MULTICLOUD SOLUTIONS WITH MASSLESS AND MASSIVE MONOPOLES

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Electric - Magnetic Duality

$N=4$ SUSY Yang-Mills

BPS solutions : Preserve $\frac{1}{2}$ of supersymmetry

Duality : $e \leftrightarrow \frac{4\pi}{e}$

$Q_M \leftrightarrow Q_E$

"elem. part" $\leftrightarrow$ "soliton"

$W^{\pm} \longleftrightarrow$ (Anti-)-monopole

$\gamma \longleftrightarrow \gamma$

$h \longleftrightarrow h$

similar pattern

"gluons" $\longleftrightarrow$ Massless Monopoles

$(m=0, Q_E \neq 0)$

WHAT ARE THESE?

HOW DO THEY INTERACT?

$G \supset SU_2$ Adjoint Φ

$\hookrightarrow$ rank r

Lie algebra: H_i $(i=1,2,\dots r)$ Cartan subalgebra

$E_{\vec{\alpha}},\ E_{-\vec{\alpha}}$ $[H_i, E_{\vec{\alpha}}] = \alpha_i E_{\vec{\alpha}}$

$$r \to \infty: \quad \boxed{\Phi \to \vec{h}\cdot\vec{H}}$$

a) $\vec{h}\cdot\vec{\alpha} \neq 0$, all $\vec{\alpha}$

$$G \to (U_1)^r$$ (Maximal symmetry breaking)

b) $\vec{h}\cdot\vec{\alpha} = 0$, some $\vec{\alpha}$

$$G \to K \times (U_1)^{r-k}$$

$$r \to \infty: \quad [B_i, \Phi] \to 0$$

$$\Rightarrow \quad \boxed{B_i \to \frac{\hat{r}_i}{4\pi r^2}\ \vec{g}\cdot\vec{H}}$$

$$e^{i\vec{g}\cdot\vec{H}} = I$$

Maximal Symmetry Breaking

$G \rightarrow (U_1)^r$

Expect: r topological charges n_j

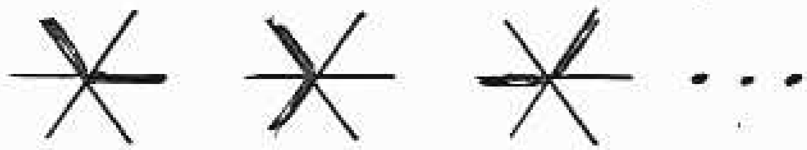
r fundamental monopoles

Simple roots

$\beta_a \ (a=1,2 \cdots r) \rightarrow$ Basis for root lattice

$\rightarrow$ Dynkin diagram

SU(3):



Require $\vec{h}\cdot\vec{\beta}_a > 0 \implies$ unique set

$$\vec{g} = \frac{4\pi}{e} \sum_a n_a \vec{\beta}_a^{\,*}$$

$\vec{\beta}_a^{\,*}$ ← $\vec{\beta}/\vec{\beta}^2$

n_a ← Topological charges

$$M = \vec{g}\cdot\vec{h} = \sum_a n_a \left(\frac{4\pi}{e}\vec{h}\cdot\vec{\beta}_a^{\,*}\right) = \sum_a n_a m_a$$

$$\#\text{ zero modes} = 4\sum_a n_a$$

Fundamental monopoles:

$\vec{\beta}_a \Rightarrow SU_2$ subgroup

$\vec{t}(\vec{\beta}_a) \sim [E_\beta + E_{-\beta},\ E_\beta - E_{-\beta},\ \beta\cdot H]$

$\Rightarrow$ Embedding of SU_2 solution

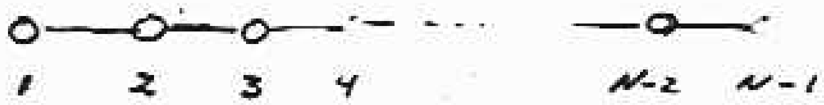
$M = m_a \qquad n_b = \delta_{ab} \qquad$ 4 zero modes

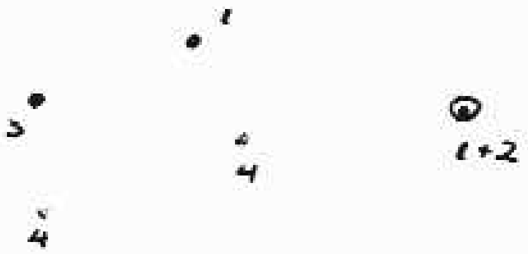
Ex: $SU(N) \rightarrow U(1)^{N-1}$

$$\Phi = \mathrm{diag}\,(\varphi_1, \varphi_2, \cdots, \varphi_N)$$

$$\varphi_1 > \varphi_2 > \cdots > \varphi_N$$

Mass $\sim \varphi_j - \varphi_{j+1}$





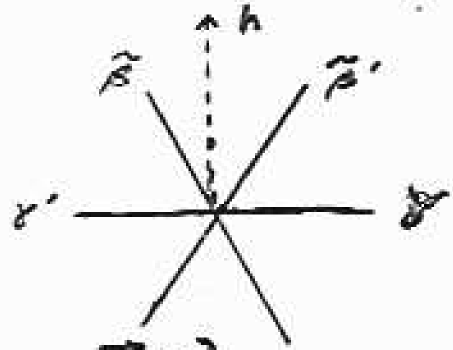
$$\text{Interaction} \sim K_{ij} = \begin{cases} 2 & i=j \\ -1 & i=j\pm 1 \\ 0 & |i-j|>1 \end{cases}$$

$G \to K \times U(1)^{r-k}$

simple roots: $\tilde\beta_1, \tilde\beta_2, \dots, \tilde\beta_{r-k}, \underbrace{\gamma_1, \gamma_2, \dots, \gamma_k}_{K}$

$\vec h \cdot \tilde\beta_a > 0, \quad \vec h \cdot \vec\gamma_i = 0$

$\{\tilde\beta_a, \gamma_i\}$ not unique



$$\vec g = \frac{4\pi}{e}\left\{ \sum \tilde n_a \vec{\tilde\beta}_a^{*} + \sum q_i \vec\gamma_i^{*} \right\}$$

$\sum \tilde n_a \vec{\tilde\beta}_a^{*}$ → topological charges, gauge-indep.

$\sum q_i \vec\gamma_i^{*}$ → gauge-dep.

