30th Oct--2nd Nov, 2012
5th KIAS workshop

Precision calculations for cosmological power spectrum in real and redshift spaces

Atsushi Taruya

RESearch **C**enter for the **E**arly **U**niverse (RESCEU), Univ. Tokyo

In collaboration with

Francis Bernardeau, Takahiro Nishimichi, Codis Sandrine

Plan of this talk

New development of cosmological tool to accurately compute power spectrum/correlation function of large-scale structure

Motivation

RegPT: new scheme of perturbation theory

Application & Extension

Summary

Motivation

Precision studies of Large-scale structure (LSS)
in the era of precision cosmology

New methodology & technique with clustering anisotropies:

Alcock-Paczynski effect ·······➤ cosmic expansion $D_A(z)$, $H(z)$

Redshift-space distortion (RSD) effect

·······➤ growth of structure $f(z) = \dfrac{d\ln D_+}{d\ln a}$

With baryon acoustic oscillation (***BAO***) as standard ruler,

➡ Nature of dark energy / cosmological test of gravity

Upcoming surveys can make a percent-level measurement:
need for a high-precision theoretical template

Q Can we really achieve percent-level accuracy ?

Regime of our interest

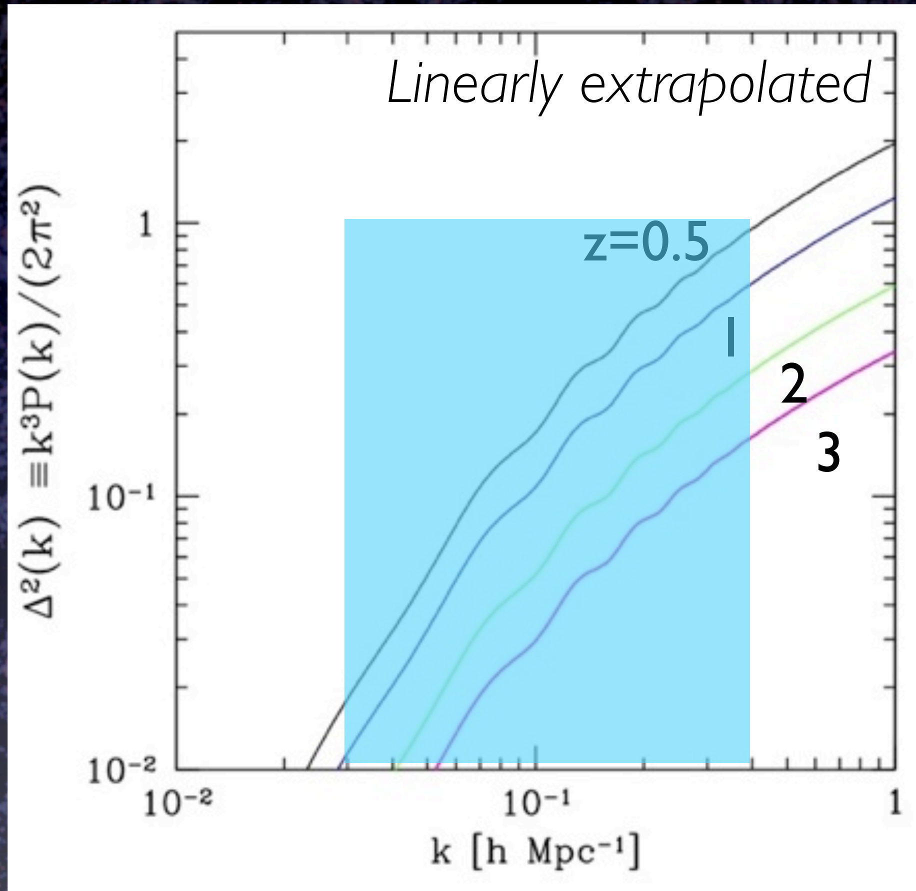
For BAO & RSD
measurements,

0.05< k < 0.4 h/Mpc

or

30 < r < 120 Mpc/h

Especially around z~1



Fully Nonlinear

Weakly nonlinear

Linear

→ weakly non-linear regime of gravitational clustering

Perturbation theory (PT) is a promising tool
as alternative to N-body-based treatments

Perturbation theory: quick review

Theory of large-scale structure based on gravitational instability

Juszkiewicz ('81), Vishniac ('83), Goroff et al. ('86), Suto & Sasaki ('91), Jain & Bertschinger ('94), ...

