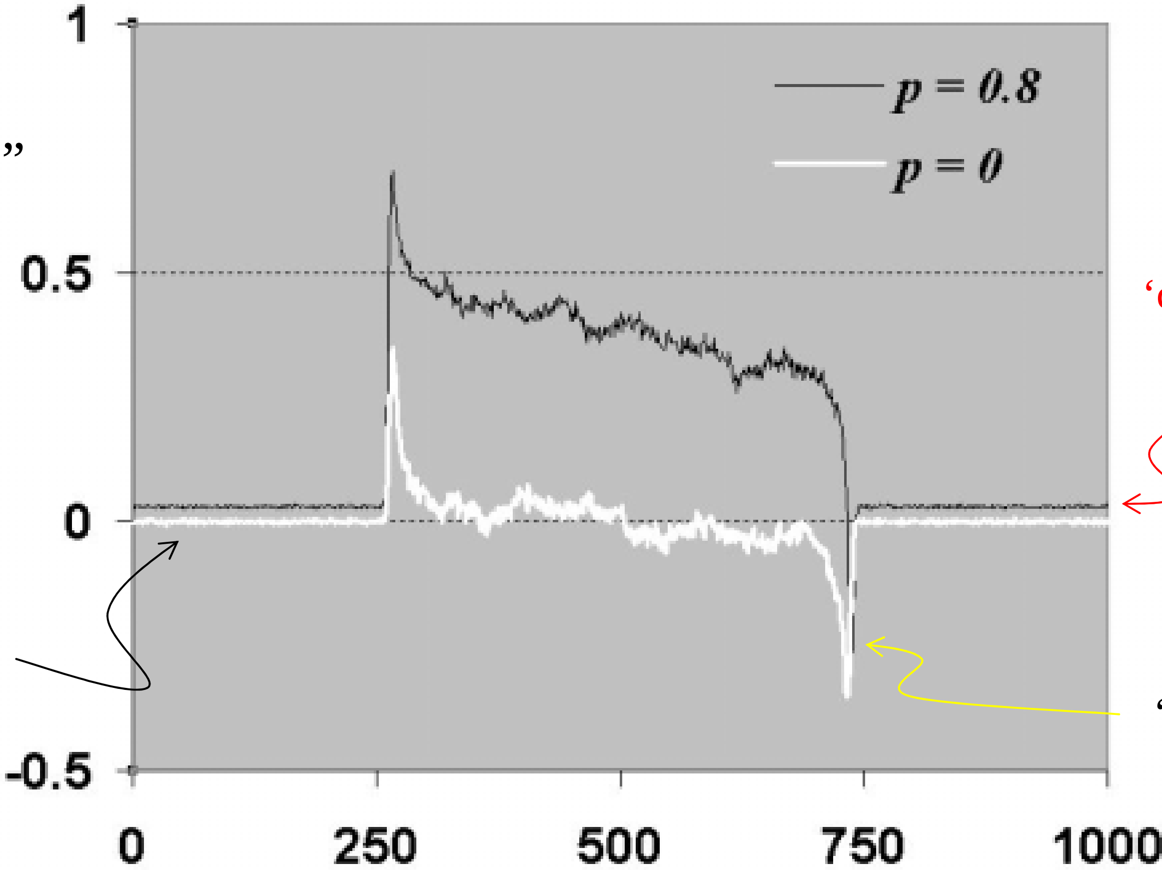
…instead, there is a twist :



Profiles at ‘small’ *p*

(CM of entire ‘jam’ centered at 500, then averaged)

density of
‘cars’ – ‘trucks”
in ‘fast lane’



excess of
‘car-travelers’ ove
‘truck-travelers’

