String Fluid

& Domain Walls

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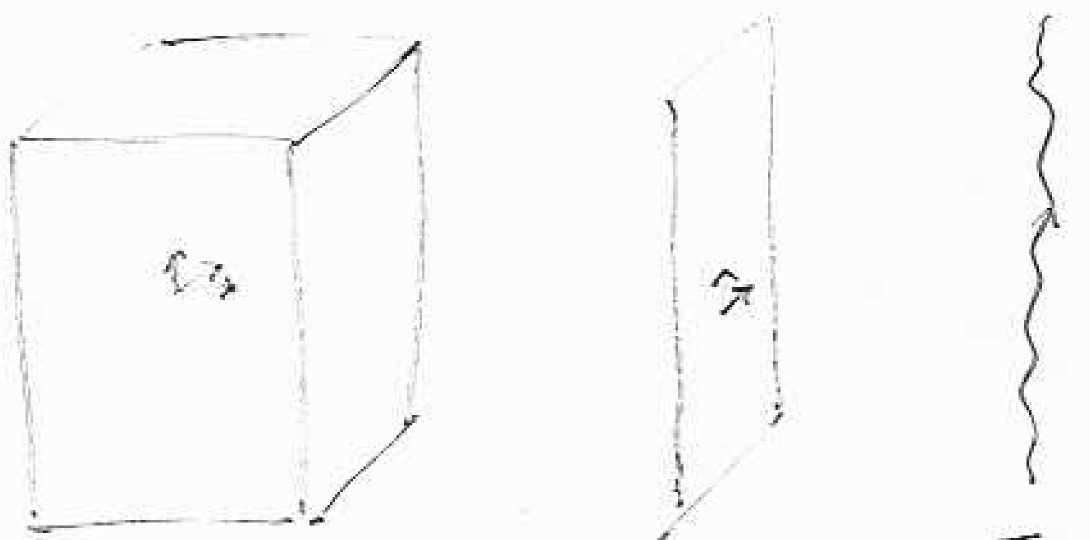
Low Energy Action on Dp

$$S = S_0 + S_1$$

Kalb-Ramond

$$S_0 = -\tau_p \int d\tau\, d\sigma^p \left(-\mathrm{Det}\left(g_{\mu\nu} + (B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}) + \partial_\mu X^I \partial_\nu X^I\right)\right)^{1/2}$$

$$S_1 = \tau_p \int e^{2\pi\alpha' F} \wedge \underbrace{\sum_q C_q}_{\text{RR-tensors}}$$



D_p $D(p-1)$ $F1$

World Volume Bosonic Field Content

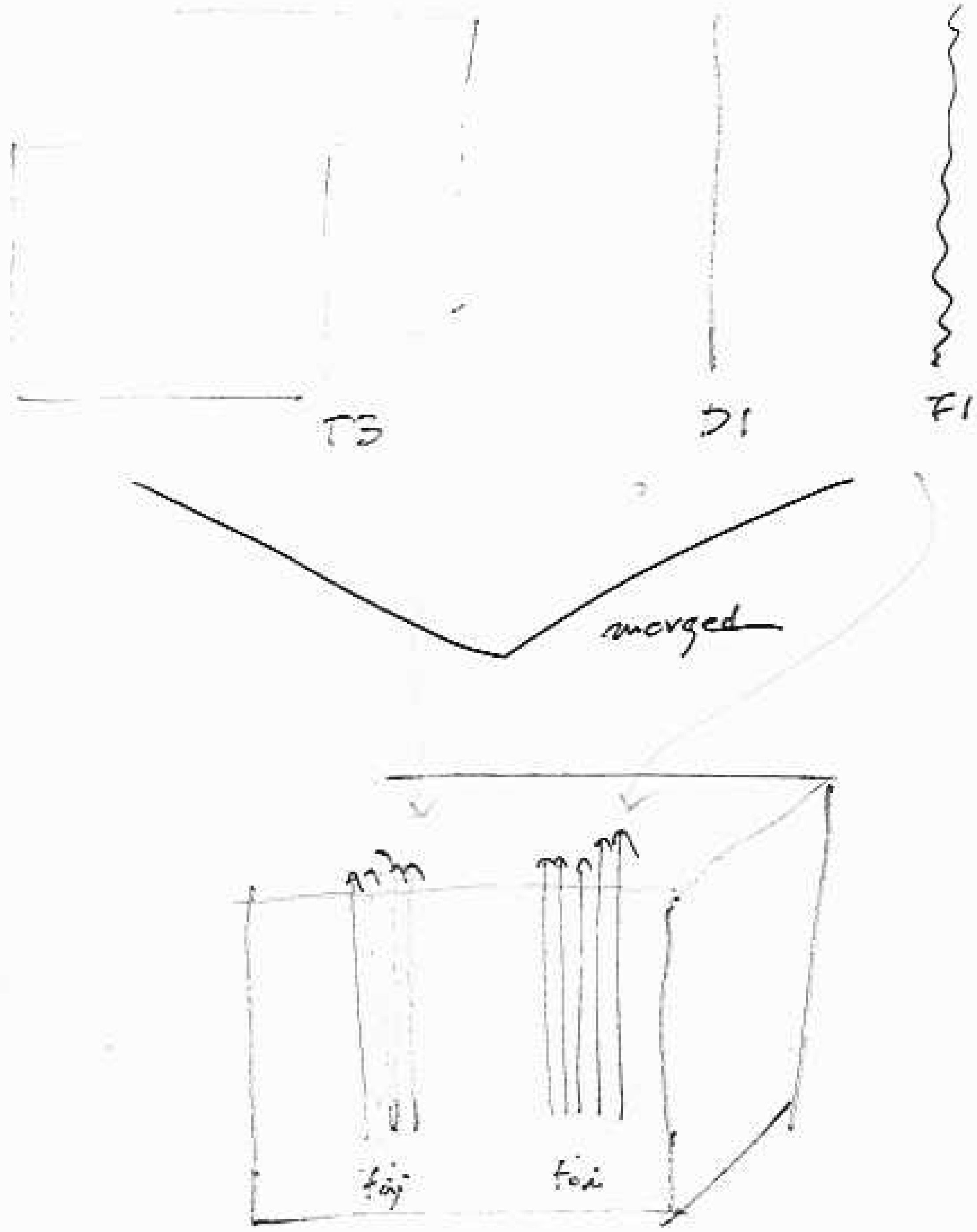
$\equiv$ Gage Field + Transverse Scalars

