# Traffic jams and ordering far from thermal equilibrium

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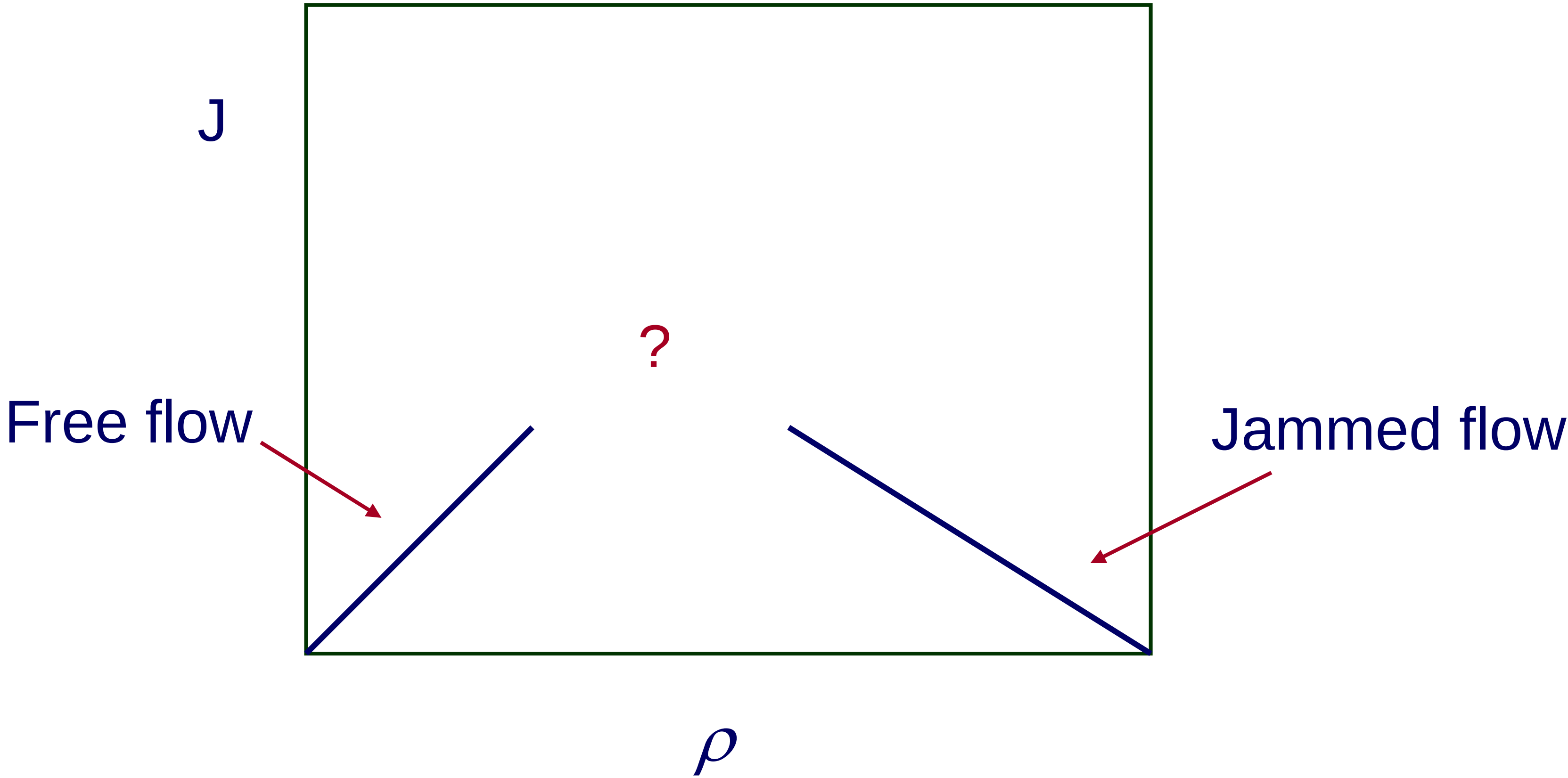
# Fundamental Diagram




Is there a jamming phase transition?
or is it a broad crossover?

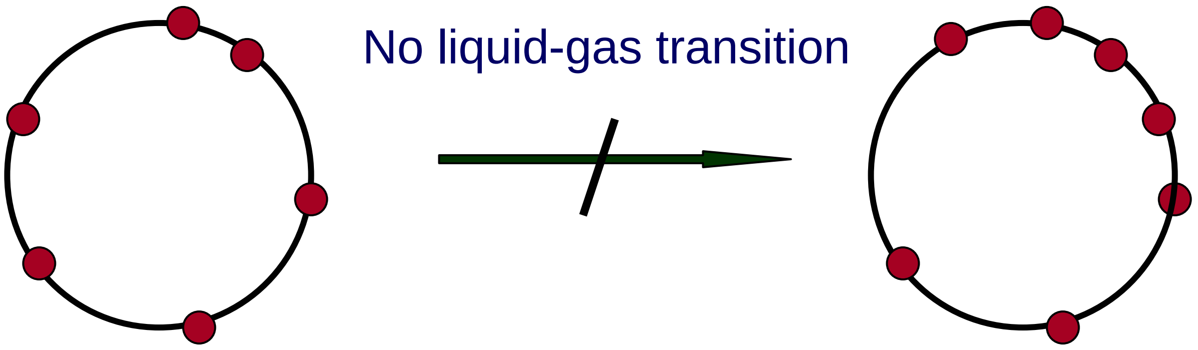
# Phase separation in 1d

In thermal equilibrium:

short range interactions
T>0

→

Density is macroscopically homogeneous



Can one have phase separation in 1d driven systems (?)

local, noisy dynamics
homogeneous, ring geometry
no detailed balance

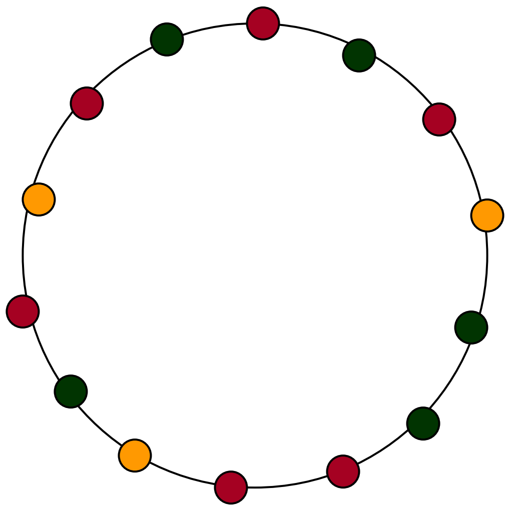
A criterion for phase separation in such systems (?)

