Cosmological tests of GR
A Principal component analysis

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AH et al arXiv:1210.6880 , AH arXiv:1210.3903

AH et al PRD (2012) , G. Zhao et al PRL (2010)

ΛCDM (+GR): Preferred model

- %70 Dark energy (DE): Λ
  - Constant energy density
  - Equation of state parameter : $\mathbf{w \equiv \frac{P}{\rho} = -1}$

However,

- **Λ**: has problems
  - Cosmological constant problem
  - Coincidence problem

- **GR:** has **not** been tested on large scales

We will have the opportunity to test !

Linear growth of structure

Need to evolve four variables : ϕ, Ψ, δ and V

$$\boxed{ds^2 = a(\tau)^2[-(1+2\Psi)d\tau^2 + (1-2\Phi)dx^2]}$$

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Theory of gravity, GR:

$$\boxed{\begin{aligned} &k^2\Psi = -4\pi G a^2 \rho \Delta \\ &\Phi = \Psi \end{aligned}}$$

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Theory of gravity, Parametrized:

GR+ΛCDM: μ=γ= 1

$$k^2\Psi = -4\pi G a^2 \mu(k,a)\rho\Delta$$

$$\Phi = \gamma(k,a)\Psi$$

Alternative gravity theory: Different evolution for perturbations

MGCAMB

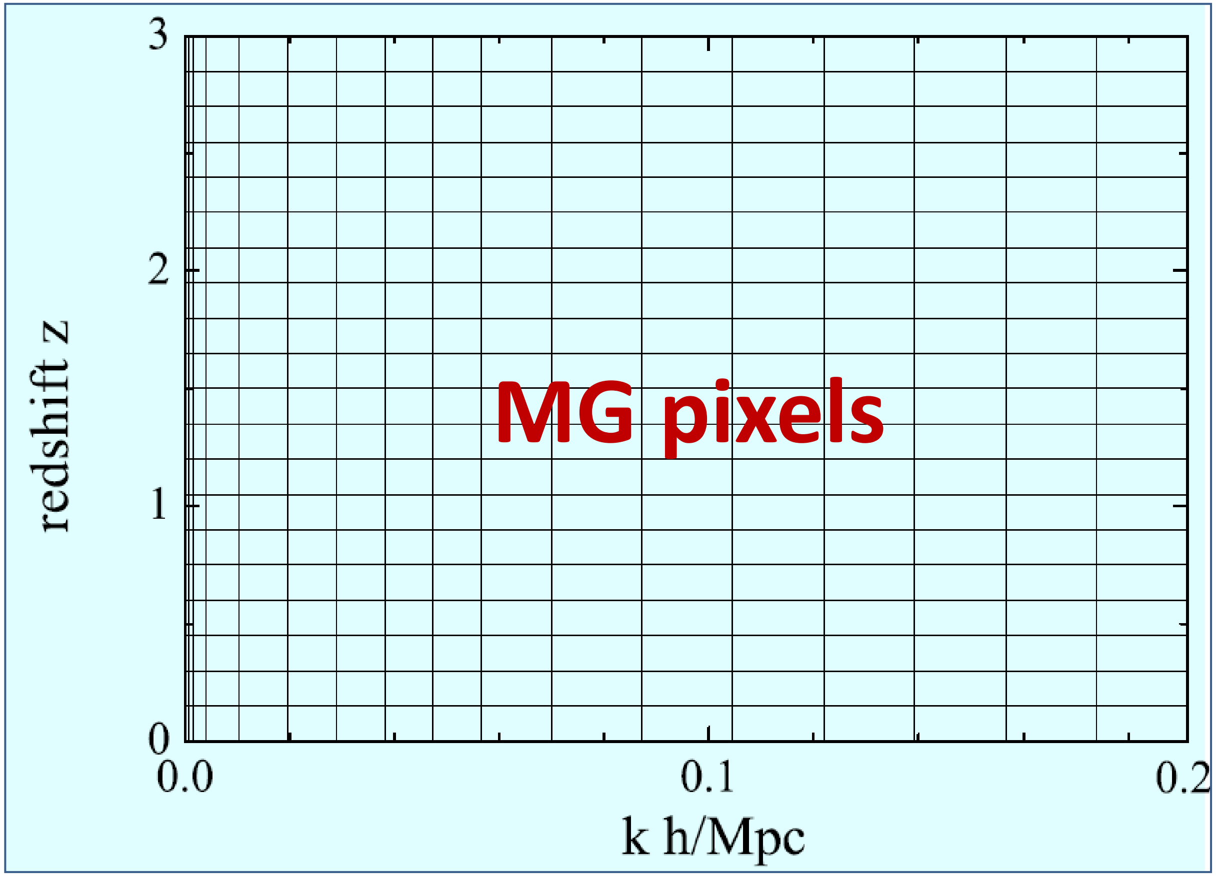
- Einstein-Boltzmann codes (CAMB,CMBFAST) are based on GR
- Parametrized equations should be implemented
- MGCAMB : Modification of Growth with CAMB
  - Works for a general parametrization (general μ,γ)
  - Several parametric forms are implemented
  - Compatible with CosmoMC

http://www.sfu.ca/~aha25/MGCAMB.html

Constraining (μ,γ), forecast

(G. Zhao et al PRL 09 – AH et al PRD 12)

- Discretize μ and γ
- Fisher analysis
- Estimate errors



Errors are large and correlated !

Principal Component Analysis (PCA)

- By diagonalizing the Covariance matrix of μ (γ)