Couples R-R tensor gauge field

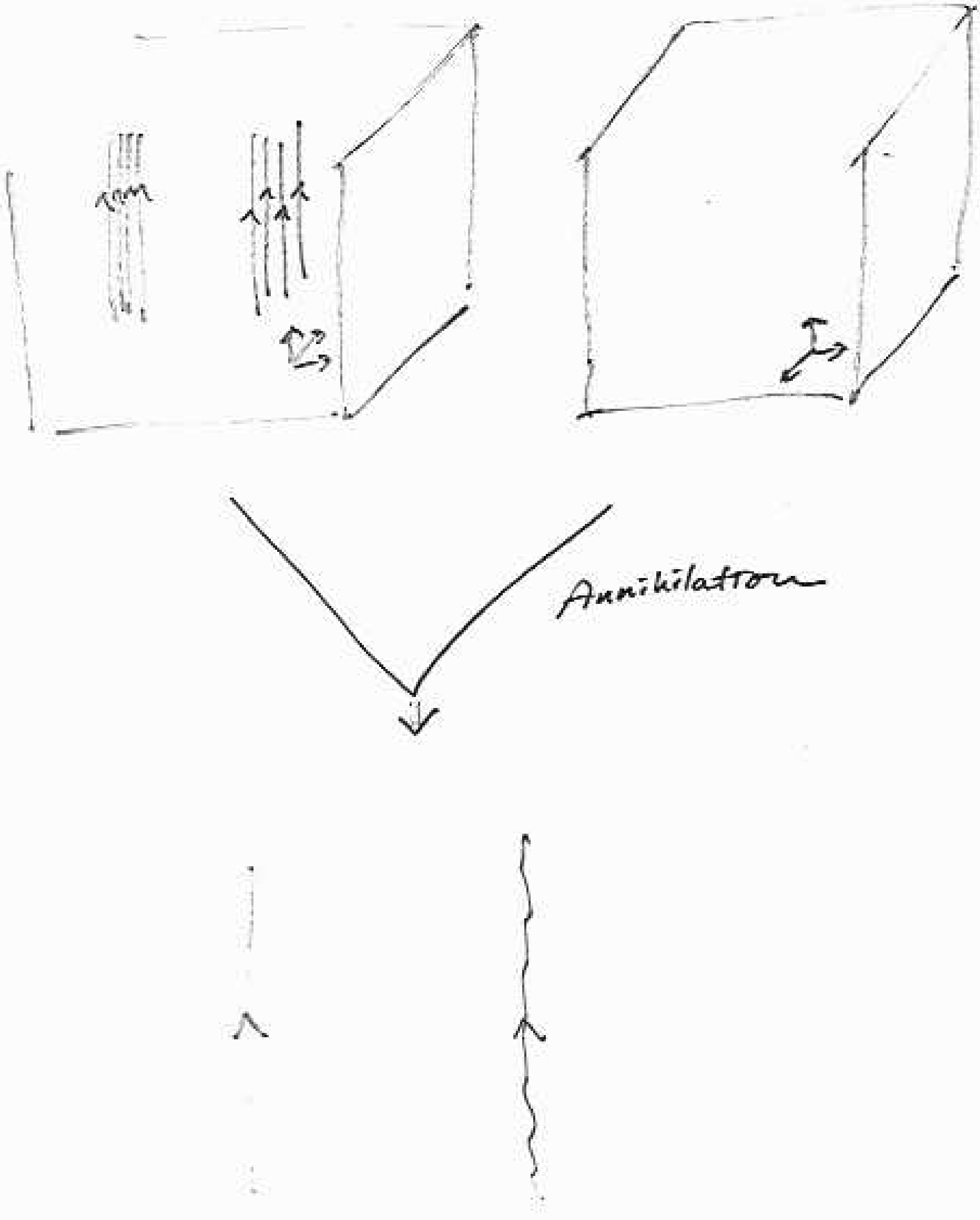
$C_{\mu_1 \cdots \mu_{p+1}}$ or $C_{\mu_1 \cdots \mu_p}$

