Supertubes & Dyonic Instantons

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Supertubes

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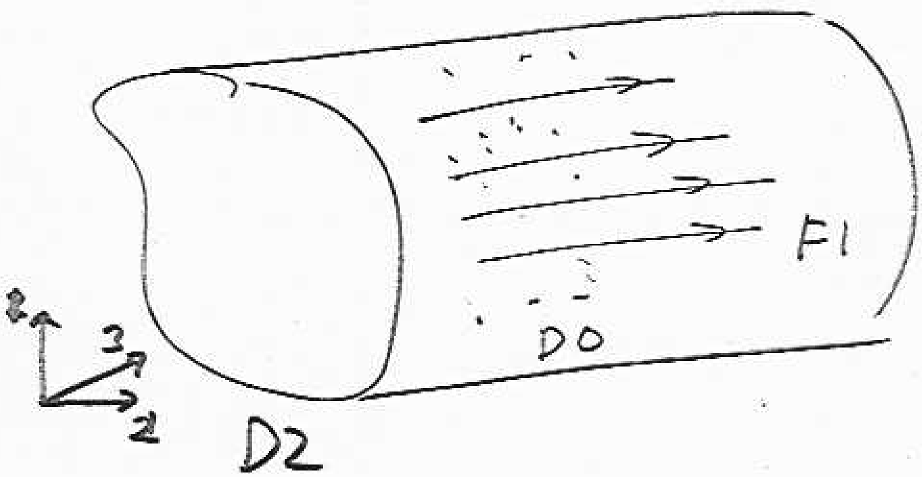
- D2 − F1 − D0



$$\begin{cases} X^0 = \sigma^0 \\ X^1 = A(\sigma^1) \\ X^2 = A(\sigma^1) \\ X^3 = \sigma^2 \end{cases}$$

$F_{02} = E(\sigma^1)$

$F_{12} = B(\sigma^1)$

induced metric $G_{ab} = \eta_{\mu\nu} \dfrac{\partial X^\mu}{\partial \sigma^a} \dfrac{\partial X^\nu}{\partial \sigma^b}$

· Action

$$S = -T\int d^3\sigma \sqrt{-\det(G_{ab} + F_{ab})}$$

$$= \int d^3\sigma$$

· $$\mathcal{L} = -\sqrt{(1-E^2)s + B^2}$$

$$s = \acute{A}^2 + \acute{B}^2$$

· $$\pi = \frac{\partial \mathcal{L}}{\partial E} = \frac{Es}{\sqrt{s(1-E^2)+B^2}}$$