Assume asymptotically abelian field

$$\vec g \cdot \vec\gamma_i = 0$$

$M = \sum_a \tilde n_a m_a$ <u>No</u> q_i terms

$\#$ zero modes $= 4 \sum_a \tilde n_a + 4 \sum_i q_i$

Fundamental monopoles: $\beta_a \leftrightarrow$ massive

$\gamma_i \leftrightarrow$ massless

Embedded SU(2) solutions: <u>only</u> for β_a

solutions with massive and massless monopoles

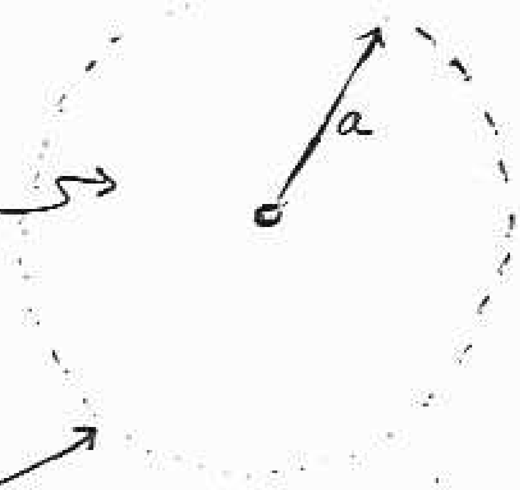
$SO(5) \to SU(2) \times U(1)$

Charge = $(1, [1])$

$SU(2) \times U(1)$ Coulomb field

$U(1)$ Coulomb field

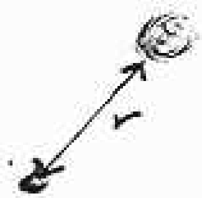
"Massless monopole cloud"



a is arbitrary, E indep. of cloud size

$SO(5) \to U(1) \times U(1)$

Charge $(1, 1)$



$r \longleftrightarrow a$

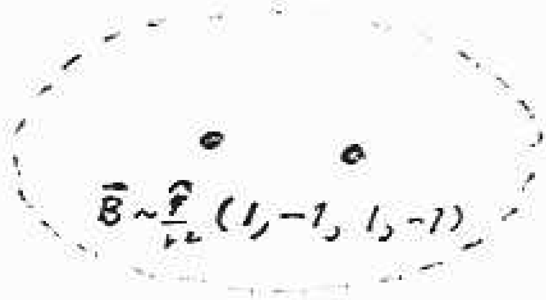
$\left.\begin{matrix} \theta, \varphi \\ \chi \end{matrix}\right\} \longleftrightarrow (\alpha, \beta, \gamma)$

$a \longleftrightarrow$ "position of massless monopole"

$SU(4) \to U(1) \times SU(2) \times U(1)$

$\vec{B} \sim \frac{\hat{r}}{r^2}(2,0,0,-2)$

charge $= (1, [1], 1)$



parameters:

massive positions	6
global $U(1) \times SU(2) \times U(1)$	5
cloud size	1

$\Updownarrow$

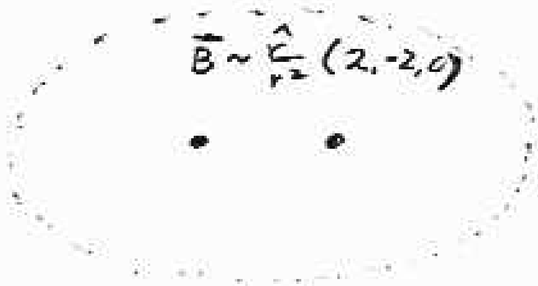
"massless mono. position"

$SU(3) \to SU(2) \times U(1)$

$\vec{B} \sim \frac{\hat{r}}{r^2}(2,-1,-1)$

charge $= (2, [1])$

"Dancer solution"



Fields not known analytically

parameters:

massive positions	6
$U(1) \times SU(2)$	4
rel $U(1) \approx$ axial rot.	1
cloud size	1

Move complex situations

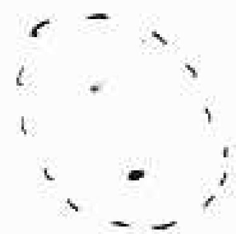
1) Does (# of massless monopoles)

= (# of clouds) ?

2) Do monopoles group into well separated clouds ?

$SU(N) \to U(1) \times SU(N-2) \times U(1)$

a) $(1, [1], [1] \cdots [1], 1)$ solutions
2 massive + $(N-3)$ massless
but 1 cloud
But: All are $SU(4)$ imbeddings $\Rightarrow$ degenerate case ?



b) $(2, [2], \cdots [2], 2)$ solutions

Do these generically split into

two $(1, [1], \cdots, 1)$ solutions?

Nahm Construction $\leftrightarrow$ D-brane construction

$SU(2)$, charge k, $\langle \phi \rangle = \mathrm{diag}(\mu/2, -\mu/2)$

1) Nahm data

$$T_i(s) \qquad -\mu/2 \le s \le \mu/2$$

$k \times k$ Hermitian

$$\frac{dT_i}{ds} = -\frac{1}{2}\epsilon_{ijk}\,[T_j, T_k]$$

$k=1 \Rightarrow$ constants

$k=2 \Rightarrow$ elliptic functions

2) Construction equation

$$v_a(s, \vec{x})$$

$2k$ components

labels indep. solutions

$$\frac{dv_a}{ds} = -(T_i - I r_i) \otimes \sigma_i \; v_a$$

$$\delta_{ab} = \langle v_a | v_b \rangle = \int_{-\mu/2}^{\mu/2} ds \; v_a^{+}(s)\, v_b(s)$$

3) Fields

$$\phi_{ab} = \langle v_a | s | v_b \rangle = \int_{-\mu/2}^{\mu/2} ds \; s\, v_a^{+}(s)\, v_b(s)$$

$$\vec{A}_{ab} = \langle v_a | -i\vec{\nabla} | v_b \rangle = -i\int_{-\mu/2}^{\mu/2} ds \; v_a^{+}\, \vec{\nabla}\, v_b$$

Behavior near poles:

$$T_i \sim \frac{J_i}{t-t_0}$$

Nahm eq $\Rightarrow$ $J_i = \frac{i}{2} \epsilon_{ijk} [J_j, J_k]$

$J_i \leftrightarrow SU(2)$ rep'n

Construction eq

$$0 = \Delta^+ v \approx -\left[\frac{d}{dt} - \frac{J_i \otimes \sigma_i}{t-t_0}\right] v$$

Suppose J_i = irreducible spin $\left(\frac{k-1}{2}\right)$

$\Rightarrow$ $J_i \otimes \sigma_i$ has eigenvalues $\begin{cases} \frac{(k-1)}{2} \\ -\frac{(k+1)}{2} \end{cases}$