Cold dark matter + baryons = pressureless & irrotational fluid

Basic eqs. (GR w/o ν)

$$\frac{\partial\delta}{\partial t}+\frac{1}{a}\vec{\nabla}\cdot\left[(1+\delta)\vec{v}\right]=0$$

$$\frac{\partial\vec{v}}{\partial t}+\frac{\dot{a}}{a}\vec{v}+\frac{1}{a}(\vec{v}\cdot\vec{\nabla})\vec{v}=-\frac{1}{a}\vec{\nabla}\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi=4\pi G\bar{\rho}_{\rm m}\delta$$

standard PT

$|\delta|\ll 1$

$$\delta=\delta^{(1)}+\delta^{(2)}+\delta^{(3)}+\cdots \qquad \langle\delta(\boldsymbol{k};t)\delta(\boldsymbol{k}';t)\rangle=(2\pi)^3\,\delta_{\rm D}(\boldsymbol{k}+\boldsymbol{k}')\,P(|\boldsymbol{k}|;t)$$

Perturbation theory: quick review

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Cold dark matter + baryons = pressureless & irrotational fluid

Modern description

Doublet $\Psi_a(\boldsymbol{k};\eta) = \begin{pmatrix} \delta_{\rm m}(\boldsymbol{k};\eta) \\ \theta(\boldsymbol{k};\eta)/f(\eta) \end{pmatrix}$

in Fourier space

Linear growth factor

$$\frac{\partial}{\partial\eta}\Psi_a(\boldsymbol{k};\eta) + \Omega_{ab}(\eta)\Psi_b(\boldsymbol{k};\eta) = \int \frac{d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2}{(2\pi)^3}\,\delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}_1-\boldsymbol{k}_2)\,\gamma_{abc}(\boldsymbol{k}_1,\boldsymbol{k}_2)\,\Psi_b(\boldsymbol{k}_1;\eta)\Psi_c(\boldsymbol{k}_2;\eta)$$

$$\eta \equiv \ln D_+(t) \qquad f = \frac{d\ln D_+}{d\ln a}$$

n-th order solution

symmetric PT kernel

$$\Psi_a^{(n)}(\boldsymbol{k};\eta) = e^{n\,\eta}\int \frac{d^3\boldsymbol{k}_1\cdots d^3\boldsymbol{k}_n}{(2\pi)^{3(n-1)}}\delta_D(\boldsymbol{k}-\boldsymbol{k}_{12\cdots n})\ F_a^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)\times\delta_0(\boldsymbol{k}_1)\cdots\delta_0(\boldsymbol{k}_n)$$

Perturbation theory : revolution

Standard PT turns out to have a poor convergence

Improved PT ('06~'08)

RPT Crocce & Scoccimarro ('06ab, '08)