Nambu-Goto (Polyakov) Action

Couples $B_{\mu\nu}$



← Process should be reversible



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Simplify the Problem

$$IIA = IIB/(-1)^{F_L}$$

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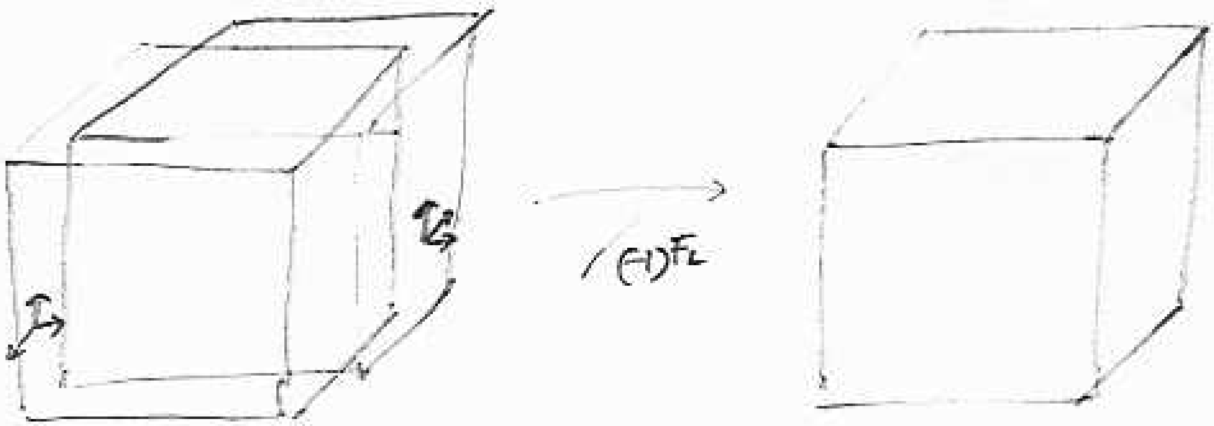
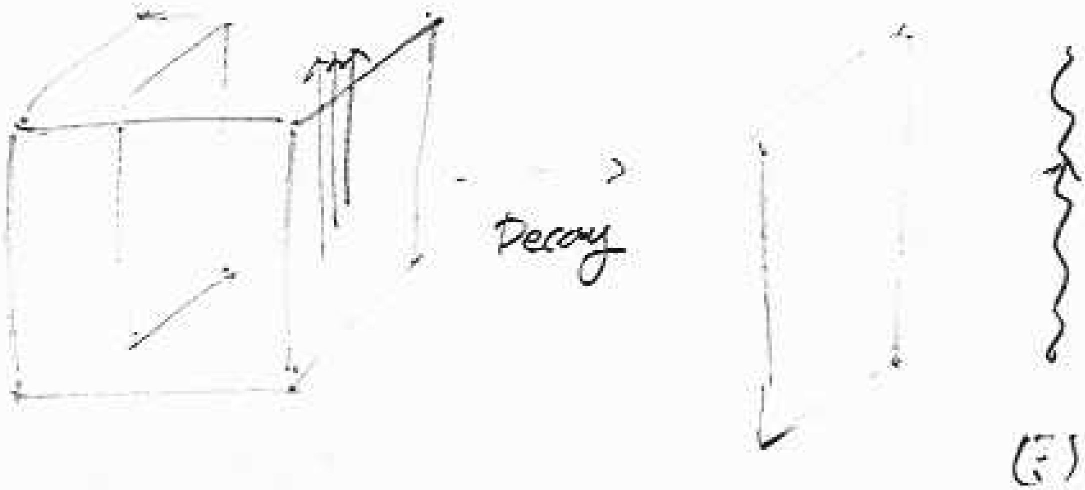


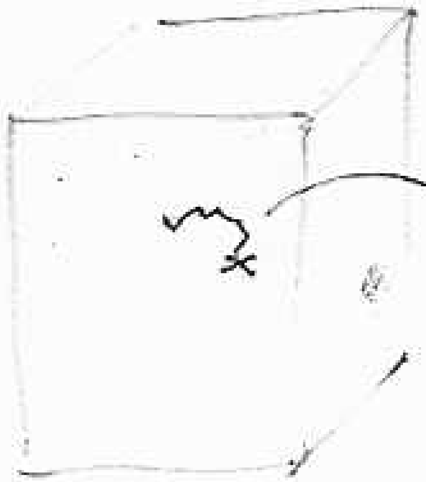
$D_p - \overline{D_p}$ Unstable D_p

embed stable $D(p-1)$
or $F1$



< Starting Point >

Low Energy Action of an Unstable Dp-Brane



$\underbrace{T(\tau,\sigma)}_{\text{tachyonic}}$, $\underbrace{A_\mu(\tau,\sigma), \; X^I(\tau,\sigma)}_{\text{massless}}$

$$S = S_0 + S_1$$

Kalb-Ramond

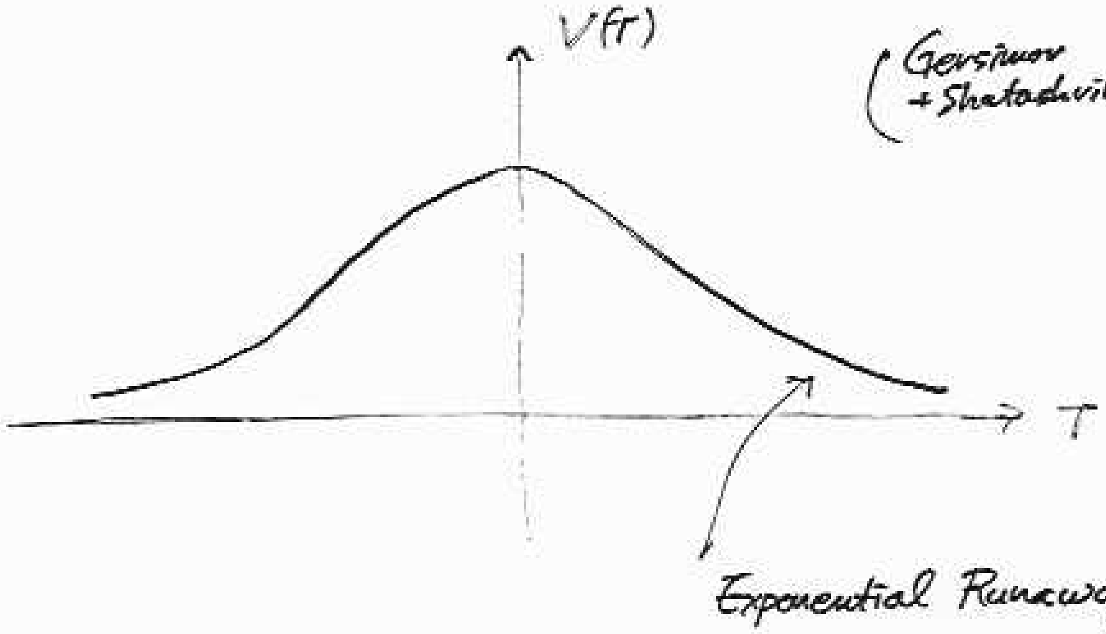
$$S_0 = -\int d\tau d\sigma \; V(T) \times \sqrt{-\mathrm{Det}\left(g_{\mu\nu} + (B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}) + \partial_\mu T \partial_\nu T + \partial_\mu X^I \partial_\nu X^I\right)}$$

(Garousi, Bergshoeff et al 1999)

$$S_1 = \int W(T)\, dT \wedge e^{2\pi\alpha' F} \wedge \sum_q \underbrace{C_{(q+1)}}_{\text{RR-tensor}}$$

(A. Sen 1999)

<Tachyon>



(Gerasimov + Shatashvilli 2000)