$\Rightarrow$ Near t_0,

$$v_a \sim \begin{cases} \eta_a (t-t_0)^{\left[\frac{k-1}{2}\right]} \\ \cancel{\eta_a (t-t_0)^{-\left[\frac{k+1}{2}\right]}} \end{cases}$$

$\Rightarrow$ $k-1$ conditions at each pole

Irreducible poles at t_{max}, t_{min} give
$2(k-1)$ conditions
$\Rightarrow$ $2k - 2(k-1) = 2$ normalizable v_a

$\Rightarrow$ $SU(2)$ fields

$k = 1$: Nahm data = constants

Construction eq $\Rightarrow$ usual BPS solution

$k = 2$: Nahm data ~ elliptic functions

Construction eq $\Rightarrow$ not soluble in closed form

$k > 2$: Nahm data not known for generic case

<u>BUT</u>

For k widely-separated monopoles, $\vec{\Phi}(\vec{x})$ & $\vec{B}(\vec{x})$ are just sums of poles + exp'ly small corrections

$SU(N)$, charge $(k_1, k_2, \ldots, k_{N-1})$

$$\langle \phi \rangle = (t_N, t_{N-1}, \ldots, t_1)$$

p^{th} interval: $\quad T_j(t) = k_p \times k_p$

$\quad v_a(t, \vec{x})$ has $2k_p$ components

<u>Boundary conditions:</u>

1) $k_n = k_{n+1} + m \quad , \quad m \geq 2$

$$T_i^{(n)} \sim \left(\begin{array}{c|c} T_i^{(n+1)} + O(t) & O[t^{(m-1)/2}] \\ \hline O[t^{(m-1)/2}] & \dfrac{J_i}{t} + O(1) \end{array} \right)$$

2) $k_n = k_{n+1} + 1$

$$T_i^{(n)} \sim \left(\begin{array}{c|c} T_i^{(n+1)} + O(t) & O(1) \\ \hline O(1) & O(1) \end{array} \right)$$

3) $k_p = k_{p+1}$: "jump data" $a_{r\alpha}$

$$(\Delta T_i)_{rs} = \frac{1}{2} a^*_{r\alpha} (\sigma_i)_{\alpha\beta} a_{s\beta}$$

$$(\Delta v_a) = S_a(\vec{x}) a$$

$$\delta_{ab} = \langle V_a | V_b \rangle = \int_{t_1}^{t_N} dt \; v_a^+(t) v_b(t) + \sum_p S_a^{(p)+} S_b^{(p)}$$

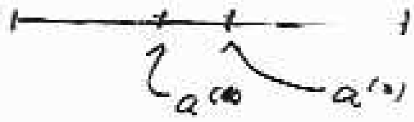
$$\Phi_{ab} = \langle V_a | t | V_b \rangle = \int_{t_1}^{t_N} dt \; t \, v_a^+ v_b + \sum_p t_p S_a^{(p)+} S_b^{(p)}$$

$$\vec{A}_{ab} = \langle V_a | -i\vec{\nabla} | V_b \rangle$$

$$= -i \int_{t_1}^{t_N} dt \; v_a^+ \vec{\nabla} v_b - i \sum_p S_a^{(p)+} \vec{\nabla} S_b^{(p)}$$

$SU(4) \to U(1) \times U(1) \times U(1)$

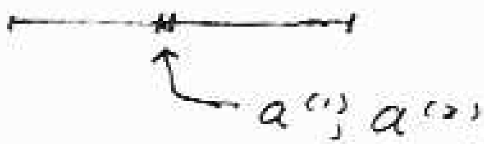
$(1, 1, 1)$



$T_i = 1 \times 1$, piecewise constant

$SU(4) \to U(1) \times SU(2) \times U(1)$

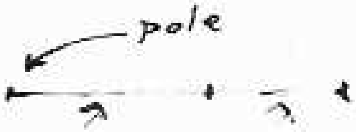
$(1, [1], 1)$



$T_i = 1 \times 1$, piecewise constant

$SU(3) \to U(1) \times U(1)$

$(2, 1)$



$SU(3) \to SU(2) \times U(1)$

$(2, [1])$

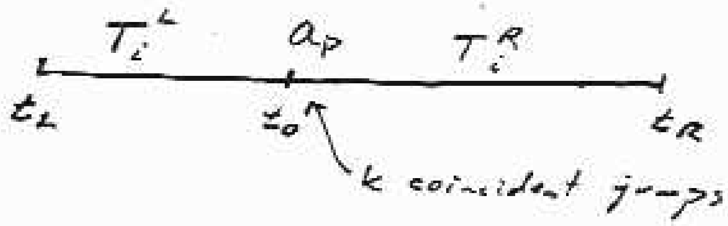
$SU(N) \to U(1) \times SU(N-2) \times U(1)$

$\langle \Phi \rangle = \mathrm{diag}(t_R, t_0, t_0 \cdots t_0, t_L)$

Charge $(k, [k], \cdots [k], k)$

$N = 2k+2 \Rightarrow$ generic
$N > 2k+2 \Rightarrow$ embedded $SU(2k+2)$ solutions

~~But~~ Choose $N = 2k+2$



$$(\delta T_i)_{rs} = \frac{1}{2} \sum_{p=1}^{k} a_{pr}^{+} \sigma_i a_{ps}$$

$$(T_4)_{rs} = \frac{1}{2} \sum_{p=1}^{k} a_{ps}^{+} a_{pr}$$

$$K \equiv T_4 \otimes I + \delta T_i \otimes \sigma_i = \sum_{p=1}^{k} a_p \otimes a_p^{+}$$

$2k \times 2k$ $\qquad$ $2k$-vector

$SU(k)$ Invariance: $a_p \to a_p' = U_{pq} a_q$

Require $a_p^{+} a_q = \lambda_p \delta_{pq}$
(reason why $N = 2k+2$ generic)

$$K^{-1} = \sum_{p=1}^{2k} \frac{1}{\lambda_p^2} a_p \otimes a_p^{+}$$

T_i^L ?