$$\mathcal{H} = \pi E - \mathcal{L}$$

$$= \sqrt{s(s+B^2)(s+\pi^2)}$$

$$= \sqrt{s\left[(s-\pi B)^2 + s(\pi+B)^2\right]}$$

Gauss law $\partial_{\sigma_2}\pi = 0$

- BPS bound on Energy density

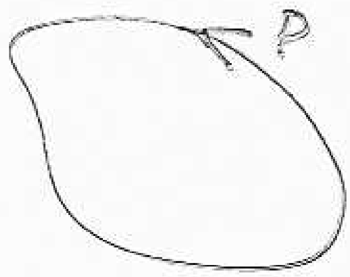
$$\mathcal{H} \geq s(\Pi + B)$$

saturated if $s - \Pi B = 0$

or $\boxed{E = 1}$

- Linear momentum Density

$P_i \sim \epsilon_{ij} F_{12} E_j$

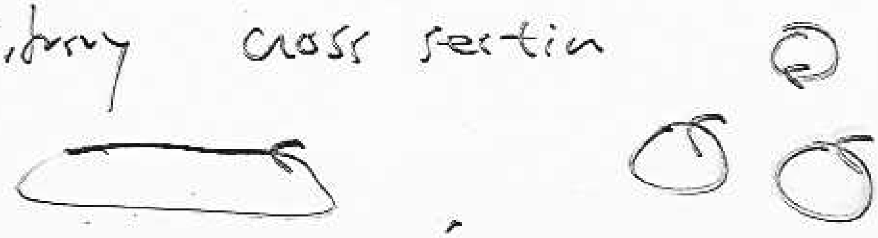


- Supersymmetry $\frac{1}{4}$ BPS   8 susy $= \frac{32}{4}$

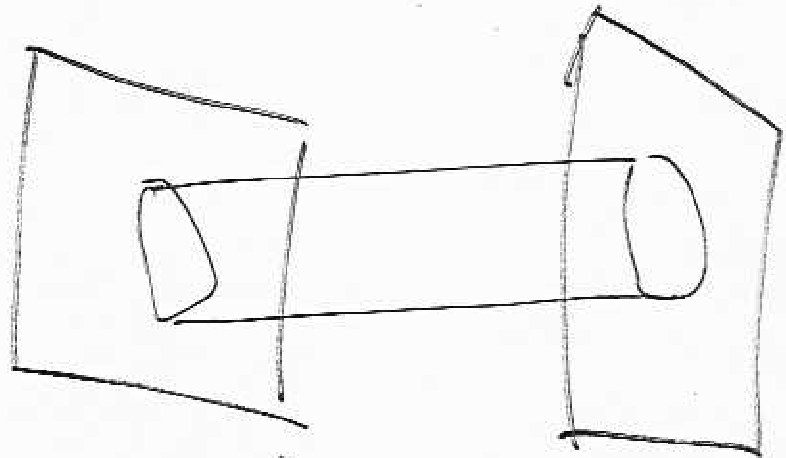
- No energy due to D2

$$\mathcal{E} = s(\underset{\substack{\uparrow \\ F1}}{\Pi} + \underset{\substack{\uparrow \\ D0}}{B})$$

- arbitrary cross section



Superstrings between "D4"s



D4 : 4+1 dim YM Higgs

D2 : monopole string

F1 : dyon

D0 : instanton

$\Rightarrow$ 'dyonic instanton + monopole string

$\frac{1}{8}$ BPS = $\frac{32}{8}$ = 4 SUSY

Dyonic Instantons

- BPS

$$F_{\mu\nu} = \tilde{F}_{\mu\nu} \longrightarrow K \text{ instantons}$$

$$\left.\begin{array}{l} E_\mu = D_\mu \phi \\ D_0 \phi = 0 \\ \text{- Gauss} \quad D_\mu E_\mu \sim [\phi, D_0 \phi] \end{array}\right\} \begin{array}{l} A_0 = \phi \\ D_\mu^2 \phi = 0 \end{array}$$

- $D_\mu^2 \phi$: the covariant laplacian

$$\phi(\infty) = v$$

$$\left(\begin{array}{l} \text{Solve} \quad F_{\mu\nu} = \tilde{F}_{\mu\nu} \xrightarrow{\text{ansatz ADHM}} \text{, then} \\ \text{solve} \quad D_\mu^2 \phi = 0 \longrightarrow \text{ansatz ADHM} \end{array} \right.$$

$E_\mu = D_\mu \phi$: dyonic

't Hooft Solution

- $A_\mu = \frac{i}{2}\sigma^a \bar{\eta}^a_{\mu\nu}\, \partial_\nu \ln H(x)$

$$H(x) = 1 + \sum_{i=1}^{K} \frac{s_i}{|x - a_i|^2} \qquad s_i \geq 0$$

K instantons $\quad$ 5K parameters + 3

$5K + 3 < 8K$ $\quad$ except $K = 1$

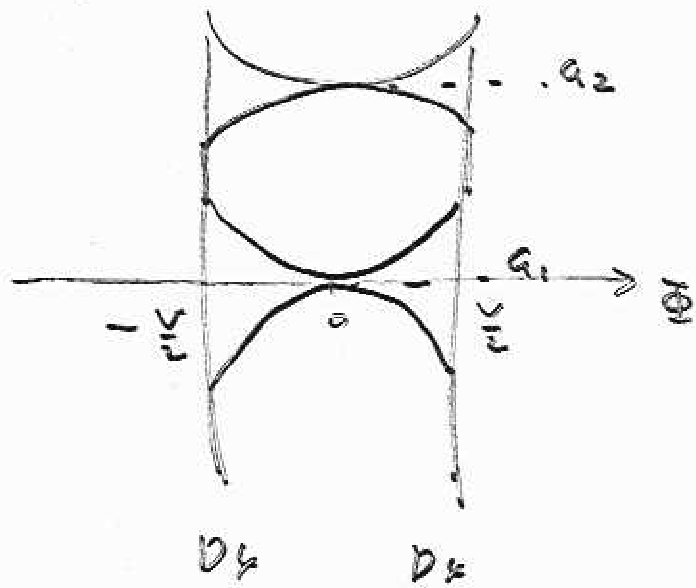
- $D_\mu^2 \phi = 0$ $\qquad$ Egress, Townsend, Zemekla

smooth

$$\phi = \frac{v}{H} \xrightarrow[x \to \infty]{} v \qquad \text{constant Tony}$$

$\phi = 0$ at $x = a_i$'s

• Φ: the transverse coordinates of D4 brane



$x^{0,1234}$

Collapsed super tubes (lines connecting D4's)

Jackiw-Nohl-Rebbi Solution

K-instanton

- $$H = \sum_{i=0}^{K} \frac{s_i}{|x-a_i|^2}$$

when a_i are on circle

parameters $5K + 5 - 1 - 1 + 3$

global gauge

$= 5K + 4 + 3 - [1]$

$K=1$ 12 not really $\to$ 8

* $K=2$ $10 + 4 + 3 - 1 = 16$ ok

$K=3$ ... $5K + 7 < 8K$ less

- $$\phi = \frac{v}{H}$$ ok for $D_\mu^2 \phi = 0$

$\to \infty$ $x \to \infty$ (not right)