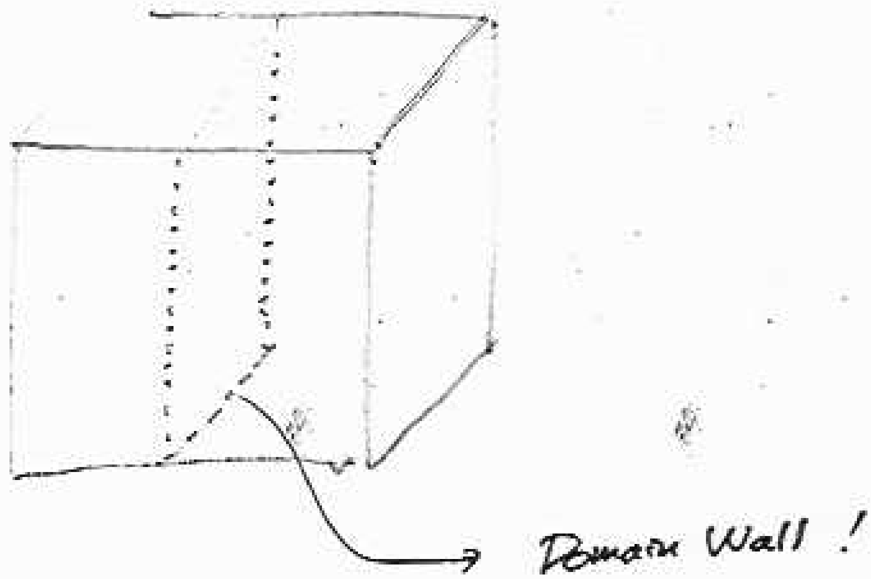
⇒ Real Scalar with Vacua at $T = \infty, -\infty$

→ Static State at $T = \infty$ or $-\infty$ has $V(T) = 0$

→ Action Density, Energy Density
Charge Density $\longrightarrow 0$

→ Disappearance of Unstable Dp

How Does a Stable $D(p-1)$ Embed itself to an Unstable Dp ?



Why? $D(p-1)$ Couples to $C_{(p)}$ RR-tensor

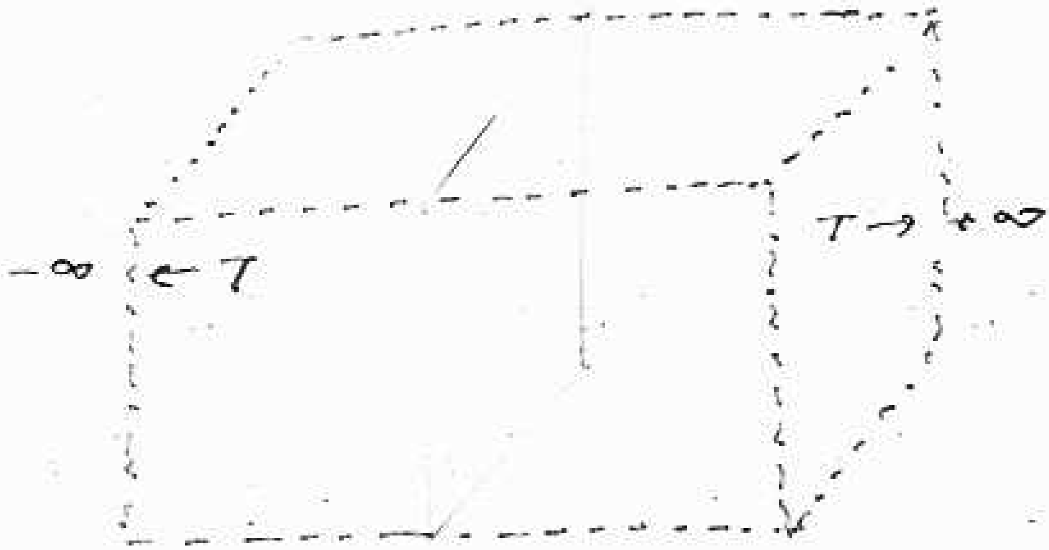
$\Rightarrow$ Whatever the configuration, it must couple to $C_{(p)}$ minimally

$$S_2 = \cdots \int_{(p+1)} w(T)\,dT \wedge e^{2\pi\alpha' F} \wedge \sum C_{(q)}$$

$$= \cdots + \underbrace{\int_{(p+1)} w(T)\,dT \wedge C_{(p)}} + \cdots$$

$\rightarrow \int_{(p)} C_{(p)}$ on a thin domain wall

Domain Wall Solution ?



$$\mathcal{L} = -V(T)\sqrt{-\mathrm{Det}(\eta + \partial T \partial T)}$$
$$= -V(T)\sqrt{1 - \dot{T}^2 + (\vec{\partial} T)^2}$$

$|\langle T \rangle| = \infty$ suggests that a domain wall solution would have "Zero" thickness !!

Try $T(x) = f(ax)$ with a monotonic function f

(Sen 2003)

Field Equation

$$\partial_x\left(\frac{V(T)\,\partial_x T}{\sqrt{1+(\partial_x T)^2}}\right) - V'(T)\sqrt{1+(\partial_x T)^2} = 0$$

$$\partial_x \left(\frac{V(T)\,\partial_x T}{\sqrt{1+(\partial_x T)}} \right) \Rightarrow \frac{V'(T)\,(\partial_x T)^2}{\sqrt{1+(\partial_x T)^2}} + V(T)\,\partial_x \left(\frac{\partial_x T}{\sqrt{1+(\partial_x T)^2}} \right)$$

Field Eq

$$0 = \frac{V'(T)\,a^2(f')^2}{\sqrt{1+a^2(f')^2}} + \underbrace{V(T)\,\partial_x \left[\frac{\partial_x T}{\sqrt{1+(\partial_x T)^2}} \right]}_{\to\, O\left(\frac{1}{a}\right)}$$

$$- V(T)\sqrt{1+a^2(f')^2}$$

$$\to O\left(\frac{1}{a}\right) \qquad \text{as } a \to \infty$$

<u>Claim</u>: The Only Domain Wall Solution of Pure Tachyon Case with $T(\pm\infty) = \pm\infty$ is Singular One.

$\to$ Not as bad as you would think!

