New ways of searching for the primordial gravitational wave from large-scale structure

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Introduction

Gravitational Wave 101

Gravitational wave (GW)

- is a traceless transverse (tensor) component of the metric perturbations:
(Einstein convention + Greek=0-4, Latin=1-3)

$$ds^2 = a^2(\eta)\left[-d\eta^2 + \{\delta_{ij} + h_{ij}(\eta, \boldsymbol{x})\}\,dx^i dx^j\right]$$

Traceless : $\mathrm{Tr}[h_{ij}] = h_i^i = g^{ij} h_{ij} = 0$

Transverse : $\nabla_i h_{ij} = 0$

- There are
6 (symmetric 3x3 spatial matrix) - 3 (transverse) - 1(traceless)
= 2 degrees of freedom = $h_\times$, h_+

Primordial Gravitational Wave

- de-Sitter space generates stochastic gravitational waves with amplitude of ($m_{pl} = \sqrt{G_N}$)

$$\Delta_h^2(k) = \frac{k^3 P_T(k)}{2\pi^2} = \frac{64\pi}{m_{\rm pl}^2}\left.\boxed{\left(\frac{H}{2\pi}\right)}^2\right|_{k=aH}$$

+ Friedmann equation: $3H^2 \sim 8\pi G\rho$

where power spectrum is defined as ($P_T = 4P_h$)

$$\langle h_{ij}(\boldsymbol{k})h^{ij}(\boldsymbol{k}')\rangle = (2\pi)^3 P_T(k)\delta^D(\boldsymbol{k}-\boldsymbol{k}')$$

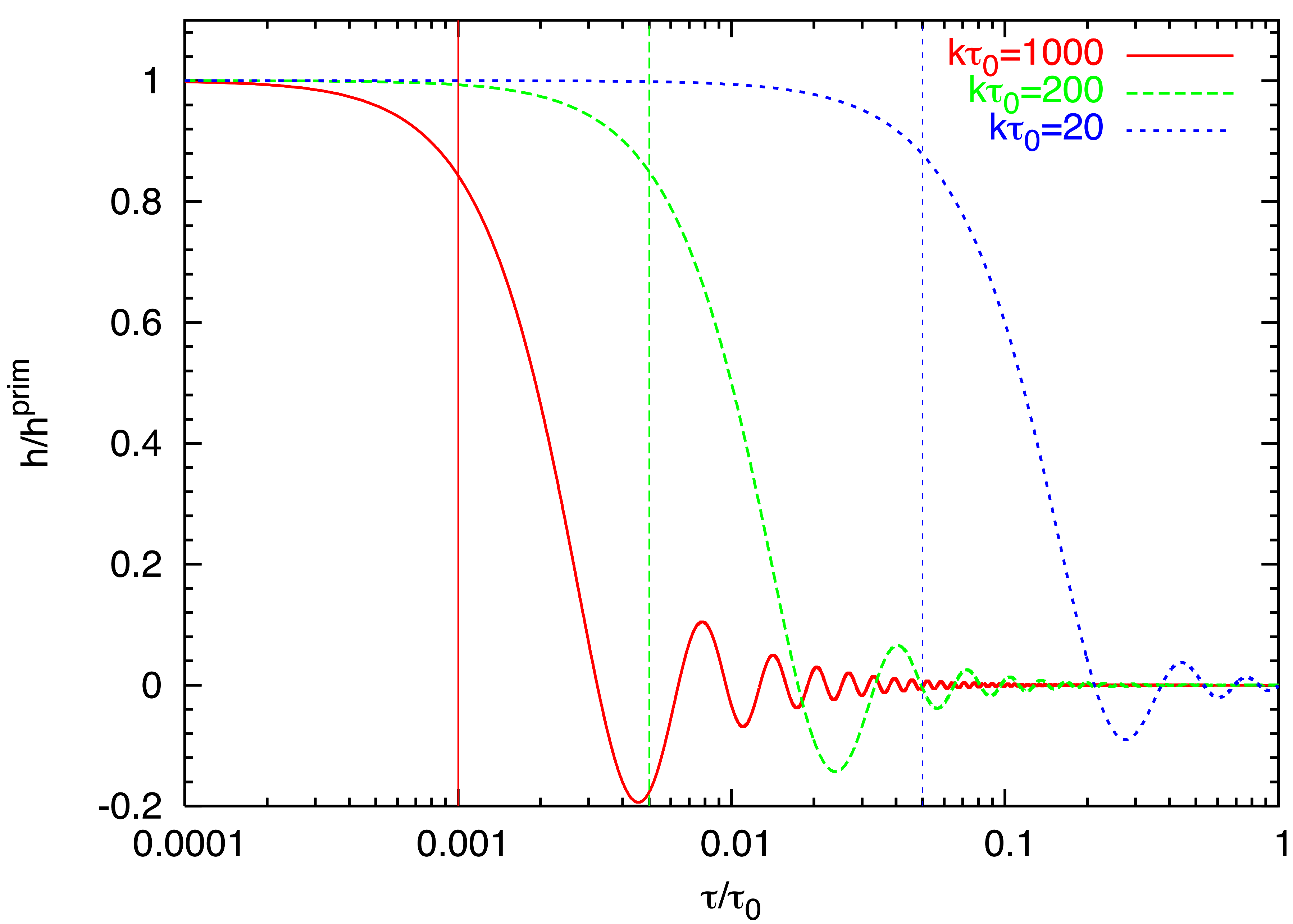
- Gravitational wave amplitude = energy scale of inflation!

Evolution of GW

- Evolution of GW(p=+,x) are described by K-G equation sourced by anisotropic stress ($\mathscr{H}$=a'/a and ' = d/dη):

$$-h_{ij;\nu}{}^{;\nu} = h_p''(\boldsymbol{k}) + \underbrace{2\mathcal{H}h_p'(\boldsymbol{k})}_{\text{Hubble damping term}} + k^2 h_p(\boldsymbol{k}) = 16\pi G a^2 \Pi_p(\boldsymbol{k})$$

Watanabe, Komatsu (2006)

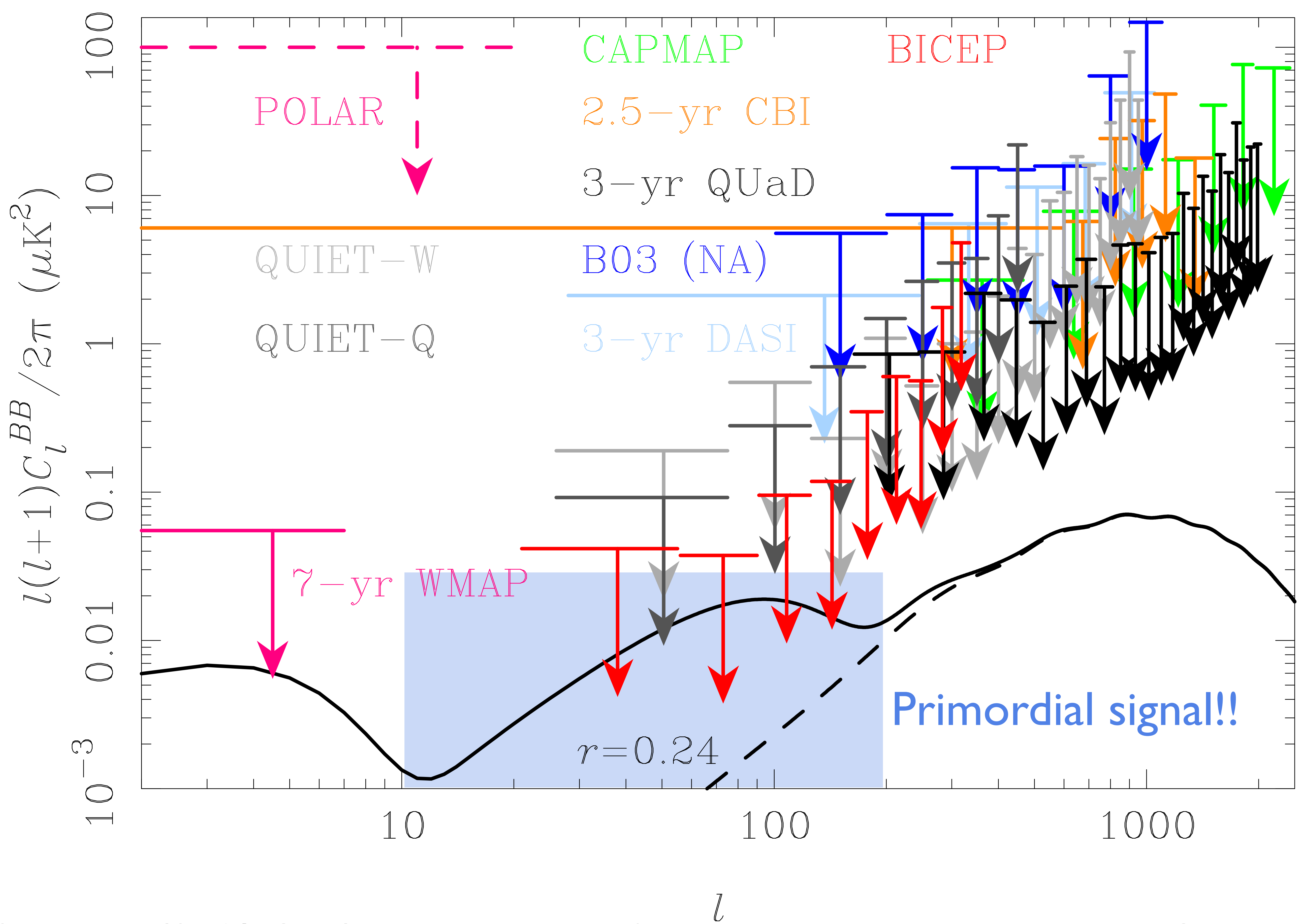


- GW decays once the mode enters the horizon; thus, it is extremely hard to detect GW from large-scale structure.

  - It always has to beat the scalar perturbations which grow inside of horizon.

r=(tensor amplitude/scalar amplitude)^2 at k=0.002 [1/Mpc]

GW from CMB polarization



2-sigma (95% C.L.) upper limit of B-mode power spectrum, Challinor (2012)

- Parity-odd (B-mode) polarization is a window to the GW (or vector) in the primordial universe!
- No B-mode yet...
- B-mode experiments: Keck array, PIPER, CLASS, LiteBIRD, PIXIE, ...
  (e.g. 5σ for r<10^{-3})

GW from Large Scale Structure

- Two effects:
  - At the location of galaxies (Source)
  - Deflection of light from galaxies (Line of sight)
- Three possible ways of detecting GW from Large Scale Structure :
  - Clustering of galaxies in large scale structure (S,L)
  - Distortion on shape of galaxies, or cosmic shear (S,L)
  - Fossil memory at the off-diagonal correlation (S)

Cosmic Rulers

or, covariant formalism for the shape distortions

Fabian Schmidt & Donghui Jeong [arXiv:1204.3625]