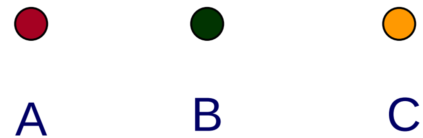
- Phase transitions do exist in one dimensional driven systems.

- Jamming is a  crossover phenomenon. Usually it does not take place via a phase transition.

# ABC Model

M. R. Evans, Y. Kafri, H. M. Koduvely and D. Mukamel
PRL 80, 425 (1998)






Random sequential update

$$AB \underset{1}{\overset{q}{\rightleftharpoons}} BA$$

$$BC \underset{1}{\overset{q}{\rightleftharpoons}} CB$$

$$CA \underset{1}{\overset{q}{\rightleftharpoons}} AC$$

# Simple argument:

ACCCC → CCCCA

CBBBB → BBBBC

BAAAA → AAAAB

$$AB \underset{1}{\overset{q}{\rightleftharpoons}} BA$$

$$BC \underset{1}{\overset{q}{\rightleftharpoons}} CB$$

$$CA \underset{1}{\overset{q}{\rightleftharpoons}} AC$$

…AACBBBCCAAABBBBCCC…

↓ fast rearrangement

…AABCCCAABCCCAAABBC…

↓ slow coarsening

…AAAAABBBBBCCCCCCAA…

- logarithmically slow coarsening

…AAAAABBBBBCCCCCCAA…

$$t \propto q^{-l} \qquad l \propto \ln t$$

- needs n>2 species to have phase separation

- strong phase separation: no fluctuation in the bulk; only at the boundaries.

…AAAAAAAAAABBBBBBBBBBBBCCCCCCCCCCCC…

Special case $N_A = N_B = N_C$

Steady state distribution obeys detailed balance with respect to

$$P(\{x\}) = q^{H(\{x\})}$$

$$H(\{x\}) = \sum_{i=1}^{N} \sum_{k=1}^{N-1} \left(1 - \frac{k}{N}\right)\left(C_i B_{i+k} + A_i C_{i+k} + B_i A_{i+k}\right)$$

summation over $(i+k)$ modulo $N$

- long range

Local dynamics

AAAAAABBBBBBCCCCC E=0

……AB….. → ……BA….. E → E+1

……BC….. → ……CB….. E → E+1

……CA….. → ……AC….. E → E+1

**A**AAAAABBBBBCCCCC → AAAABBBBBCCCCC**A**

$E \longrightarrow E+N_B-N_C$

$N_B = N_C$

Partition sum: $$Z(q) = N\left[\frac{1}{(1-q)(1-q^2)......}\right]^3$$

Correlation function: $$\langle A_1 A_r \rangle \approx 1/3$$

with $$\langle A_1 \rangle \langle A_r \rangle = 1/9$$

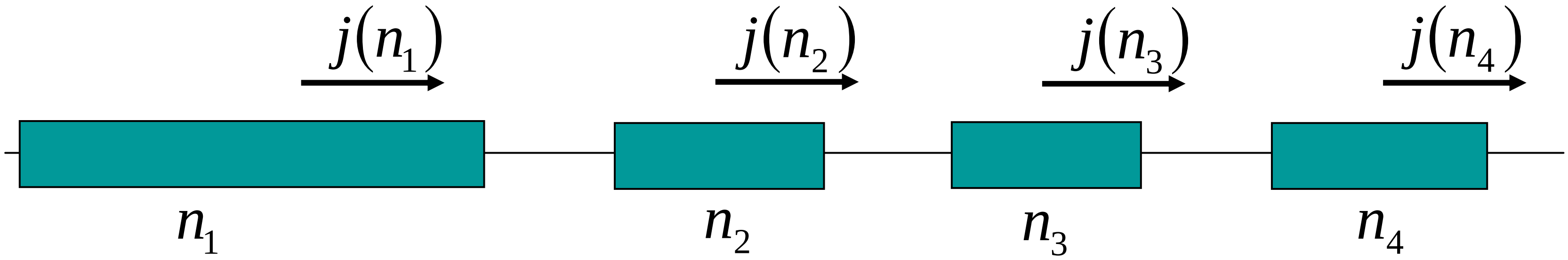
In the ABC model one has:
Strong phase separation, no fluctuation in the bulk.
Phase separation takes place at any finite density.

Find a more general framework to characterize dynamical processes which could lead to liquid-gas like transition at finite density with possibly fluctuating phases.

Y. Kafri, E. Levine, D. Mukamel. G. M. Schutz and J. Torok, Phys. Rev. Lett. 89, 035702 (2002).
Y. Kafri, E. Levine, D. Mukamel and J. Torok, J. Phys. A 35, L459 (2002).
Y. Kafri, E. Levine, D. Mukamel, G. M. Schutz and R. D. Willmann, PRE 68, 035101 (2003)

Question: Given a 1d driven process, does it exhibit phase separation?

## Criterion for phase separation




domains exchange particles via their current $j(n)$

if $j(n)$ decreases with n: coarsening may be expected to take place.

quantitative criterion?

Phase separation takes place in one of two cases:

- Case A: if $j(n) \to 0$ as $n \to \infty$

  e.g. $\exp(-n)$

- Case B: if $j(n) \approx j_\infty \left(1 + b/n^s\right)$

  with s<1 at any b>0
  or for s=1 with b>2

  Namely: if the decay is slower than 2/n

Case A: strong phase separation at any density. No bulk fluctuations

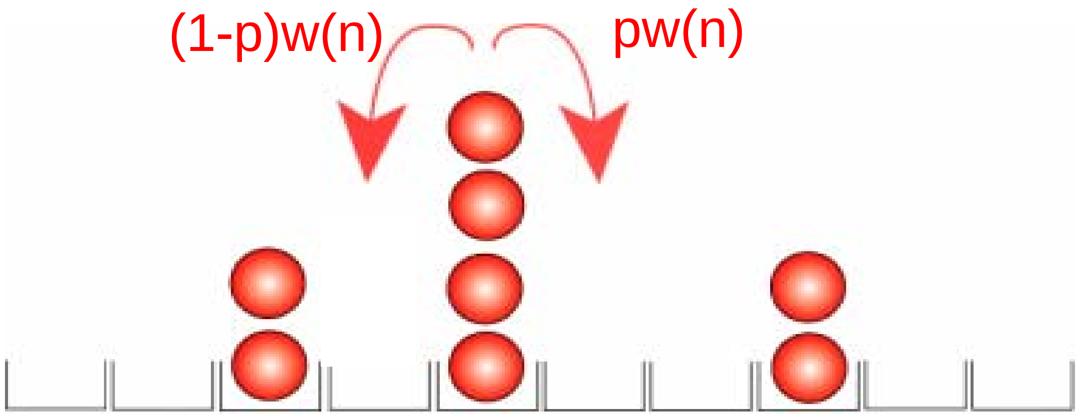
Case B: condensation transition at high density. Bulk fluctuations

- ABC model: type A $j(n) \approx e^{-n}$

Are there models which exhibit type B transition?