⟨ What Action Governs the Domain Wall Solution? ⟩

$$\int d\tau\, d\sigma^p \; - V(T)\sqrt{-\mathrm{Det}\left(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T + \cdots\right)}$$

"Solutions" $\quad T = f\left(a\left(\sigma^p - X(\tau, \sigma^1 \ldots \sigma^{p-1})\right)\right)$

with $a \to \infty$ $\quad$ Domain Wall along $\sigma^{1,\cdots,p-1}$

with Motion along σ^p

$$-\mathrm{Det}\left(\eta + \partial_\mu T \partial_\nu T\right) = 1 + \eta^{\mu\nu}\partial_\mu T \partial_\nu T \qquad (\mu,\nu = 0,\cdots,p)$$

$$= \underbrace{1}_{\text{negligible as } a \to \infty} + a^2 (f')^2 \cdot \left(1 + \eta^{\alpha\beta}\partial_\alpha X \partial_\beta X\right) \qquad (\alpha,\beta = 0,\cdots,(p-1))$$

$$-\underbrace{\int V(T)\, a f'\, d\sigma_p}_{= \int_{-\infty}^{\infty} V(T)\, dT} \times \underbrace{\int \sqrt{-\mathrm{Det}\left(\eta_{\alpha\beta} + \partial_\alpha X \partial_\beta X\right)}\, d\tau\, d\sigma^{p-1}}_{\text{DBI of } D(p-1)}$$

Similar Consideration Carries Through

When we include X^I's & A_μ's

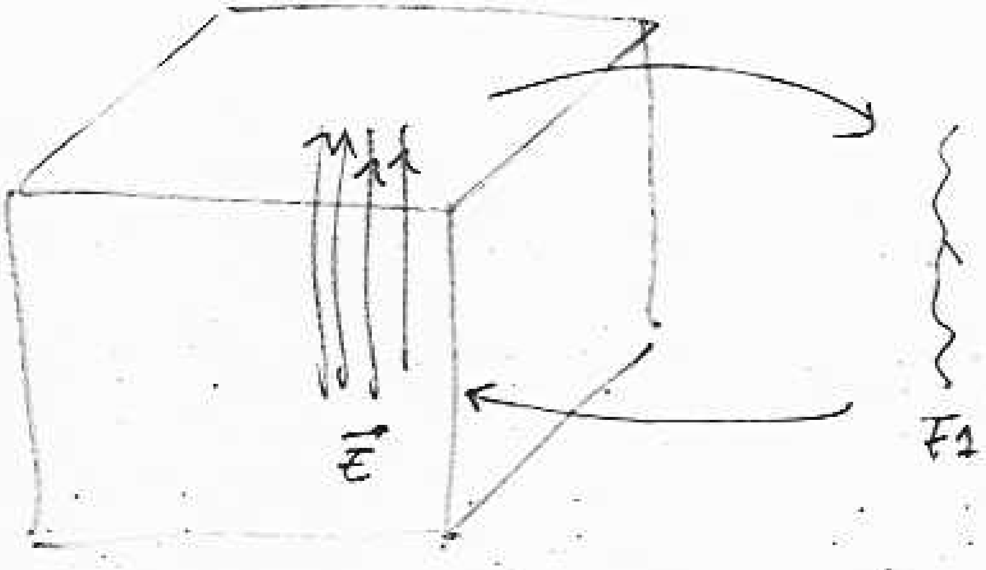
$\rightarrow$ Singular Domain Wall Solution Obeys the Same Classical Dynamics as Stable D(p-1) !!!

N.B. "$\sqrt{\cdots\cdots}$" form of Lagrangian makes this possible !!!

< How does a fundamental string embed itself to an unstable D_p ? >

$$\frac{\delta}{\delta B_{\mu\nu}}(S_0+S_1) = \frac{\delta}{\delta F_{\mu\nu}}(S_0+S_1) \Big/ 2\pi\alpha'$$

String Flux = Gauge Flux



⟨Solutions with Electric Flux?⟩

$$\mathcal{L} = -V(T)\left(-\mathrm{Det}\left(g_{\mu\nu} + \partial_\mu T \partial_\nu T + F_{\mu\nu}\right)\right)^{1/2}$$

Electric Flux

$$\pi_i \equiv \frac{\delta\mathcal{L}}{\delta E_i} = +\underbrace{\frac{V(T)}{(-\mathrm{Det}\cdots)^{1/2}}} \times \frac{\delta}{\delta E_i}\mathrm{Det}(\cdots)$$

Must Survive $V(T) \to 0$

$\therefore (\mathrm{Det}\cdots)^{1/2} \sim V(T)$ on solutions with fluxes

$\mathcal{L} \to 0$ as $V^2(T)$

if we wish to consider electric flux lines.

$\Rightarrow$ Euler-Lagrange E.O.M. is too delicate.

$\Rightarrow$ Switch To <u>Canonical</u> Formulation

$$\text{ie} \begin{pmatrix} \pi_i = \dfrac{\delta \mathcal{L}}{\delta E_i} \\ \\ \pi_T = \dfrac{\delta \mathcal{L}}{\delta \dot{T}} \end{pmatrix} \quad \text{in place of} \quad \begin{pmatrix} \dot{A}_i - \partial_i A_0 \\ \\ \dot{T} \end{pmatrix}$$

$$\rightarrow \quad \mathcal{L} = \pi_i E_i + \pi_T \dot{T} - H + A_0 \underbrace{(\partial_i \pi_i)}_{\substack{\text{Gauss'} \\ \text{Constraint}}}$$

Canonical E.O.M.

$$\dot{\pi}_i = - \frac{\delta}{\delta A_i} \int H \, d\sigma$$

$$\dot{\pi}_T = - \frac{\delta}{\delta T} \int H \, d\sigma$$

$$H^2 = \underbrace{\Pi_i \Pi_i}_{\substack{\text{conserved} \\ \text{electric flux} \\ = \text{string flux}}} + \underbrace{\Pi_T \Pi_T}_{\text{tachyon matter}}$$

$$+ \left(F_{ij}\Pi_j + \partial_i T\, \Pi_T\right)\underbrace{\left(F_{ij}\Pi_j + \partial_i T\, \Pi_T\right)}_{P_i \,:\, \text{Noether momentum}}$$

$$+ \left(\Pi_i \partial_i T\right)^2$$

$$+ V(T)^2 \, \mathrm{Det}\left(\delta_{ij} + \partial_i T \partial_j T + F_{ij}\right)$$