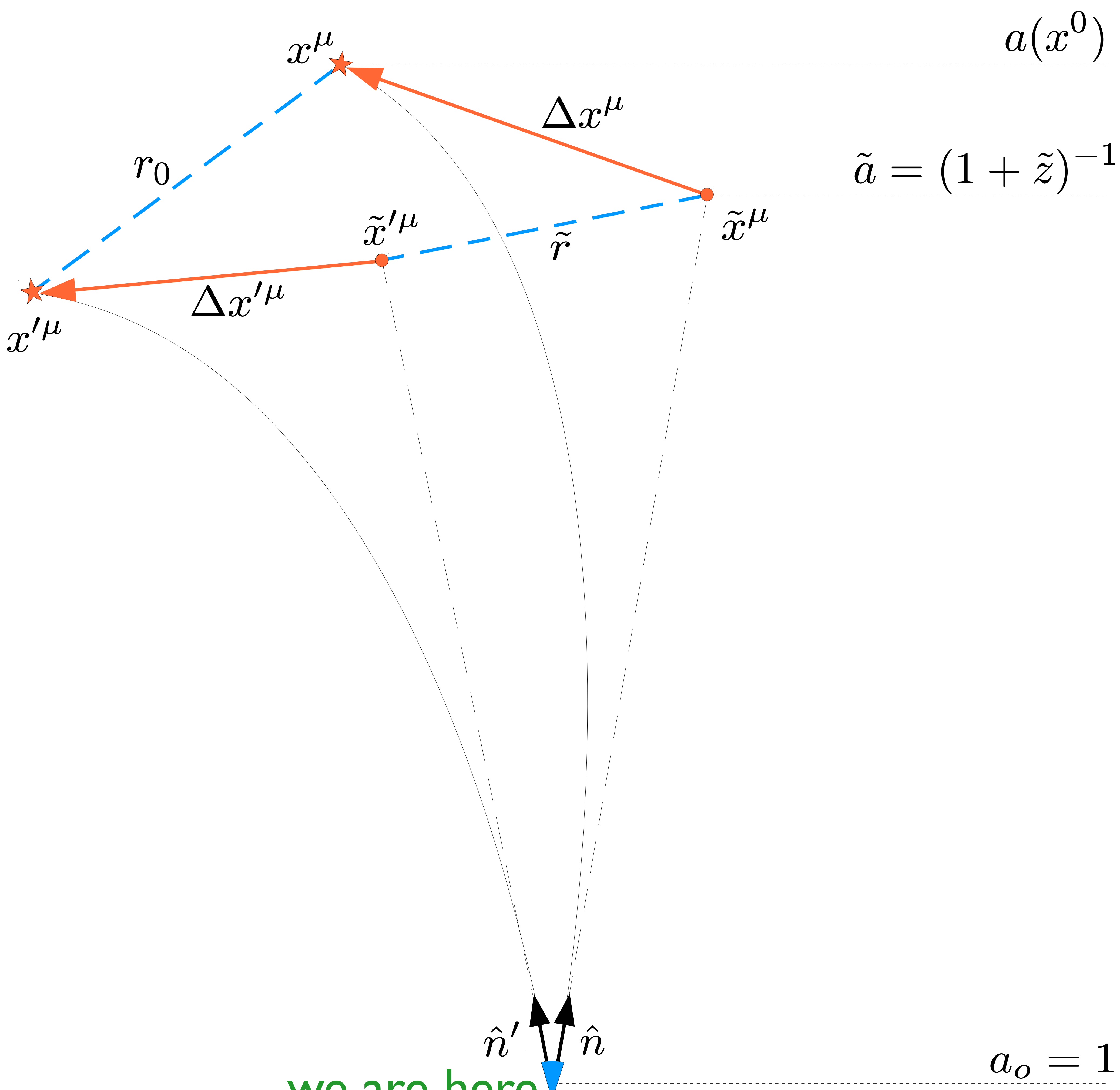
Shape distortion (lensing) with GR

(as of March 2012)

- Do we have a covariant, relativistic formula for weak lensing?
  - Yes, all the lensing literature are relativistic. But, with only scalar perturbations.
  - To our best knowledge, other than scalars, there was only one PRL article [Dodelson, Rozo and Stebbins (2003)] with somewhat mysterious term of "metric shear"
- We need a covariant formula describing the shape distortion! *Cosmic Rulers* (Schmidt & Jeong 1204.3625)

$$ds^2 = a^2(\eta)\left[-(1+2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij}+h_{ij})\,dx^i dx^j\right]$$

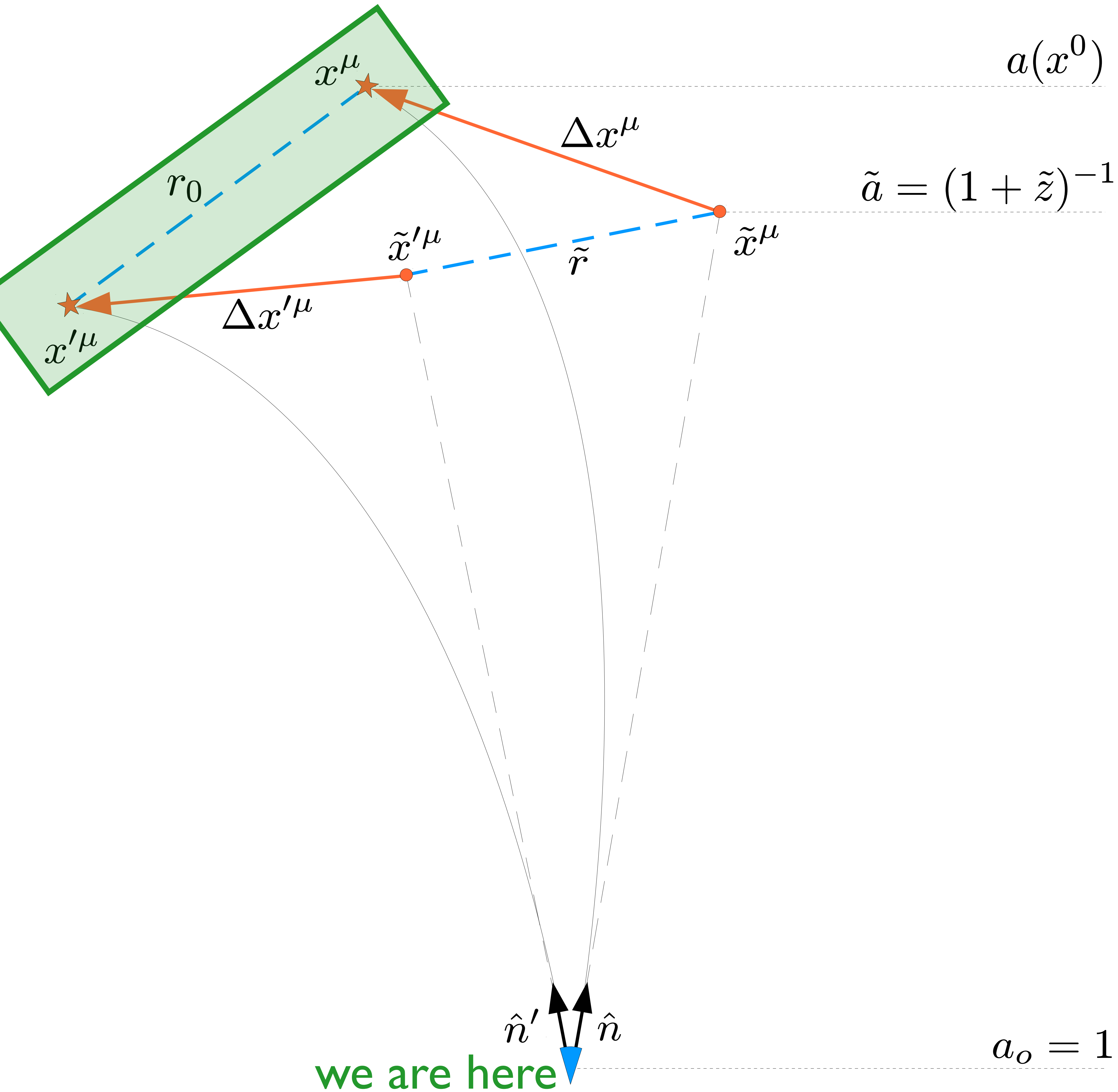
Cosmology with a high-z yardstick



- Consider a shining yardstick at high redshift, whose proper length is somehow known : r_0
- We observe (RA,DEC,z) for both ends of the stick, infer the length of the stick from them : $\tilde{r}$
- Due to perturbations, $\boxed{\tilde{r} \neq r_0}$ such a distortion to the size is an important tool to study perturbations!

$$ds^2 = a^2(\eta)\left[-(1+2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij}+h_{ij})\,dx^i dx^j\right]$$

Who measures r_0?



- An (imaginary) observer moving with the stick measures the length of the ruler:

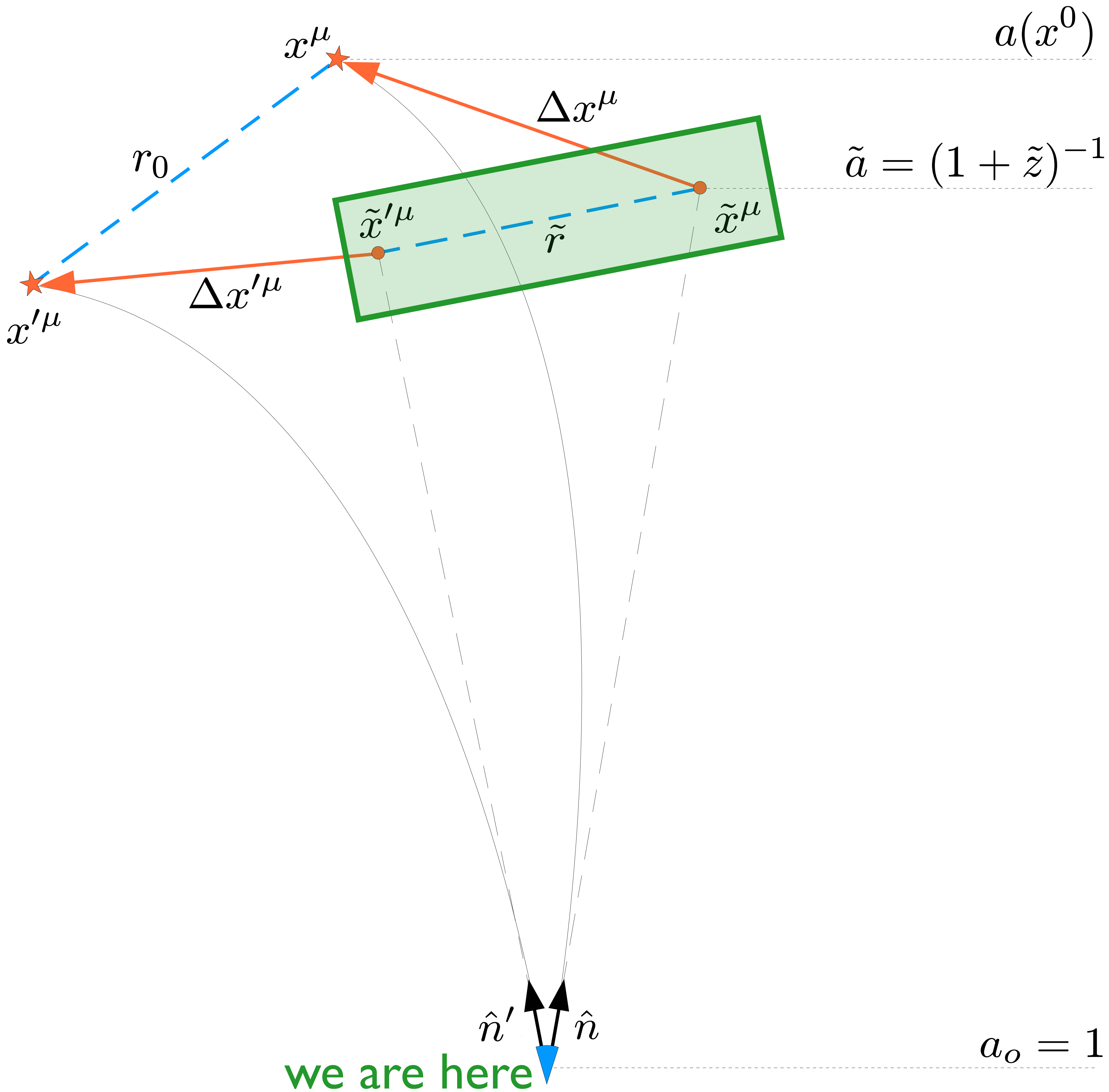
$$r_0^2 = \underbrace{\left[g_{\mu\nu} + u_\mu u_\nu\right]}(x^\mu - x'^\mu)(x^\nu - x'^\nu)$$

metric projected onto the constant-proper time hyper-surface of the comoving observer

$$g_{\mu\nu} + u_\mu u_\nu = a^2 \begin{pmatrix} 0 & -v_i \\ -v_i & \delta_{ij} + h_{ij} \end{pmatrix}$$