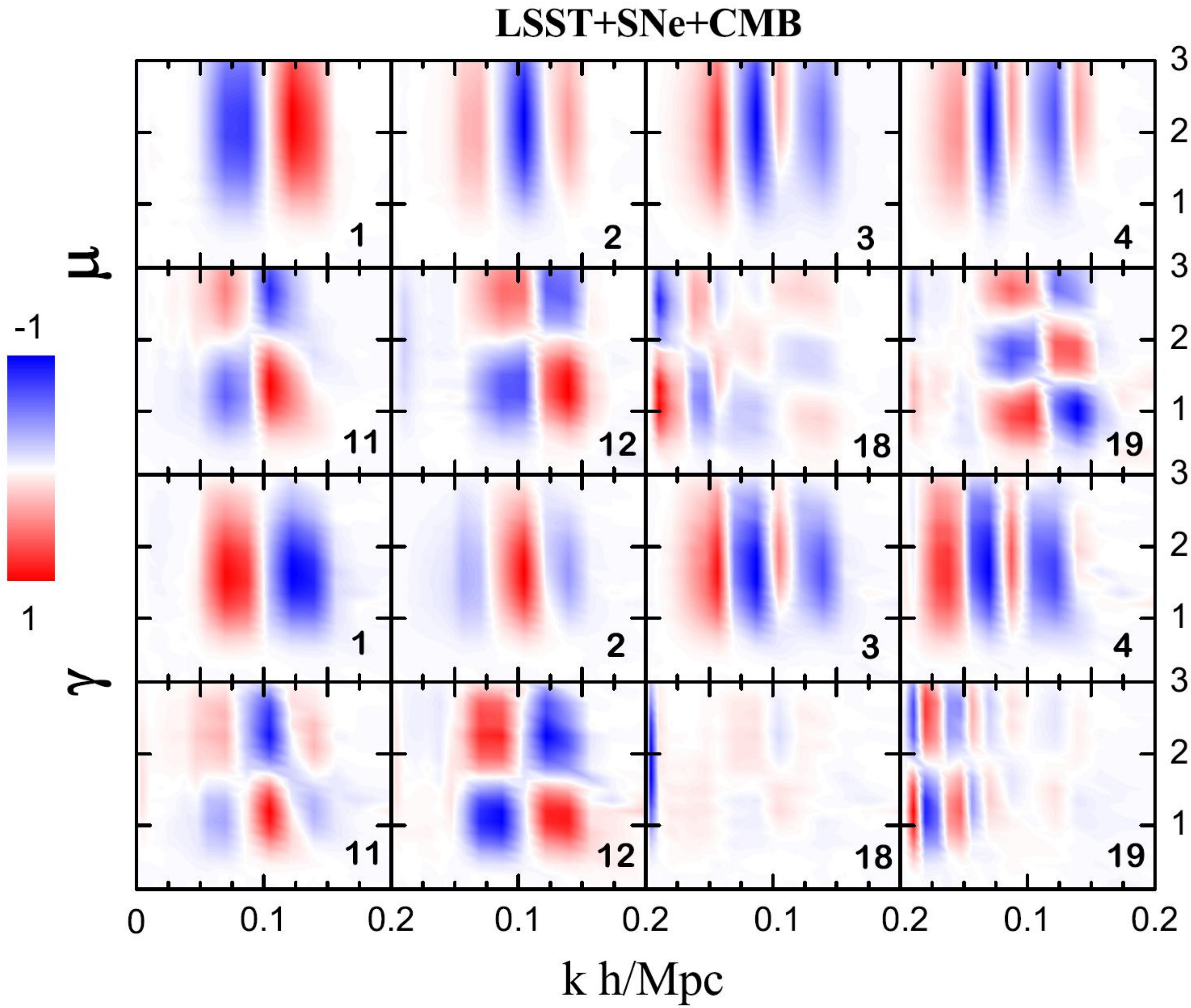
$$\boxed{C = W^T \Lambda W \;;\quad \Lambda_{ij} = \lambda_i \delta_{ij}}$$

- Expanding μ (γ) in terms of eigenmodes:

$$\boxed{\mu(z,k) - 1 = \sum_m \alpha_m e_m(z,k)}$$

- Expansion coefficients (α_m) are uncorrelated
- Working with (relatively) few best constrained eigenmodes

Eigenmodes of μ and γ



Systematics

- photo-z errors H. Zhan et al, Astrophys. J. 09
- PSF uncertainties D. Huterer et al, Mon. Not. Roy. Astron. Soc. 06

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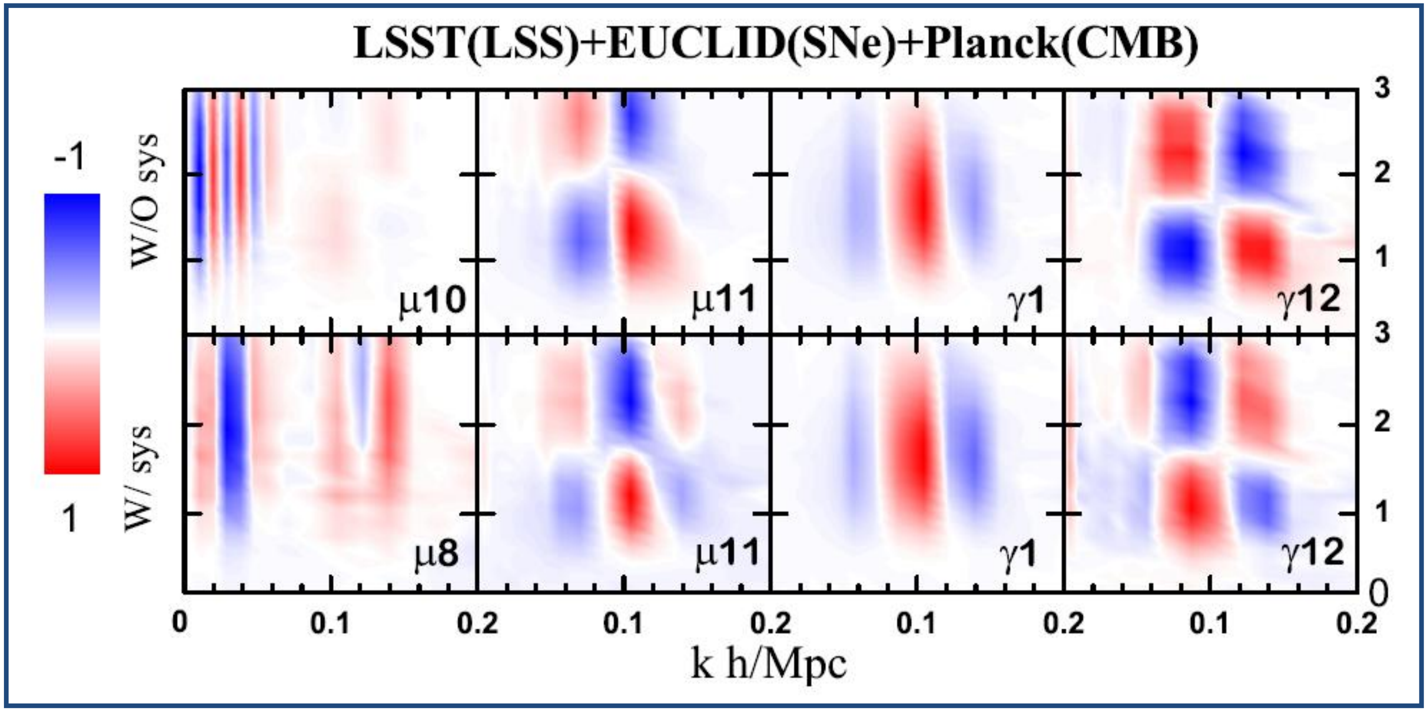

Erasing information about the evolution of perturbations

PCA of w(z)