Measures Change of T along electric flux lines

Gibbons + Hori + P.Y.
(2000)

⟨ Note that A_i's & T are on equal footing ⟩

⟨Lemma⟩

$$\mathrm{Det}\left(g_{\mu\nu} + F_{\mu\nu} + \partial_\mu T \partial_\nu T\right)$$

$$\equiv \mathrm{Det}\left(\begin{array}{c|c} g_{\mu\nu}+F_{\mu\nu} & \partial_\mu T \\ \hline -\partial_\mu T & 0 \end{array}\right)$$

$$\equiv \mathrm{Det}\left(g_{MN} + F_{MN}\right)$$

with $A_T \equiv T \quad \partial_T \equiv 0$

(Gibbons 1997)

In $V=0$ Regime, canonical EOM may be distilled into Fluid E.O.M. + Integrability Condition

$$\Pi_m = H n_m \qquad m = 1, 2, \cdots, p.T$$

$$P_m = H v_m$$

with $\left(\begin{array}{l} n^2 + v_y^2 = 1 \\ n \cdot v = 0 \end{array} \right)$

$$\frac{\partial}{\partial t} n^m + (v^i \partial_i) n^m = (n^i \partial_i) v^m$$

$$\frac{\partial}{\partial t} v^m + (v^i \partial_i) v^m = (n^i \partial_i) n^m$$

$$\frac{\partial}{\partial t} H + \partial_i (H v^i) = 0$$

(Gibbons + Hori + P.Y. 2000)

integrability conditions that says, there are underlying A_i, T as elementary fields

(Kim + P.Y. 2003)

⟨ Most General Static Solution without Domain Walls ⟩

Start with $P_i = 0 \longrightarrow v_i = 0$

No Domain Wall $\longrightarrow v_\tau = 0$

$$\Rightarrow \quad \frac{\partial}{\partial t} H = 0 \quad \frac{\partial}{\partial t} n^i = 0 \quad \frac{\partial}{\partial t} n^\tau = 0$$

$$(n^i \partial_i) n^k = 0$$

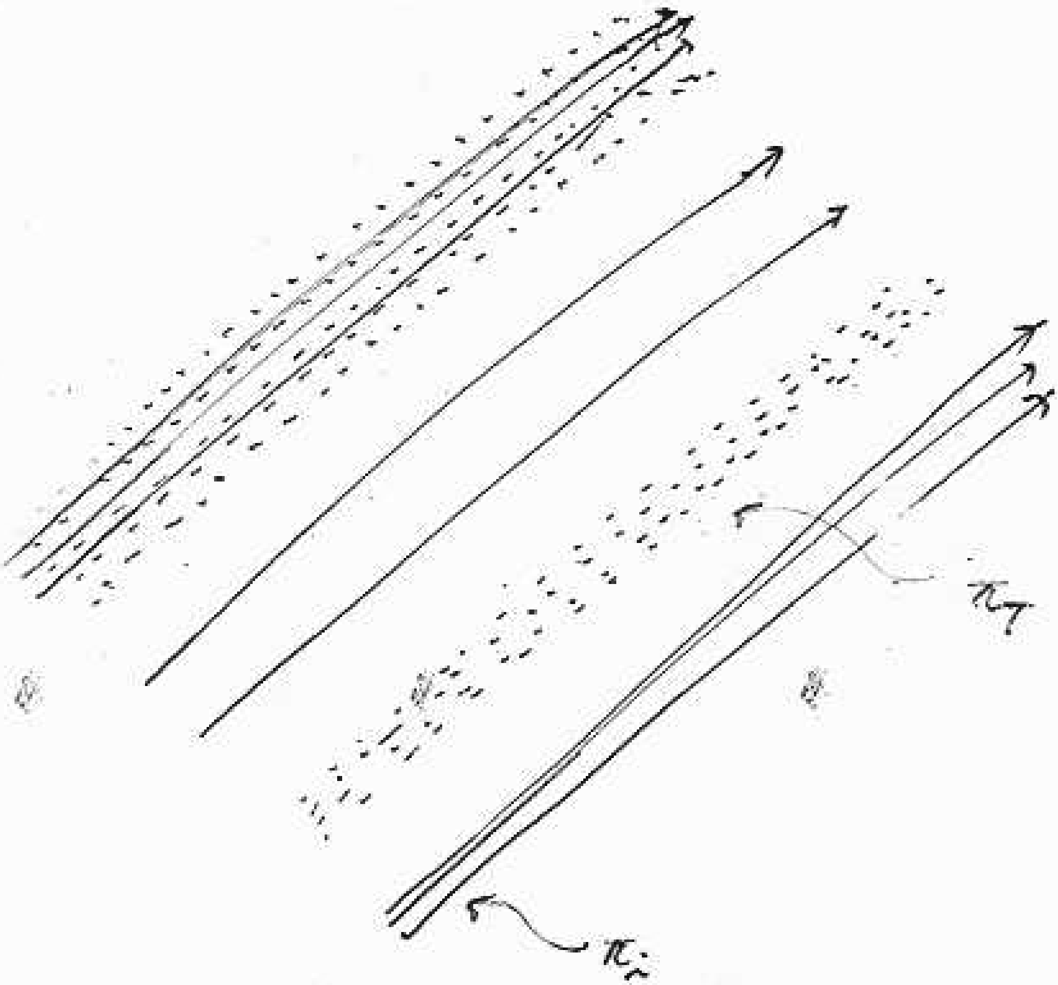
$$(n^i \partial_i) n^\tau = 0$$

$$\partial_i (H n^i) = 0$$

Straight Flux Lines
+ Homogeneous Fields Along this Direction

$$\pi^i (\sigma^2, \cdots, \sigma^p)$$
$$\pi^\tau (\sigma^2, \cdots, \sigma^p)$$

Unrelated !!!