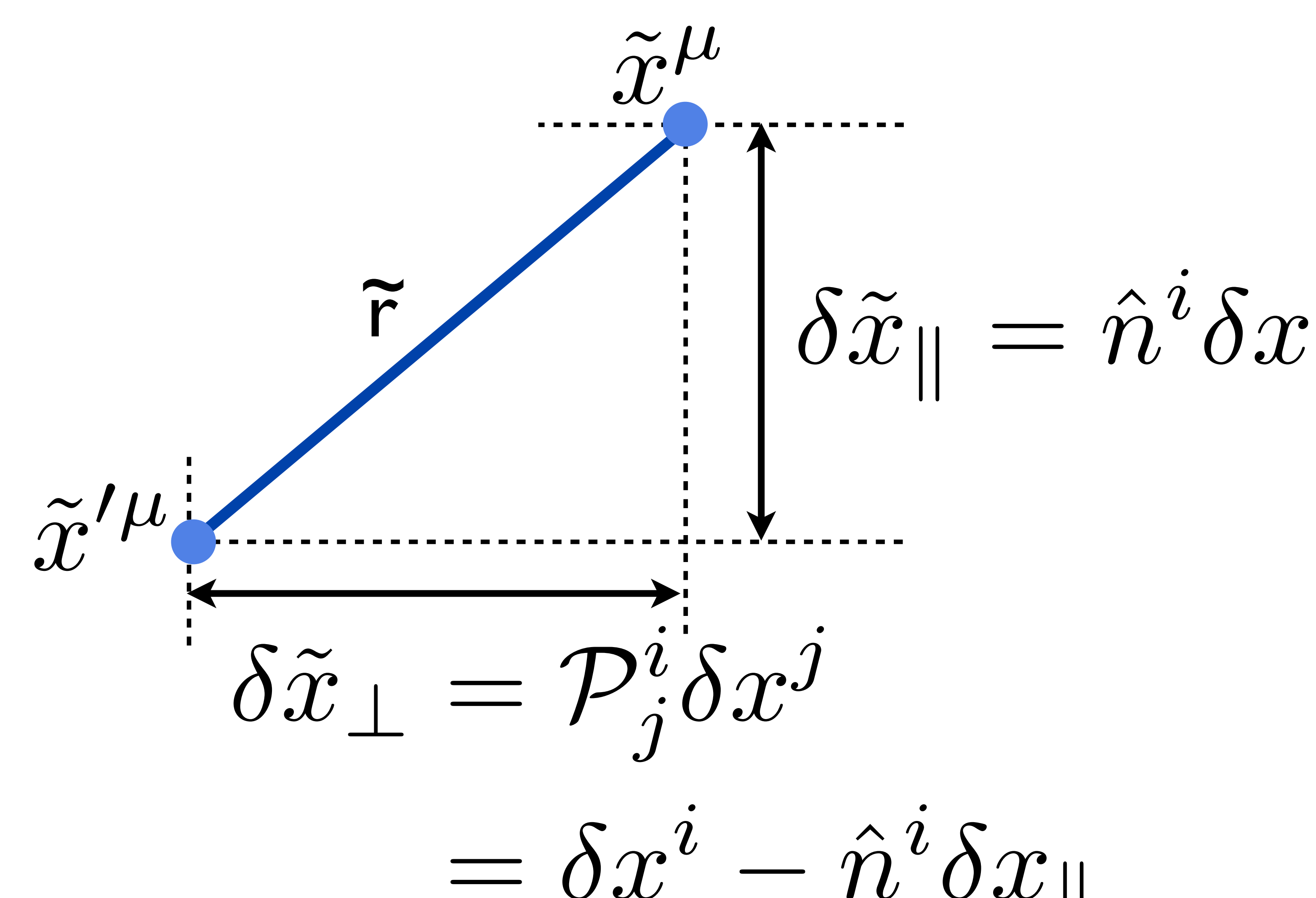
We assume a small ruler.

We measure r̃ !



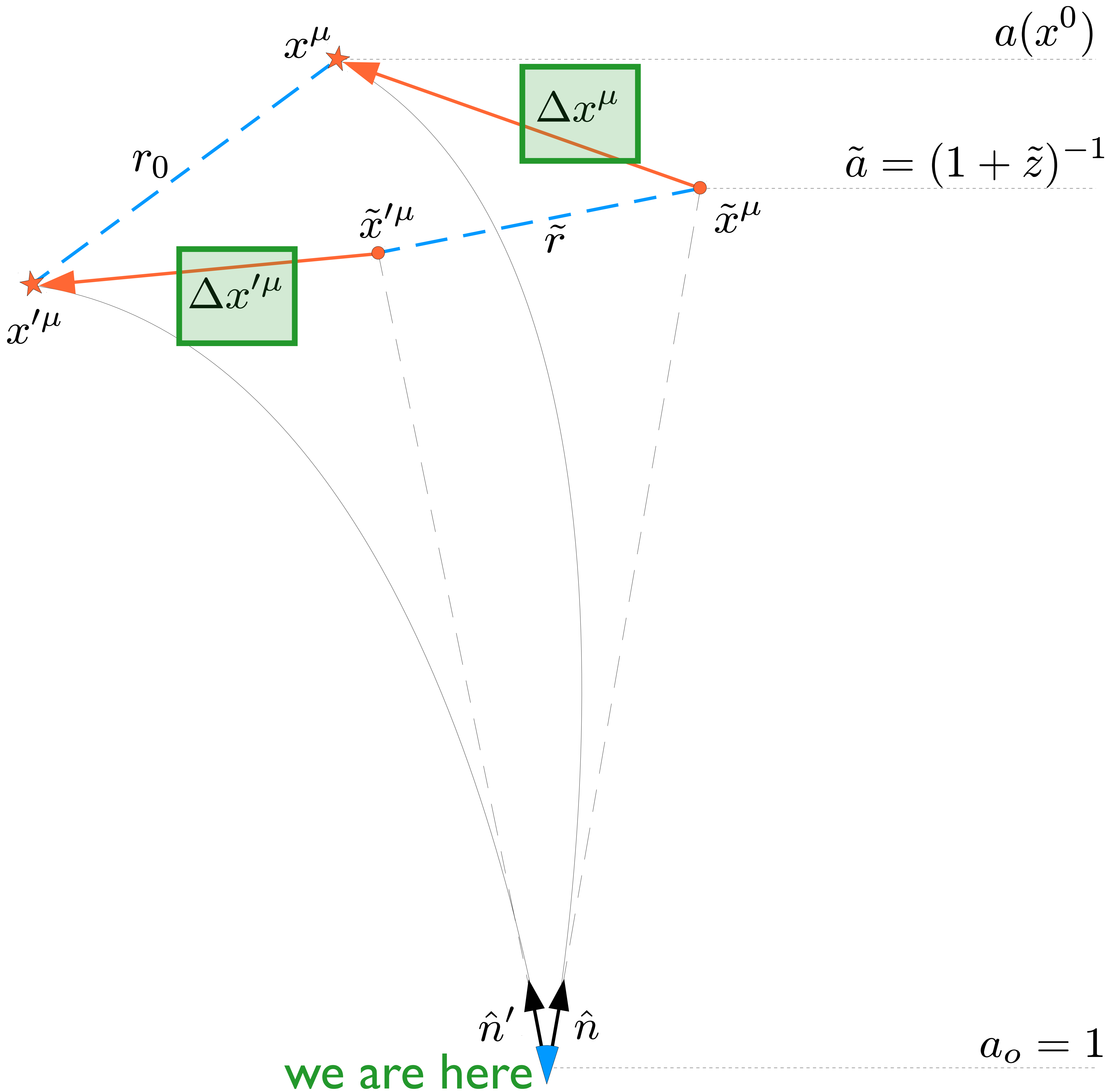
- We measure the angular and radial separations by using the unperturbed metric:

$$\tilde r^2 = \tilde a^2 \delta_{ij}\left(\tilde x^i - \tilde x'^i\right)\left(\tilde x^{-} \tilde x'^j\right)$$



$$ds^2 = a^2(\eta)\left[-(1+2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij}+h_{ij})\,dx^i dx^j\right]$$

Δx is from geodesic equations



Shift along the line of sight direction

$$\Delta x_\parallel = \int_0^{\tilde{\chi}} d\chi\left[A - B_\parallel - \frac{1}{2}h_\parallel\right] - \frac{1+\tilde{z}}{H(\tilde{z})}\Delta\ln a$$

Shift along the perpendicular direction

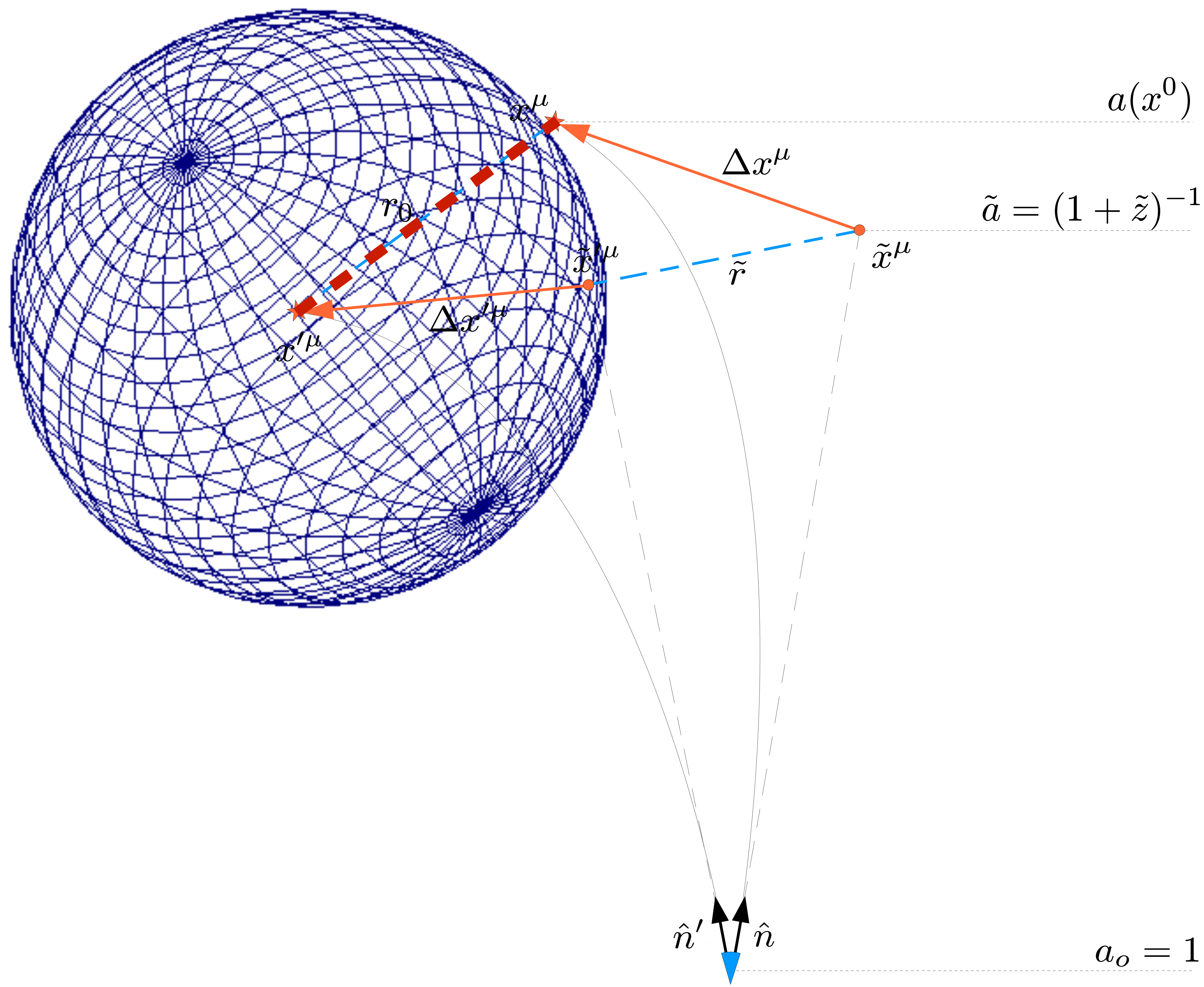
$$\Delta x_\perp^i = \left[\frac{1}{2}\mathcal{P}^{ij}(h_{jk})_o\,\hat{n}^k + B^i_{\perp o} - v^i_{\perp o}\right]\tilde{\chi} - \int_0^{\tilde{\chi}} d\chi\left[\frac{\tilde{\chi}}{\chi}\left(B^i_\perp + \mathcal{P}^{ij}h_{jk}\hat{n}^k\right) + (\tilde{\chi}-\chi)\partial^i_\perp\left(A - B_\parallel - \frac{1}{2}h_\parallel\right)\right]$$

perturbation to the scale factor at emission

$$\Delta\ln a = A_o - A + v_\parallel - v_{\parallel o} - \int_0^{\tilde{\chi}} d\chi\left[A - B_\parallel - \frac{1}{2}h_\parallel\right]'$$