- Expand w(z) in terms of eigenmodes

$$\boxed{1 + w(z) = \sum_{i=1}^{N} w_i s_i(z)}$$

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- Expand w(z) in terms of eigenmodes

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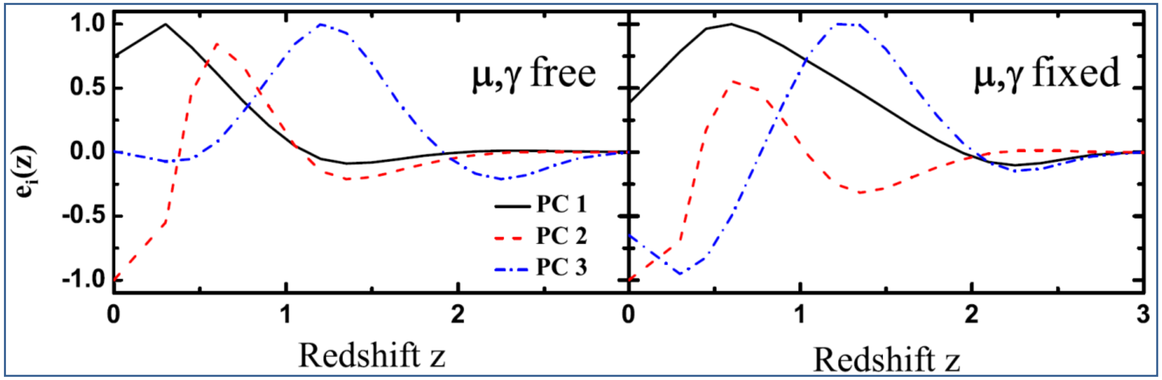
- Best eigenmodes



PCA of w(z)

- Expand w(z) in terms of eigenmodes

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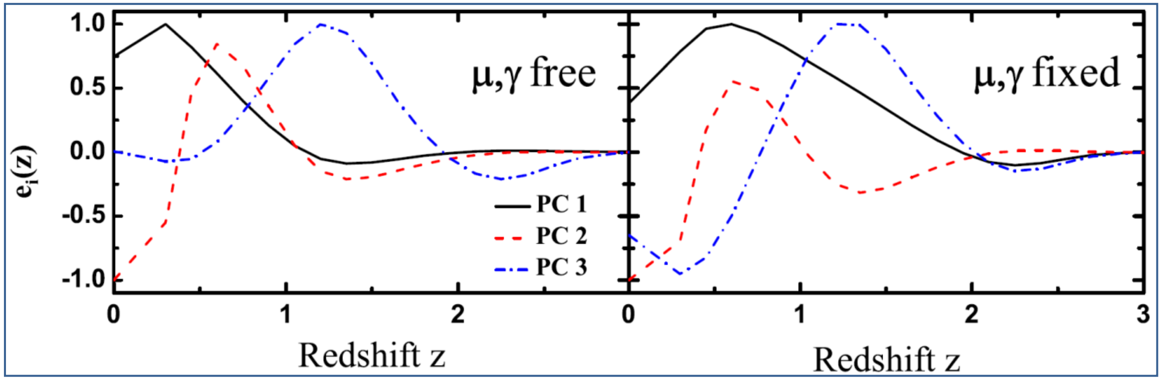
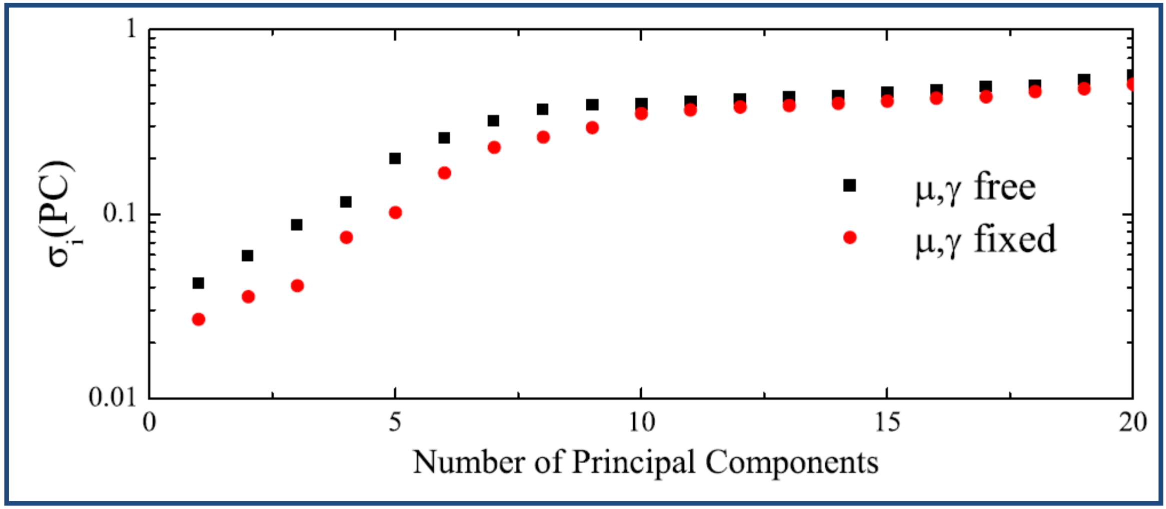
- Best eigenmodes



- We can still measure and constrain w(z)



(AH et al, PRD 85 (2012) 043508)

Degeneracies

- f(R)-class models parametrized:

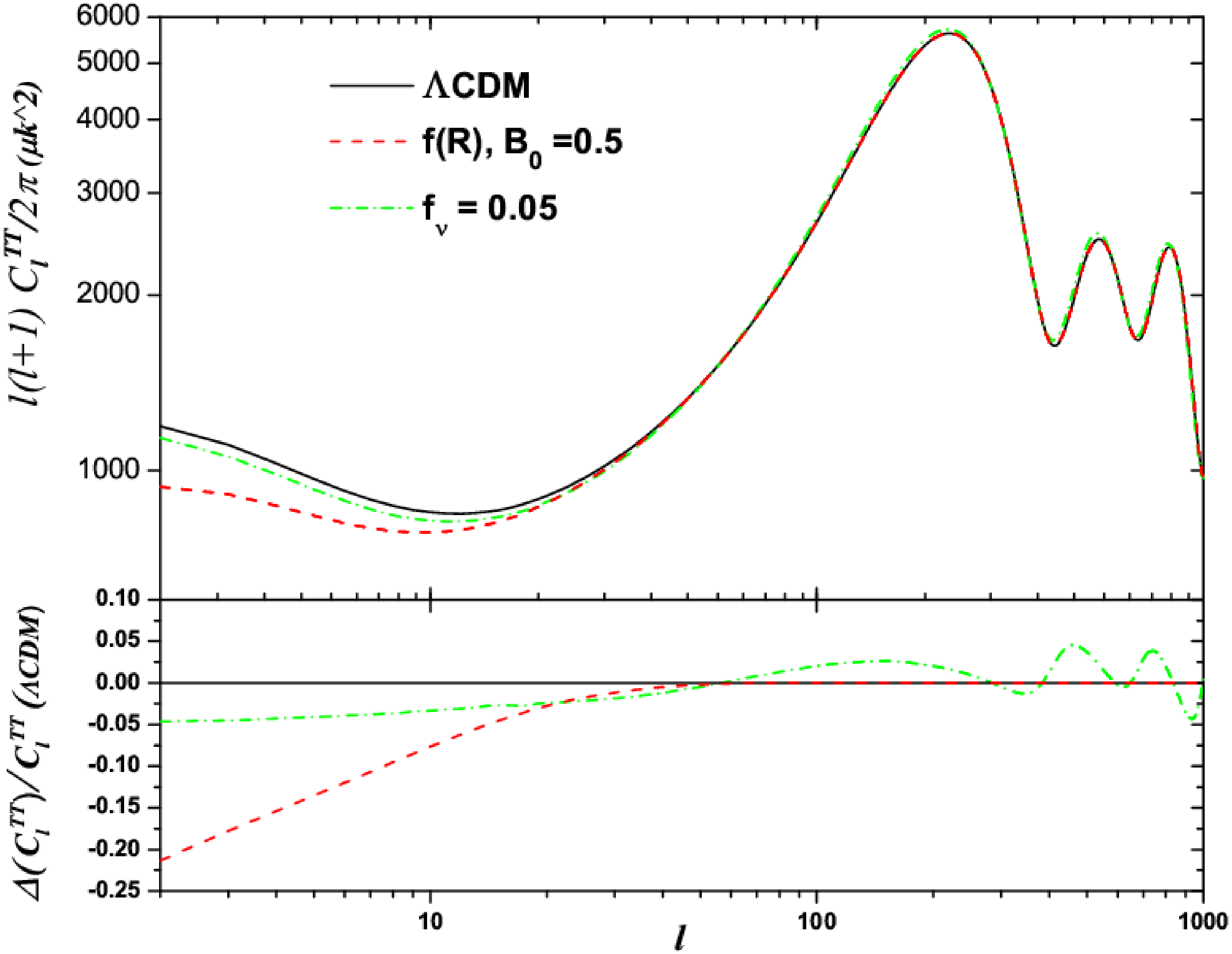
$$\boxed{\mu(k,a) = \frac{1+\frac{4}{3}B_0 C}{1+B_0 C} \qquad \gamma(a,k) = \frac{1+\frac{2}{3}B_0 C}{1+\frac{4}{3}B_0 C}}$$

$$\boxed{C \equiv (cka^2)^2/2H_0^2}$$

$$\boxed{B \equiv \frac{f_{RR}}{1+f_R}\frac{dR}{d\ln a}\left(\frac{d\ln H}{d\ln a}\right)^{-1}}$$

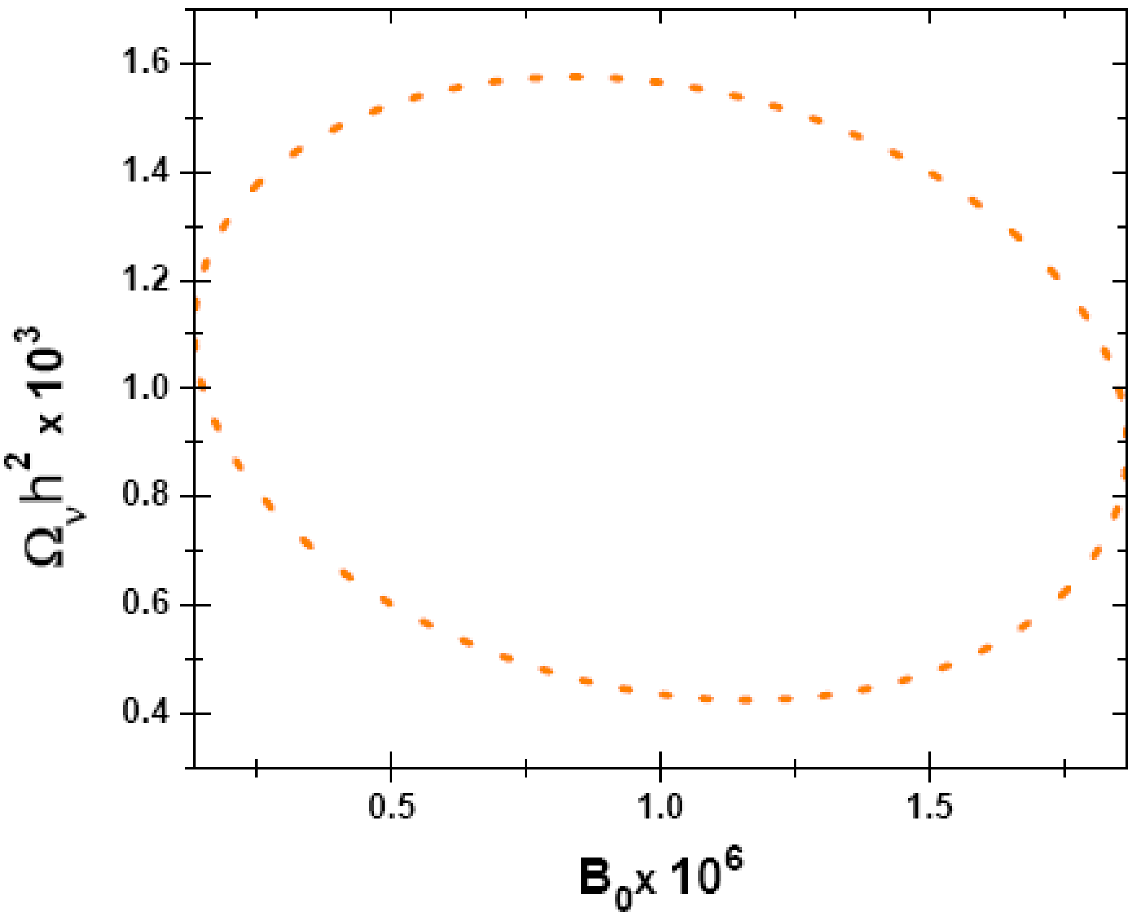
(AH et al, arXiv:1210.6880)