# Zero Range Processes

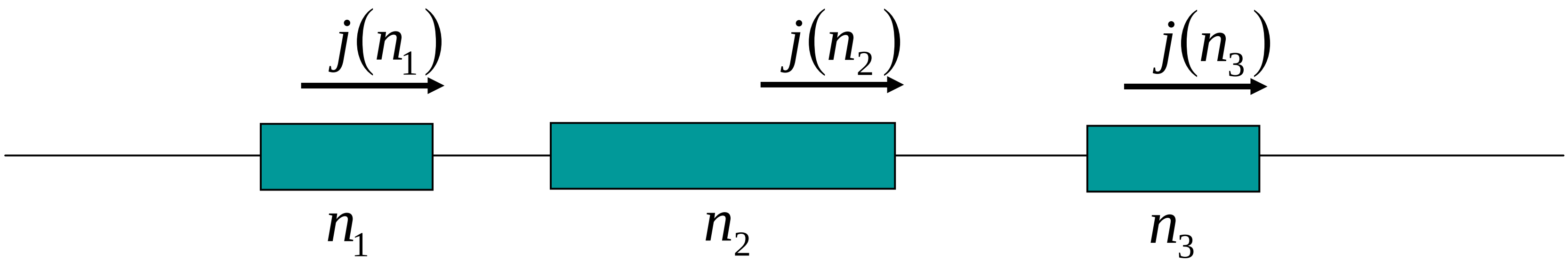
Particles in boxes with the following dynamics:

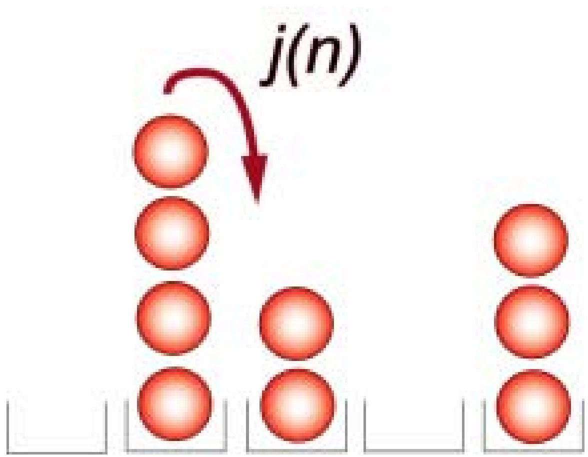


## Steady state distribution of ZRP processes:

- product measure
- $$f(n) = \prod_{k=1}^{n} \frac{1}{w(k)}$$

# Correspondence between the driven model and the ZRP




$w(n)=j(n)$

number of balls in a box → size of domain
empty box → low density region
rate of particles leaving box → current leaving domain

Phase separation in the driven system corresponds to a macroscopic occupation of one of the boxes in the ZRP .

Example:

$$j(n) = j(\infty)(1 + b/n) \qquad w(k) = j(k)$$

$$f(n) = \prod_{k=1}^{n} \frac{1}{w(k)}$$

$$\ln(f(n)) = -\sum \ln w(k) = -n \ln J(\infty) - \sum_{k=1}^{n} \ln(1 + \frac{b}{k})$$

$$\approx -n \ln J(\infty) - b \ln n$$

$$z^n f(n) = \frac{(z / J(\infty))^n}{n^b} = \frac{e^{-n/\xi}}{n^b}$$

The correlation length is determined by

$$\int nf(n)\,dn = \phi \qquad f(n) = \frac{e^{-n/\xi}}{n^b}$$

$$\xi \to \infty: \qquad \int \frac{1}{n^{b-1}} dn \to \infty \qquad \text{for } b \le 2$$

- For b<2 there is always a solution with a finite $\xi$ for any density. Thus no condensation.
- For b>2 there is a maximal possible density, and hence a transition of the Bose Einstein Condensation type.

# Example of two-species driven models

$$+0 \xrightarrow{\alpha} 0+$$

$$0- \xrightarrow{\alpha} -0$$

$$-+ \underset{1}{\overset{p}{\rightleftarrows}} +-$$

$$AB \underset{1}{\overset{q}{\rightleftarrows}} BA$$

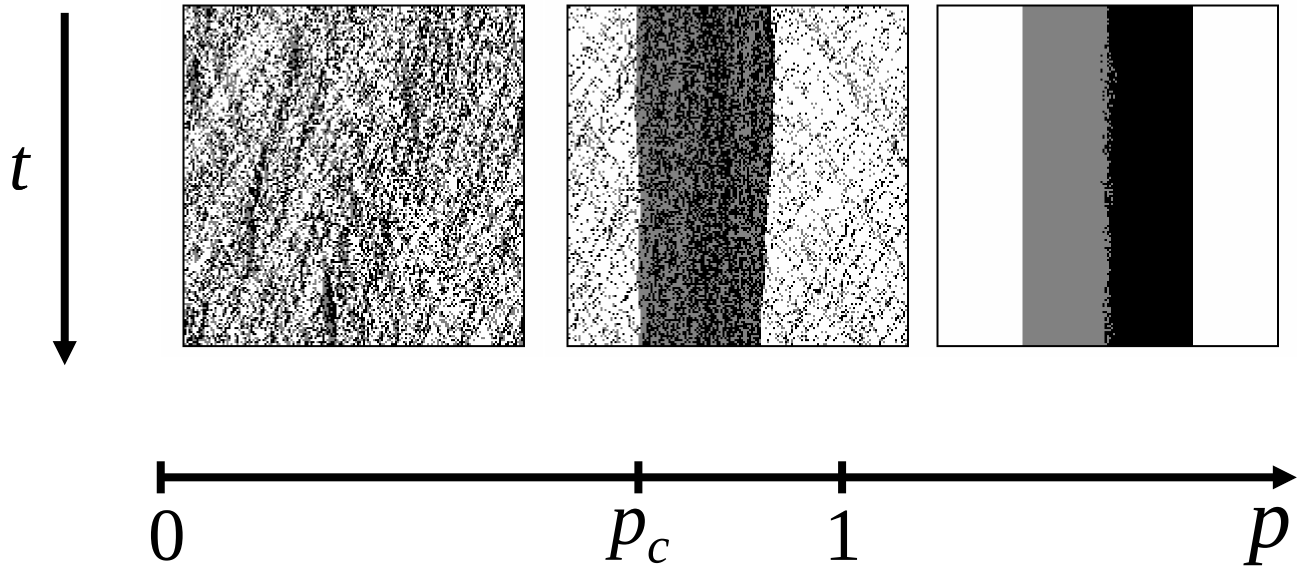
$$BC \underset{1}{\overset{q}{\rightleftarrows}} CB$$

$$CA \underset{1}{\overset{q}{\rightleftarrows}} AC$$

For p >1 the model has features similar to those of the ABC model.

