

Application of the statistical physics of evolving surfaces to the growth and form of microbial mats and laminated structures

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Non-equilibrium Statistical Physics of Complex Systems
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Examples of laminated structures

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- paper burning fronts

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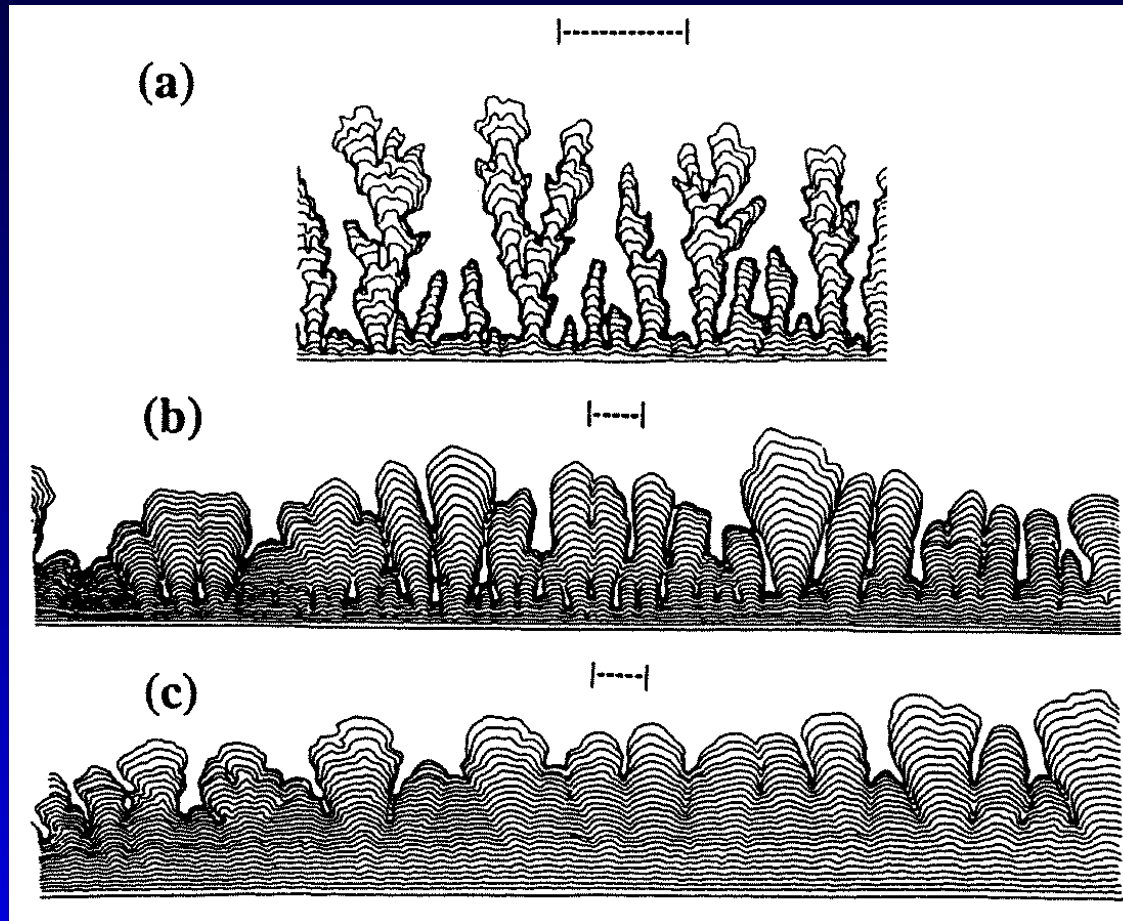
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- paper burning fronts
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- yeast colonies
- rocks