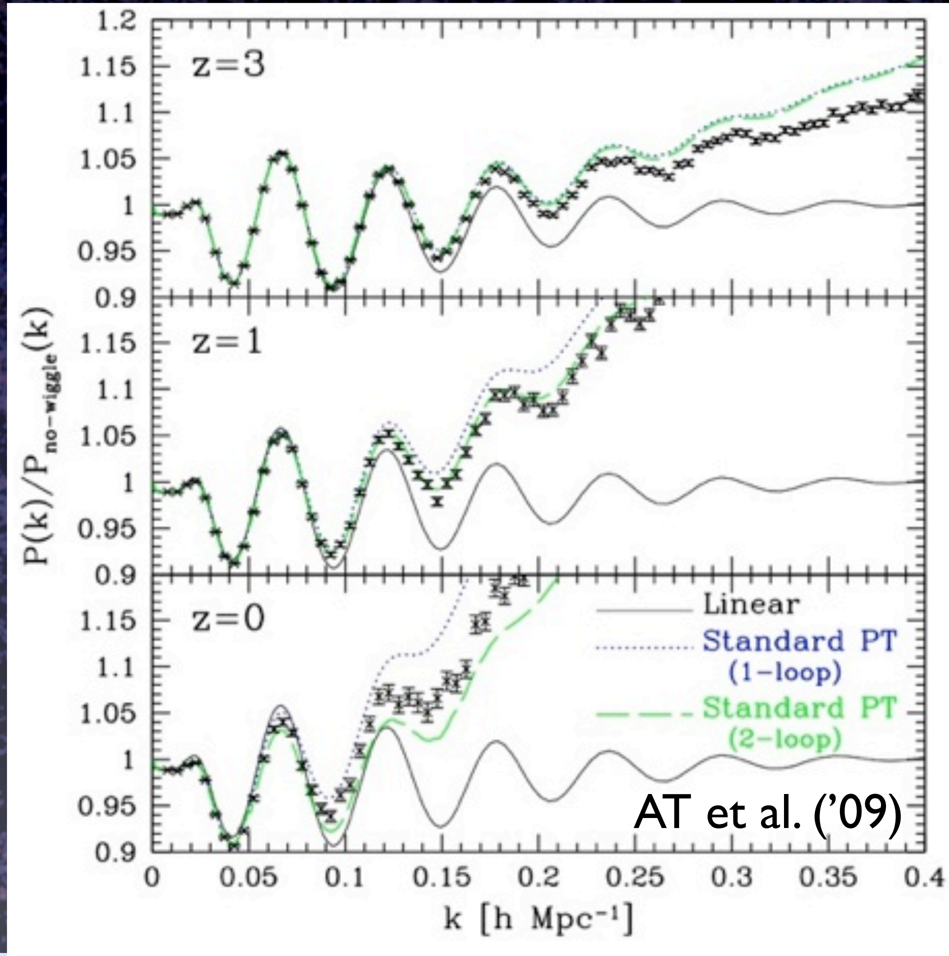
Large-N Valageas ('07)

Closure theory AT & Hiramatsu ('08)

LRT Matsubara ('08ab), Okamura et al. ('11)

Γ-expansion Bernardeau et al. ('08,'12)

Time-RG Pietroni ('08)



Good convergence of improved PT is ensured by re-organizing standard PT expansion by means of non-perturbative quantities

Improved PT in 2nd generation

Implementing more sophisticated treatment or technique, applicable range becomes wider, or calculation becomes faster

MPTbreeze	(Crocce et al. '12)
RegPT	(AT et al. '12)
WH expansion	(Sugiyama & Futamase '12)
iPT	(Matsubara '11)
CLPT	(Carlson et al. '12)

In this talk,

RegPT

Regularized perturbation theory

by means of multi-point propagator expansion

(Γ expansion)

A construction of PT expansion is easy for power spectrum,

and is straightforward for the higher-order statistics

(bispectrum, trispectrum, ...)

Γ-expansion

- A non-perturbative PT expansion formulated by Bernardeau, Crocce & Scoccimarro ('08)
- Standard PT expansion is re-organized by *multi-point propagators*

(n+1)-point propagator

$$\left\langle \frac{\delta\,\delta_{\rm m}(\boldsymbol{k};\eta)}{\delta\,\delta_0(\boldsymbol{k}_1)\cdots\delta\,\delta_0(\boldsymbol{k}_n)} \right\rangle = (2\pi)^{3(1-n)}\delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}_{12\cdots n})\Gamma^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;\eta)$$

$\cdots\cdots\cdot$ multi-point correlations btw. initial (δ_0) & evolved density fields ($\delta_{\rm m}$)

Power spectrum at *2-loop level*

$P_0(k)$: initial P(k)

$$P(k;\eta) = \left[\Gamma^{(1)}(k;\eta)\right]^2 P_0(k) + 2\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\left[\Gamma^{(2)}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q};\eta)\right]^2 P_0(q)P_0(|\boldsymbol{k}-\boldsymbol{q}|)$$

$$+\,6\int\frac{d^6\boldsymbol{p}d^3\boldsymbol{q}}{(2\pi)^6}\left[\Gamma^{(3)}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q};\eta)\right]^2 P_0(p)P_0(q)P_0(|\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}|)$$

Γ-expansion

- A non-perturbative PT expansion formulated by
  Bernardeau, Crocce & Scoccimarro ('08)
- Standard PT expansion is re-organized by *multi-point propagators*

