Improving inflation using Non-canonical scalars[a]

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The role of inflation

- **INFLATION** $\Rightarrow$ $\left\{ \begin{array}{l} \bullet \text{ Explains Large Scale Homogenity} \\ \bullet \text{ A mechanism for generating density perturbations} \\ \bullet \text{ Explanation for nearly flat universe} \end{array} \right.$

⋆ **What drives inflation $\Rightarrow$ Scalar fields**

Classification of scalar field models

- **Canonical Scalar Field** $\Longrightarrow \mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$

  $*$ $V(\phi) = V_0\phi^n \rightarrow$ **Chaotic inflation models (Linde 1983)**

  $*$ $V(\phi) = V_0 \exp\left[-\sqrt{2/p}(\phi/M_{_{pl}})\right] \rightarrow$ **Power law inflation** $a(t) \propto t^p$

- **Non canonical Scalar Field** $\Longrightarrow \mathcal{L}_\phi = \mathcal{L}(X,\phi)$ where $X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$

$$\Downarrow$$

$*$ $\mathcal{L}(X,\phi) = F(X) - V(\phi)$

or

$*$ $\mathcal{L}(X,\phi) = V(\phi)F(X)$

  $*$ **These class of models are also known as K-inflation models**

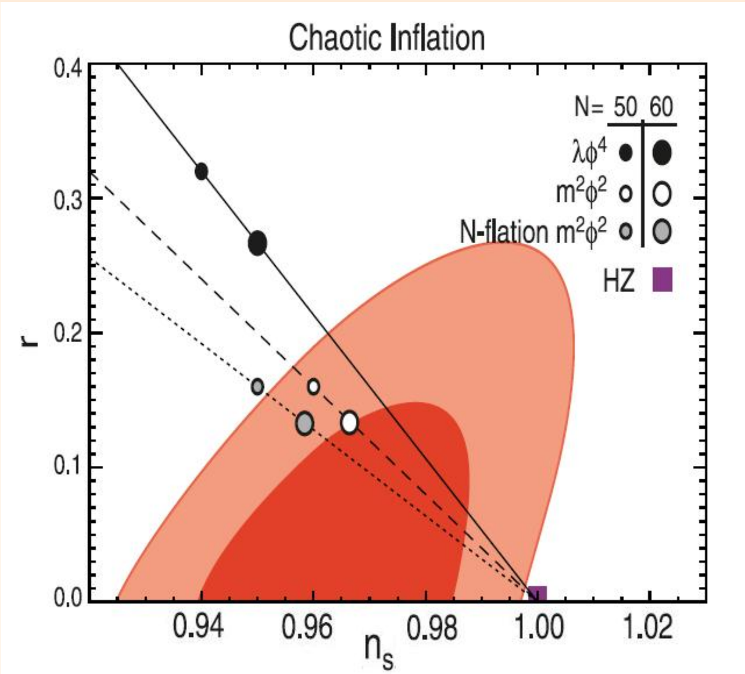
Aim of this talk

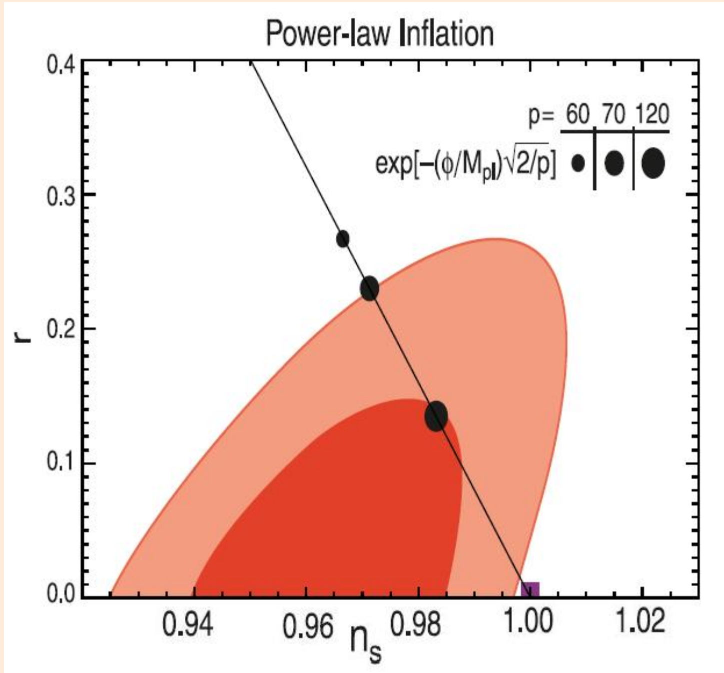
- **How good is non canonical inflation in comparison to the canonical inflation ?**
- **What class of potentials allows inflation in the non canonical setting ?**
- **How can one distinguish non canonical inflation from the canonical inflation ?**