ADHM method

$$a = \begin{pmatrix} \overset{n \times 2k}{\lambda} \\ \underset{k\times k}{\mu_\alpha} \underset{\substack{2\times 2 \\ (i\vec{\sigma},1)}}{e_\alpha} \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Delta = \begin{pmatrix} \lambda \\ (\mu_\alpha + x_\alpha) e_\alpha \end{pmatrix}, \quad \Delta^+ \Delta \sim \underset{k \times k}{f^{-1}} \, \mathbb{I}_{2\times 2}$$

$$\Delta^+ \underset{(n+2k)\times n}{v} = 0 \qquad v^+ v = 1$$

$$A_\mu = v^+ \partial_\mu v \qquad = -i A_\mu^{\text{hermitian}}$$

$$F_{\mu\nu} = \text{self dual}$$

$$\phi = v^+ \begin{pmatrix} \delta & 0 \\ 0 & Q \end{pmatrix} v$$

$$-[\mu\alpha, [\mu\alpha, Q]] - \frac{1}{2}\{W, Q\} + \Lambda = 0$$

$$W = \mathrm{tr}_2 \lambda^+ \lambda, \quad \Lambda = \mathrm{tr}_2 \lambda^+ \delta \lambda$$

ADHM & JNR

- $$a = \begin{pmatrix} a_0 \left(\frac{\lambda_1}{\lambda_0}, \ldots \frac{\lambda_k}{\lambda_0} \right) \\ a_1 & & 0 \\ & \ddots & \\ 0 & & a_k \end{pmatrix}^{(2+2k)\times 2k}$$ $a_i = a_i^\mu e_\mu$

- $$b = \begin{pmatrix} \frac{\lambda_1}{\lambda_0}, \ldots \frac{\lambda_k}{\lambda_0} \\ I_{k\times k} \end{pmatrix}$$

- $v_0 = -\dfrac{\lambda_0}{\bar{x} - \bar{a}_0}$, $v_i = \dfrac{\lambda_i}{\bar{x}_i - \bar{a}_i}$ $\qquad \bar{x} = x^\mu \bar{e}_\mu$

- go to the standard form $b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- Solve Q in terms of g

  possible for $k=2$.

2-instanton Case

$$\phi = \frac{1}{s_\Sigma H(x)} \left[Z q \bar{Z} + \mathcal{P} \mathcal{S} \right]$$

$$s_\Sigma = s_0 + s_1 + s_2 \qquad q = \tilde{q}_{\mu\nu} \tilde{\sigma}_{\mu\nu}$$

$$Z = \sum_0^2 \frac{s_i y_{i\mu}}{y_i^2} e_\mu \qquad y_i = x - a_i$$

$$i\,\sigma_{\mu\nu} = \frac{e_\mu \bar{e}_\nu - e_\nu \bar{e}_\mu}{2}$$

$$H = \sum_0^2 \frac{s_i}{y_i^2}$$

$$\mathcal{P} = \frac{8 s_1 s_2 s_3 \, \tilde{q}_{\rho\sigma} (a_{0\rho} a_{1\sigma} + a_{1\rho} a_{2\sigma} + a_{2\rho} a_{0\sigma})}{s_0 |a_1 - a_2|^2 + s_1 |a_2 - a_0|^2 + s_2 |a_0 - a_1|^2}$$

$$\mathcal{S} = 3\,\sigma_{\mu\nu} \cdot \left(\frac{y_{0\mu} y_{1\nu}}{y_0^2 y_1^2} + \frac{y_{1\mu} y_{2\nu}}{y_1^2 y_2^2} + \frac{y_{2\mu} y_{0\nu}}{y_2^2 y_0^2} \right)$$

Properties

1) $x \to \infty$

$$\phi = U g U^+ , \qquad U U^+ = U^+ U = 1$$

$$U = \frac{x_\mu e_\mu}{x^2}$$

2) Symmetric under permutation of y^0, y^1, y^2
$\xi^0\ \xi^1\ \xi^2$

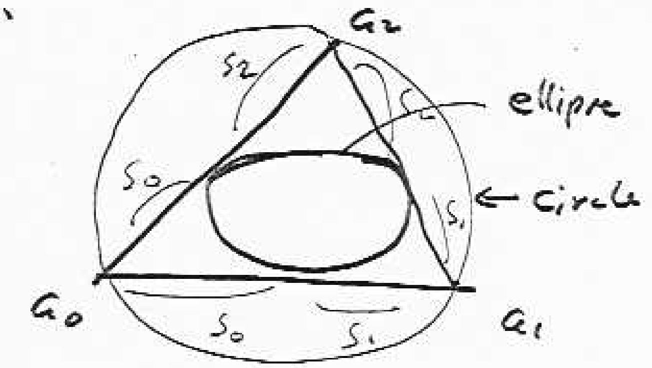
3) Say $a_1^\mu, a_2^\mu, a_3^\mu$ on x^3, x^4 plane

$$\Rightarrow \mathcal{P} \sim \underline{\underline{\xi^3}}\ \bar{\eta}^3_{34} \cdots$$