Chemical electrodeposition

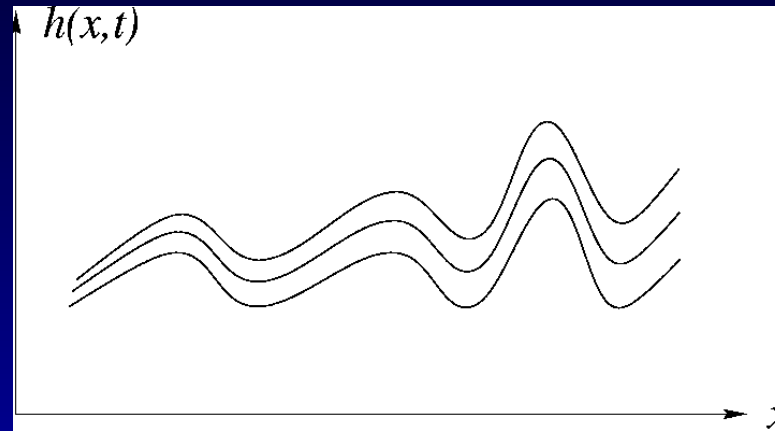


Laplacian regime

KPZ regime

From Kahanda et al. PRL **68** (1992) 3741

Kardar-Parisi-Zhang eqn [PRL 56 (1986) 889]



$$\frac{\partial h(x, t)}{\partial t} = v \frac{\partial^2 h(x, t)}{\partial x^2} + \frac{\lambda}{2} \left(\frac{\partial h(x, t)}{\partial x} \right)^2 + \eta(x, t)$$

$h(x, t)$ — the height of an interface at substrate position x and time t

v — growth parameter related to surface tension

λ — growth parameter related to lateral growth

$\eta(x, t)$ — white noise term

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- “living rocks”

Modern stromatolites

Bahamas



Survive in strong currents

Modern stromatolites

Shark Bay, Western Australia



Survive in high salinity



Ancient stromatolites

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- cf *Homo sapiens* evolved only in the last 2 million years

Ancient stromatolites

Vast array of shapes

- domes, organ-pipes
- branches
- cones

Controversy

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Oldest evidence of life on Earth??

