QFT on D-branes in PP-Wave Backgrounds

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1. Introduction

- QFT on D-branes : generalities

2. D-branes on pp-wave

3. World volume theory of D-branes on pp-wave

4. Conclusion

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· Study of worldvolume theory for D-branes

enhanced our understanding for QFT

and sometimes

gave new QFT

ex) SYM, (2, 0) theory, NCSYM, ...

String theory on pp-wave

- often exactly solvable (in light-cone gauge fixed action)

- string modes are all massive

$$E_n = \sqrt{\mu^2 + \frac{n^2}{(\alpha' p^+)^2}}$$

- some pp-wave geometry have large # of SUSY.

  IIB : 32 SUSY, ...

  IIA : 24 SUSY, ...

D-brane action

Even the lightest modes in strings on pp-wave are massive.

⇒ The low energy effective theory of D-brane world-volume is expected to be 'massive' Super Yang-Mills theory.

Question

How to implement massive vector ~~[illegible]~~. multiplet while maintaining gauge invariance?

11 D pp-wave geometry

Kowalski-Glikman
Güven
Figueroa-O'Farrill &
Papadopoulos
Penrose

$$ds^2 = -2\,dx^+ dx^- - A(x^I)(dx^+)^2 + \sum_{I=1}^{9} (dx^I)^2$$

$$F_{+123} = \mu$$

$$A(x^I) = \sum_{i=1}^{3} \frac{\mu^2}{9}(x^i)^2 + \sum_{i'=4}^{9} \frac{\mu^2}{36}(x^{i'})^2$$

- maximally supersymmetric - 32 Killing spinors
- can be derived from the 'Penrose limit' of $AdS_4 \times S^7$ or $AdS_7 \times S^4$
- $SO(3) \times SO(6)$ bosonic symmetry
- $SU(4|2) \circledS h^{18,16}$ superalgebra

(Fernando, Günaydin, Pavlyk)

Matrix Model on pp-wave

Action

$$L = \mathrm{Tr}\, \mathcal{L}$$

$$\mathcal{L} = \frac{1}{2R} D_0 X^I D_0 X^I + \frac{R}{4} [X^I, X^J]^2$$

$$+ \frac{i}{2} \theta^\alpha D_0 \theta^\alpha + \frac{R}{2} \theta^\alpha \gamma^I_{\alpha\beta} [X^I, \theta^\beta]$$

mass $\Rightarrow$ $- \frac{1}{2R} \left(\frac{\mu}{3}\right)^2 (X^i)^2 - \frac{1}{2R} \left(\frac{\mu}{6}\right)^2 (X^{i'})^2 - \frac{i\mu}{8} \theta^\alpha \Pi_{\alpha\beta} \theta^\beta$

Myer's term $\Rightarrow$ $- \frac{i\mu}{3} \epsilon_{ijk} X^i X^j X^k$

$$D_0 = \partial_0 - i [A_0, \cdot\,]$$

$$\Pi = \gamma^{123}$$

$$X^- \sim X^- + 2\pi R$$

$\Rightarrow$ (0+1)D 'massive' SYM QM.

Supersymmetry (in the $A_0 = 0$ gauge)

16 Kinematical SUSY

$$\tilde{\delta} X^I = 0$$

$$\tilde{\delta}\theta = \tilde{\eta} = e^{\frac{\mu}{4}\Pi t}\,\tilde{\epsilon} \qquad \Pi = \gamma^{123}$$

16 Dynamical SUSY

$$\delta X^I = i\theta\gamma^I\eta$$

$$\delta\theta = \Big(P^I\gamma^I + \tfrac{i}{2}[X^I, X^J]\gamma^{IJ} \Rightarrow \text{usual SUSY in SYM}$$

massive extension $\Rightarrow$ $$+ \frac{\mu}{3R}X^i\Pi\gamma^i - \frac{\mu}{6R}X^{i'}\gamma^{i'}\Pi\Big)\eta$$

$$\eta = e^{-\frac{\mu}{12}\Pi t}\,\epsilon$$

NOTE: η & $\tilde{\eta}$ have different time dependence.