(Kwon + P.Y. 2003)

⟨ Dynamical Stability? ⟩

$$\vec{n} = \vec{n}_0 + \vec{n}_1 + \vec{n}_2 + \cdots$$

$$\vec{v} = \vec{v}_0 + \vec{v}_1 + \vec{v}_2 + \cdots$$

Perturb Fluid E.O.M.

$$\left[\left(\frac{\partial}{\partial t}\right)^2 - \left(\vec{n}_0 \cdot \vec{\partial}\right)^2 \right] \begin{pmatrix} \vec{n}_1 \\ \vec{v}_1 \end{pmatrix} = 0$$

$\longrightarrow$ Free Propagation of Disturbance along $\pm\vec{n}_0$ direction with speed $\left(\sqrt{v_0^2 n_0^2}\right)$

Essentially the Same Result at 2nd order.

G. Gibbons + K. Hashimoto + P.Y. 2002

K. Kwon + P.Y. 2003

< Another Miracle >

Take $\pi_\tau = 0$

Consider Generic Motion of π_i Concentrated to a narrow string

Low Energy Effective Action is Derived !!!

$$\int d\tilde{\tau}\, d\tilde{\sigma} \sqrt{-\mathrm{Det}\left(\partial_a \vec{X}\, \partial_b \vec{X}\right)}$$

$\vec{X}(\tau, \sigma)$: Position of the String in World-Volume (or Spacetime)

Nambu - Goto !!!

Gibbons + Hori + Y. 2000

Sen 2000

(Nielsen + Olesen 1973)

⟨ How Does Domain Wall Interact with String Fluid? ⟩

EOM with $V(T)$ Kept! (case of Unstable D2)

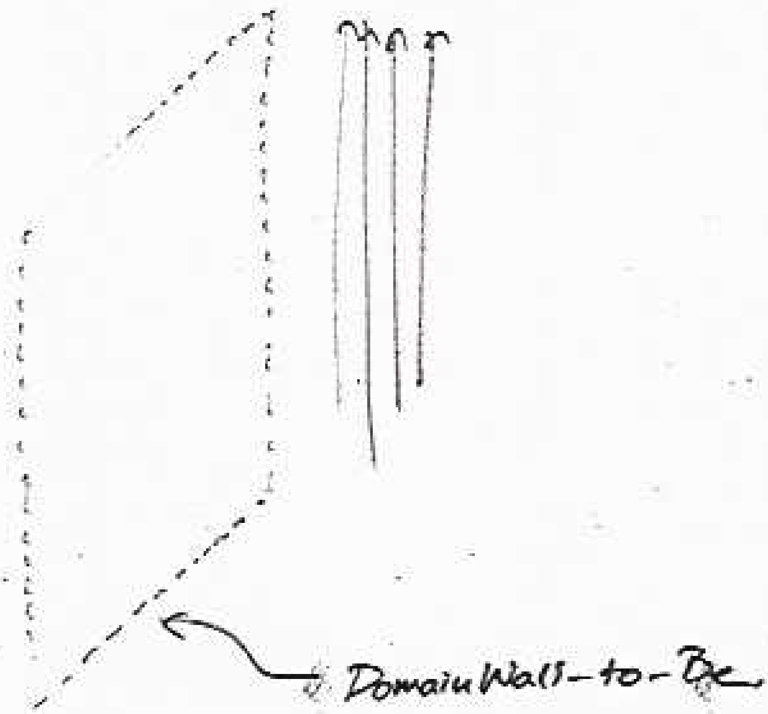
$$\dot{\pi}_i + \partial_j \left(\frac{\pi_i P_j - \pi_j P_i + V^2 B \epsilon_{ij}}{H} \right) = 0$$

$$\dot{\pi}_T + \partial_j \left(\frac{\pi_T P_j - \pi_j (\pi_i \partial_i T) - V^2 \partial_j T}{H} \right) = -\frac{V V' (1 + (\partial T)^2)}{H}$$

$$\vec{E} = \frac{1}{H} (\vec{\pi} - \vec{B} \times \vec{P})$$

$$\vec{B} = (\partial_2 T, -\partial_1 T, B)$$

$$\vec{P} = (P_1, P_2, \pi_i \partial_i T)$$



$$\ddot{\pi}_x\Big|_{t=0} = \Bigg[\underbrace{\frac{V^2}{H}\partial_y^2\left(\frac{\pi_x}{H}\right) - \frac{\pi_x}{H}\partial_y^2\left(\frac{V^2}{H}\right)}\Bigg]_{t=0}$$

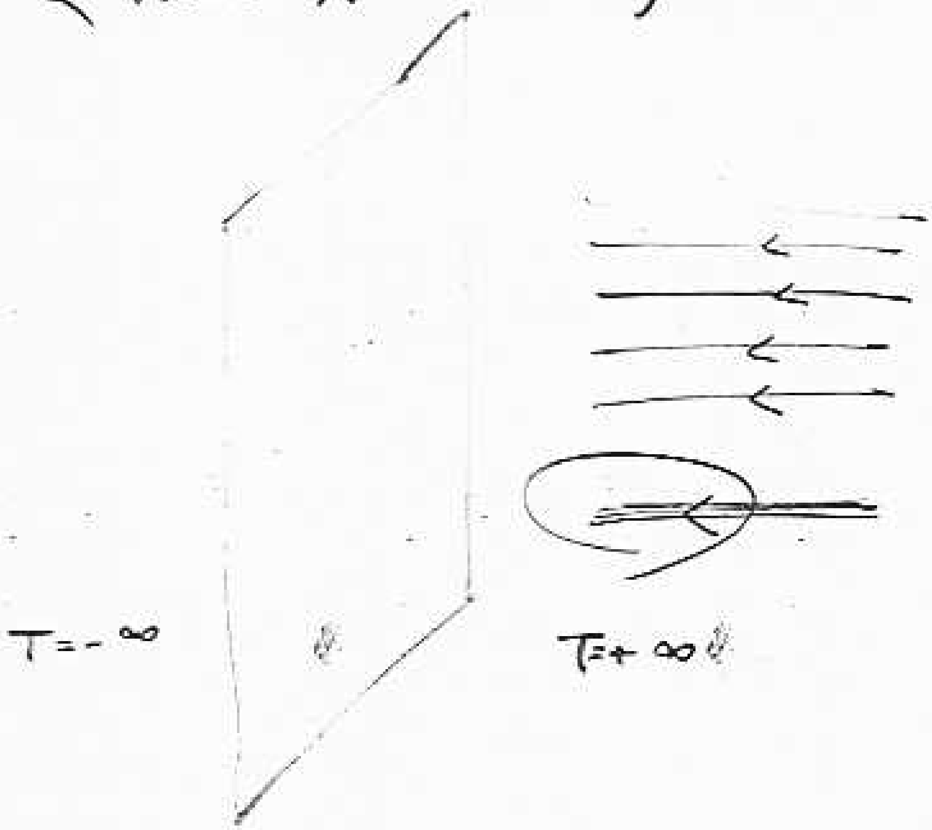
> 0 near V_{max}

< 0 away from Domain Wall

$\longrightarrow$ Net Aggregation of Parallel Electric Flux toward domain Wall during initial phase.