Outback Australia



Branching



Jay Creek, Northern Territory

Organ pipes



N/W Queensland







Vologdin's dream

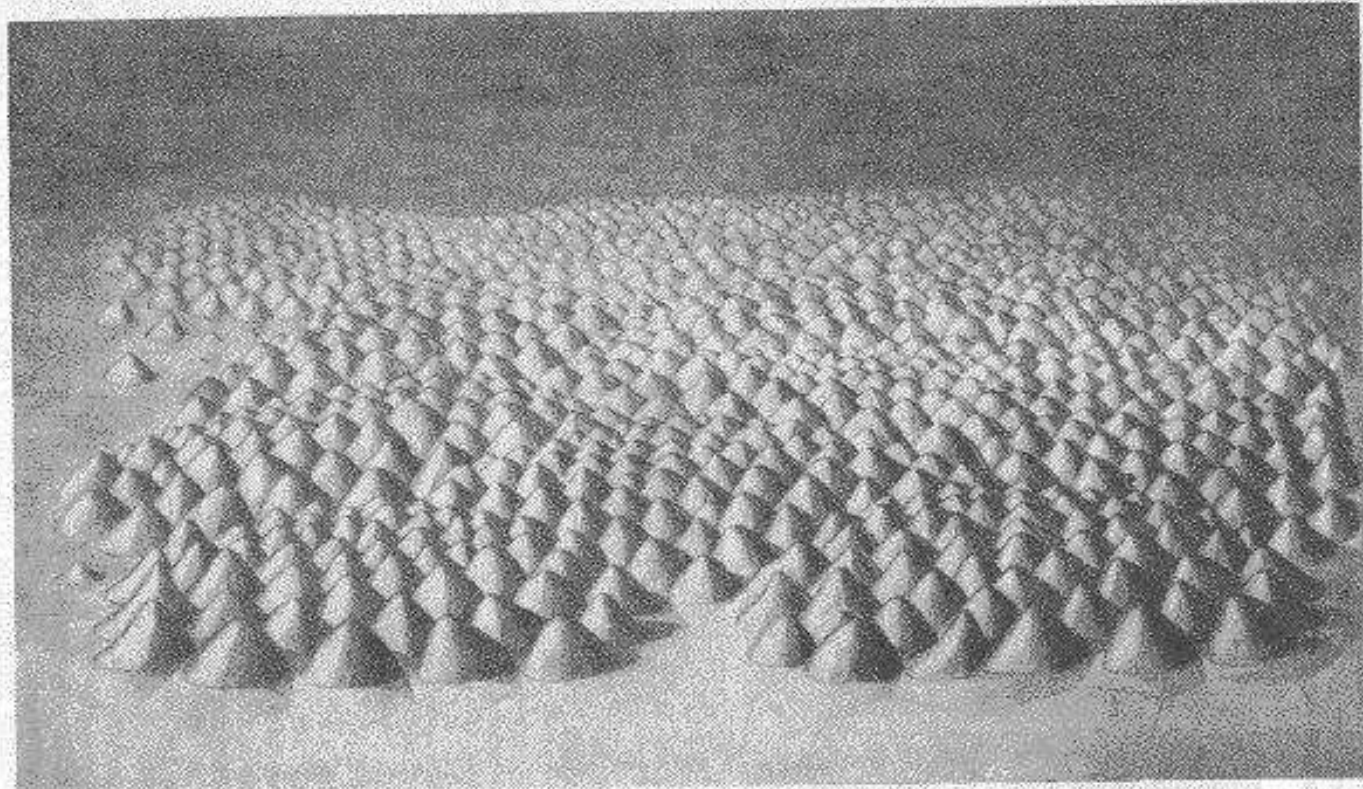


Рис. 22. Вид строматолитовой банки, образованной жизнедеятельностью водоросли *Stromatolites vulgaris* Vologdin, кр. пов.; реконструкция ($\times 10$); ажурная света, нижний синий.