$k=1$: $T_i^L = \text{const.}$

$k=2$: T_i^L = Dancer T_i, pole at t_L

$k>2$: $T_i^L \longleftrightarrow$ "generalized Dancer"

$\leftrightarrow (k, [k-1], [k-2], \ldots, [1])$
for $SU(k+1) \to U(1) \times SU(k)$

Pole at t_L, ~~no~~ finite at t_0

T_i^R : similar, pole at t_R

Assume that solution for "Dancer" problem known

$$T_i^L(t) \Rightarrow W_a^L(\vec{r}, t)$$

$$\delta_{ab} = \int_{t_L}^{t_0} dt\, W_a^L(\vec{r}, t)^+ W_b^L(\vec{r}, t)$$

$$\varphi_{ab}^L = \int_{t_L}^{t_0} dt\, (t-t_0)\, W_a^L(\vec{r}, t)^+ W_b^L(\vec{r}, t)$$

$$\vec{A}_{ab}^L = -i \int_{t_L}^{t_0} dt\, W_a^L(\vec{r}, t)^+ \vec{\nabla} W_b^L(\vec{r}, t)$$

+ similar for $L \longleftrightarrow R$

Full $SU(2k+2)$ problem:

$T_i^L, T_i^R, \underline{T_4} \Rightarrow$ solution

Need orthonormal $V_a = (v_a^L, v_a^R, S_a^P)$

First, find linearly indep., but **NOT** orthonormal, $\tilde{V}_a$

$$\tilde{V}_a = \begin{cases} (0, -w_a^R, \tilde{S}_a^P) \\ \\ (w_{a-k+1}^L, 0, \tilde{S}_a^P) \end{cases}$$

$$V_a^0(\vec{r}) \equiv \tilde{v}_a^L(t_0,\vec{r}) - \tilde{v}_a^R(t_0,\vec{r}) = \sum_P \tilde{S}_a^P(\vec{r})\, a_P$$

$$\Rightarrow \tilde{S}_a^P(\vec{r}) = \frac{1}{\lambda_P}\, a_P^\dagger V_a^0(\vec{r})$$

$$\begin{aligned} \langle \tilde{V}_a | \tilde{V}_b \rangle &= \delta_{ab} + \sum_P \frac{1}{\lambda_P^2} (V_a^{0\dagger} a_P)(a_P^\dagger V_b^0) \\ &= \delta_{ab} + V_a^{0\dagger} K^{-1} V_b^0 \\ &\equiv B_{ab} \end{aligned}$$

$$V_a = \tilde{V}_b \, (B^{-1/2})_{ba}$$

$$\Phi_{ab} = \langle V_a | t | V_b \rangle$$

$$= \langle V_a | (t - t_0) | V_b \rangle + t_0 \delta_{ab}$$

$$= (B^{-1/2})_{ac} \langle \tilde{V}_c | (t - t_0) | \tilde{V}_d \rangle (B^{-1/2})_{db} + t_0 \delta_{ab}$$

$$\Phi = B^{-1/2} \varphi B^{-1/2} + t_0 I$$

$$\varphi = \begin{pmatrix} \varphi^R_{\text{Dancer}} & 0 \\ 0 & \varphi^L_{\text{Dancer}} \end{pmatrix}$$

Far from massive cores, can choose:

one	W_a^L	concentrated	near	t_L
$(k-1)$	"	"	"	t_0
one	W_a^R	"	"	t_R
$(k-1)$	W_a^R	"	"	t_0

$$\Rightarrow \quad \varphi^R = \begin{pmatrix} \phi^R & 0 \\ 0 & \hat{\phi}^R \end{pmatrix} + \phi^R_\infty$$

$$\varphi^L = \begin{pmatrix} \hat{\phi}^L & 0 \\ 0 & \phi^L \end{pmatrix} + \phi^L_\infty$$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \hat{B} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \phi^R & 0 & 0 \\ 0 & \hat{B}^{-1/2}\,\hat{\varphi}\,\hat{B}^{-1/2} & 0 \\ 0 & 0 & \phi^L \end{pmatrix}$$

$$B = I + V^{o}(\bar{r})^{+} K^{-1} V^{o}(\bar{r})$$

$V^{o}(\bar{r}) \sim |\bar{r} - \bar{r}_{mon}|$, (Dancer cloud size)

$K^{-1} \sim \frac{1}{\lambda_{P}}$

All λ_p large $\Rightarrow K^{-1}$ small, $B \approx I$

$\rightarrow$ Unshielded Dancer fields "inside cloud"

All λ_p small $\Rightarrow K^{-1}$ large, $\hat{B}$ large, $\hat{B}^{-1/2}$ small

$\Rightarrow$ Non-Abelian fields shielded "outside cloud"

$SU(6) \longrightarrow U(1) \times SU(4) \times U(1)$

$$\beta_1 - \gamma_1 - \delta_2' - \delta_3' - \beta_2$$

charge $(2, [2], [2], [2], 2) \Rightarrow$ 4 massive + 6 massless

$10 \times 4 - (17$ global gauge$) \Rightarrow 23$ non-gauge parameters

Massive positions	$\vec{x}_1 \ldots \vec{x}_4$	12
Dancer cloud size	a_L, a_R	2
Dancer relative $U(1)$ [weak dependence]		2
Dancer $SU(2)$ orientation	6	
$SU(2)$ angles	2	5
"$SU(4)$"-cloud" parameters	p, q	2

$T_4 = p + \vec{q}\cdot\vec{\tau}$

Ingredients for construction:

Left Dancer: $T_i^L(t_0), v_a^L(t_0), \varphi^L(\vec{x})$

Right Dancer: $T_i^R(t_0), v_a^R(t_0), \varphi^R(\vec{x})$

SU(4)-cloud: $p + \vec{q} \cdot \vec{T}$

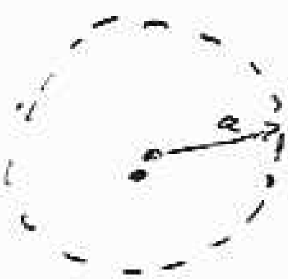
[illegible]

[illegible] $(p + \vec{q} \cdot \vec{T}) \otimes I + \Sigma$ [illegible] positive

Tractable Dancer cases:

1) "Minimal cloud"

2) "Large cloud"