BPS branes in matrix model on pp-wave

$\tilde{\delta}(\text{field}) = 0$

rotating flat membrane solutions spanning $x^{i'}$

$$X_4 = r_1 \cos\frac{\mu t}{6} \qquad X_7 = r_1 \sin\frac{\mu t}{6}$$

$$X_5 = r_2 \cos\frac{\mu t}{6} \qquad X_8 = r_2 \sin\frac{\mu t}{6}$$

with $[r_1, r_2] = i F_{12} I$

$F_{12} = - F_{21}$

I : unit matrix in $SU(N)$

NOTE : 1. The solution exists only in the large N limit

2. It has $\frac{1}{8}$ SUSY (4 supercharges)

$$\Pi \epsilon = \gamma_{47} \epsilon = \gamma_{58} \epsilon = \pm i \epsilon$$

$$\Rightarrow \delta\theta = \frac{i}{2} [r_1, r_2] e^{\pm \frac{i\mu}{4} t} \gamma_{45} \epsilon$$

$$\Pi \tilde{\epsilon} = \pm i \tilde{\epsilon}$$

$$\tilde{\delta}\theta = e^{\pm \frac{i\mu}{4} t} \tilde{\epsilon} \qquad (\delta + \tilde{\delta})\theta = 0$$

longitudinal five brane stretched along $X^{i'}$

$$X^4 = x^4 \cos\frac{\mu t}{6} - x^7 \sin\frac{\mu}{6}t$$

$$X^7 = x^4 \sin\frac{\mu t}{6} + x^7 \cos\frac{\mu t}{6}$$

$$X^5 = x^5 \cos\frac{\mu t}{6} - x^8 \sin\frac{\mu t}{6}$$

$$X^8 = x^5 \sin\frac{\mu t}{6} + x^8 \cos\frac{\mu t}{6}$$

$x^{i'}$: time independant and satisfy

$$[x^{i'}, x^{j'}] = \frac{1}{2}\epsilon_{i'j'k'l'}[x^{k'}, x^{l'}]$$

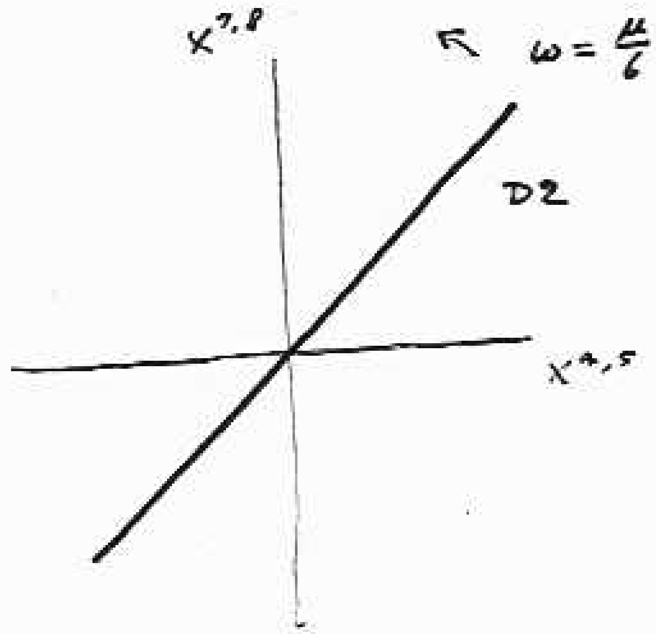
$$\Rightarrow \quad \delta\theta = \frac{i}{2}[X^{i'}, X^{j'}]\gamma_{i'j'}\epsilon$$

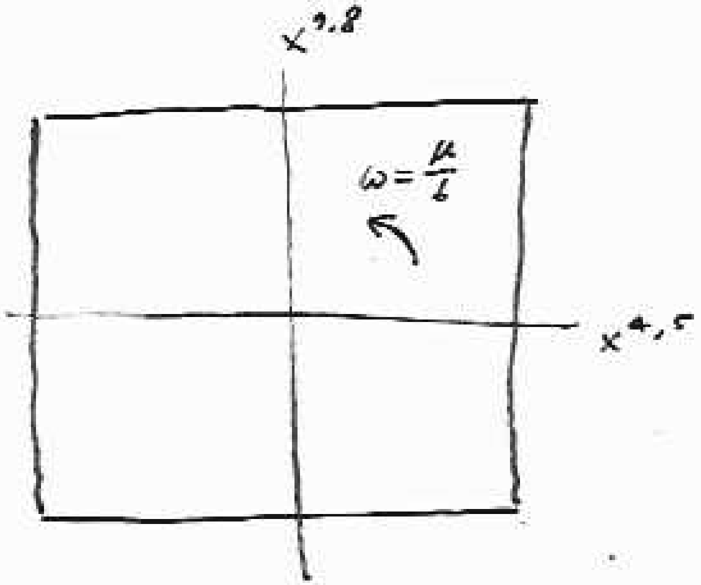
$\Rightarrow$ 4 supercharges

- There are other rotating solutions
- The only static solution is fuzzy sphere

$$[X^i, X^j] = i\frac{\mu}{3R}\epsilon_{ijk}X^k \qquad i=1,\cdots 3$$

- Meaning of flat rotating branes?