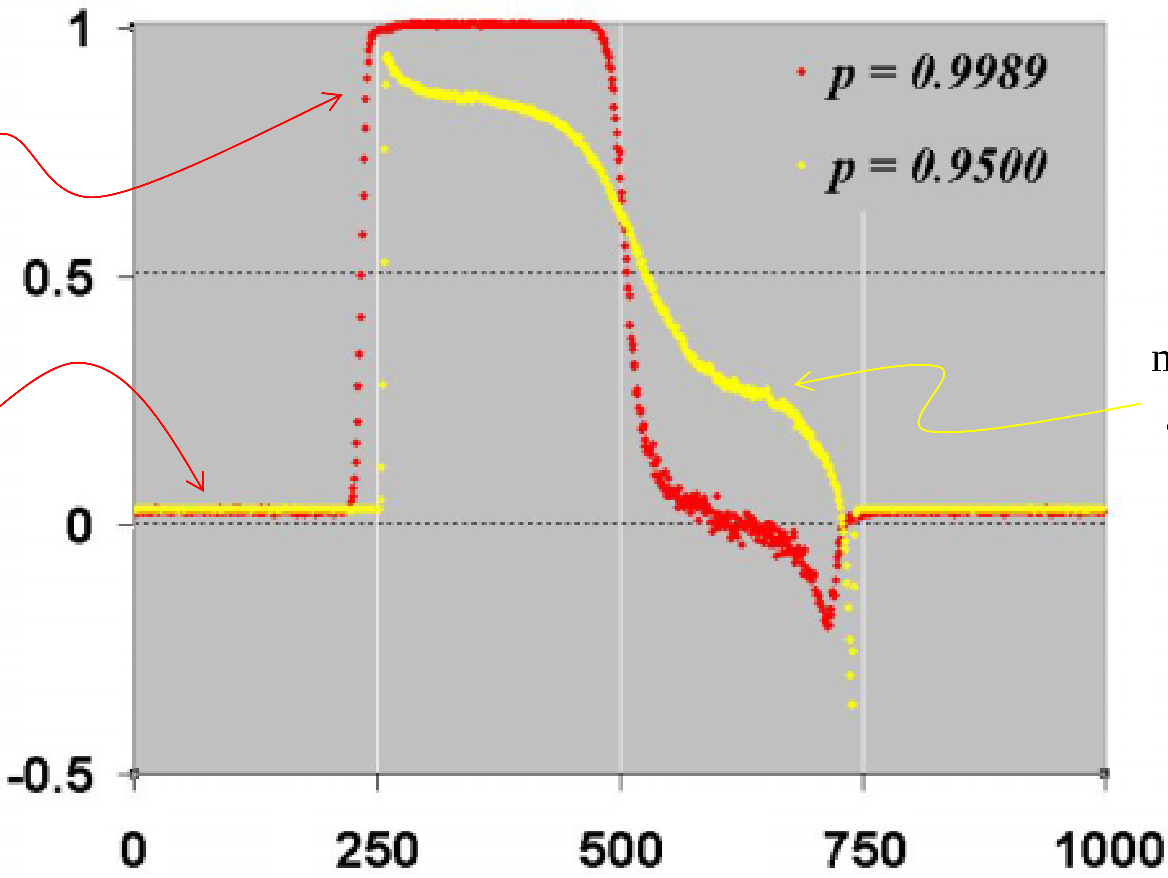
jams align;
Q=0; profiles
anti-symmetric

‘jam’ got longer

Profiles at ‘large’ *p*

offset ‘jams’;

travelers density lower!



profile shows more structure ;

‘jams’ are still aligned

brief remarks on 2×*L* systems

- Simple model seems to show effects we might see (should expect?) on two-lane roads
- Numerical integration of MFT display qualitatively same behavior
- Need a better *understanding* of ‘negative response’
- Need better theories for quantitative predictions

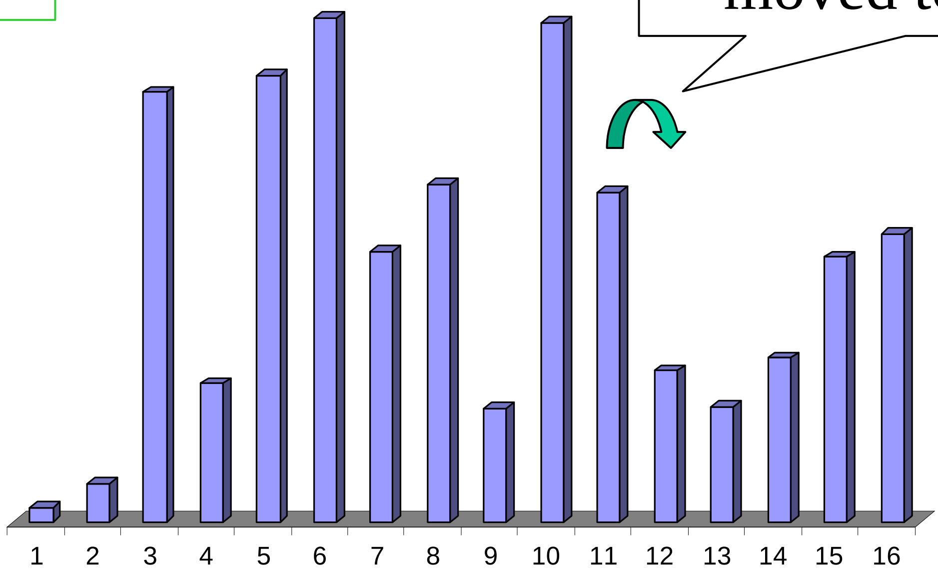
Solution for a class of Mass Transport Models