Conophyton

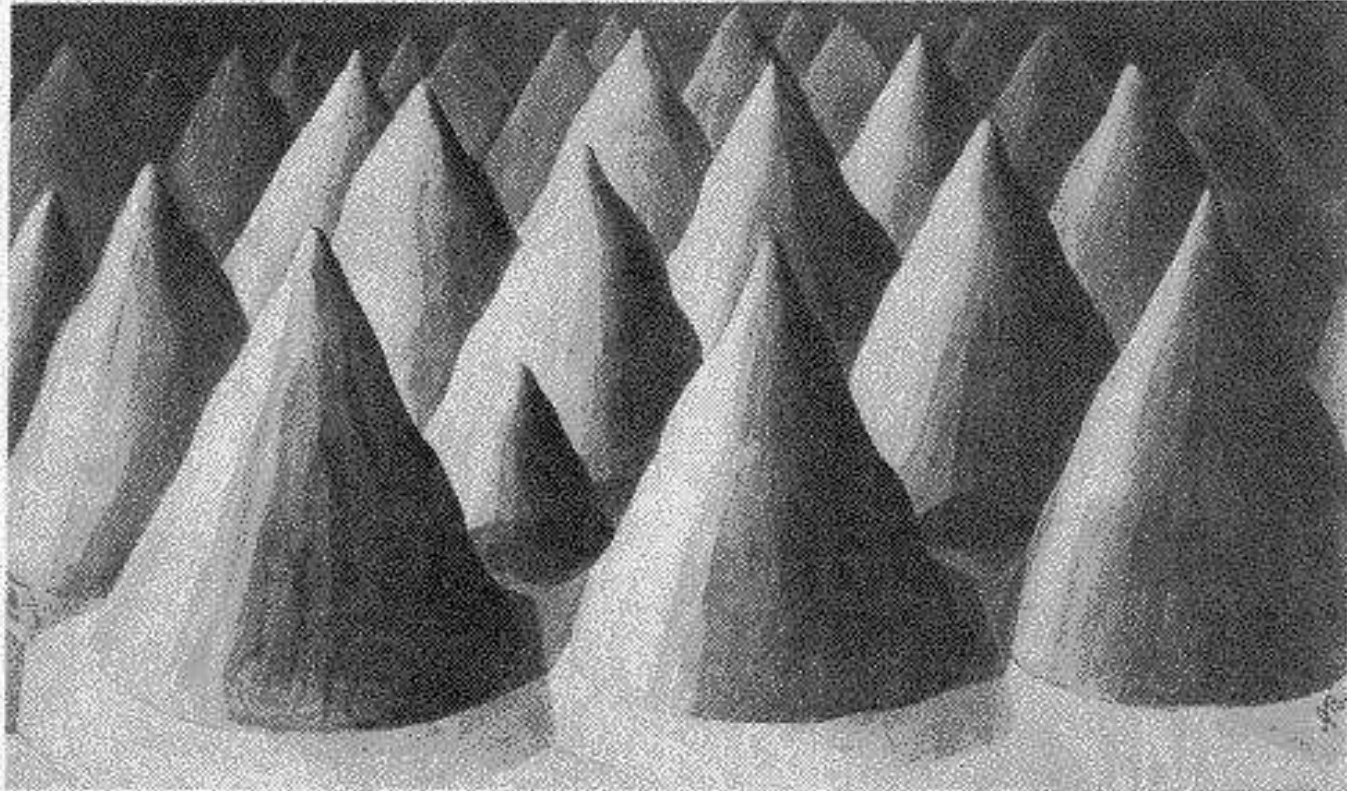


Рис. 31. Форма строматолитов, образованных бактериями *Gratiifera djuricensis* Vologdin, sp. nov.; обр. 1898—50/42-6; реконструкция (X¹⁰); джурская слита, нижний сеней.



Longitudinal section of Conophyton from the 1,700 million year old Dungaminnie Formation, Northern Territory, Australia (grid ref. 53K NB 0578973 8154545).

Coin 28mm for scale





Transverse section of Conophyton in the >1,670 million year old Amelia Dolomite, Macarthur Group, near the Kilgour River Junction with Archie Creek, Northern Territory, Australia (grid reference 53K NB 0596496 8109117).

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Can be solved for various initial profiles using the method of characteristics

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Radially symmetric solutions reduce to almost exactly the (1+1)D case.

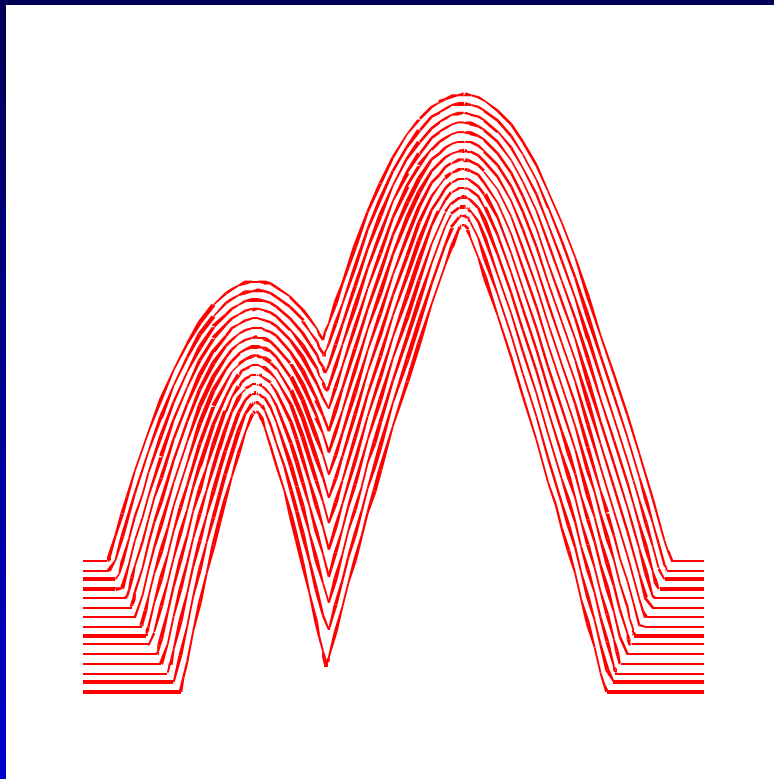
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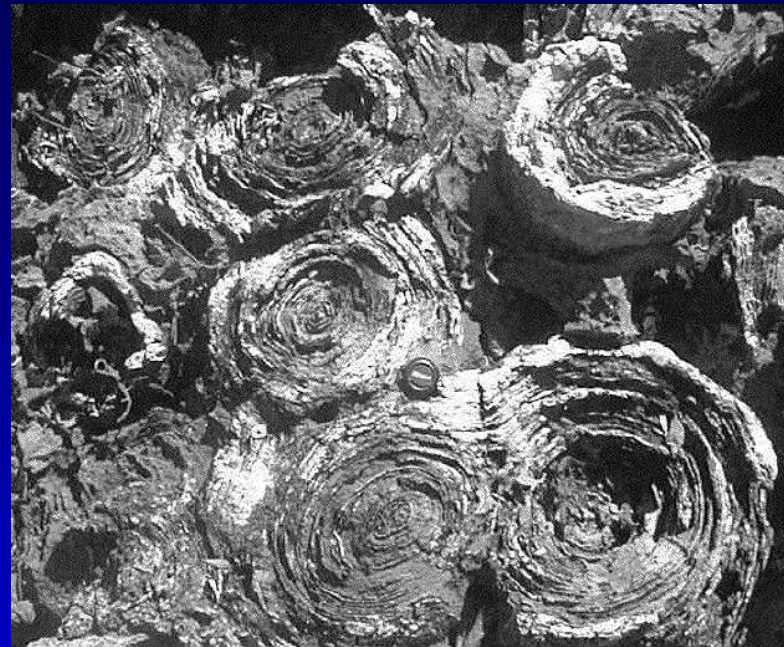
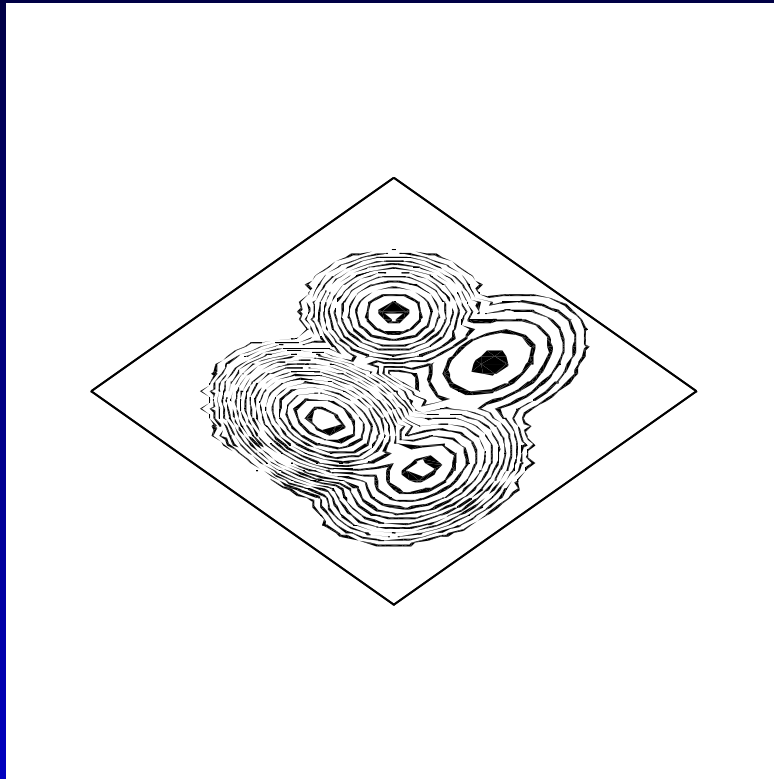
Radially symmetric solutions reduce to almost exactly the (1+1)D case.