$\Rightarrow$ chose wrong coordinates

To study the worldvolume theory of these branes, one may use comoving coordinates.

5D action for longitudinal five branes on a pp-wave

matrix model in the rotating coord. system

$$x^1 \to \cos\frac{\mu x^+}{6}\, x^1 + \sin\frac{\mu x^+}{6}\, x^2$$

$$x^2 \to \cos\frac{\mu x^+}{6}\, x^2 - \sin\frac{\mu x^+}{6}\, x^1$$

$$x^3 \to \cos\frac{\mu x^+}{6}\, x^3 + \sin\frac{\mu x^+}{6}\, x^4$$

$$x^4 \to \cos\frac{\mu x^+}{6}\, x^4 - \sin\frac{\mu x^+}{6}\, x^3$$

bosons X^i : the same as above.

fermions ψ : $\psi \to e^{\frac{\mu}{12}(\Gamma^{12}+\Gamma^{34})t}\,\psi$

(Lorentz tranf.)

$$S = \frac{l_p^6}{R^3}\int dt\left(\mathcal{L}_0 + \mu\mathcal{L}_1 + \mu^2\mathcal{L}_2\right)$$

$$\mathcal{L}_0 = \mathrm{Tr}\Big(\tfrac{1}{2}D_t X^A D_t X_A + \tfrac{1}{4}[X^A, X^B]^2 + \tfrac{i}{2}\psi^\dagger D_t\psi - \tfrac{1}{2}\psi^\dagger\Gamma[X,\psi$$

$$\mathcal{L}_1 = \mathrm{Tr}\Big(-\tfrac{1}{6}J^{ij}X_i D_t X_j - i\tfrac{1}{3}\epsilon^{rst}X_r X_s X_t + \tfrac{i}{24}\psi^\dagger(\Gamma^{12}+\Gamma^{34} + 3\Gamma^{789})\psi\Big)$$

$$\mathcal{L}_2 = -\tfrac{1}{2}\mathrm{Tr}\Big(\tfrac{1}{36}(X_5^2 + X_6^2) + \tfrac{1}{9}(X_7^2 + X_8^2 + X_9^2)\Big)$$

$i, j = 1 \cdots 4$ $\quad r, s = 7, 8, 9$ $\quad A, B = 1 \cdots 9$ $\quad J^{12} = J^{34} = 1$

static flat longitudinal 5-brane configurations

$$[X^1, X^2] + [X^3, X^4] = 0$$

$$[X^1, X^3] + [X^4, X^2] = 0$$

$$[X^1, X^4] + [X^2, X^3] = 0$$

longitudinal 5-branes with stacks of D2 branes in them :

$$X^i = i\,\hat{\partial}_i \qquad i = 1, 2, 3, 4$$

where $\hat{\partial}_i$ is related to the coord. of 4D non-commutative space, x^i

$$x^i = i\,\theta^{ij}\,\hat{\partial}_j$$

$$[x^i, x^j] = i\theta^{ij} \qquad [\hat{\partial}_i, \hat{\partial}_j] = i\theta^{-1}_{ij} \qquad [\hat{\partial}_i, x^j] = \delta_i^j$$

with anti-self-duality condition

$$\theta^{ij} + \frac{1}{2}\epsilon^{ijkl}\,\theta^{kl} = 0$$

NOTE:

$Dp - D(p-2)$ bound state

$=$ Dp with magnetic flux

decoupling limit of $D(p-2)$

$=$ NCSYM limit of Dp

ex) M2/D2 in matrix model

$=$: D2 - D0 bound state

decoupling limit of matrix model

$=$ NCSYM limit of D2

5D NCSYM with Kähler-Chern-Simons and Myers term

$$X_i = \underline{\underline{i\hat{\partial}_i}} + A_i \qquad i = 1, 2, 3, 4$$

background 5-brane

fluctuation

$$X_a = \Phi_a \qquad a = 5, 6, 7, 8, 9$$

$\Rightarrow$

$$D_t X_i = F_{0i} \qquad [X_i, X_j] = i(F_{ij} - \theta^{-1}_{ij})$$

$$[X_i, \Phi] = iD_i\Phi \qquad [X_i, \Psi] = iD_i\Psi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$$

$$J^{ij}\,\mathrm{Tr}(X_i D_t X_j) = -\frac{1}{2}\epsilon^{\lambda\mu\nu ij}\,\mathrm{Tr}\left(A_\lambda\partial_\mu A_\nu - i\frac{2}{3}A_\lambda A_\mu A_\nu\right) J_{ij}$$

$$+ \underbrace{J^{ij}\theta^{-1}_{ij}}_{=0}\,\mathrm{Tr}\,A_0 + \frac{d}{dt}\mathrm{Tr}(iJ^{ij}\hat{\partial}_i A_j$$