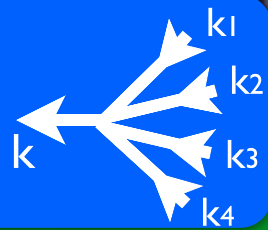
(n+1)-point propagator

$$\left\langle \frac{\delta\,\delta_{\rm m}(\boldsymbol{k};\eta)}{\delta\,\delta_0(\boldsymbol{k}_1)\cdots\delta\,\delta_0(\boldsymbol{k}_n)} \right\rangle = (2\pi)^{3(1-n)}\delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}_{12\cdots n})\Gamma^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;\eta)$$

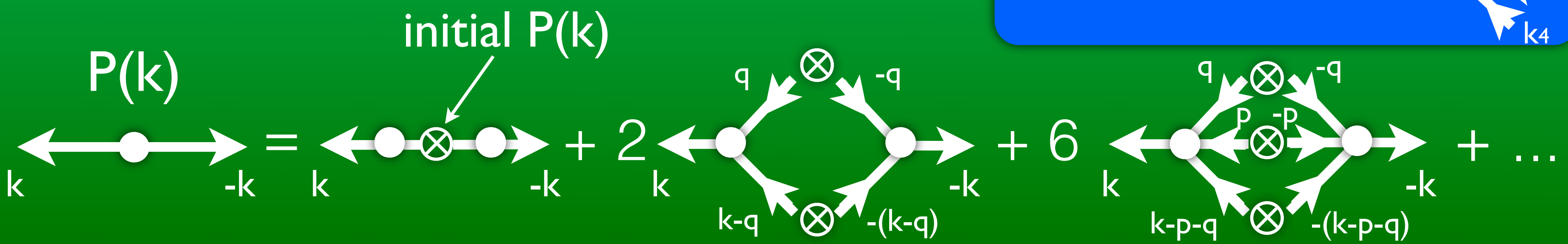
······· multi-point correlations btw. initial (δ_0) & evolved density fields ($\delta_{\rm m}$)