…thanks to Satya Majumdar and Martin Evans… who taught me everything I know about ARAP's and ZRP's, last September in Dresden.

Solution for a class of Mass Transport Models

mass m_i
on site i

mass μ "chipped off" from m
moved to next site



. . .

L sites on a ring

model specified by
"chipping kernel"

$\phi(\mu\,;m)$

= probability that μ
is moved, given m.

e.g., ARAP, ZRP

What's stationary state: P^* ?

- Master equation can be written for $P(\{m_i\},t)$
- Given $\phi(\mu;m)$, is $P^*(\{m_i\})$ a product measure?" i.e.,

$$P^*(m_1, m_2, \ldots m_L) \propto \prod_{i=1}^{L} f(m_i) \quad ?$$

M.R. Evans, Satya N. Majumdar, and R.K.P. Zia,

J. Phys. **A37**, L257 (2004) = cond-matt/0406524

What's stationary state: P^* ?

- Master equation can be written for $P(\{m_i\},t)$
- Given $\phi(\mu;m)$, is $P^*(\{m_i\})$ a product measure?"
- Criterion (necessary and sufficient):

m-μ : mass that "stayed"

$$\phi(\mu,\sigma) = \frac{v(\mu)w(\sigma)}{[v * w](m)}$$

convolution

v , w are *arbitrary* functions;
normalizability not needed!!

M.R. Evans, Satya N. Majumdar, and R.K.P. Zia,

J. Phys. **A37**, L257 (2004) = cond-matt/0406524

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$$\phi(\mu,\sigma) = \frac{v(\mu)w(\sigma)}{[v*w](m)}$$

Note: These two functions "generates" *all* ϕ's which admit product measure!!

M.R. Evans, Satya N. Majumdar, and R.K.P. Zia,

J. Phys. **A37**, L257 (2004) = cond-matt/0406524

Simple test and Explicit construction of $f(m)$

Test and Construction

- Given $\phi(\mu;m)$, compute

$$[\partial_\mu \partial_m + \partial_m^2] \ell n \phi(\mu;m)$$

Test and Construction

- Given $\phi(\mu;m)$, compute …
- P^* is factorizable if and only if this is a function of m alone.

Test and Construction

- Given $\phi(\mu;m)$, compute …
- P^* is factorizable if and only if this is a function of *m* alone.
- If yes, define

$$[\partial_\mu \partial_m + \partial_m^2]\ell n\phi(\mu;m) \equiv h(m)$$

Test and Construction

- Given $\phi(\mu;m)$, compute …
- P^* is factorizable if and only if this is a function of m alone.
- If yes, define $h(m)$ ...
- Then,

$$f(m) = e^{-\int^{m} dm' \int^{m'} dm'' h(m'')}$$

Note: Two arbitrary constants here are irrelevant for P^* !!

Test and Construction

- Method easily generalized to discrete masses (as in ZRP).
- All previous known cases are recovered with a few simple lines of math.
- This approach generates *all* chipping kernels which allow factorizable P^*'s.
- Full implications yet to be explored.

Conclusions

- Lots of exciting things *yet* to be discovered and understood:
  - in driven lattice gases (just tip of iceberg here)
  - in other non-equilibrium steady states (e.g., reaction diffusion)
  - in full dynamics
- Many possible applications
- A range of methods (from simple MC to sophisticated SFT)

…come, join the party!