Can follow the evolution of conical or paraboloid initial profiles

Longitudinal axial section of two evolving contiguous cones



Transverse cross-section of an evolving paraboloid field



Long term solutions

An isolated protrusion centred at the origin and evolving under the KPZ equation approaches the smooth radially symmetric paraboloid solution

$$h(x, y, t) = h(0, 0, 0) - \frac{r^2}{2\lambda t} + (\nu + \lambda)t$$

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$$h(x, y, t) = h(0, 0, 0) - \frac{r^2}{2\lambda t} + (\nu + \lambda)t$$

For an initial profile consisting of a field of protrusions, the long-term solution will therefore consist of smooth radially symmetric paraboloids separated by shocks.

Surface diffusion

The more general deterministic KPZ equation

$$\frac{\partial h}{\partial t} = \nu + \lambda + \frac{\lambda}{2} \left[\left(\frac{\partial h}{\partial x} \right)^2 + \left(\frac{\partial h}{\partial y} \right)^2 \right] + v \frac{\partial^2 h(x, t)}{\partial x^2}$$

has a radially symmetric paraboloid solution given by

$$h(r, t) = -\frac{r^2}{2\lambda t} - \frac{2v}{\lambda} \log(2\lambda t) + (\lambda + \nu)t + A$$

where v is the surface diffusion.

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If $\lambda \ll \nu$, then even after a long period of time, as measured by the amount of upward growth, the paraboloid will still closely resemble its initial form.