$\longrightarrow$ Tendency to produce $D(p-1) + F1$ Bound State.

⟨ What About Orthogonal Flux? ⟩

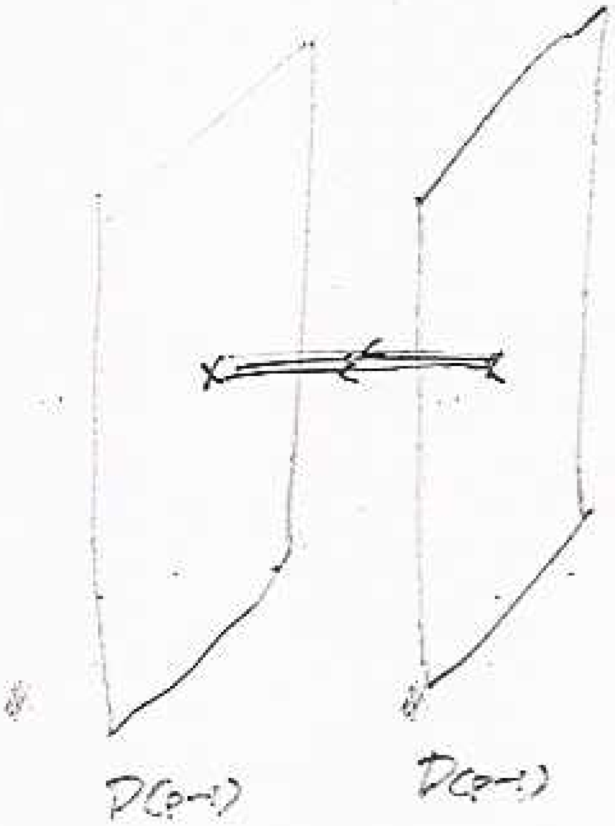


Energy of Flux Line Bundle

$$= \left(\pi_i^2 + \cdots + (\pi_i \partial_i T)^2 + V^2(\cdots) \right)^{1/2}$$

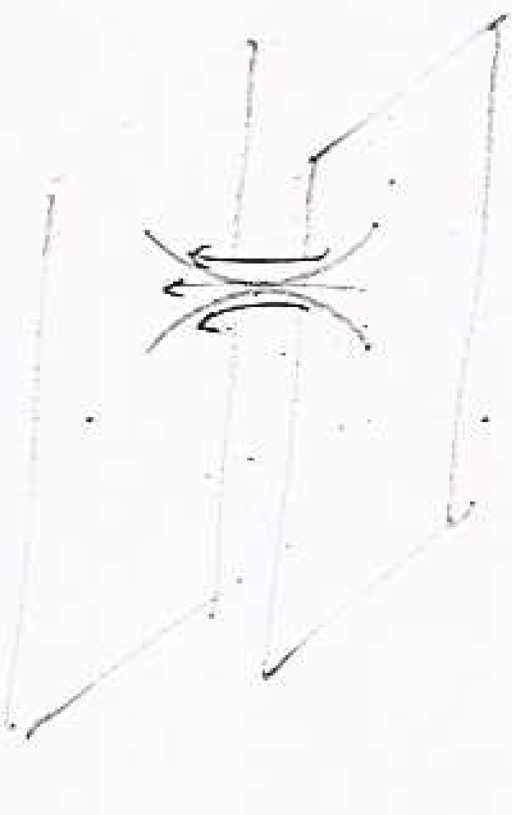
$$\geq | \pi_i \partial_i T | \to (\text{Net Flux}) \times (\Delta T)$$

$\to$ String Fluid of $V(T)=0$
Cannot Be Continued into
the Region of $V(T) \neq 0$!



NOT POSSIBLE

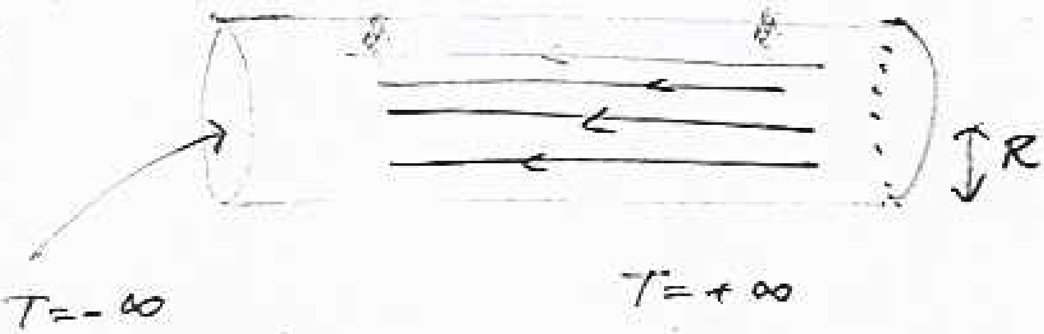
Sen 2003



ALLOWED
Because $T=0$ is maintained along the flux line.

New Fundamental String

with $V \neq 0$ Core?



Only Stable Solution with Pure

String Charge is $R \longrightarrow 0$

Reliable ????

Q1. Is Domain Wall Essential
in Recovery of Fundamental String ?
??

Q2. Is Domain Wall Enough
for Recovery of Fundamental String ?
NOT

Summary

Successful Recovery of Lower Dim D-Branes in Open String Viewpoint

⇓

More Confusions & Tantalizing Hints with Reproduction of Fundamental String

Closed String from Open String?