P. F. Arndt, T. Heinzel, V. Rittenberg, AHR model
J. Phys. A. 31, L45 (1998)

Numerical simulations of the model suggested the following Phase diagram:



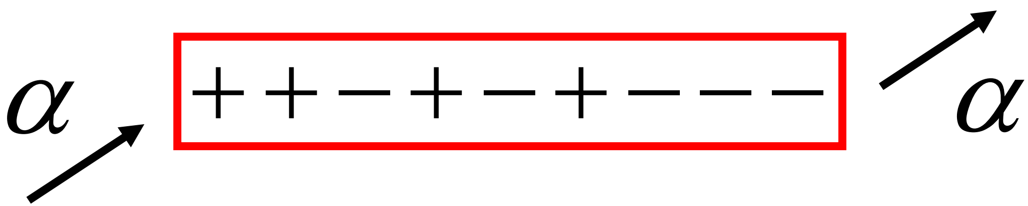

Phase Separation Fluctuating phases

Strong phase separation

Analytical solution has shown that no fluctuating phase separated state exists
But rather an anomalously large correlation length.

N. Rajewsky, T. Sassamoto, E. R. Speer, Physica A279, 123 (2000).

# AHR model




$$+0 \xrightarrow{\alpha} 0+$$
$$0- \xrightarrow{\alpha} -0$$
$$-+ \underset{1}{\overset{p}{\rightleftharpoons}} +-$$

The current is known to be

T Sasamoto (1999), RA Blythe *et al*,(2000)

$$j(n) = \frac{p-1}{4p}\left(1 + \frac{b}{n} + O\left(\frac{1}{n^2}\right)\right)$$

with $b = \frac{3}{2} < 2$

NO phase separatio

# A driven model with b>2

(Y. Kafri, E. Levine, D. Mukamel, G.M. Schutz, R. Willmann PRE 68 035101 (2003))

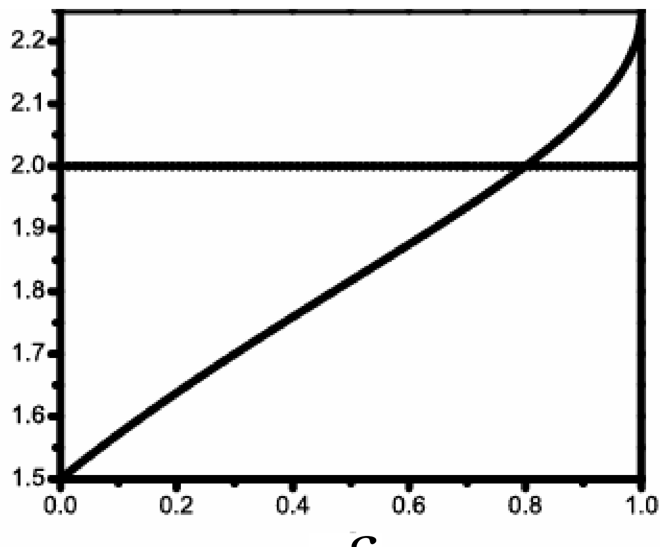
$$+0 \xrightarrow{\alpha} 0+$$

$$0- \xrightarrow{\alpha} -0$$

$$+- \xrightarrow[1+\Delta h]{} -+$$

with $$h = -\frac{\varepsilon}{4}\sum_i s_i s_{i+1} \qquad 0 \le \varepsilon \le 1$$

$b$

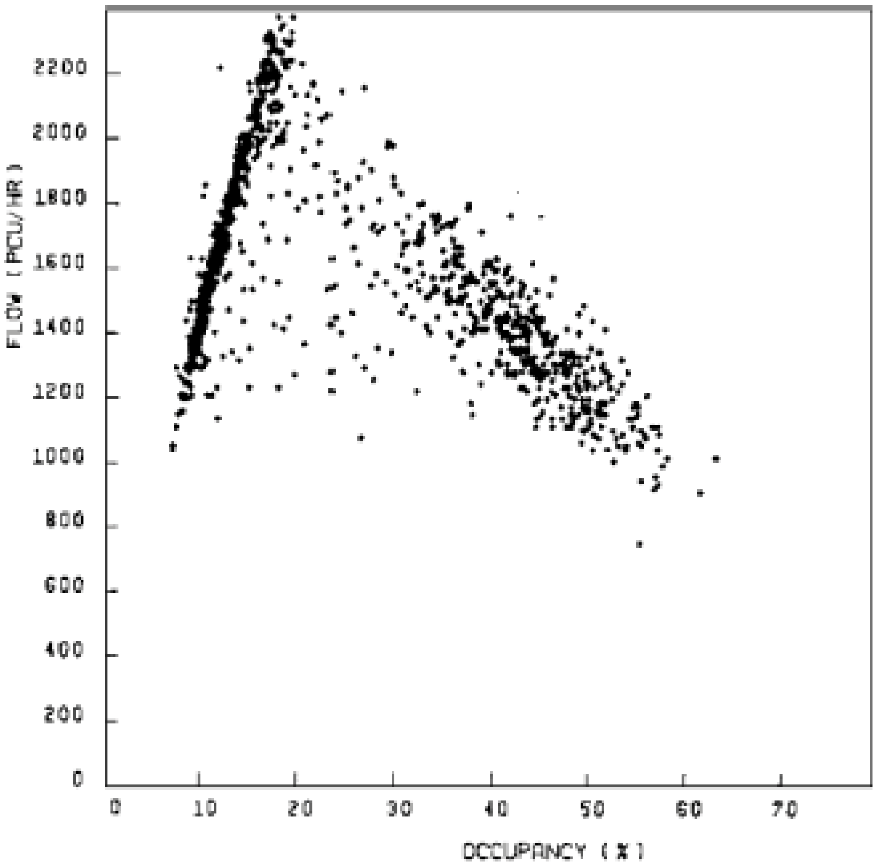


The b curve may be evaluated analytically

$$b(\varepsilon) = \frac{3}{2}\frac{(2+\varepsilon)v + 2\varepsilon}{2(v+\varepsilon)}$$