$\Rightarrow$ Kähler-Chern-Simons term

J_{ij} : Kähler form

$$J_{ij} = -J_{ji} \qquad J_{12} = J_{34} = 1$$

$$S = \frac{1}{g_{YM}^2} \int d^5x \; \mathcal{L}_0 + \mu \mathcal{L}_1 + \mu^2 \mathcal{L}_2$$

$$g_{YM}^2 = \frac{(2\pi\theta)^2 R^3}{l_p^6} = \frac{(2\pi\theta)^2 g_s}{l_s^3}$$

$$\mathcal{L}_0 = \mathrm{tr}_N \Big[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_\mu \Phi_a D^\mu \Phi_a + \frac{1}{4} [\Phi_a, \Phi_b]^2$$
$$- \frac{i}{2} \Psi^+ \Gamma^\mu D_\mu \Psi - \frac{1}{2} \Psi^+ \Gamma^a [\Phi_a, \Psi] \Big]$$

$$\mathcal{L}_1 = \mathrm{tr}_N \Big[+ \frac{1}{12} \epsilon^{\lambda\mu\nu ij} \, \mathrm{Tr} \, (A_\lambda \partial_\mu A_\nu - i \frac{2}{3} A_\lambda A_\mu A_\nu) J_{ij}$$
$$- \frac{i}{3} \epsilon^{rst} \Phi_r \Phi_s \Phi_t + \frac{i}{48} \Psi^+ (\Gamma^{ij} J_{ij} + 6 \Gamma^{789}) \Psi \Big]$$

$$\mathcal{L}_2 = -\frac{1}{2} \mathrm{tr}_N \Big[\frac{1}{36} (\Phi_5^2 + \Phi_6^2) + \frac{1}{9} (\Phi_7^2 + \Phi_8^2 + \Phi_9^2) \Big]$$

from

$$\mathrm{Tr} \, \mathcal{O}(x) = \frac{1}{(2\pi\theta)^2} \int d^4x \; \mathrm{tr}_N \, \mathcal{O}(x)$$

In matrix model

$$g_s \sim \epsilon^{\frac{3}{4}} \quad , \quad l_s \sim \epsilon^{\frac{1}{4}}$$

so that $\quad (g_{YM}^{(D0)})^2 = \dfrac{g_s}{l_s^3} \quad$: fixed

We take the same limit in 5D NCSYM

with $\quad \theta \sim$ fixed

so that $\quad (g_{YM}^{(5D)})^2 = \dfrac{(2\pi\theta)^2 g_s}{l_s^3} \quad$: fixed

Now, in going commutative limit, we take

$\theta^{ij} \to 0 \quad$ while $\underset{\text{keeping}}{\wedge}$ g_{YM}^2 fixed.

$\Rightarrow$ 5-brane, without the stacks of D2 branes.

Indeed, charge densities become

$$\frac{l_p^6}{R^3} [X^i, X^j] = \frac{1}{g_{YM}^2} \mathcal{O}(\epsilon) \to 0$$

$$\frac{l_p^6}{R^3} \epsilon_{0ijkl} X^i X^j X^k X^l = \frac{1}{g_{YM}^2} \times (\text{const.})$$

longitudinal 5 brane action on a pp-wave

$$S = \frac{1}{g_{YM}^2} \int dx^5 \left(\mathcal{L}_0 + \mu \mathcal{L}_1 + \mu^2 \mathcal{L}_2 \right)$$

$$\mathcal{L}_0 = \mathrm{tr}_N \Big[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2} D_\mu \Phi^a D^\mu \Phi_a + \tfrac{1}{4} [\Phi_a, \Phi_b]^2$$

$$+ \text{ fermions} \Big]$$

$$\mathcal{L}_1 = \mathrm{tr}_N \Big[\tfrac{1}{12} \epsilon^{\lambda\mu\nu ij} \, \mathrm{Tr} \left(A_\lambda \partial_\mu A_\nu - i \tfrac{2}{3} A_\lambda A_\mu A_\nu \right) J_{ij}$$

$$+ \tfrac{1}{6} \epsilon^{pq} \Phi_p D_0 \Phi_q - i \tfrac{1}{3} \epsilon^{rst} \Phi_r \Phi_s \Phi_t \Big]$$

$$\mathcal{L}_2 = -\tfrac{1}{2} \mathrm{tr}_N \left[\left(\tfrac{1}{3}\right)^2 \left(\Phi_7^2 + \Phi_8^2 + \Phi_9^2 \right) \right]$$