Degeneracies: MG with massive neutrinos
CMB+SNe + ISW

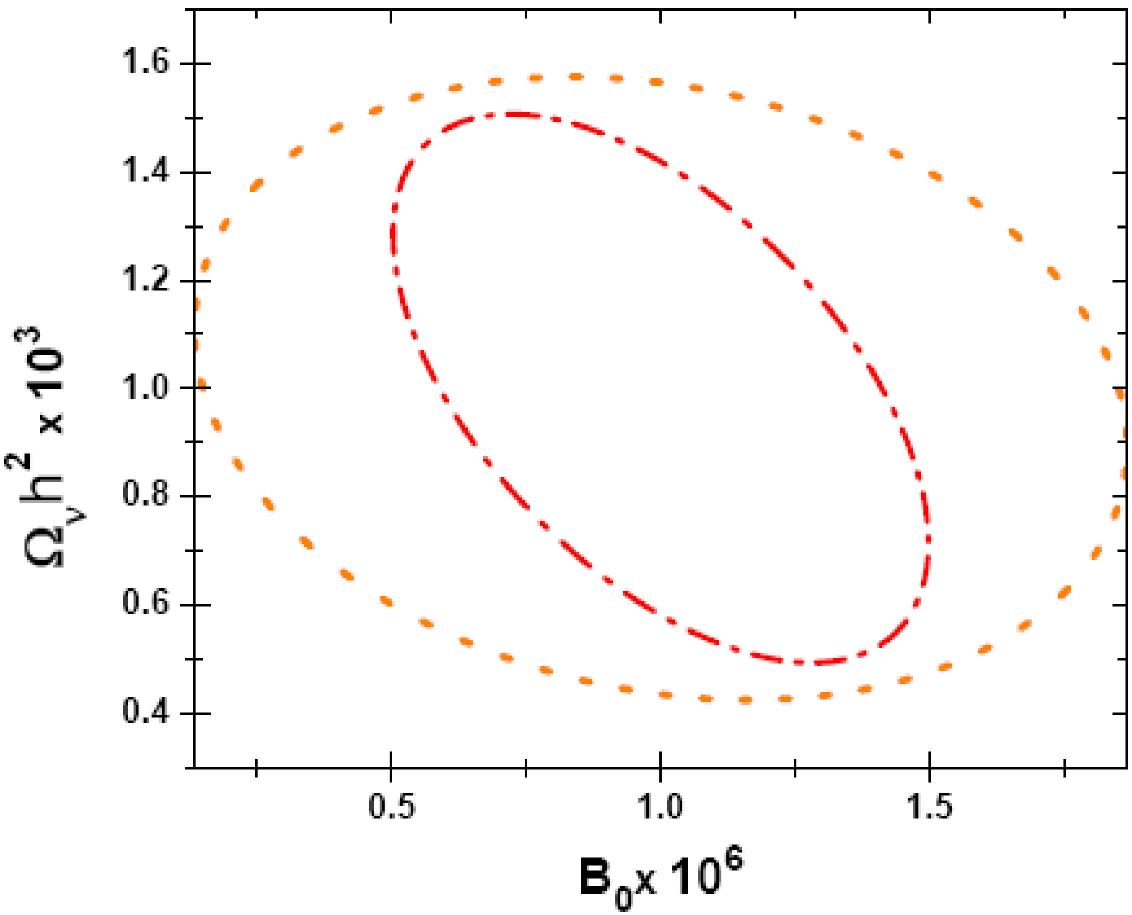


AH, G.B. Zhao & L. Pogosian, JCAP 1108:005 (2011)

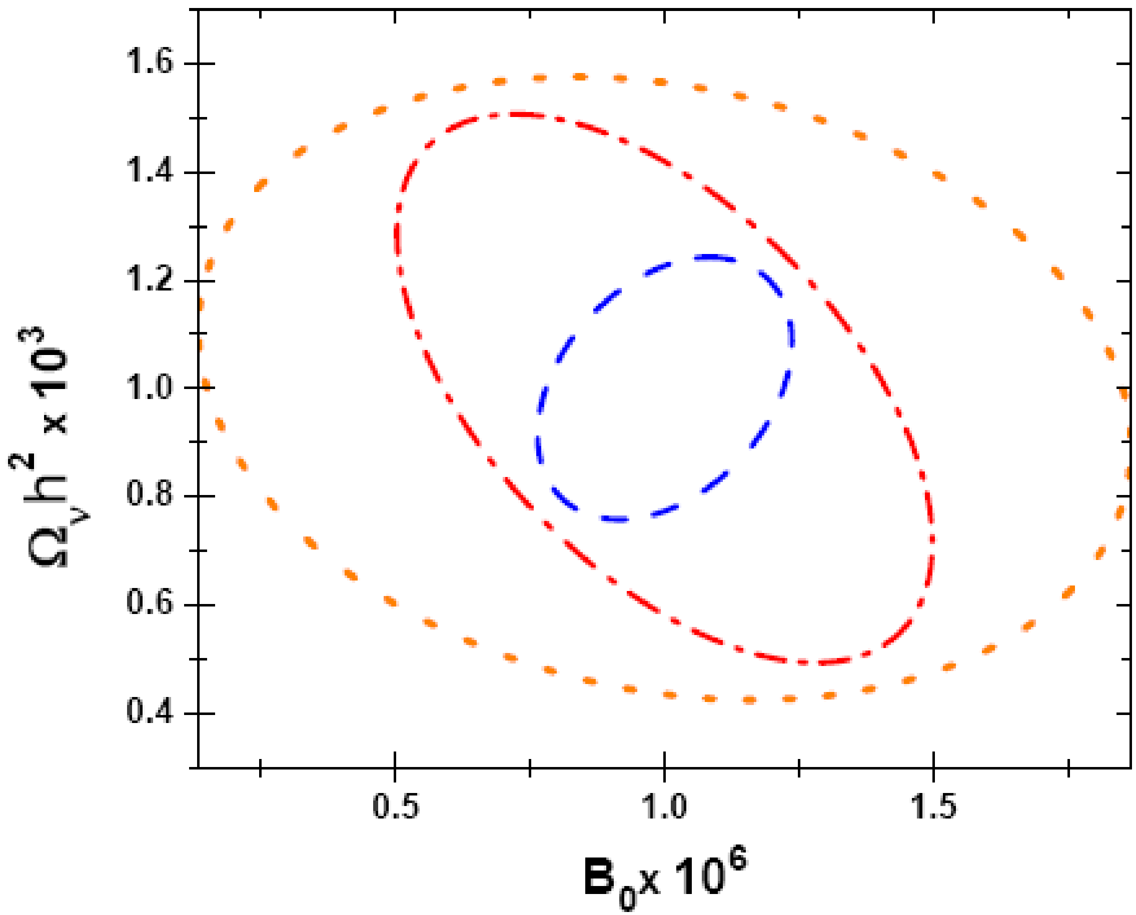
Degeneracies: MG with massive neutrinos
CMB+SNe + GC