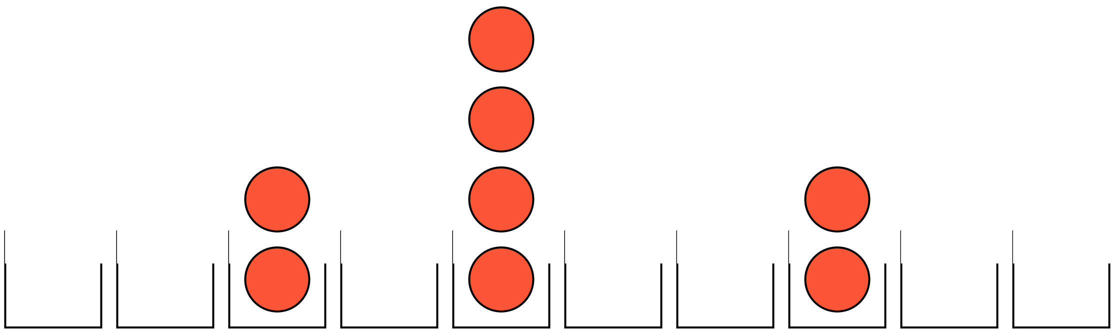
$$v = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}} + 1$$

# Traffic flow: fundamental diagram




Hall *et al*, Trans. Res. A: Gen. **20**, 197 (1986)

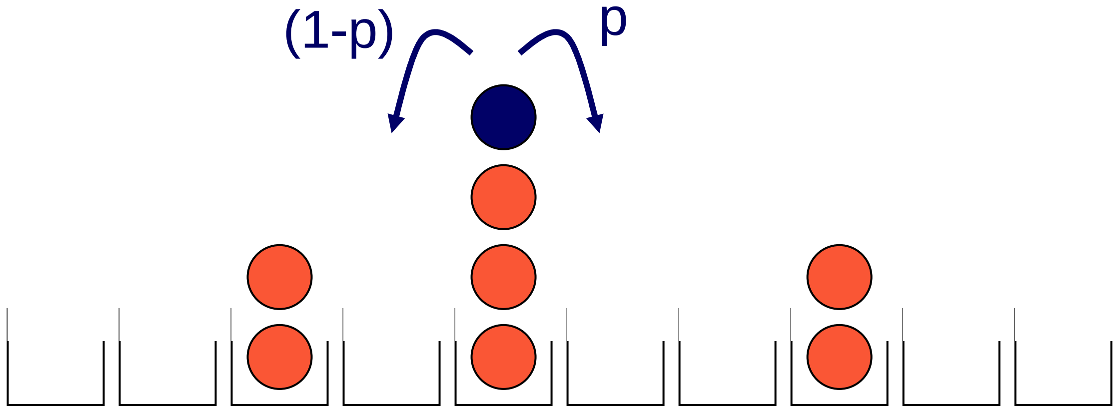
# The Chipping Model



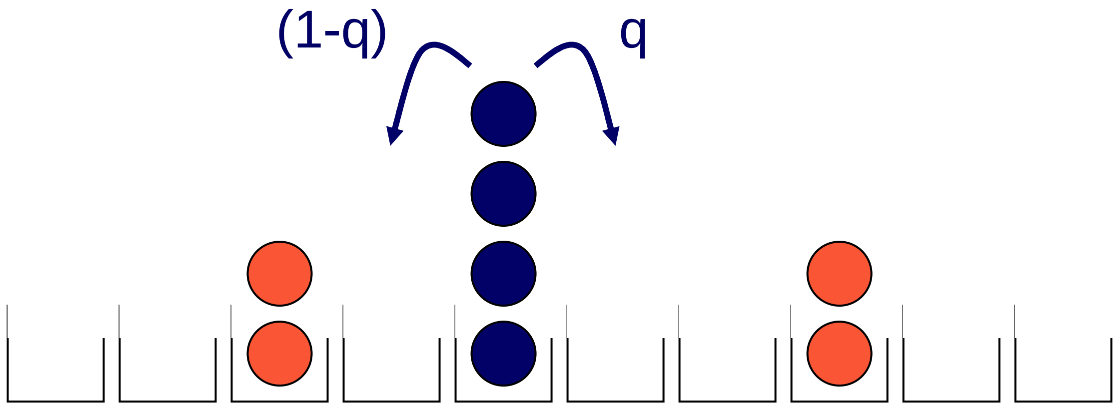
SN Majumdar, S Kirshnamurthy, and M Barma, *Phys. Rev. Lett.* **81**, 3691 (1998); R Rjesh and SN Majumdar , *Phys. Rev. E* **63**, 036114 (2001); R Rajesh and S Kirshnamurthy, *Phys. Rev. E* **66**, 046132 (2002).

# chipping model: dynamical processes

chipping:
(ZRP w(n)=1)




diffusion:



# The chipping model

Probability distribution of site occupation $$P(k) \propto \frac{e^{-k/\xi}}{k^{\tau}}$$

Asymmetric chipping: $\tau = 2$ (no condensation)

Symmetric chipping: $\tau = 5/2$ (condensation)

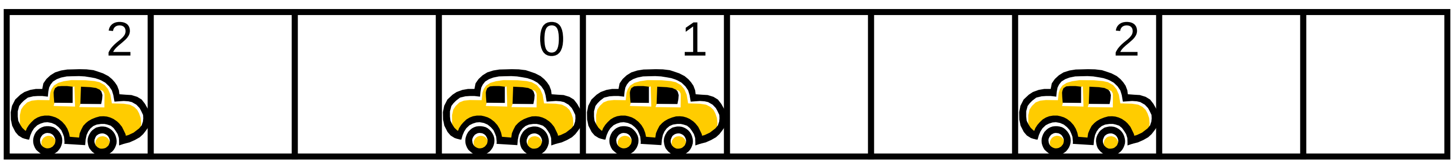
will argue:
coarse grained traffic dynamics is describes
by the asymmetric chipping model.
Hence NO jamming transition in traffic models.