Also, see Yoo et al. (2010)

Now, consider a spherical ruler

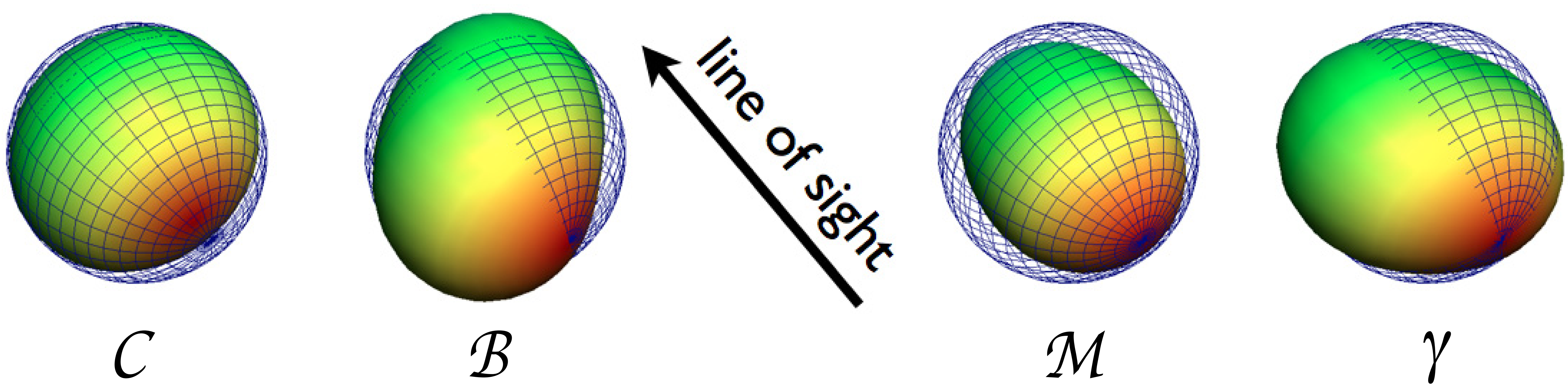


Classification of distortion

- We decompose the distortion as Scalar, Vector and Tensor according to their rotational property on sphere:

$$\frac{\tilde{r}-r_0}{\tilde{r}} = \mathcal{C}\frac{(\delta\tilde{x}_\parallel)^2}{\tilde{r}_c^2} + \mathcal{B}_i\frac{\delta\tilde{x}_\parallel\delta\tilde{x}^i_\perp}{\tilde{r}_c^2} + \mathcal{A}_{ij}\frac{\delta\tilde{x}^i_\perp\delta\tilde{x}^j_\perp}{\tilde{r}_c^2}$$

longitudinal scalar | Vector | Magnification (trace) + shear (spin-2)



New!!

$$ds^2 = a^2(\eta)\left[-(1+2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij}+h_{ij})\,dx^i dx^j\right]$$

Covariant formula for γ!!

- First fully relativistic, covariant expression for the cosmic shear!!

$$\begin{aligned} {}_{\pm2}\gamma \;=& -\frac{1}{2}h_{\pm} - \frac{1}{2}(h_{\pm})_o - \int_0^{\tilde\chi} d\chi\Bigg[\left(1-2\frac{\chi}{\tilde\chi}\right)\left[m^k_{\mp}\partial_{\pm}B_k + (\partial_{\pm}h_{lk})m^l_{\mp}\hat n^k\right] - \frac{1}{\tilde\chi}h_{\pm} \\ & + (\tilde\chi-\chi)\frac{\chi}{\tilde\chi}\left\{ - m^i_{\mp}m^j_{\mp}\partial_i\partial_j A + \hat n^k m^i_{\mp}m^j_{\mp}\partial_i\partial_j B_k + \frac{1}{2}m^i_{\mp}m^j_{\mp}(\partial_i\partial_j h_{kl})\hat n^k \hat n^l\right\}\Bigg] \end{aligned}$$

Here, ${}_{\pm2}\gamma(\hat{\boldsymbol n}) \equiv m^i_{\mp}m^j_{\mp}\mathcal{A}_{ij}$ is a spin ±2 component of the shear, where

$\boldsymbol m_{\pm} = \frac{1}{\sqrt2}(\boldsymbol e_1 \mp i\boldsymbol e_2)$ are spin ±1 vector field on sphere in the sense that it transforms $\boldsymbol m_{\pm} \to \boldsymbol m'_{\pm} = e^{\pm i\psi}\boldsymbol m_{\pm}$ under the rotation $\boldsymbol e_i \to \boldsymbol e'_i$ with angle ψ.

- Conformal Newtonian gauge: ${}_{\pm2}\gamma(\hat{\boldsymbol n}) = \int_0^{\tilde\chi} d\chi(\tilde\chi-\chi)\frac{\chi}{\tilde\chi}m^i_{\mp}m^j_{\mp}\partial_i\partial_j(\Psi-\Phi)$

Large-Scale Structure with GW II : Shear

Fabian Schmidt & Donghui Jeong [arXiv:1205.1514]

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

Cosmic shear with GW

- With only tensor perturbation, shear expression becomes

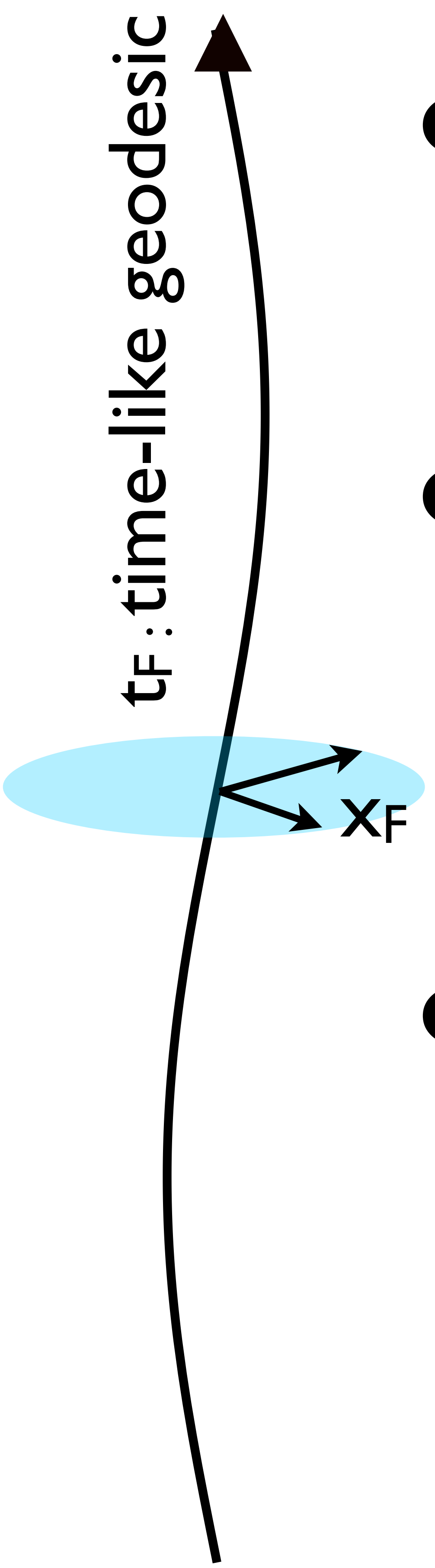
$$_{\pm 2}\gamma(\hat{\boldsymbol{n}}) = \underbrace{-\frac{1}{2}h_{\pm o} - \frac{1}{2}h_{\pm}}_{\text{Metric Shear}} - \int_0^{\tilde{\chi}} d\chi \left\{ \frac{\tilde{\chi}-\chi}{2}\frac{\chi}{\tilde{\chi}}(m^i_{\mp} m^j_{\mp}\partial_i\partial_j h_{kl})\hat{n}^k\hat{n}^l + \left(1 - 2\frac{\chi}{\tilde{\chi}}\right)\hat{n}^l m^k_{\mp} m^i_{\mp}\partial_i h_{kl} - \frac{1}{\tilde{\chi}}h_{\pm} \right\}$$

- Dodelson, Rozo & Stebbins (2003)
"Assuming physical isotropy, we must add a 'metric shear' caused by the shearing of the coordinates with respect to physical space, i.e. $\Delta\gamma_{ij}$, which is just the traceless transverse projection of $-h_{ij}/2$"

???

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

What "metric shear" really is



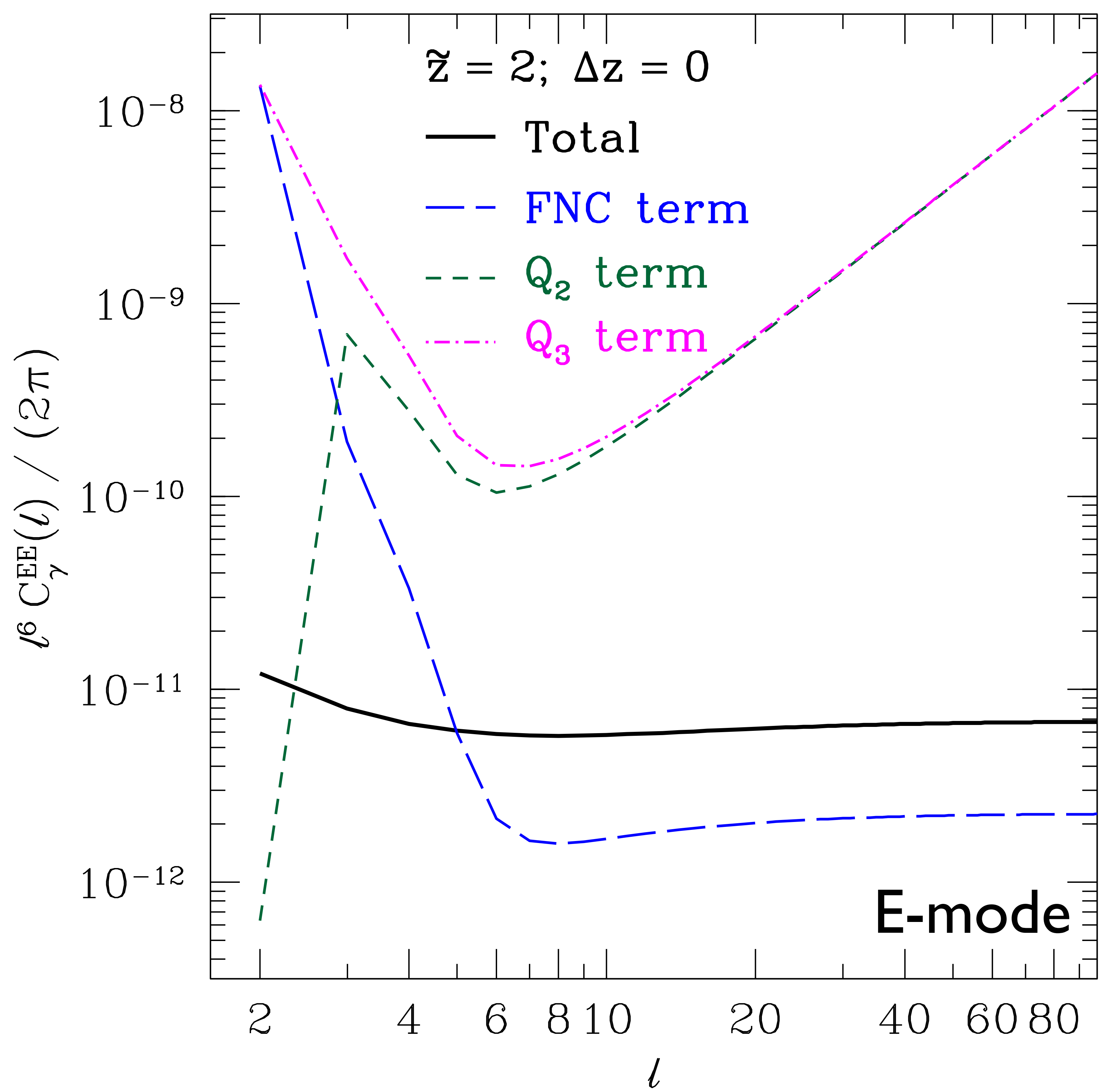
- The cosmic shear measurement are referenced to the frame within which galaxies are statistically round.
- The most natural choice of such coordinate is the local inertial frame defined along the time-like geodesic of the galactic center, or so called Fermi Normal Coordinate (FNC)!
- Coordinate transformation from FRW to FNC coordinate:

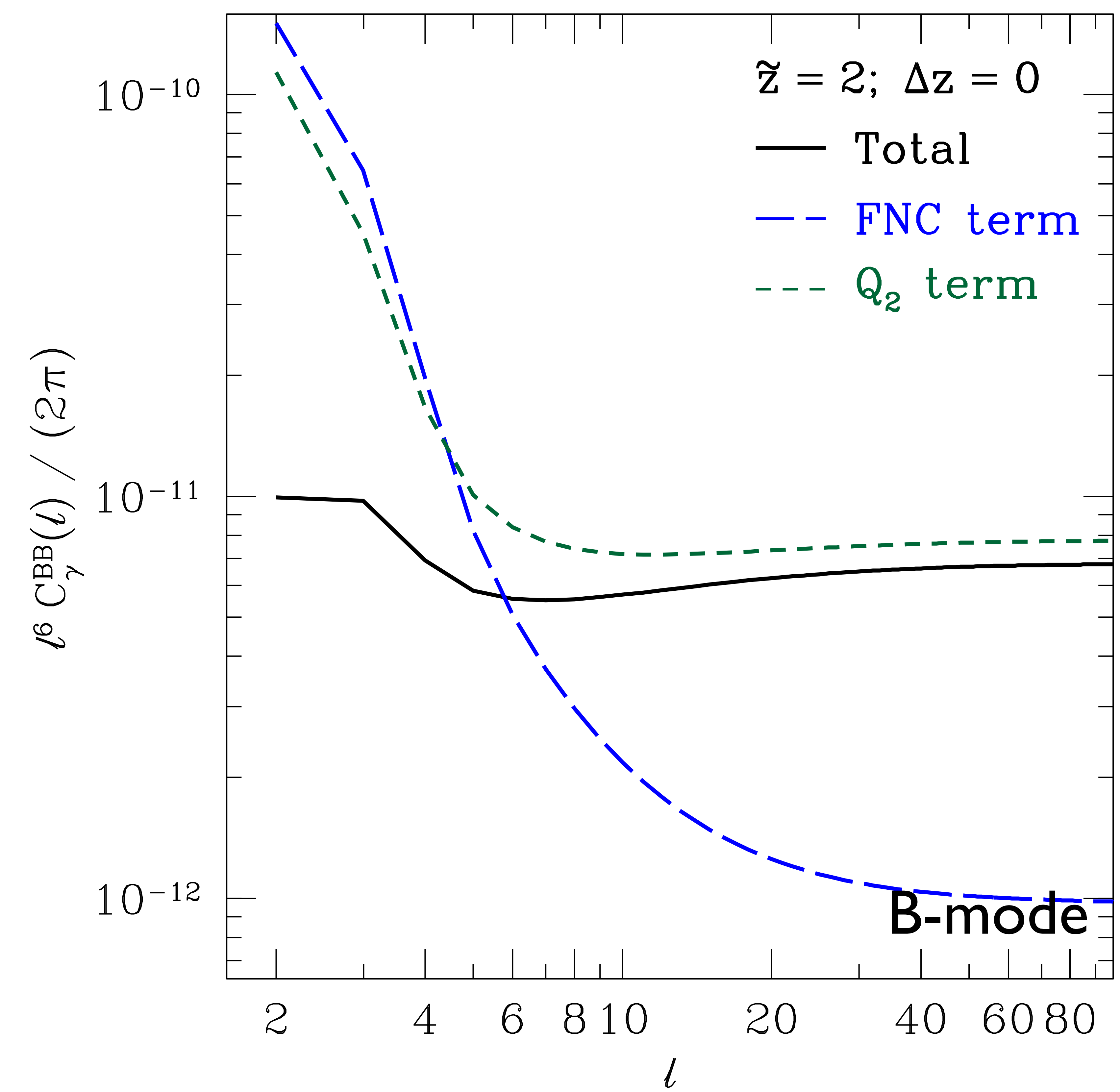
$$x_F^i = x^i - \frac{1}{2}h_{ij}x^j - \frac{1}{2}\Gamma^i_{jk}x^j x^k + \mathcal{O}(x^3)$$

FNC term

leads to an additional shear of $$\underbrace{\partial_{\perp(i}\Delta x_{\perp j)}}_{\gamma_{ij}} \to \underbrace{\partial_{\perp(i}\Delta x_{\perp j)}}_{\gamma_{ij}} + \boxed{\frac{1}{2}\mathcal{P}_i^k\mathcal{P}_j^l h_{kl}} + \cdots$$

Metric shear vs. l.o.s. integral





- They are about the same order of magnitude, but with opposite sign...

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

FNC metric and tide

- The metric in the Fermi Normal Coordinate is given by

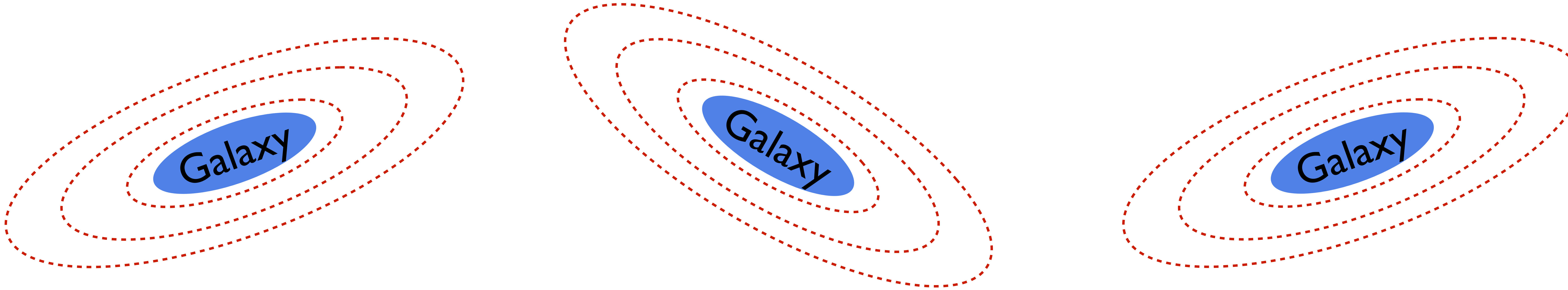
$$\begin{aligned}
g^F_{00} &= -1 + \left(\dot{H} + H^2\right) r_F^2 + \left[\frac{1}{2}\ddot{h}_{lm} + H\dot{h}_{lm}\right] x^l_F x^m_F . \\
g^F_{0i} &= \frac{1}{3}\left(\nabla_i \dot{h}_{lm} - \nabla_m \dot{h}_{li}\right) x^l_F x^m_F \\
g^F_{ij} &= \delta_{ij} + \frac{H^2}{3}\left[x^i_F x^j_F - r_F^2 \delta_{ij}\right] + \frac{1}{6}\left(\nabla_i\nabla_j h_{ml} + \nabla_l \nabla_m h_{ij} - \nabla_l \nabla_j h_{im} - \nabla_i \nabla_m h_{jl}\right) x^l_F x^m_F \\
&\quad + \frac{H}{6}\left(\dot{h}_{lj} x^l_F x^i_F + \dot{h}_{im} x^m_F x^j_F - \dot{h}_{ij} r_F^2 - \dot{h}_{lm} x^l_F x^m_F \delta_{ij}\right) .
\end{aligned}$$

- Equation of motion for non-relativistic body in FNC is determined by the effective gravitational potential $\Psi_{\rm eff} = -\delta g_{00}/2$.

- Ψ generates tidal force: $t_{ij} = \left(\partial_i \partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\Psi^F = -\left(\frac{1}{2}\ddot{h}_{lm} + H\dot{h}_{lm}\right)$

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

Intrinsic alignment (IA) model



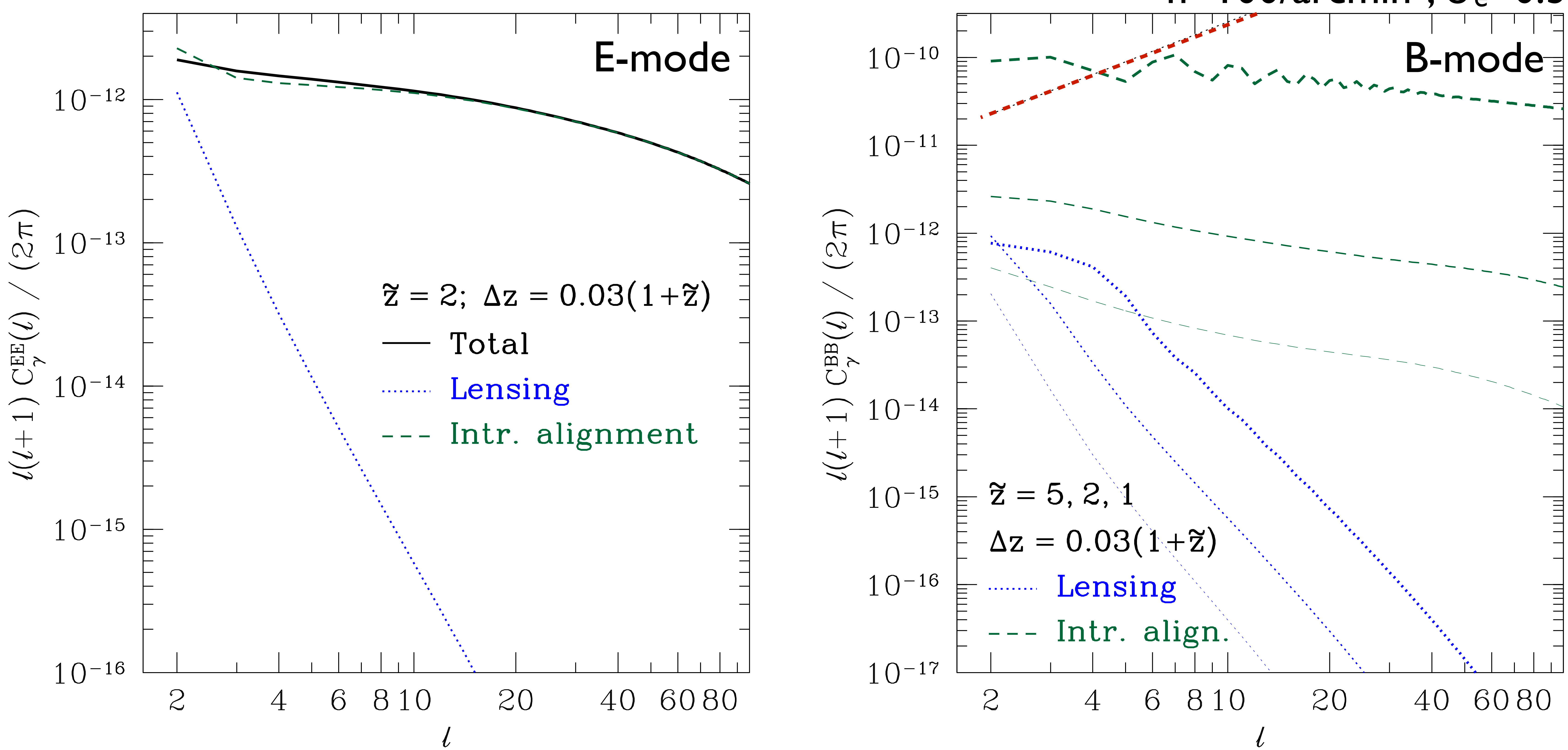
- Intrinsic alignment: tidal fields (anisotropic gravitational potential) tends to align galaxies
- Linear alignment model $\gamma_{ij}^{IA}(\boldsymbol{n}) = -\frac{C_1}{4\pi G}\mathcal{P}_{ik}\mathcal{P}_{jl}t^{kl} = -\frac{2}{3}\frac{C_1\rho_{\rm cr0}}{H_0^2}\mathcal{P}_{ik}\mathcal{P}_{jl}t^{kl}$
  - consistent with observations on large (>10 [Mpc/h]) scales