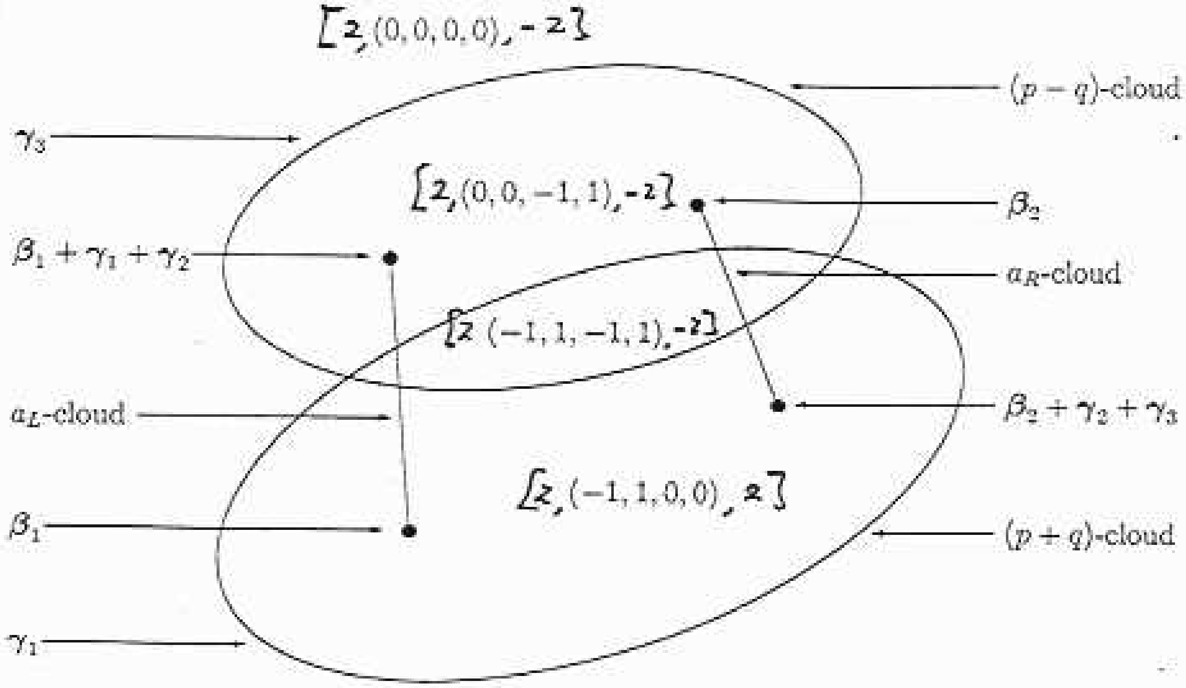


FIG. 2. A solution with two minimal Dancer clouds and all cloud SU(2) orientations aligned. The clouds are labelled both by the relevant distance scale and by the associated massless monopole. As described in Sec. VII, the location of the massless monopoles at the end of the Dancer clouds is a gauge-dependent choice. The diagonal elements of Q_{NA}, which is assumed to be a diagonal matrix, are shown for each of the regions defined by the clouds.

$$\vec{B} \sim \frac{\text{"}\hat{r}\text{"}}{r^2} \times \mathrm{diag}[2, (\;,\; ,\; ,\;), -2] + O(1/r^3)$$

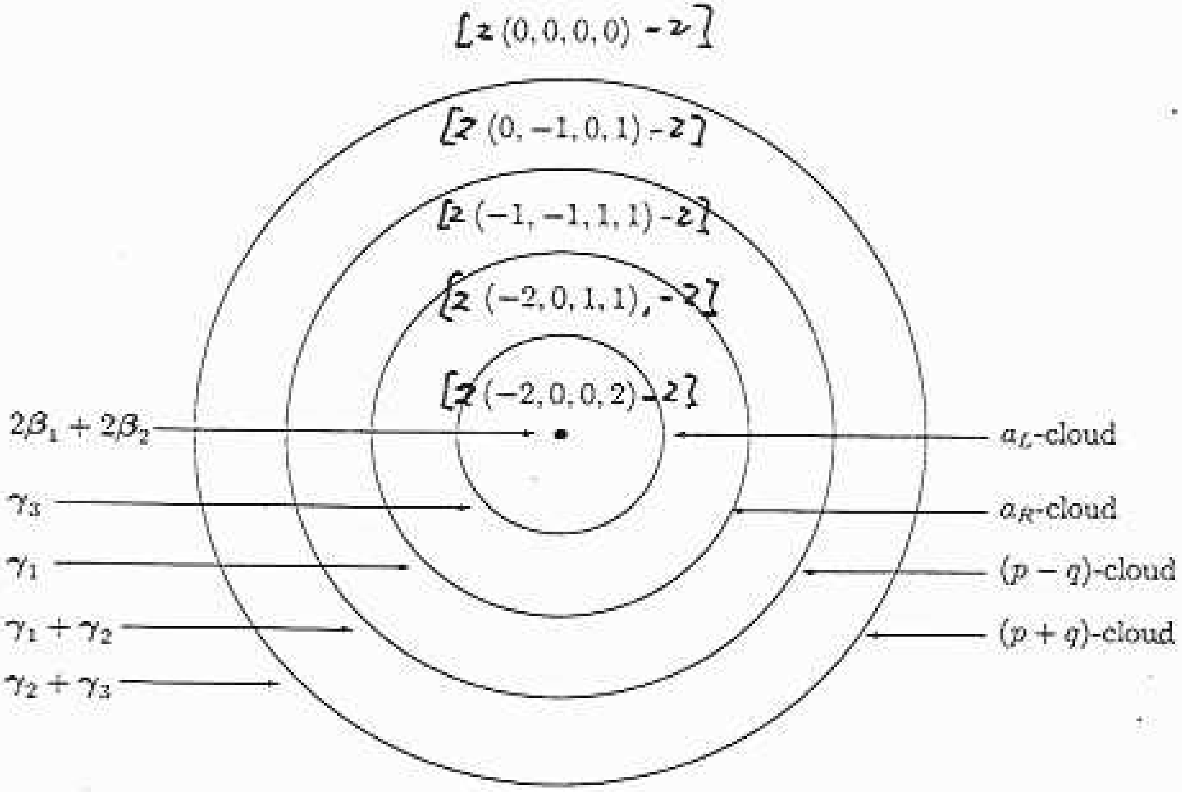


FIG. 5. Schematic illustration of a solution with two concentric large Dancer clouds. The clouds are labelled both by the relevant distance scale and by the associated massless monopole. The diagonal elements of Q_{NA}, which is assumed to be a diagonal matrix, are shown for each of the regions defined by the clouds.