Inflation using canonical scalar fields

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

- **Chaotic inflation models (Linde 1983)** $\rightarrow V(\phi) = V_0\phi^n$
- **Power law inflation** $a(t) \propto t^p \Rightarrow V(\phi) = V_0 \exp\left[-\sqrt{2/p}(\phi/M_{pl})\right]$





From Komatsu et al (2011)

Inflation using non canonical scalar fields

A specific model

$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - \,V(\phi)$$

where $X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$

- M **is a constant with dimension of mass**
- α **is the dimensionless parameter of the theory.**
- $\alpha = 1$ **corresponds to canonical scalar field.**

The above Lagrangian can be viewed as a generalization of the usual Lagrangian for the canonical scalar field

Slow roll parameters

- **The slow roll parameters ϵ and δ are defined as**

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} \qquad \text{and} \qquad \delta \equiv \varepsilon - \frac{\dot{\varepsilon}}{2\,H\,\varepsilon}$$

- **It follows from the Friedmann equation that**

$$\frac{\ddot{a}}{aH^2} = 1 - \varepsilon$$

- **Therefore, inflation ($\ddot{a} > 0$) occurs when $\varepsilon < 1$ and ends at $\varepsilon = 1$**
- **EOS parameter w_ϕ is related to ε as $\rightarrow w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \left(\frac{2\varepsilon}{3}\right) - 1$**
- **Slow roll inflation occurs when $\varepsilon << 1$ which gives $p_\phi \simeq -\rho_\phi$**
- **Slow roll approximation is defined as**

$$\varepsilon << 1 \quad \text{and} \quad |\delta| << 1$$

Solution in the slow roll limit

The slow roll assumptions ($\varepsilon \ll 1$ and $|\delta| \ll 1$) leads to

$$\dot{\phi} = -\theta \left\{ \left(\frac{M_{_{pl}}}{\alpha\,\sqrt{3}}\right) \left(\frac{\theta\, V'(\phi)}{\sqrt{V}}\right) \left(2\, M^4\right)^{\alpha-1} \right\}^{\frac{1}{2\alpha-1}}$$

where

$$\theta = +1 \text{ when } V'(\phi) > 0$$
$$\theta = -1 \text{ when } V'(\phi) < 0.$$

- **In which regime of the potential $V(\phi)$ is the above solution valid ?**

Potential slow roll parameter

- **The slow roll condition $\varepsilon \ll 1$ and $|\delta| \ll 1$ implies that**

$$\varepsilon_{\mathrm{V}} \ll 1 \quad \text{and} \quad \delta_{\mathrm{V}} \ll 1$$

$$\begin{aligned} \varepsilon_{\mathrm{V}} &\equiv \left\{ \left(\frac{1}{\alpha}\right) \left(\frac{3\,M^4}{V(\phi)}\right)^{\alpha-1} \left(\frac{M_{pl}\,V'(\phi)}{\sqrt{2}\,V(\phi)}\right)^{2\alpha} \right\}^{\frac{1}{2\alpha-1}} \\ \delta_{\mathrm{V}} &\equiv \left(\frac{\alpha}{2\alpha-1}\right)(\eta_{\mathrm{V}} - \varepsilon_{\mathrm{V}}) \end{aligned}$$

where

$$\eta_{\mathrm{V}} \equiv 2\,\varepsilon_{\mathrm{V}} \left(\frac{V(\phi)V''(\phi)}{V'(\phi)^2}\right)$$