Thus when $\xi^1, \xi^2 \neq 0$, $\xi^3 = 0 \Rightarrow \mathcal{P} = 0$

$$\Phi = 0 \quad \text{at} \quad Z = 0$$

Porism



local gauge transformation ϵ paramts

$$\delta \zeta_i, \ \delta a_i \sim \qquad \epsilon$$

reduce $16 \to 13$ parameters

$\Phi = 0 \quad \leftarrow$ gauge invariant

points where $\Phi = 0$ is same under local gauge transf

$q_0, q_2, q_3 = 0 \qquad a_0, a_1, a_2 \sim x^3 - x^4$ plane

$\Phi = 0 \to \mathcal{Z} = 0$ or

$$A\, S_\Sigma x^2 - \overbrace{\left(\sum_i (S_\Sigma - S_i) y_i\right)}^{C_0} x + \overbrace{\left(a_0 s_\Sigma a_2 \sum_i \frac{S_i}{a_i}\right)}^{C_1} = 0$$

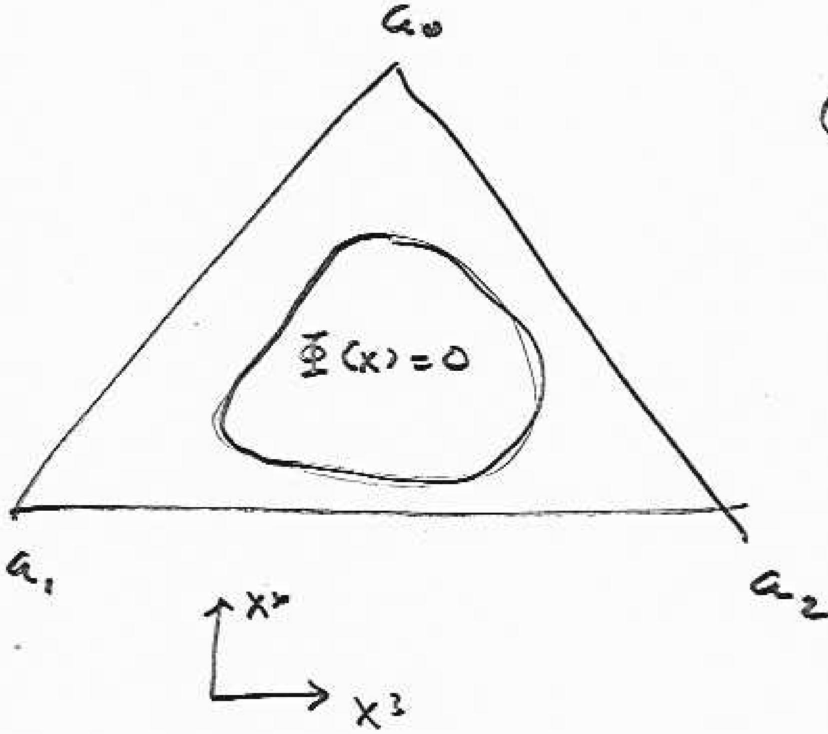
$x = x^3 e^3 + x^4 e^4, \quad y_i = y_i^3 e^3 + y_i^4 e^4$

Cross Section $\Phi = 0$

$a_i^1 = a_i^2 = 0,\quad q^1 = q^2 = 0$

$\rightarrow\quad X^1 = X^2 = 0$

indep of ξ^3



$(X_{3,4})^k + \cdots$

$S_1 = S_2 = S_3 = 1$ equitriangle of $\sqrt{3}$ 

$\rightarrow$ circle of radius $\sqrt{\frac{\sqrt{13}-1}{6}}$

- $a_i = R\,e^{i\sigma_3 \theta_i}$, $\sum \lambda_i = \lambda_\Sigma$ fixed

$\delta\theta_i$, $\delta\lambda_i$ leaving C_0, C_1 invariant

one parameter family

Conclusion

- Supertubes and its Field theoretic realizations are fun to study
- More general case ($K \geq 3$) needs to be solved
- Deeper implications may appear.
  - 5-dim Yang Mills on $R^{1+4} \times S^1$
  - (2,0) theory
  - String & M-theory
  - Black holes.