Blazek+(2011), Joachimi+(2011)

$$_{\pm2}\gamma^{\rm IA}(\hat{\boldsymbol{n}}) = \frac{1}{3}\frac{C_1\rho_{\rm cr0}}{a^2H_0^2}\left(h''_{\pm} + aHh'_{\pm}\right)$$

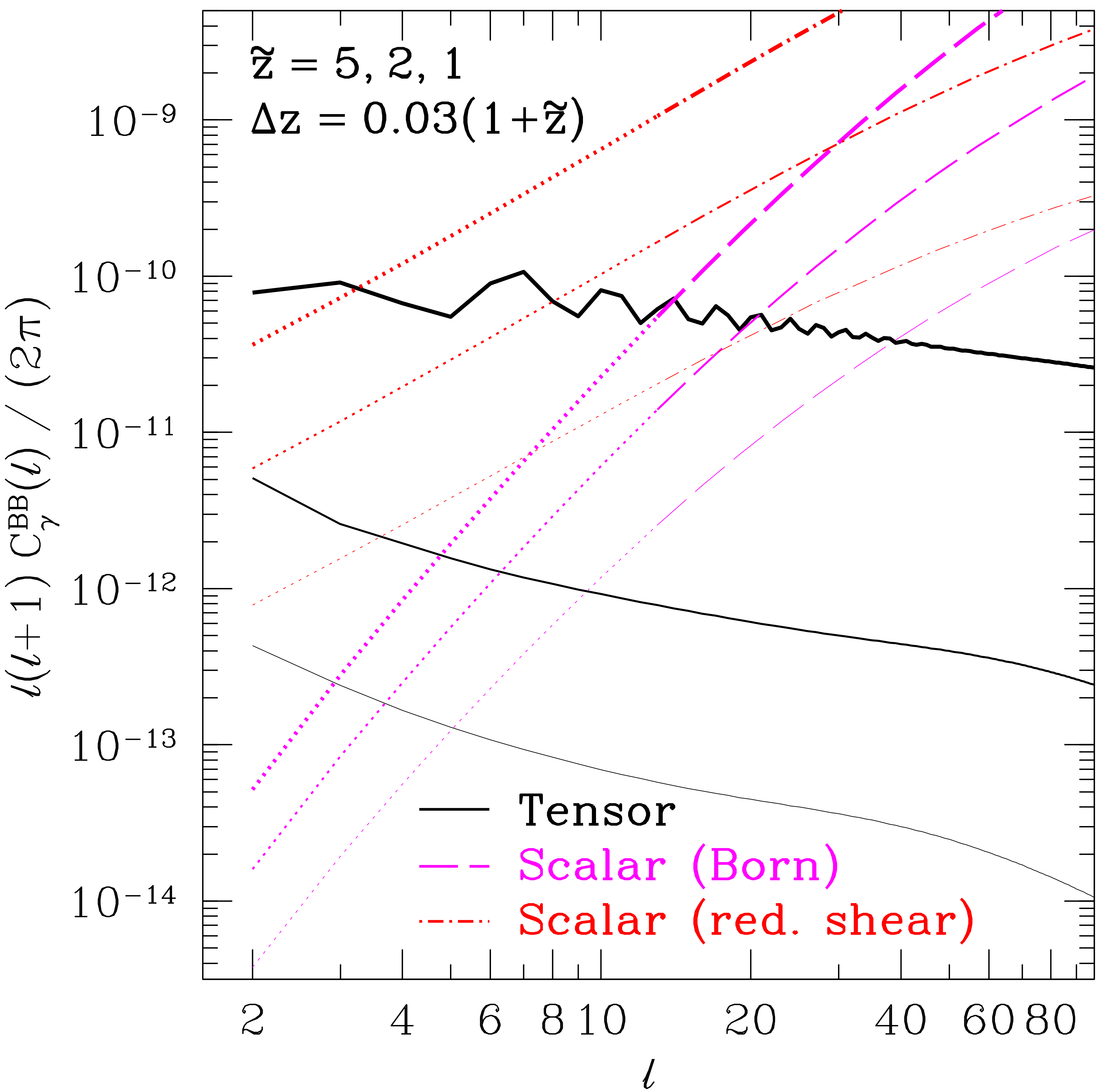
Shear vs. intrinsic alignment

noise for a half sky survey with
n=100/arcmin^2, σ_e=0.3



- Intrinsic alignment dominates over the lensing signal, and IA signal increases at higher redshifts!

What about 2nd'ary B-modes?

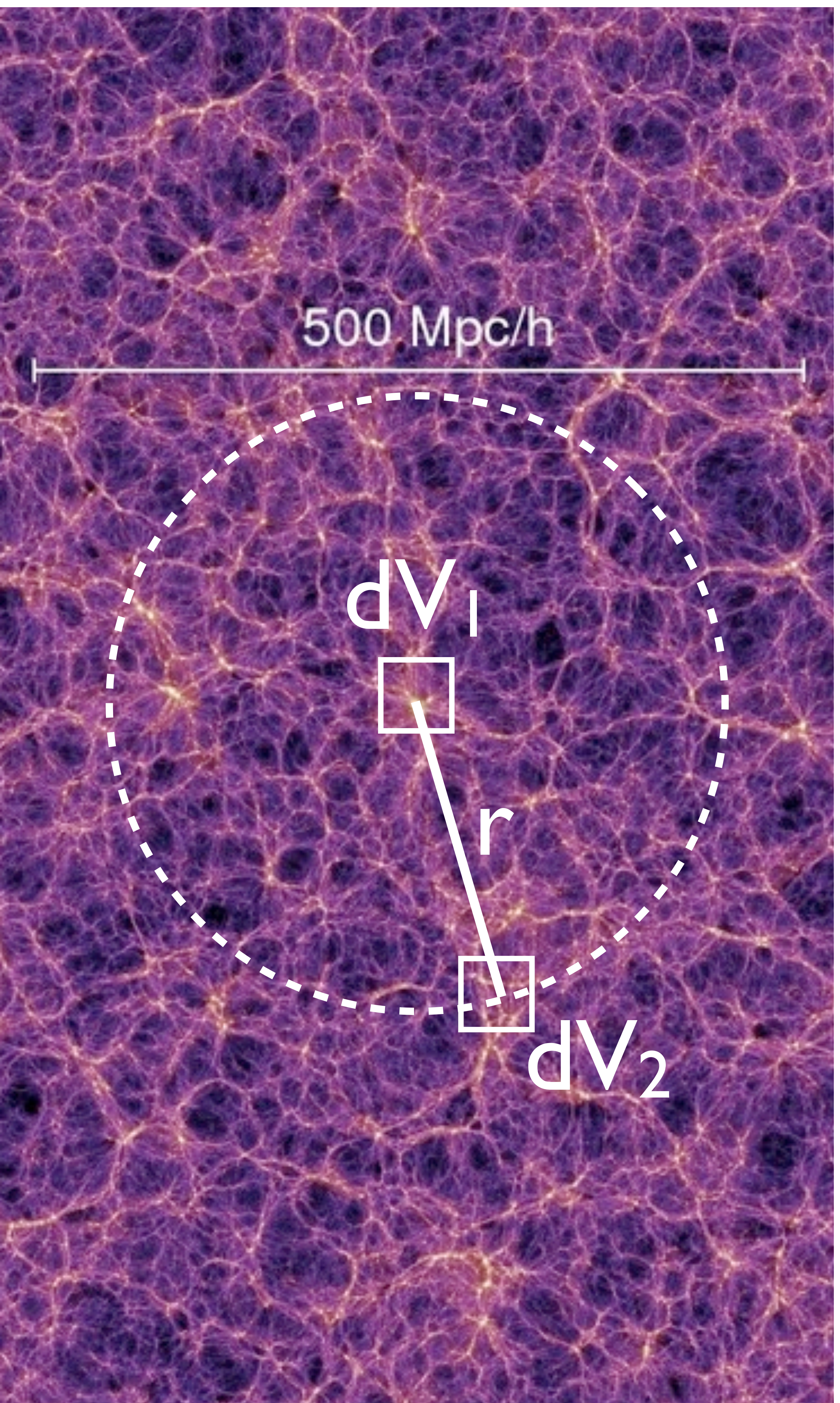


- The second order scalar perturbations can also generate parity odd (B-mode) lensing signal
  - Induced GW ~10^{-14} (Mollerach+2004;Bauman+2007)
  - 2nd-order geodesic eqn. (Hirata&Seljak 2003)
  - reduced shear + lensing bias (Schneider+1997,Dodelson+2006, Schmidt+2009)

Clustering Fossils from the Early Universe

Donghui Jeong & Marc Kamionkowski [arXiv:1203.0302]

Two-point correlation functions



- Probability of finding two galaxies at separation r is given by the two-point correlation function:

$$P_2(\boldsymbol{r}) = \bar{n}^2[1+\xi(\boldsymbol{r})]dV_1 dV_2$$

$$\xi(\boldsymbol{r}) \equiv \langle \delta(\boldsymbol{x})\delta(\boldsymbol{x}+\boldsymbol{r})\rangle$$

statistical homogeneity (translational invariance)

- Power spectrum is the Fourier transform of it:

$$P(\boldsymbol{k}) = \int d^3r\,\xi(\boldsymbol{r})e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$

or in terms of density contrast,

$$\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\rangle = (2\pi)^3 P(\boldsymbol{k})\boxed{\delta^D(\boldsymbol{k}+\boldsymbol{k}')}$$

Jeong & Kamionkowski (2012)

Non-Gaussianity and (local) ~~homogeneity~~

- IF we have a following non-linear coupling between primordial density fluctuations and new field h_p (JK coupling):

(e.g. Maldacena, 2003)

$$\langle \delta_i(\boldsymbol{k}_1)\delta_i(\boldsymbol{k}_2)h_p(\boldsymbol{K})\rangle = (2\pi)^3 \overbrace{P_p(K)}^{\text{power spectrum of new field}} \underbrace{f_p(\boldsymbol{k}_1,\boldsymbol{k}_2)}_{\text{coupling amplitude}} \underbrace{\epsilon^p_{ij}}_{\text{polarization basis (scalar, vector, tensor)}} k_1^i k_2^j \delta^D(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{K})$$

- THEN, density power spectrum we observe now has ***non-zero off-diagonal*** components: **Fossil equation**

$$\langle \delta_i(\boldsymbol{k}_1)\delta_i(\boldsymbol{k}_2)\rangle|_{h_p(\boldsymbol{K})} = h_p(\boldsymbol{k}_1+\boldsymbol{k}_2) f_p(\boldsymbol{k}_1,\boldsymbol{k}_2)\epsilon^p_{ij}k_1^i k_2^j \delta^D_{\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{K}}$$

Why worrying about new fields?