In the canonical limit

$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi)$$

- $\alpha = 1 \;\Rightarrow\;$ **Canonical Scalar field** $\rightarrow \mathcal{L}(X,\phi) = X - V(\phi)$
- ε_{V} **and** δ_{V} **becomes**

$$\varepsilon_{\mathrm{V}} = \left(\frac{M_{pl}^2}{2}\right)\left(\frac{V'(\phi)}{V(\phi)}\right)^2$$

$$\delta_{\mathrm{V}} = \eta_{\mathrm{V}} - \varepsilon_{\mathrm{V}}$$

where

$$\eta_{\mathrm{V}} = M_{pl}^2\left(\frac{V''(\phi)}{V(\phi)}\right)$$

Potentials in the canonical setting

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

- **PSR parameter** $\rightarrow \varepsilon_{\rm V} = \left(\frac{M_{pl}^2}{2}\right)\left(\frac{V'(\phi)}{V(\phi)}\right)^2$
- **Slow roll inflation occurs when** $\varepsilon_{\rm V} << 1$ **and ends when** $\varepsilon_{\rm V} \simeq 1$**.**
- **Therefore, for inflation it is required that as the scalar field rolls down the potential** $\varepsilon_{\rm V}$ **to evolve from** $\varepsilon_{\rm V} << 1$ **to** $\varepsilon_{\rm V} \simeq 1$**.**
- **For** $V(\phi) = V_0\, \phi^n \Rightarrow \varepsilon_{\rm V} \propto \phi^{-2}$**. Therefore, the above criteria is satisfied only when** $n > 0$**.**
- **Consequently, inverse power law potential won't work in the canonical framework**
- **Power law inflation** $\rightarrow V(\phi) = V_0\, \exp\left[-\lambda(\phi/M_{pl})\right] \Rightarrow \varepsilon_{\rm V} =$ **const.**

Inflationary potentials for non canonical scalars

For our non-canonical model

$$\varepsilon_{\rm V} \equiv \left\{\left(\frac{1}{\alpha}\right)\left(\frac{3\,M^4}{V(\phi)}\right)^{\alpha-1}\left(\frac{M_{pl}\,V'(\phi)}{\sqrt{2}\,V(\phi)}\right)^{2\alpha}\right\}^{\frac{1}{2\alpha-1}}$$

- **For** $3M^4 \ll V \Rightarrow \varepsilon_{V} < \varepsilon_{V}^{(c)}$
- **Inverse power law potential can drive inflation in the non canonical setting.**
- **For exponential potential** $V(\phi) = V_0\,\exp\left[-\lambda(\phi/M_{pl})\right]$ **it turns out that** $\varepsilon_{\rm V}$ **evolves from** $\varepsilon_{\rm V} << 1$ **to** $\varepsilon_{\rm V} \simeq 1$.

$\Rightarrow$ **Non-Canonical scalar fields widens the domain of potentials which can drive inflation.**

Power spectra for non canonical model

- **For the model**

$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi)$$

in the slow roll limit it turns out that

$$\mathcal{P}_{_S}(k) = \left(\frac{1}{72\pi^2 c_{_s}}\right)\left\{\left(\frac{\alpha\, 6^\alpha}{\mu^{4(\alpha-1)}}\right)\left(\frac{1}{M_{_{pl}}^{14\alpha-8}}\right)\left(\frac{V(\phi)^{5\alpha-2}}{V'(\phi)^{2\alpha}}\right)\right\}^{\frac{1}{2\alpha-1}}$$