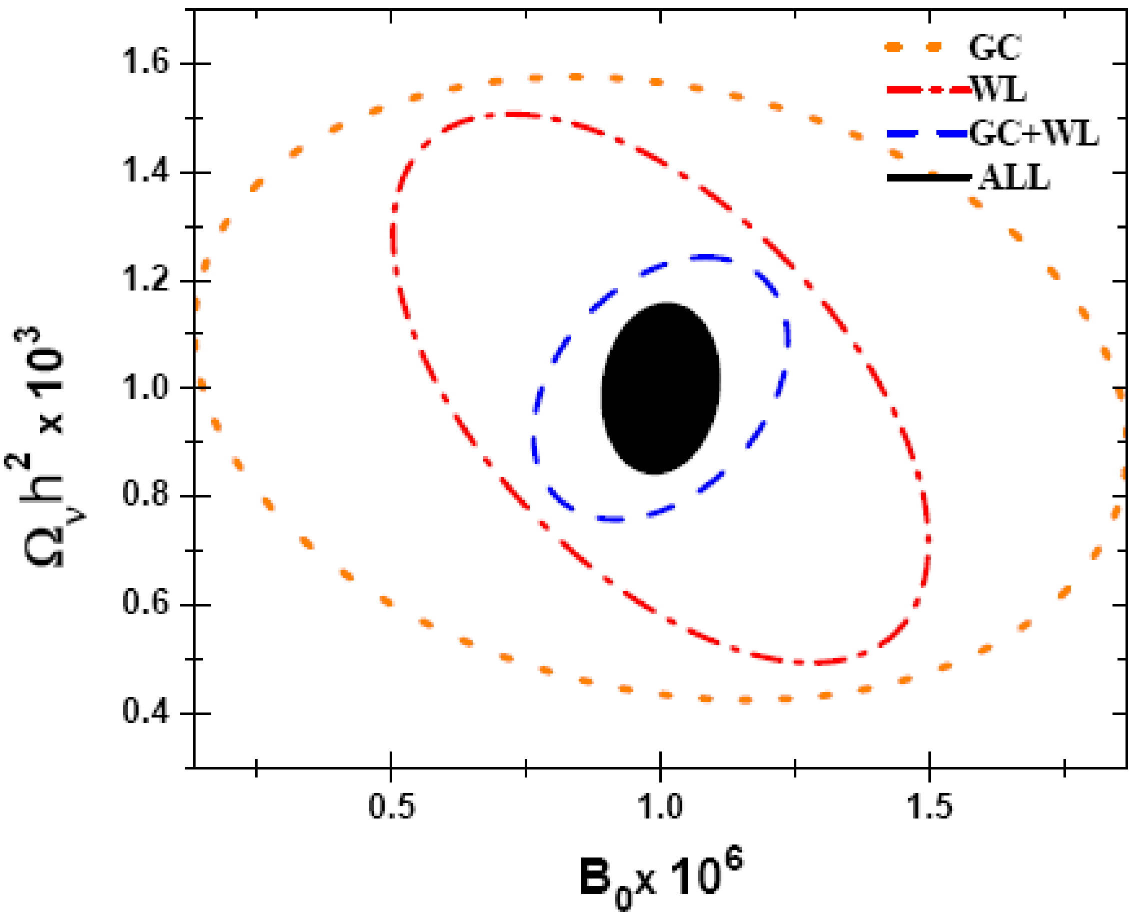
Degeneracies: MG with massive neutrinos CMB+SNe + WL