SUSY (4 SUSY or "$N = \frac{1}{2}$" SUSY)

$$\delta A_\mu = i \Psi^\dagger \Gamma_\mu \varepsilon \,, \qquad \delta \Phi_a = i \Psi^\dagger \Gamma_a \varepsilon$$

$$\delta \Psi = \Big[\tfrac{1}{2} F_{\mu\nu} \hat{\Gamma}^\mu \Gamma^\nu + D_\mu \Phi_a \hat{\Gamma}^\mu \Gamma^a$$

$$- \tfrac{i}{2} [\Phi_a, \Phi_b] \Gamma^{ab} + \tfrac{\mu}{3} \left(\Phi_p \Gamma^p - \Phi_r \Gamma^r \right) \Gamma^{789} \Big] \varepsilon$$

$$\underset{\text{const spinor}}{\varepsilon} = \Omega \varepsilon \,, \qquad \Omega = \tfrac{1}{4} \left(1 - \Gamma^{1234} - \Gamma^{3456} - \Gamma^{5612} \right)$$

Energy spectra (U(1) case) from EOM.

E	ψ	A_μ	$\Phi_{5,6}$	$\Phi_{7,8,9}$
$E_k = \sqrt{\left(\frac{\mu}{3}\right)^2 + \vec{k}^2}$	4	1	0	3
$E_k^+ = \sqrt{\left(\frac{\mu}{6}\right)^2 + \vec{k}^2} + \frac{\lvert\mu\rvert}{6}$	2	1	1	0
$E_k^- = \sqrt{\left(\frac{\mu}{6}\right)^2 + \vec{k}^2} - \frac{\lvert\mu\rvert}{6}$	2	1	1	0.

3D $\mathcal{N}=2$ massive SYM

on M2/D2 in pp-wave background

By similar reasoning, we get

$$S = \frac{1}{g_{YM}^2} \int d^3x \; \mathcal{L}_0 + \mu \mathcal{L}_1 + \mu^2 \mathcal{L}_2$$

$$\mathcal{L}_0 = \mathrm{tr}_N \Big[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_\mu \phi_a D^\mu \phi_a + \frac{1}{4} [\phi_a, \phi_b]^2$$
$$+ \text{fermions} \Big]$$

$$\mathcal{L}_1 = \frac{1}{\sqrt{2}} \mathrm{tr}_N \Big[\frac{1}{3} (\phi_4 F_{01} + \phi_3 F_{02}) + \frac{1}{6} \epsilon^{pq} \phi_p D_0 \phi_q$$
$$- i \frac{1}{3} \epsilon^{rst} \phi_r \phi_s \phi_t + i \frac{1}{24} \psi^\dagger \Pi \psi \Big]$$

$$\mathcal{L}_2 = -\frac{1}{2} \left(\frac{1}{3\sqrt{2}} \right)^2 \mathrm{tr}_N (\phi_7^2 + \phi_8^2 + \phi_9^2)$$

$\mu, \nu = 0, 1, 2 \qquad a = 3, 4, \cdots 9 \qquad p = 5, 6 \qquad r = 7, 8, 9$

$\epsilon^{56} = \epsilon^{789} = 1$

$\Pi = (\gamma^{14} + \gamma^{23} - \gamma^{56} + 3 \gamma^{789})$

D2 vs. M2

$$\partial_0 \phi^9 = - F_{12} \qquad \phi^9 = X^{11}$$

$$\partial_1 \phi^9 = F_{20} - \frac{\mu}{3\sqrt{2}} \phi^a$$

$$\partial_2 \phi^9 = F_{01} - \frac{\mu}{3\sqrt{2}} \phi^3$$

(generalization of $F_{\mu\nu} = \epsilon_{\mu\nu\rho} \partial^\rho \phi^9$)

Comments :

1. No Lorentz inv.
 world-volume

2. Chern-Simons term can not appear
 in single D2 from M2

3. One can obtain the same U(1) action
 by considering DBI action (for D2)
 or supermembrane action (for M2)
 in pp-wave.

Conclusion.

- By studying M/D-branes on the pp-wave, we have obtained new massive SYM. theories.

- Due to the lack of world-volume Lorentz inv. we could have $\underset{5D}{\wedge}$ '$N=\frac{1}{2}$' SYM theory.

- massive vector multiplets are realized by Kähler-Chern-Simons term in 5D and ϕF term in 3D theory. They maintain gauge invariance.

- It would be interesting to study other dim'nal case as well.