$$\mathcal{P}_{_T}(k) = \left(\frac{2\,V(\phi)}{3\,\pi^2 M_{_{pl}}^4}\right)$$

where

$$c_{_S}^2 = \frac{1}{2\,\alpha-1}$$

- $\alpha \geq 1$ **ensures that** $c_{_S}^2 \leq 1$

Scalar spectral index and T-to-S ratio

- **Scalar spectral index $n_{_S}$ is defined as**

$$n_{_S} - 1 \equiv \frac{\mathrm{d}\,\ln \mathcal{P}_{_S}}{\mathrm{d}\,\ln k}$$

- **Tensor to scalar ratio**

$$r \equiv \frac{\mathcal{P}_{_T}}{\mathcal{P}_{_S}}$$

- **For chaotic inflationary model $V(\phi) = V_{_0}\,\phi^n$, it turns out that**

$$n_{_S} = 1 - 2\left(\frac{\gamma + n}{2N\gamma + n}\right) \quad \text{and} \quad r = \left(\frac{1}{\sqrt{2\,\alpha - 1}}\right)\left(\frac{16\,n}{2\,N\gamma + n}\right)$$

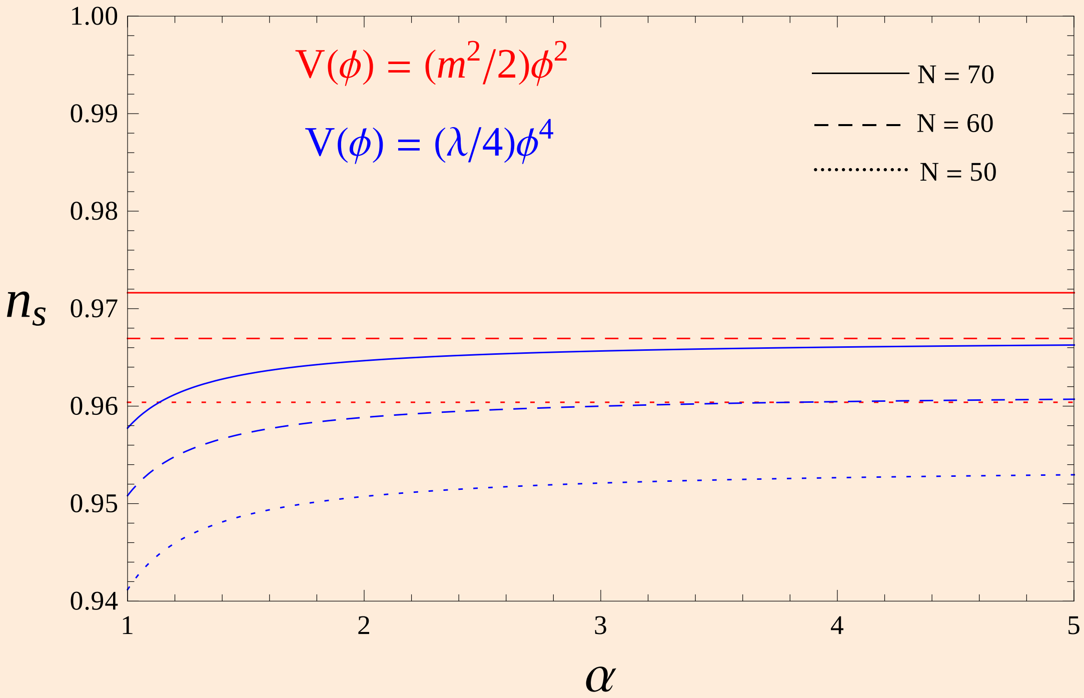
where

$$\gamma \equiv \frac{2\alpha + n\,(\alpha - 1)}{2\alpha - 1}$$

⋆ This result was also independently obtained by Sheng and Liddle (arXiv:1204.6214) !