Degeneracies: MG with massive neutrinos CMB+SNe +GC+ WL

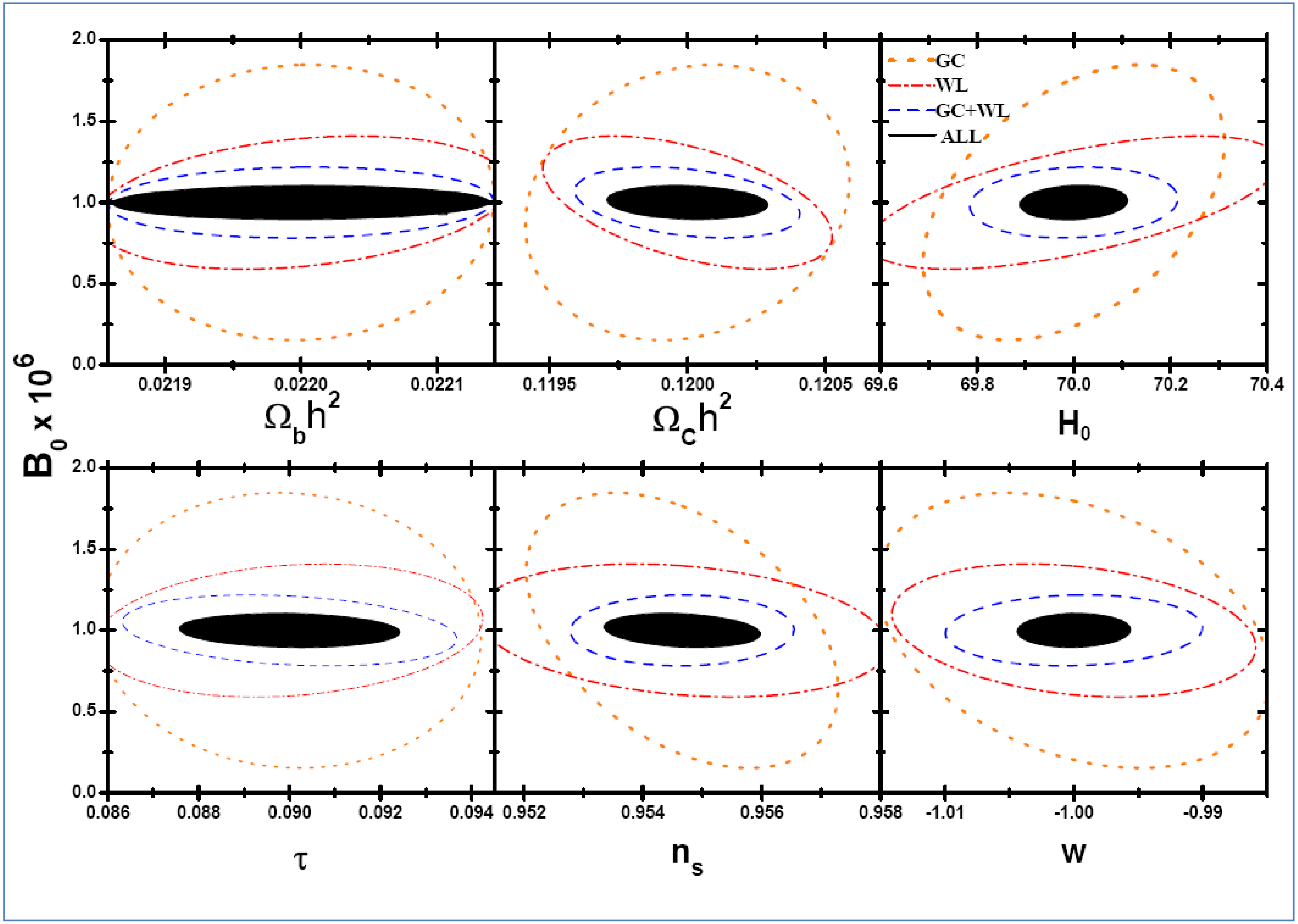


Degeneracies: MG with massive neutrinos CMB+SNe + GC+WL+GCxWL



AH, arXiv:1210.3903

Degeneracies: MG with basic parameters



AH, arXiv:1210.3903

Summary

- **Upcoming experiments** are able test GR
- **Model-independent approaches:** useful in the absence of a compelling DE or MG theory
- **Linear growth of perturbations : PCA**
  - Estimate the # of MG parameters well constrained by data
  - Learn about the MG effects experiments can constrain
- **Degeneracies :**
  - Estimate the power of different observables in breaking the degeneracies
  - Evaluate the impact of other degenerate effects (massive neutrinos)
  - DE and MG can be constrained **simultaneously**

Summary

- **Upcoming experiments** are able test GR
- In the absence of a compelling Dark energy or Modified gravity theory, **model-independent approaches** are useful
- **PCA :**
  - Estimate the # of well constrained MG parameters by data
  - Learn about the MG effects experiments can constrain, the scales and redshifts
  - Shows that experiments are more sensitive to scales dependent modifications
- DE and MG can be constrained **simultaneously**

Current Picture

Isotropy, Homogeneity (FRW metric) + Initial Conditions (inflation) + Contents (Particle Physics)

$$\boxed{ds^2 = a^2(\tau)\{d\tau^2 + r^2 d\Omega^2\}}$$

$\Downarrow$

Evolve :

Conservation (Boltzmann) equations

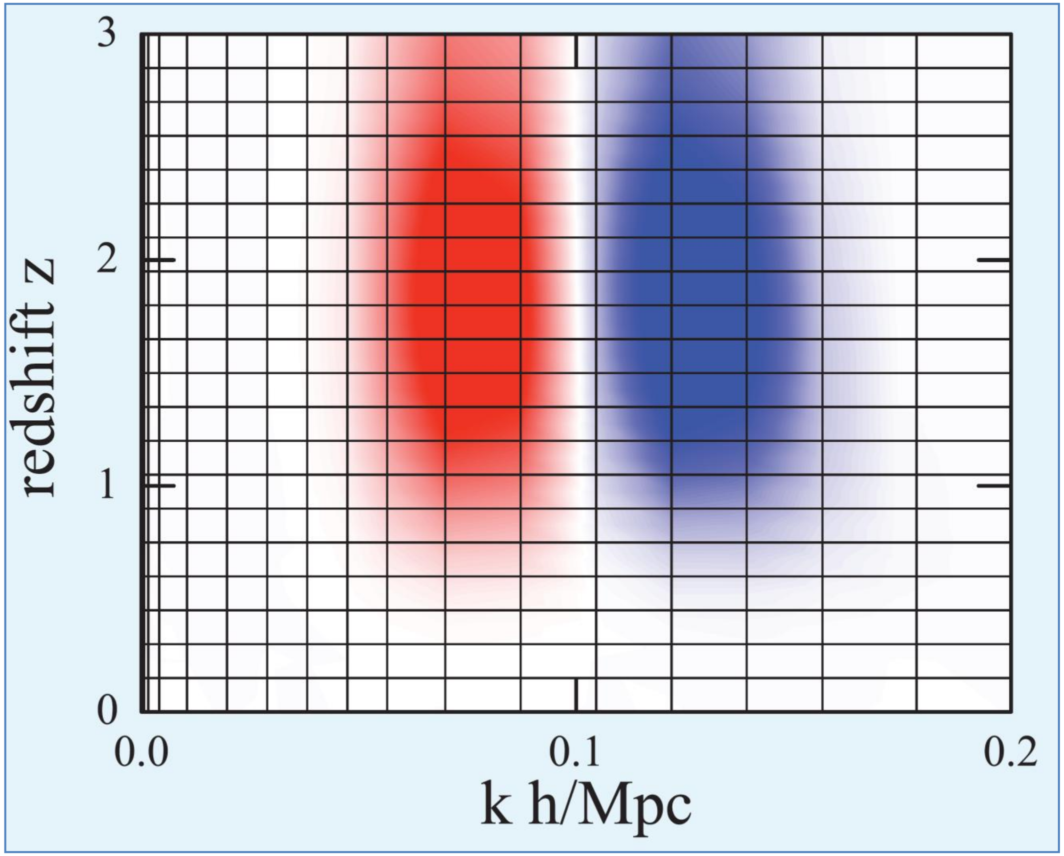
Gravity Theory (GR)

$\Downarrow$

Cosmological Observables

What do we get by doing PCA ?