# Cellular automata traffic models

Dynamical variables: position x(n) ; velocity v(n) v=0,1,…,

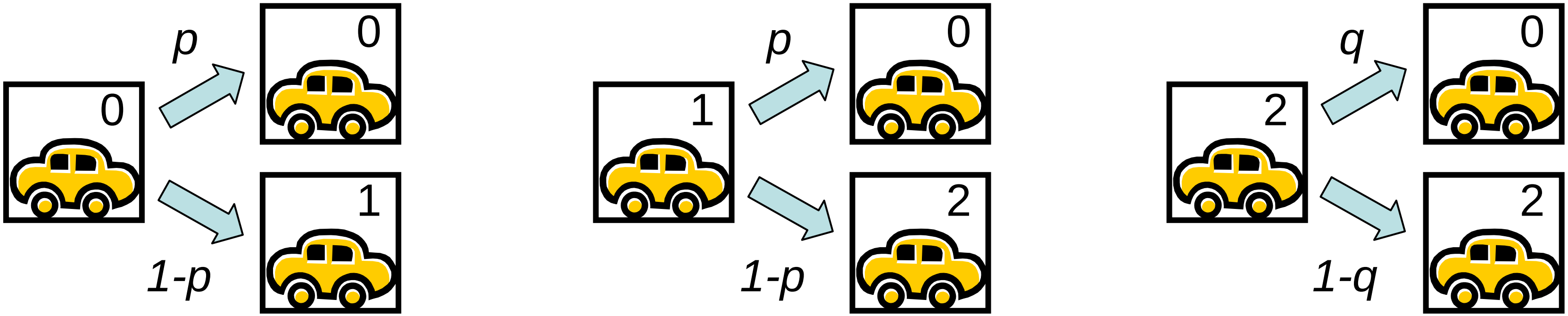
**update scheme**

- in parallel
- no accident
- randomization

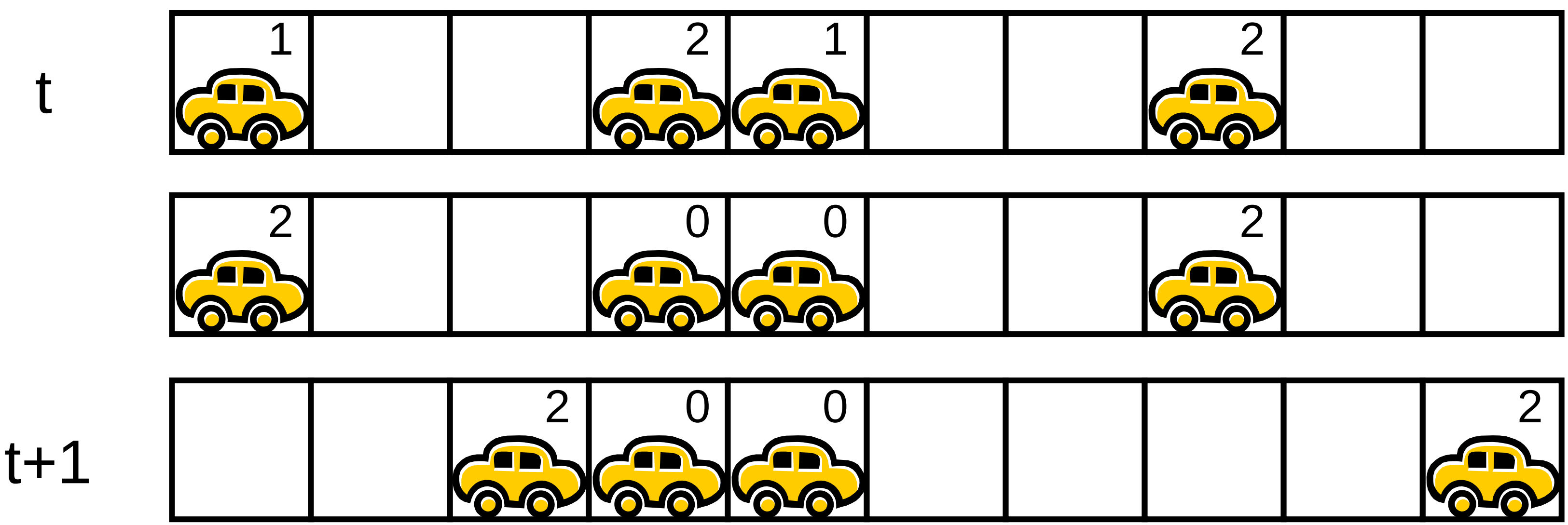



Nagel & Schreckenberg J. Physique I, 2, 2221 (1992)
A recent review: Chowdhury *et al*, Phys. Rep. **329**,199 (2000)