Scalar spectral index n_S

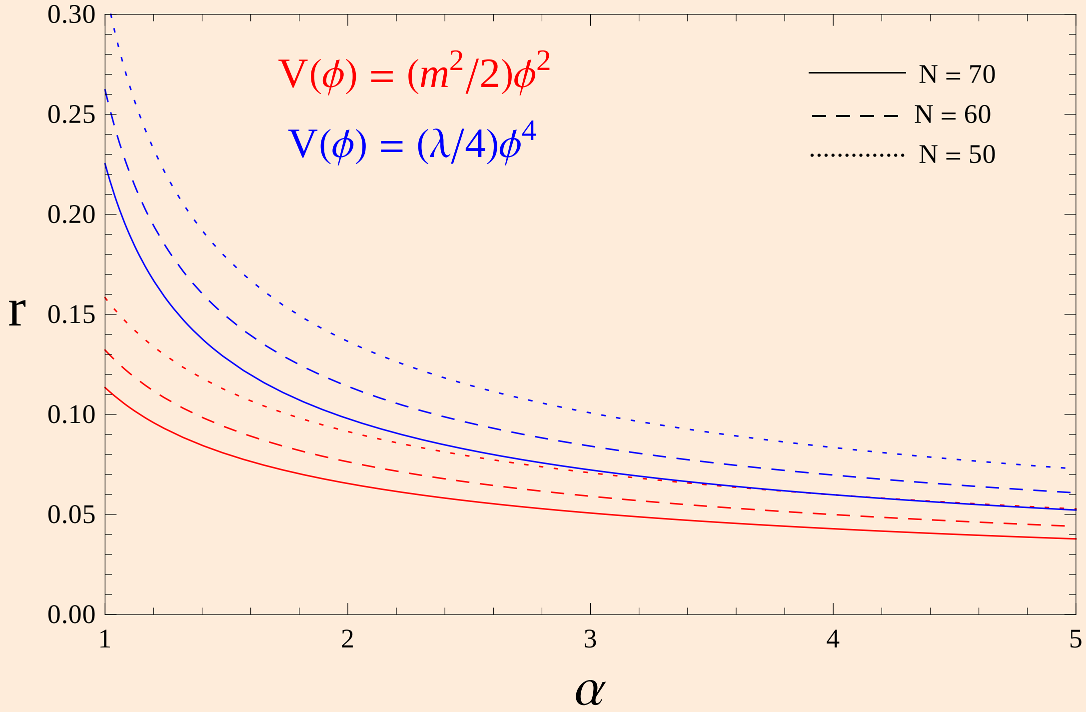
$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi) \qquad \text{where} \qquad X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$



- **The value of n_S for $m^2\phi^2$ potential is independent of α !**

Tensor-to-Scalar ratio r

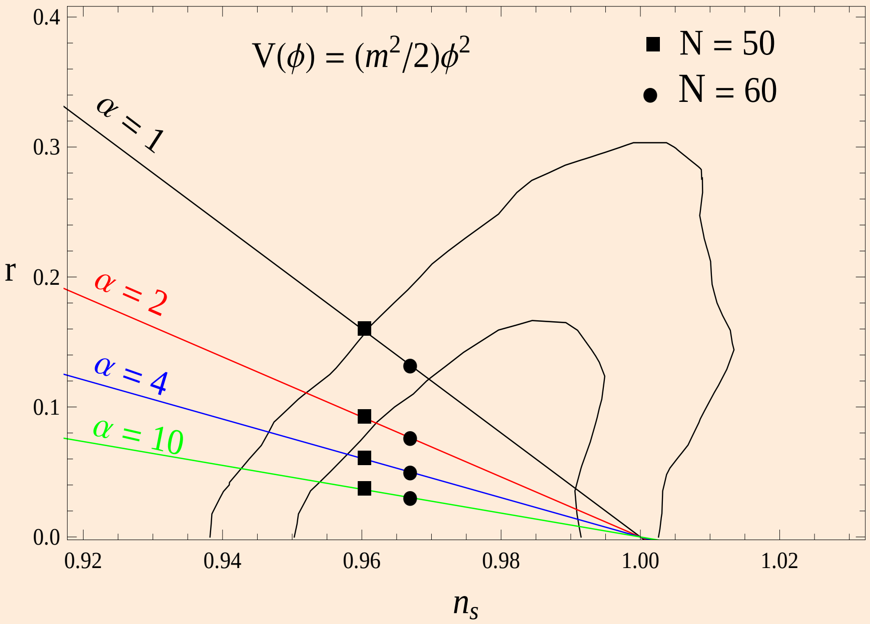
$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi) \qquad \text{where} \qquad X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