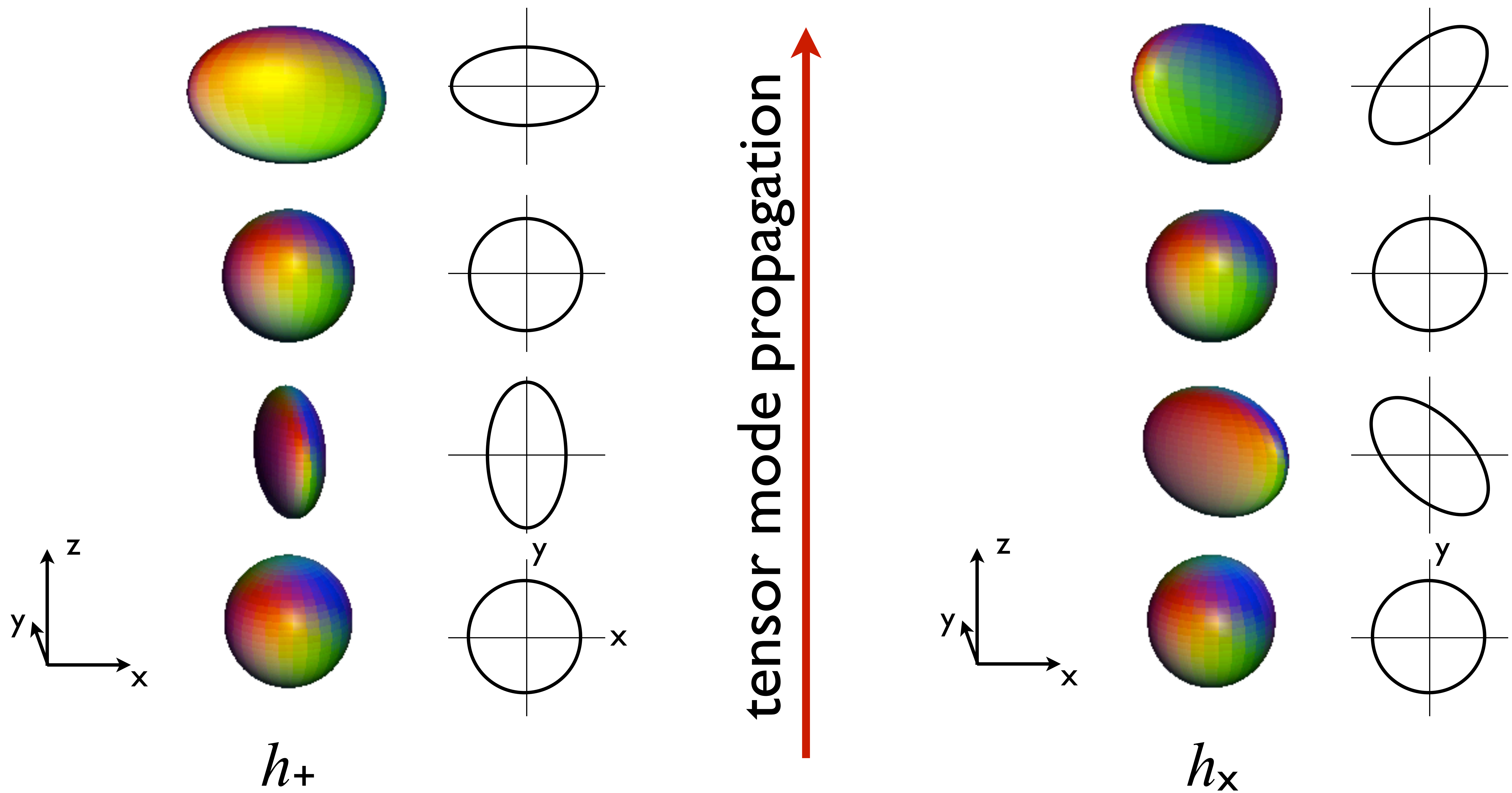
- Inflaton(s) : a scalar field(s) responsible for inflation
- But, inflaton might not be alone. Many inflationary models need/introduce additional fields. But, direct detection of such fields turns out to be very hard:
  - Additional Scalar: not contributing to seed fluctuations
  - Vector: decays as 1/[scale factor]
  - Tensor: decays after coming inside of comoving horizon
- Off-diagonal correlation (Fossil equation) opens new way of detecting them!

$\varepsilon^{p}{}_{ij}$: six independent modes

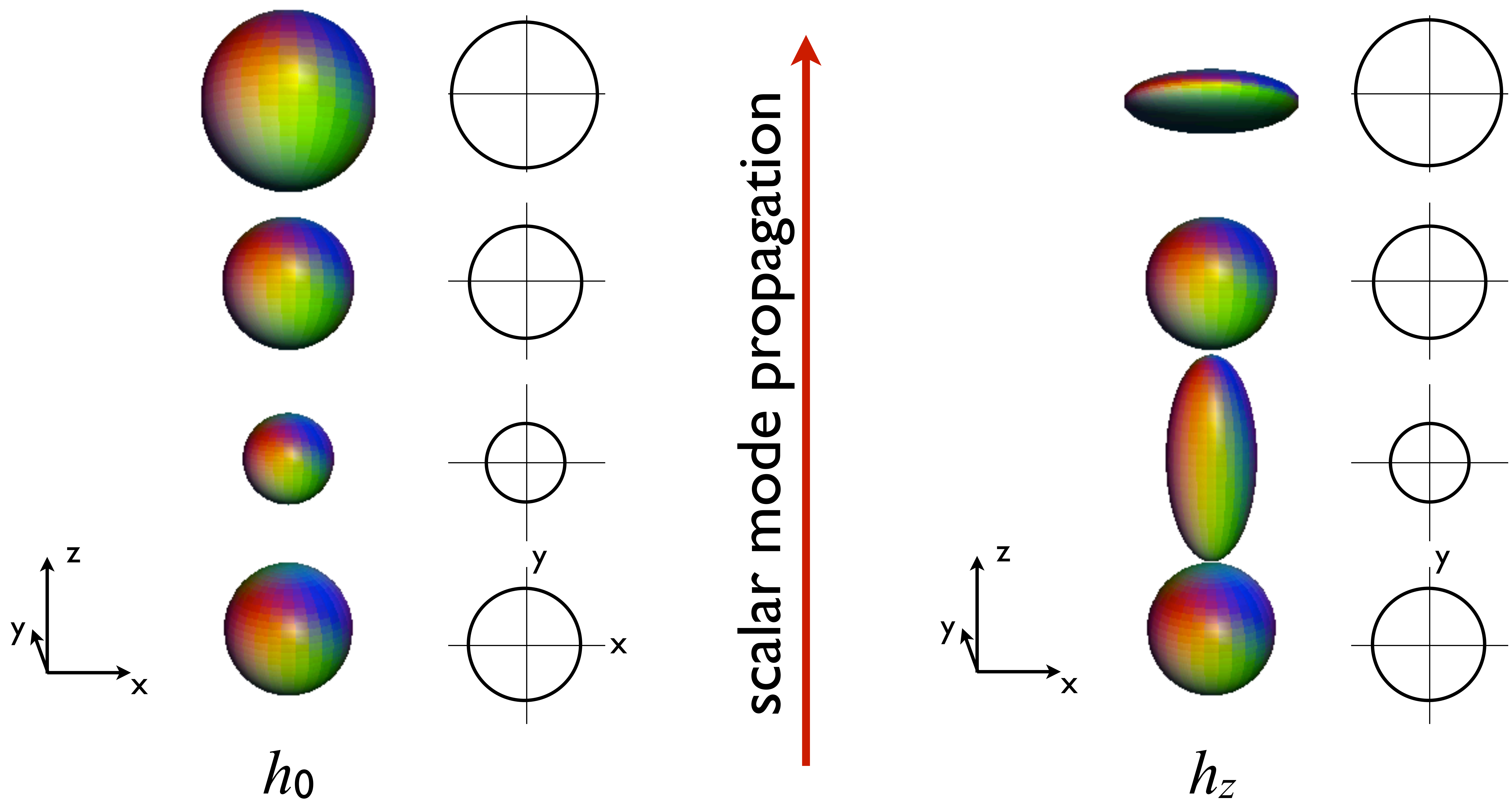
- In a symmetric 3x3 tensor, we have 6 degrees of freedom, which are further decomposed by Scalar, Vector and Tensor polarization modes.
- They are orthogonal: $\epsilon^{p}_{ij}\epsilon^{p',ij} = 2\delta_{pp'}$
  - Scalar (p=0,z): $\epsilon^{0}_{ij} \propto \delta_{ij}$ $\quad \epsilon^{z}_{ij}(\boldsymbol{K}) \propto K_i K_j - K^2/3$
  - Vector (p=x,y): $\epsilon^{x,y}_{ij}(\boldsymbol{K}) \propto \frac{1}{2}(K_i e_j + K_j e_i)$ where $K_i e_i = 0$
  - Tensor (p=x,+): transverse and traceless

$$K_i \epsilon^{+,\times}_{ij}(\boldsymbol{K}) = 0 \qquad \delta_{ij}\epsilon^{+,\times}_{ij}(\boldsymbol{K}) = 0$$