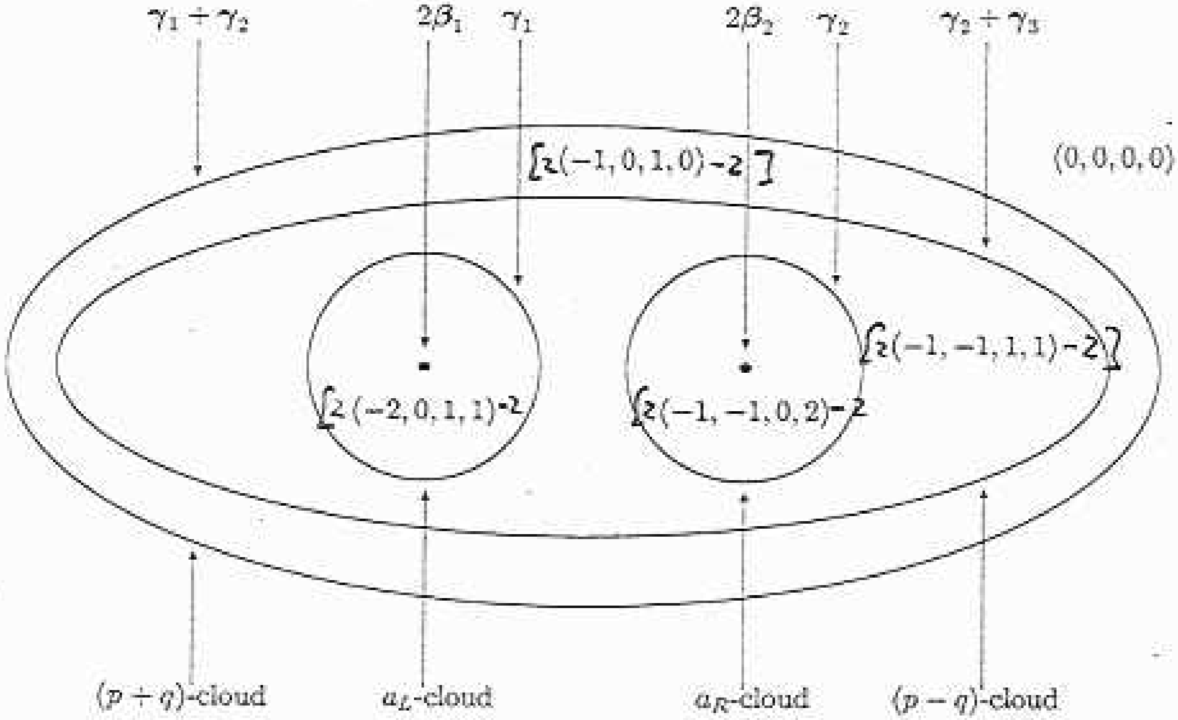


FIG. 4. Schematic illustration of a solution with two widely-separated large Dancer clouds. The clouds are labelled both by the relevant distance scale and by the associated massless monopole. The diagonal elements of Q_{NA}, which is assumed to be a diagonal matrix, are shown for each of the regions defined by the clouds.

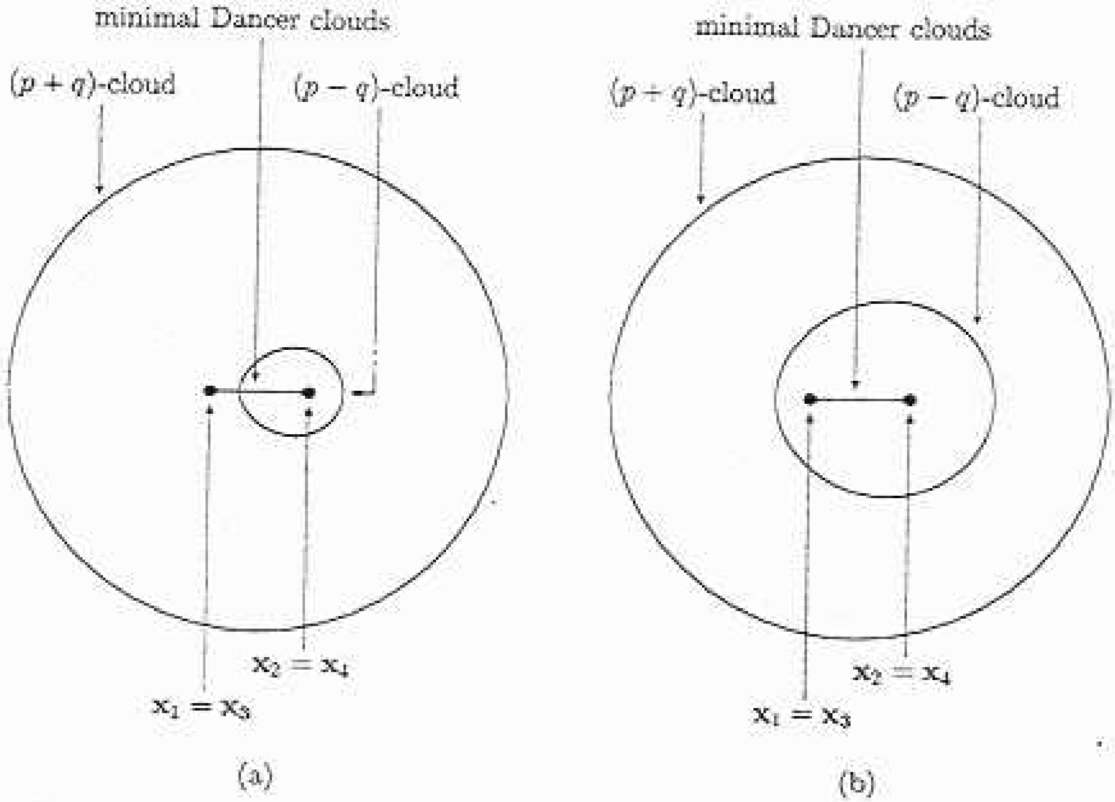


FIG. 3. Schematic illustration of a solution with minimal Dancer clouds and coincident massive monopoles, as discussed in Subsec. VI A 2. In both figures the $(p+q)$-cloud is roughly defined by the curve $\lambda_+ = 1$ with $p+q = 5D$ and the $(p-q)$-cloud by the curve $\lambda_- = 1$, where in (a) $p-q = 1.2D$ and in (b) $p+q = 2.2D$. In each case $\alpha = 4\pi/3$ and there are coincident β_1- and β_2-monopoles at $\mathbf{x}_1 = \mathbf{x}_3$ and at $\mathbf{x}_2 = \mathbf{x}_4$. The massless monopole locations are not shown, but calculating Q_{NA} in different regions indicates that there are γ_1- and γ_3-monopoles on the minimal Dancer clouds, a $(\gamma_2+\gamma_3)$-monopole on the $(p-q)$-cloud and a $(\gamma_1+\gamma_2)$-monopole on the $(p+q)$-cloud.