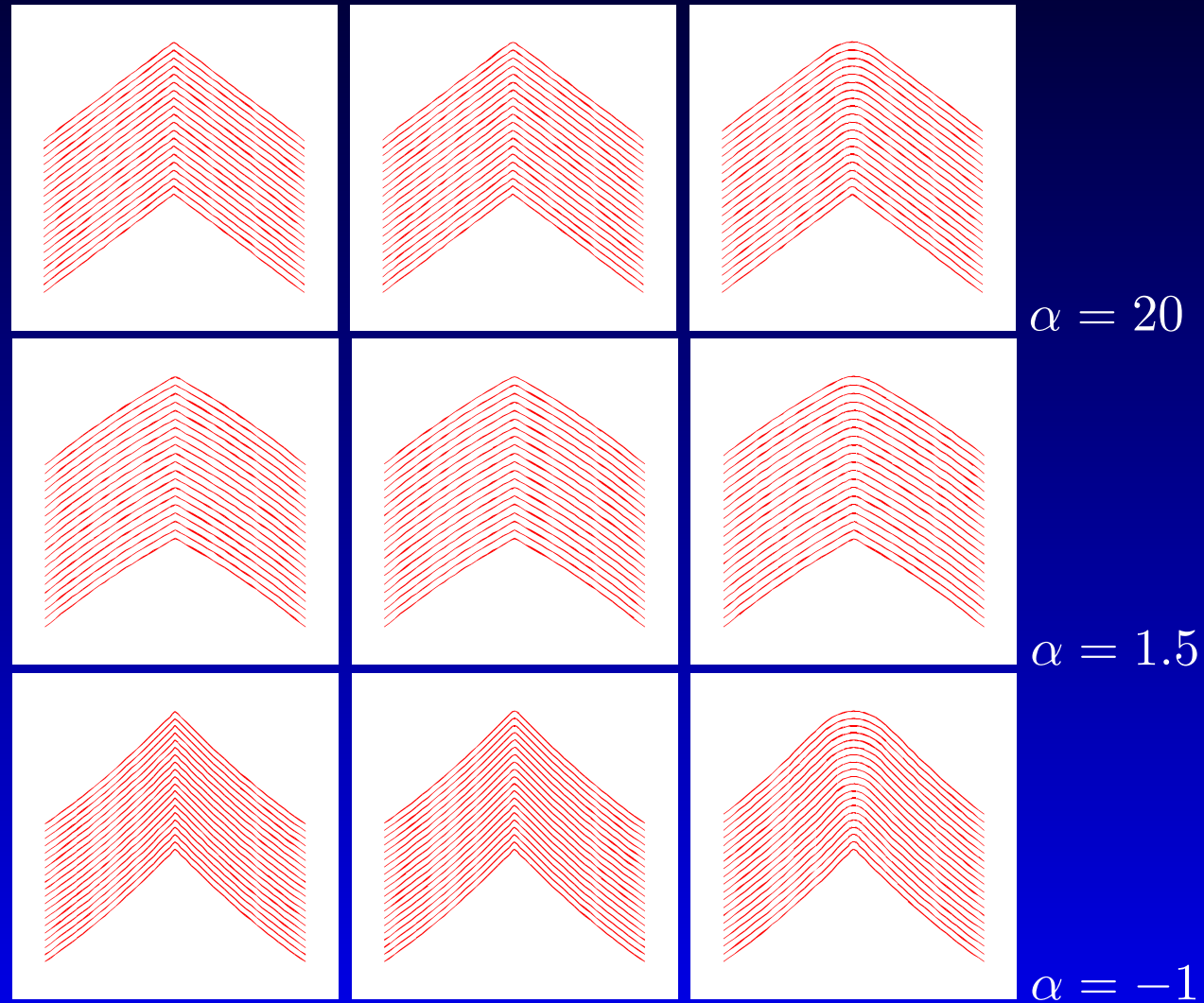
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If $\lambda \geq \nu$, the transition from the initial profile to a smooth radially symmetric paraboloid takes place over a much shorter period of vertical growth.

Equal time snapshots



$\lambda = 0.01, \nu = 1.99$ $\lambda = 0.1, \nu = 1.9$ $\lambda = 1, \nu = 1$

Addendum

More angular coniform structures, similar to the stromatolites in the 3450 million year old Warrawoona Group, Western Australia, claimed as the oldest macroscopic evidence of life, form when the photic effects dominate over mineral accretion.

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in preparation