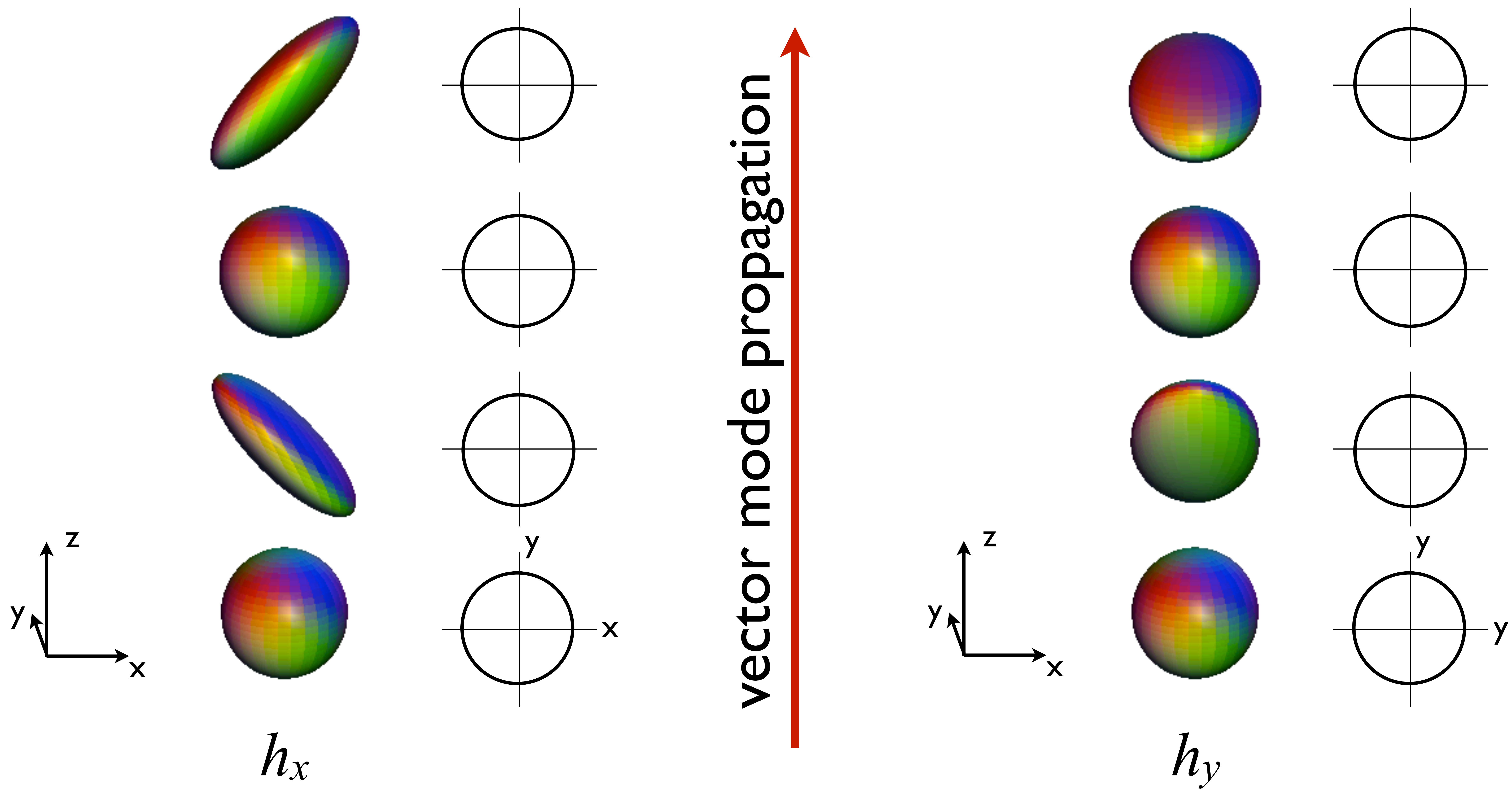
$\xi(\mathbf{r})$ with single tensor mode (p=+,x)



$\xi(\mathbf{r})$ with single scalar mode (p=0,z)



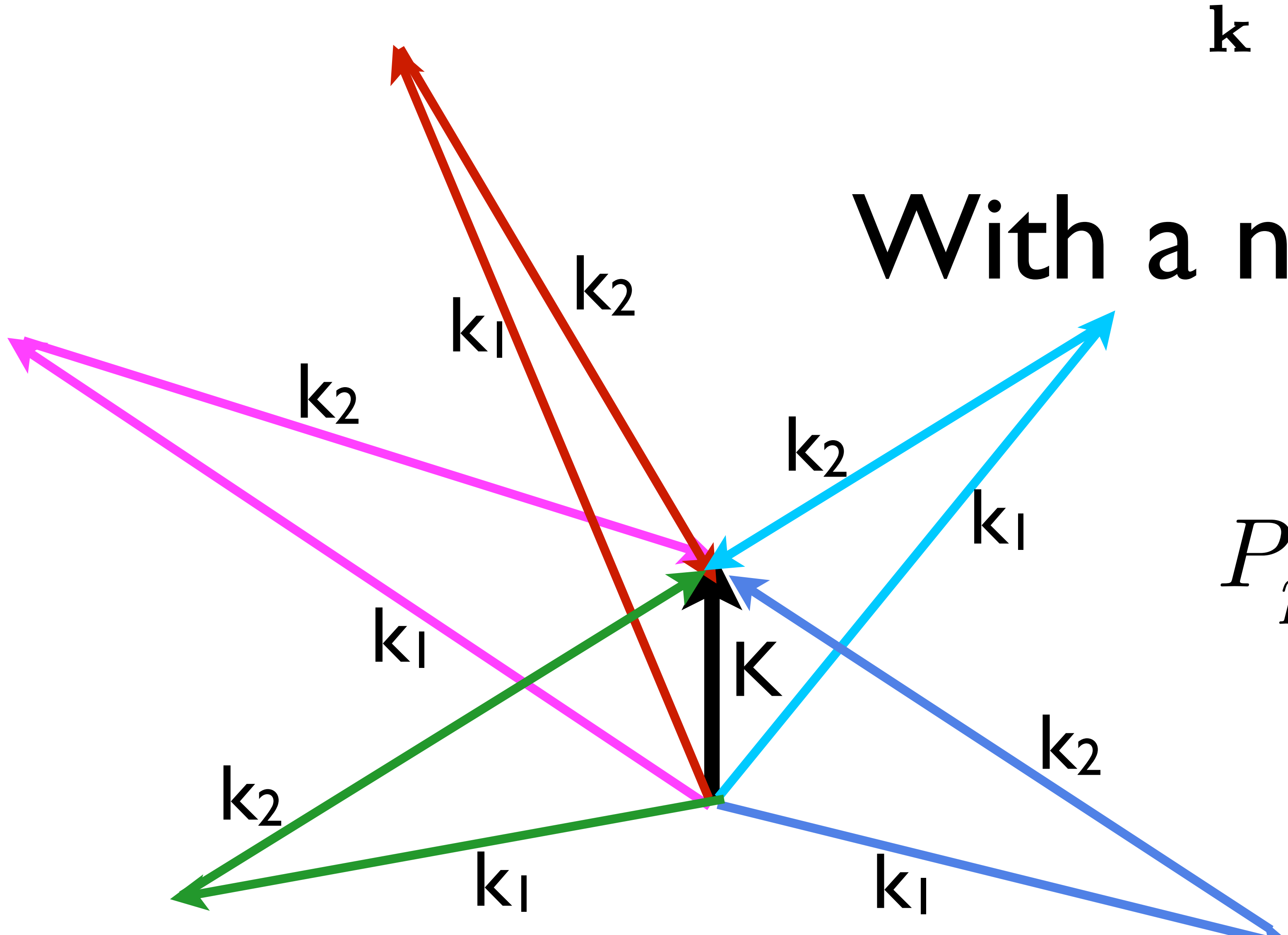
$\xi(\mathbf{r})$ with single vector mode (p=x,y)



Optimal estimator for a single mode

- Inverse-variance weighting gives an optimal estimator for a single mode

$$\widehat{h_p(\mathbf{K})} = P_p^n(\mathbf{K}) \sum_{\mathbf{k}} \frac{f_p^*(\mathbf{k}, \mathbf{K}-\mathbf{k})\epsilon_{ij}^p k^i (K-k)^j}{2V P^{\rm tot}(k) P^{\rm tot}(|\mathbf{K}-\mathbf{k}|)} \delta(\mathbf{k})\delta(\mathbf{K}-\mathbf{k})$$

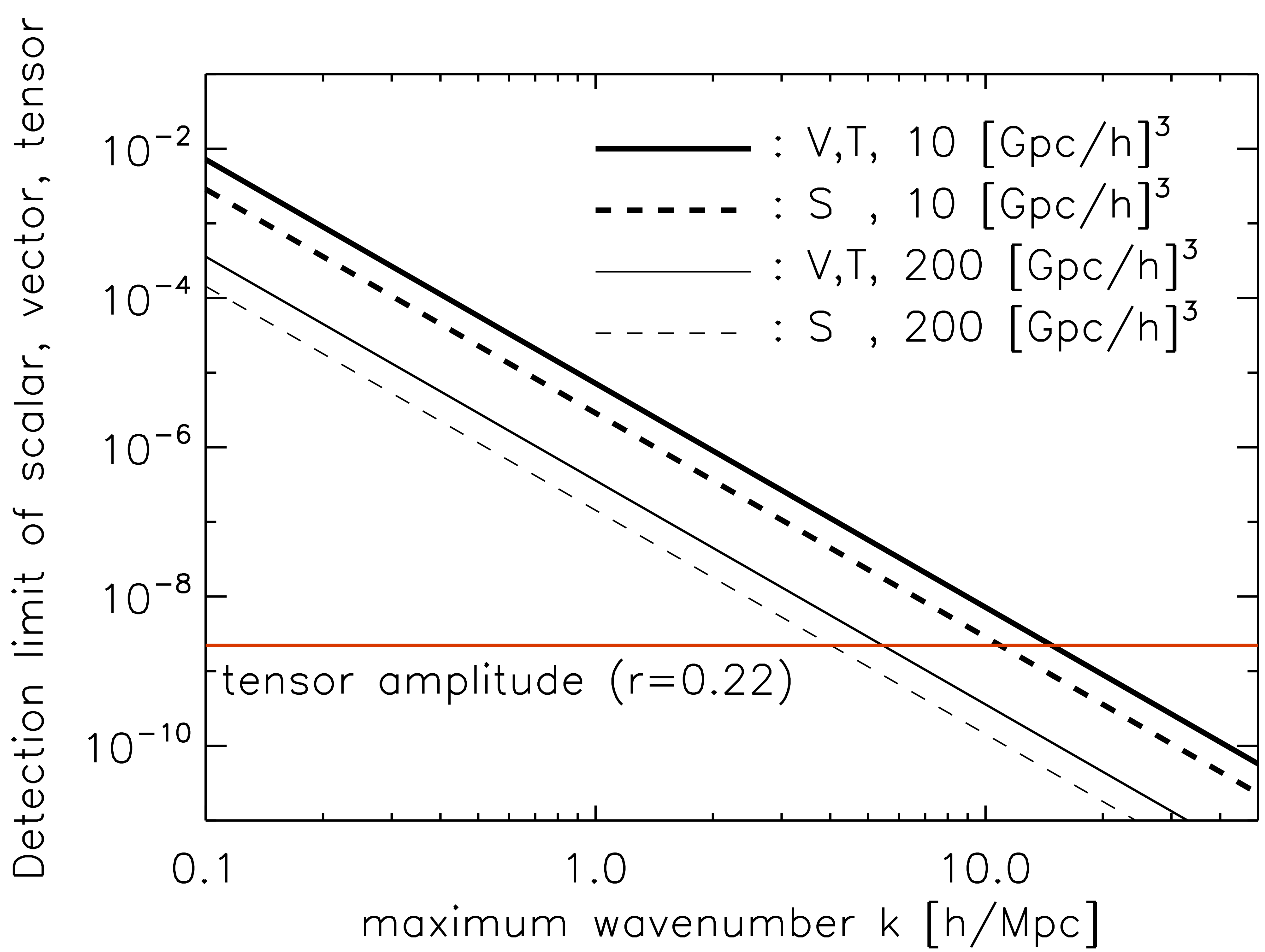


With a noise power spectrum ($P_{tot} = P_{galaxy} + P_{noise}$)

$$P_p^n(K) = \left[\sum_{\mathbf{k}} \frac{\left| f_p(\mathbf{k}, \mathbf{K}-\mathbf{k})\epsilon_{ij}^p k^i (K-k)^j \right|^2}{2V P^{\rm tot}(k) P^{\rm tot}(|\mathbf{K}-\mathbf{k}|)} \right]^{-1}$$

When new “fields” are usual metric fluctuations

- Then, new field only rescales the wave-vector $k^2 \rightarrow k^2 - h_{ij}k_ik_j$, which reads $f_p=-3/2P(k)/k^2$ (Maldacena, 2003)



- projected 3-sigma (99% C.L.) detection limit with galaxy survey parameters
- To detect the gravitational wave, we need a large dynamical range
- Current survey (e.g. SDSS) should set a limit on primordial V and T!

Conclusion

- We present three different ways of detecting primordial GW. For all three methods, effect at the source location is important as GW itself decays in time.
  - Galaxy clustering: impossible to probe as the signal is too weak compared to that of scalar perturbations
  - Cosmic shear: a bit challenge, but possible to detect GW on large scales thanks to the intrinsic alignment effect!
  - Fossil equation: requires large dynamical range to beat the small signal (21cm map?). Interesting potential to detect primordial vector fields as well.