Massive monopoles:

Gauge equiv. sets:

a)	β_1	$\beta_1+\gamma_1$	$\beta_1+\gamma_1+\gamma_2$	$\beta_1+\gamma_1+\gamma_2+\gamma_3$
Q_{NA}:	$(-1,0,0,0)$	$(0,-1,0,0)$	$(0,0,-1,0)$	$(0,0,0,-1)$
b)	β_2	$\beta_2+\gamma_3$	$\beta_2+\gamma_3+\gamma_2$	$\beta_2+\gamma_3+\gamma_2+\gamma_1$
Q_{NA}	$(0,0,0,1)$	$(0,0,1,0)$	$(0,1,0,0)$	$(1,0,0,0)$

Massless monopoles

γ_1	γ_2	γ_3	$\gamma_1+\gamma_2$	$\gamma_2+\gamma_3$
$(1,-1,0,0)$	$(0,1,-1,0)$	$(0,0,1,-1)$	$(1,0,-1,0)$	$(0,1,0,-1)$

$\gamma_1+\gamma_2+\gamma_3$
$(1,0,0,-1)$

Adding massless monopole to give cloud is possible only if

1) $q = Q_{NA}$(massless mono.) is <u>not</u> in the subgroup that leaves Q_{NA} invariant

2) $Q'_{NA} = q + Q_{NA}$ is not gauge-equiv to Q_{NA}

Find: 6 "massless monopoles"

$2 \times (\gamma'_1, \gamma'_2, \gamma'_3)$

must be added in no more than 4 steps

6 massless monos $\Longrightarrow$ 4 clouds

Time-dependent solutions

Use moduli space approximation (MSA)

Only massive monopoles $\Rightarrow$ valid for low E

Apply to $SU(4)$ $(1,[1],1)$

$r = |\vec{x}_1 - \vec{x}_2| \rightarrow vt$

cloud size $\rightarrow kt^2$

MSA must fail at large $|t|$.

