Buildings and Non-Associative Algebras

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We give a brief overview of Tits's classification of spherical buildings, emphasizing the role of certain non-associative division algebras: octonions, quadratic Jordan algebras of degree three and a third exotic type we call "quadrangular" algebras. These are precisely the algebraic structures that arise in the study of the spherical buildings which are associated to exceptional groups of relative rank at least two. The remaining spherical buildings are classified by classical algebraic structures: fields, skew-fields, anisotropic quadratic spaces, etc.

We will also say a few words about the problem of determining when a given spherical building is the "building at infinity" of an affine building. This, it turns out, is roughly equivalent to asking to what extent the corresponding algebraic structures exist over a field which is complete with respect to a discrete valuation.