- # of well-constrained eigenmodes
- “Sweet spots”
- Impact of observables

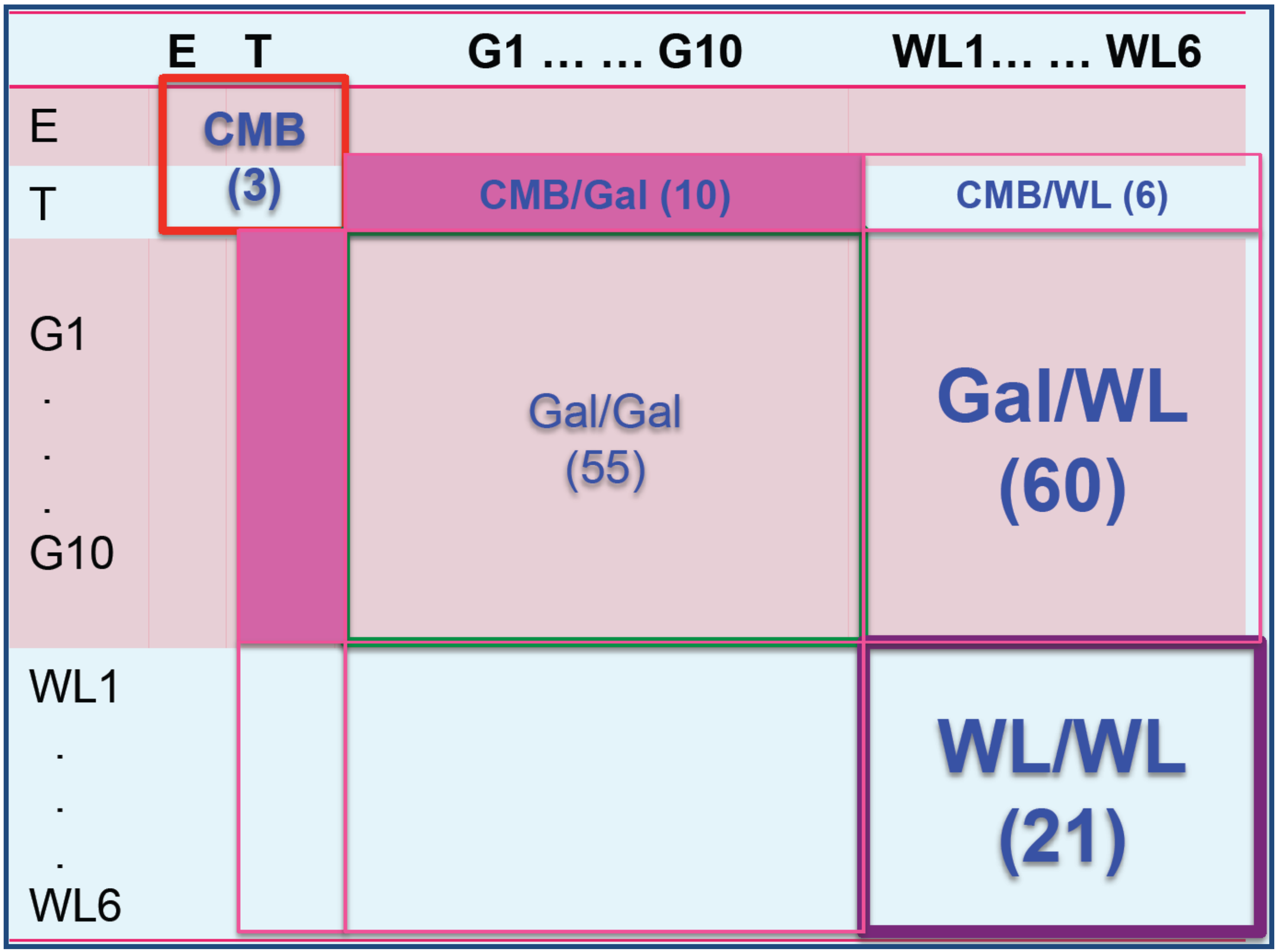


Summary

- **Future experiments** are able test GR
- In the absence of a compelling Dark energy or Modified gravity theory, **model-independent approaches** are useful
- **MG pixels** can store information about linear growth
- **PCA :** Gives insight about experiments

Observables: power spctra

Planck + LSST



(μ,γ) and similar parametrizations: What *they are* and what *they are not*

- **Are:** Model-independent parametrizations describing possible departures from GR (+ ΛCDM) in linear growth of structure

- **Are not:** Unique (will see)

- **Are not:** Theories of modified gravity
  - Can describe solutions of a theory (considering the initial conditions)
  - These solutions can be used to calculate observables for a theory

Constraining DE *and* MG

- *"w"* is a background parameter

- $$\rho_{\rm DE}(z) = \rho_{\rm DE}^{(0)} \exp\left\{ -3 \int d{\rm ln} z' [1 + \mathbf{w}(z')] \right\}$$

- *"w" m*ight be time-varying

- How well can it be constrained in the presence of MG, **simultaneously**?

- PCA of *w(z)* ?

"Measurement" of μ and γ

(G.B. Zhao et al , PRD 09, PRL 09)

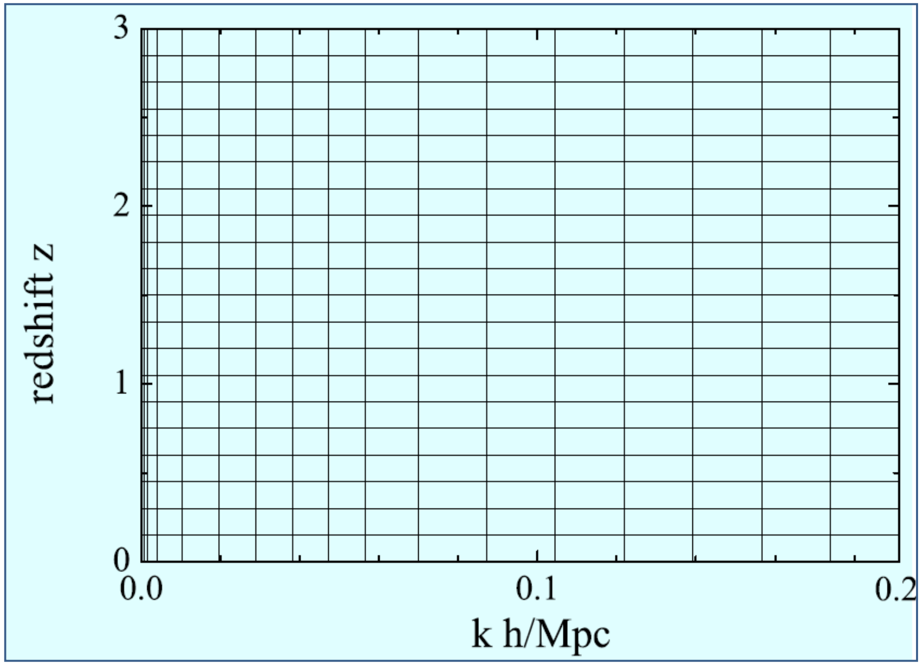
- Discretize μ and γ
- Current data: Not good enough !
- Future data (forecasting) :
  - **Treat pixels as parameters**
  - **Calculate Fisher matrix :**

Cosmological

(Ωc, Ωb, h, ns, As, τ, *bias,SN nuis.*)

+

pixels



$$(C^{-1})_{ab} = F_{ab} = \sum_{O} \frac{\partial O_i}{\partial p_a} \frac{1}{Cov(i,j)} \frac{\partial O_j}{\partial p_b}$$

$$\sigma_{p_a} \geq \sqrt{(F^{-1})_{aa}}$$

What do we get by doing PCA ?

- # of well-constrained eigenmodes for each experiment
(Potentially, # of model parameters)

- “Sweet spots” of experiments

- Learn by studying the behavior of eigenmodes

- PC’s can “store information”