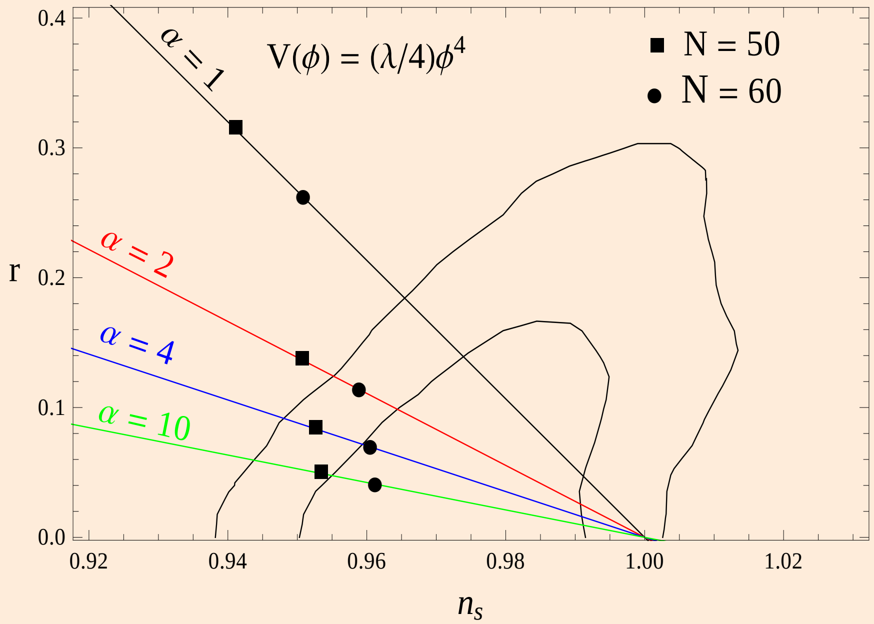


- **Tensor-to-scalar ratio decreases as the parameter α is increased.**

n_s-r Plain

$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi) \qquad \text{where} \qquad X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$





Inflationary consistency relation

- **For canonical scalar field**

$$r = -8n_T$$

where

$$n_T \equiv \frac{\mathrm{d}\ln\mathcal{P}_T}{\mathrm{d}\ln k}$$

- **For the model**

$$\mathcal{L}(X,\phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi)$$

It turns out that

$$r = -\frac{8\,n_T}{\sqrt{2\alpha-1}}$$

- **For** $\alpha > 1 \Rightarrow r < -8\,n_T$

$\Rightarrow$ **Non-canonical scalar fields violates the standard consistency relation**

Summary and Conclusions

We considered a non-canonical model of inflation with

$$\mathcal{L}_\phi = \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right)^\alpha - V(\phi)$$

- **The tensor-to-scalar ratio decreases considerably as the parameter α is increased.**
- **Therefore non-canonical scalars can accommodate a wider class of potentials for driving inflation.**
- **The non-canonical version of $V(\phi) \sim \lambda\phi^4$ inflation model, is found to agree with observations.**
- **This model violates the standard consistency relation $r = -8n_{_T}$.**
- **When $\alpha >> 1$, it turns out that $f_{_{NL}} \simeq 0.65 \times \alpha \Rightarrow$ it can lead to large non-Gaussianity**

$\Rightarrow$ **Non-canonical scalars can significantly improve the viability of inflationary models**

감사합니다
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