# Example of CA traffic update scheme
## (velocity-dependent-braking model VDB)

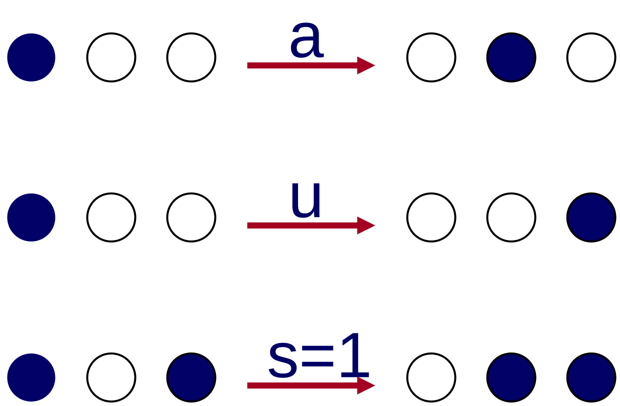



## Avoid Accident !



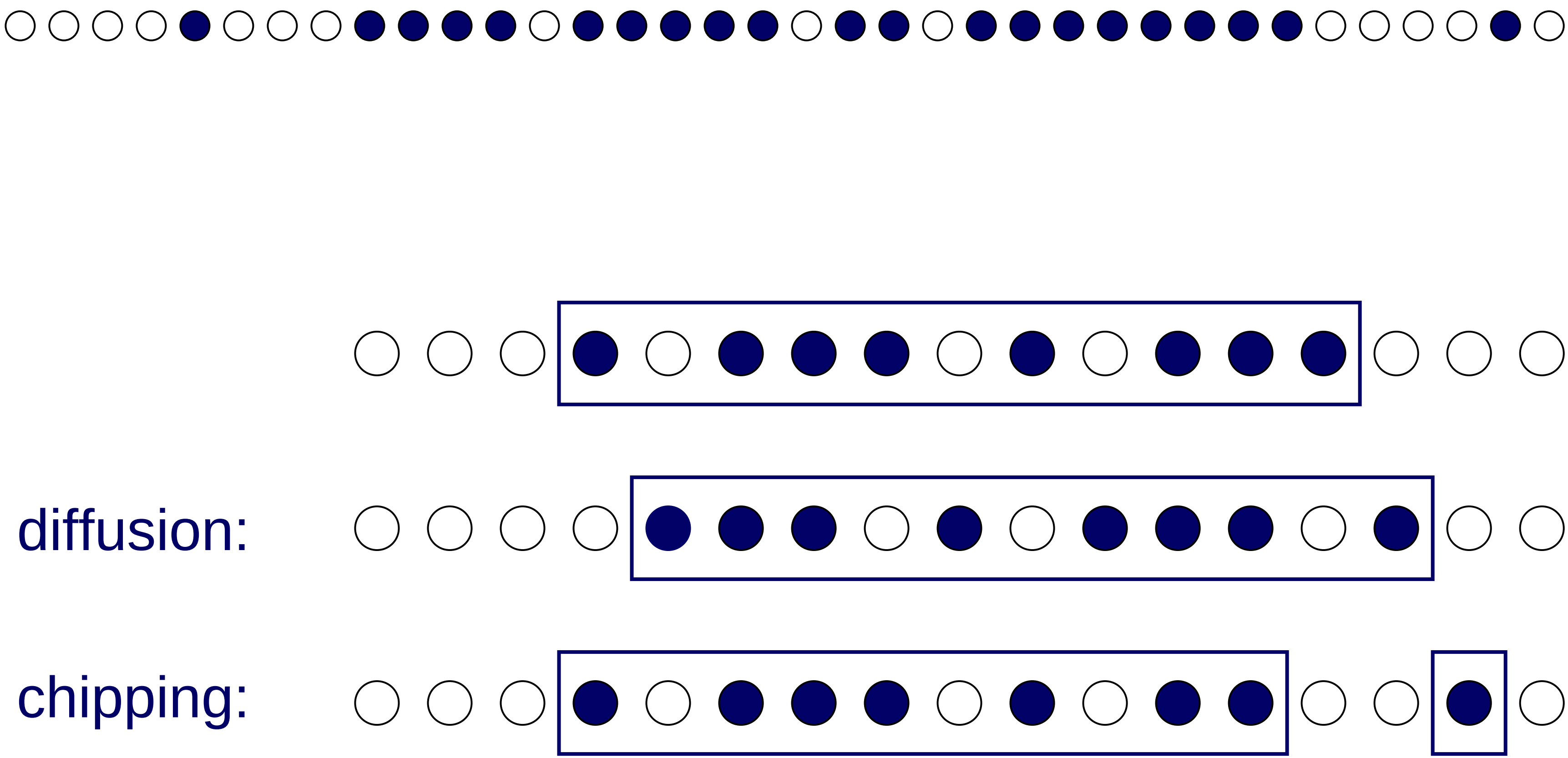
# A simple traffic cellular automaton model

● car ○ vacancy

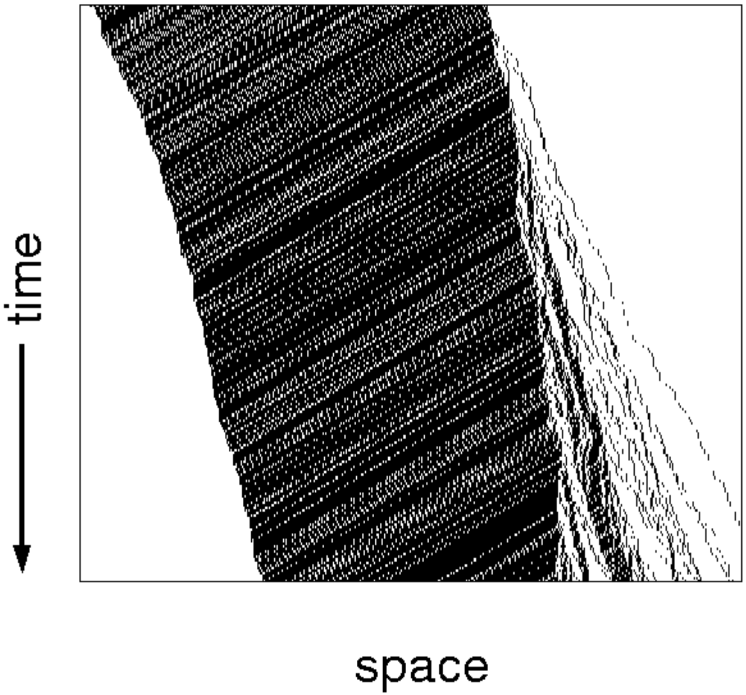



E Levine, G Ziv, L Gray and D Mukamel, **cond-mat/0312459**
**cond-mat/0402071**

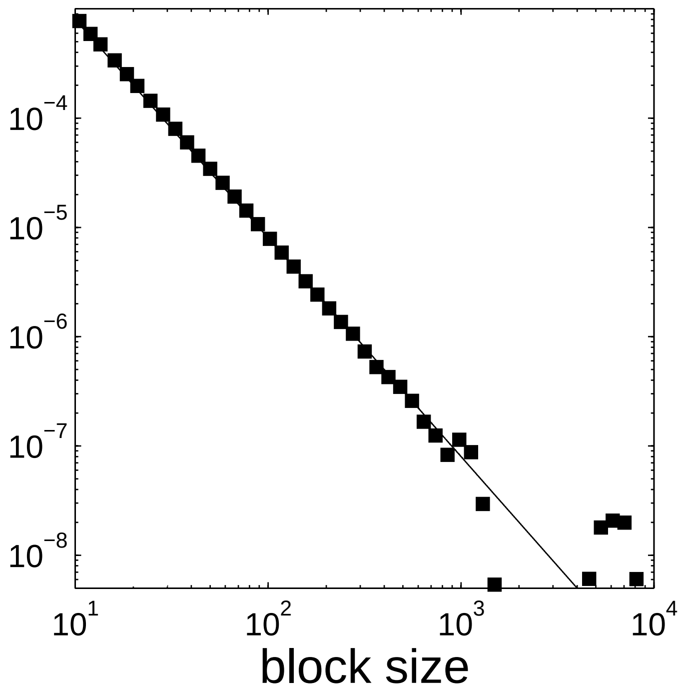
# Jams (domains): cars with isolated vacancies