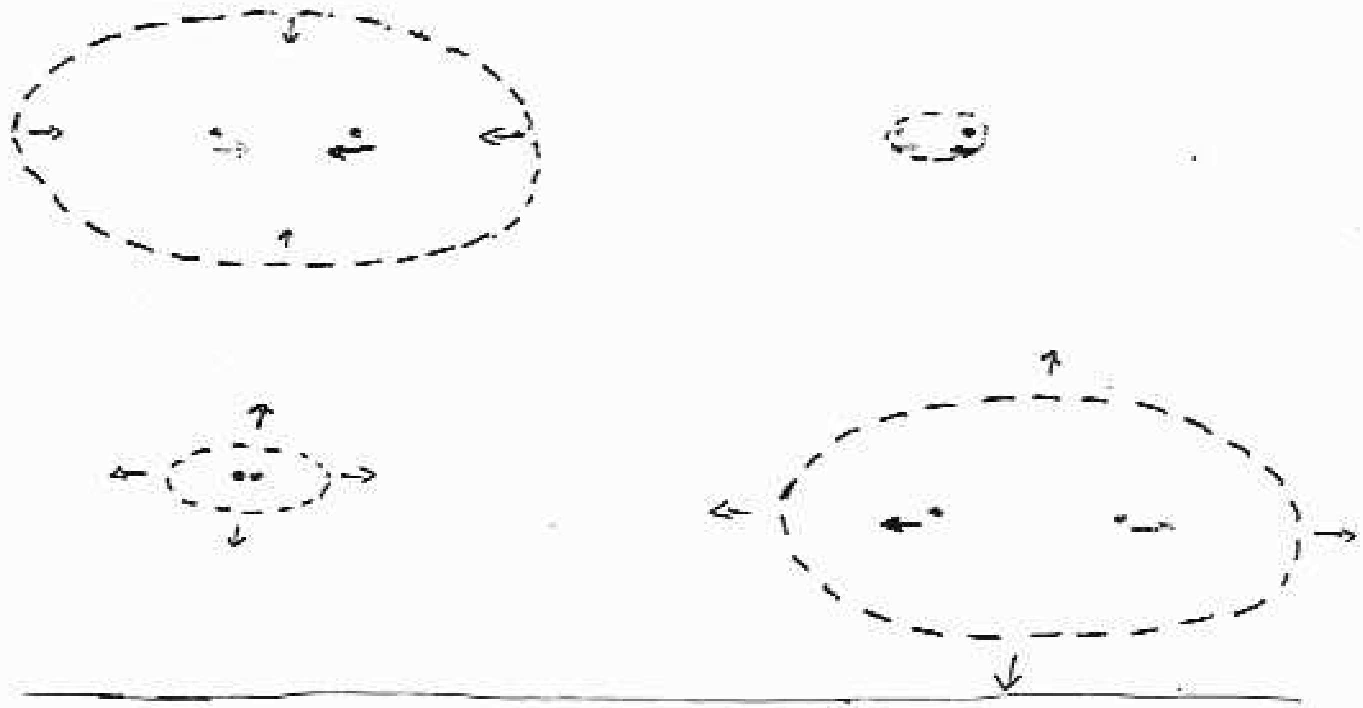
Analytic, numerical arguments

$\Rightarrow$ Breakdown when

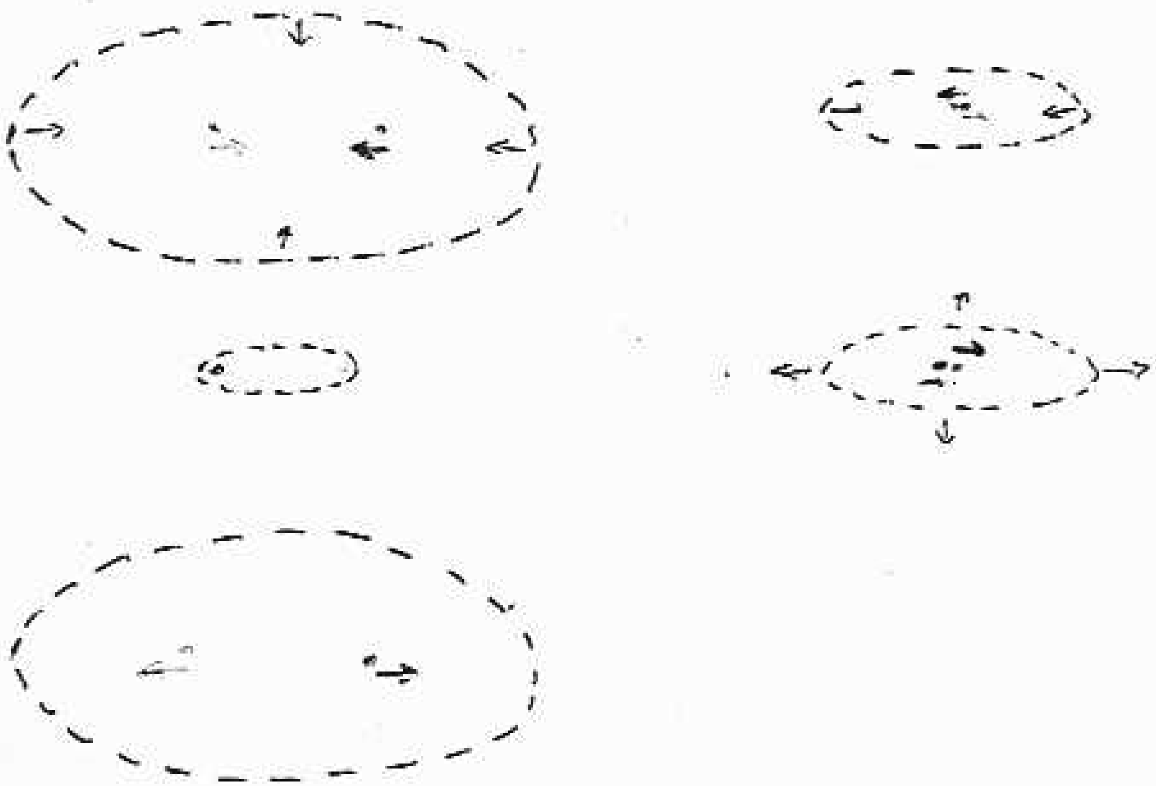
$$\frac{d}{dt}[b_{MSA}(t)] = O(1)$$

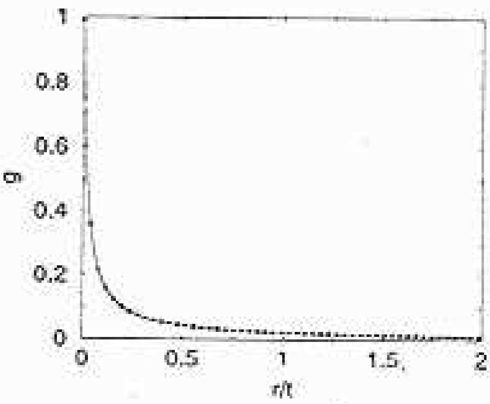
$$\longleftrightarrow t \sim t_{crit} = \frac{1}{e^2 E}$$

d)



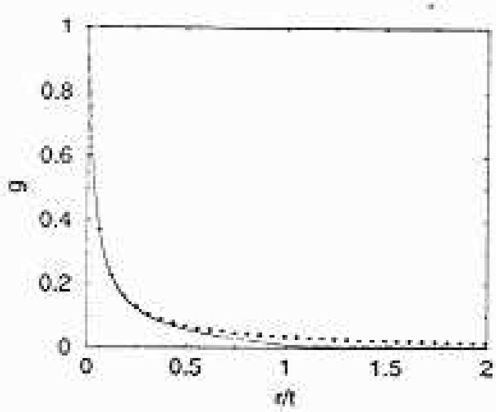
e)





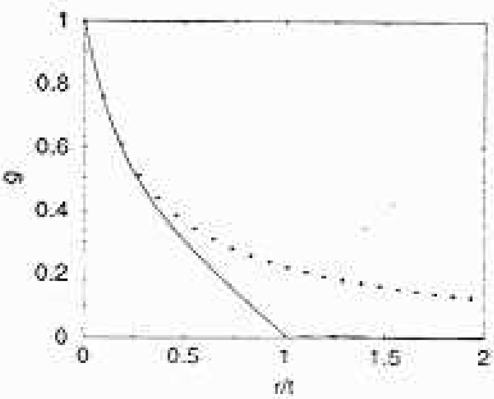
$t = 101\, a_0$

(a)



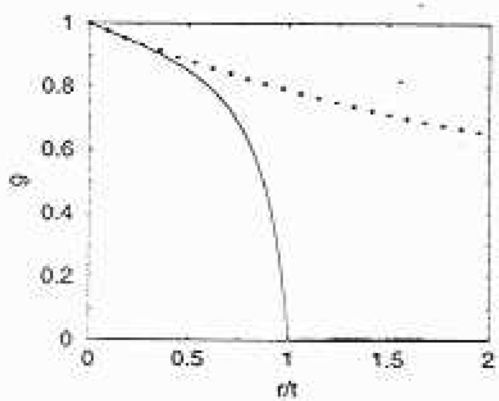
$t = 1030\, a_0$

(b)



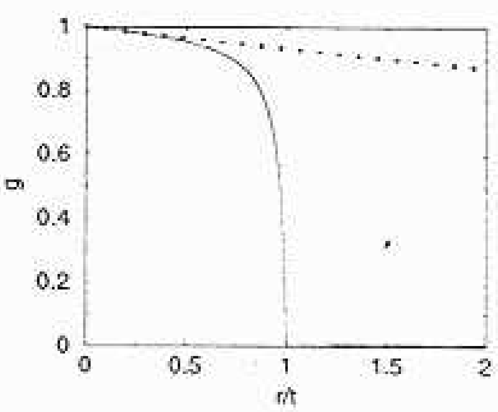
$t = 1.11 \times 10^4\, a_0$

(c)



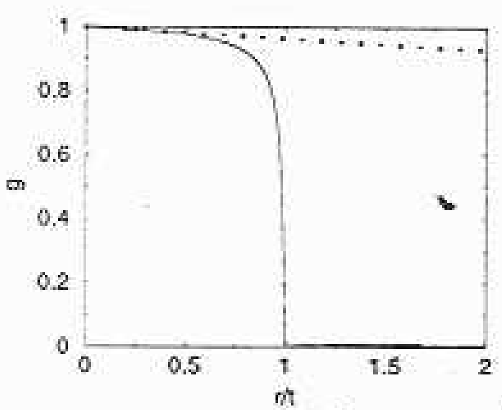
$t = 1.49 \times 10^5\, a_0$

(d)



$t = 5.45 \times 10^5\, a_0$

(e)



$t = 1.07 \times 10^6\, a_0$

(f)

$$e \longleftrightarrow \frac{4\pi}{e}$$

Magnetic	solitons $\leftrightarrow$ elementary	
Electric	elementary $\leftrightarrow$ solitons	

Small e solitons

Massive monopoles

$$r_{core} \sim \frac{1}{e^2 M} \gg (\Delta r) \sim \frac{1}{M}$$

Massless monopoles

$$t_{crit} \sim \frac{1}{e^2 E} \gg (\Delta t) \sim \frac{1}{E}$$

Summary / Conclusions

1) # clouds < # massless monopoles

2) Clouds do not generically split into disjoint subsets

3) Soliton picture valid only for a time $\Delta t \sim \frac{1}{e^2 E}$