# Time evolution of a jam (domain)

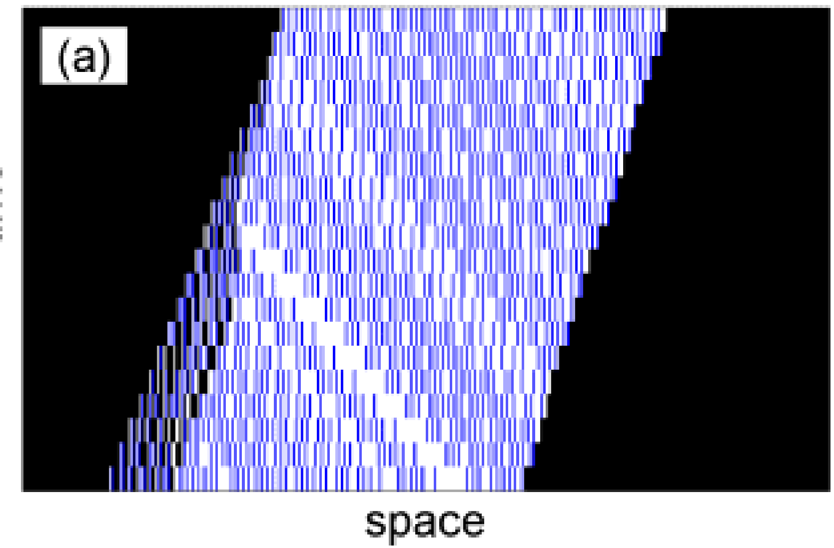


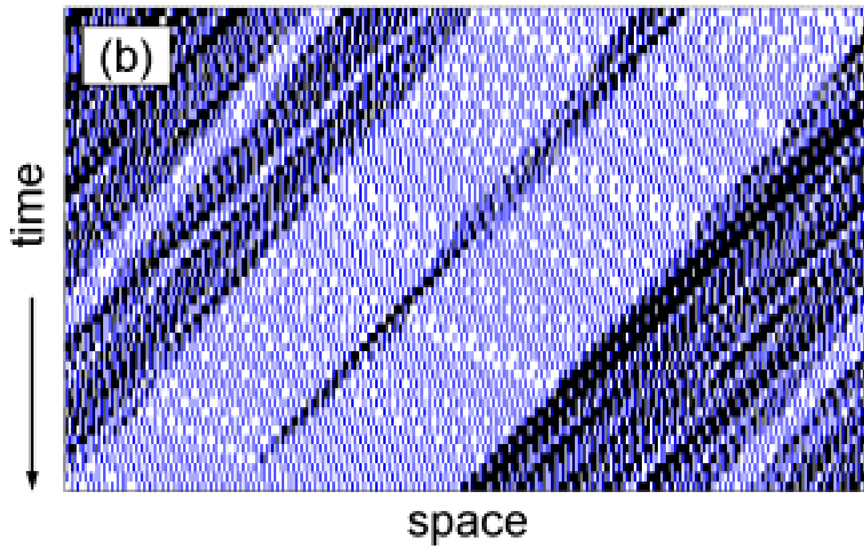

dynamical processes: asymmetric chipping + diffusion domain size distribution $P(k) \propto 1/k^{\tau}$ $\tau = 2$




no jamming transition

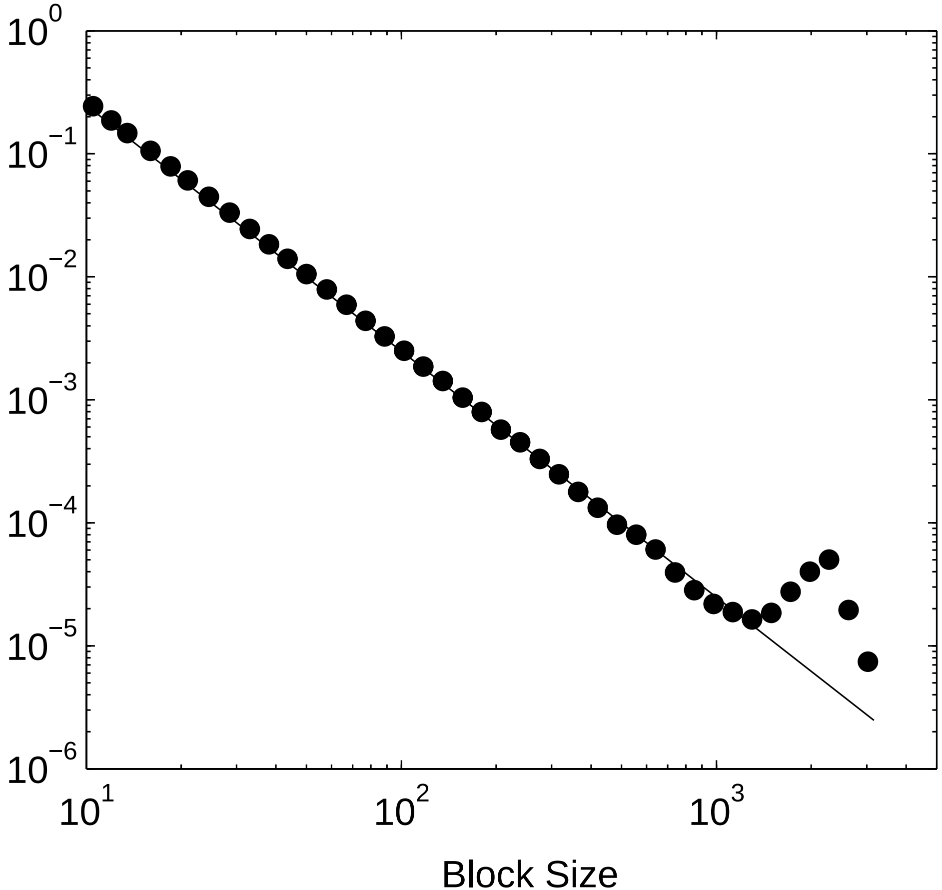
# Domain evolution of the VDB model



domain size distribution
(VDB model)

$$P(k) \propto 1/k^{\tau} \qquad \tau = 2$$



# Summary

- A framework is introduced within which one can analyze the coarse-grained dynamics of driven models.
- A correspondence between driven 1d systems and ZRP is suggested.
- Based on this correspondence a criterion for phase separation in 1d driven systems is suggested.
- It is suggested that traffic models correspond to an asymmetric chipping model which exhibits no condensation transition as $\tau = 2$ .
- This correspondence holds for a large class of traffic models.
- Time evolution and the approach to the steady state?