e.g., 5-pt propagator

$\Gamma^{(4)}(k_1,\cdots,k_4;\eta)$



Diagrammatic representation for P(k) in RegPT

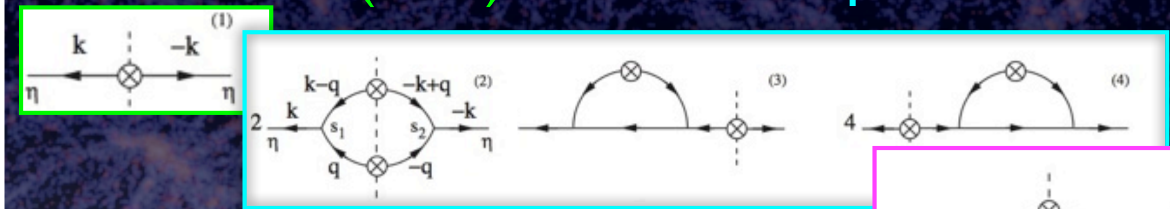


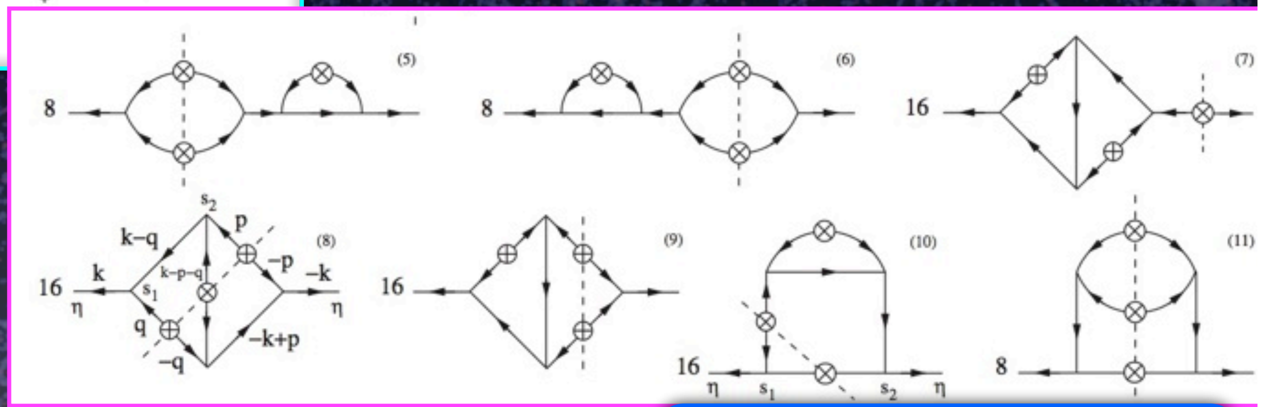
Standard PT vs. Γ-expansion

Standard PT

$$P^{(mn)} \simeq \langle \delta^{(m)} \delta^{(n)} \rangle$$

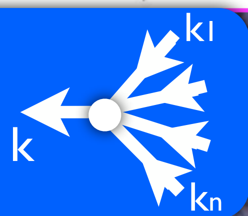
$$P(k) = \underbrace{P^{(11)}(k)}_{\text{Linear (tree)}} + \underbrace{\Big(P^{(22)}(k) + P^{(13)}(k) \Big)}_{\text{1-loop}} + \underbrace{\Big(P^{(33)}(k) + P^{(24)}(k) + P^{(15)}(k) \Big)}_{\text{2-loop}} + \cdots$$

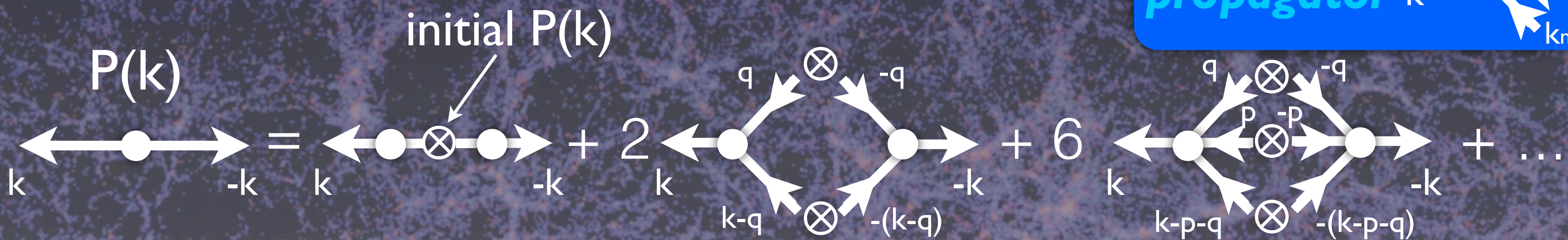




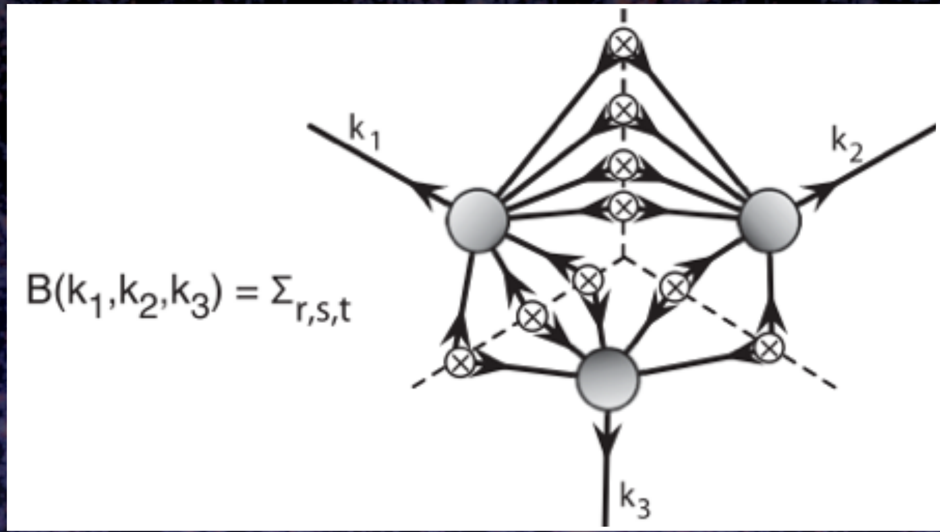
RegPT (Γ-expansion)

multi-point propagator





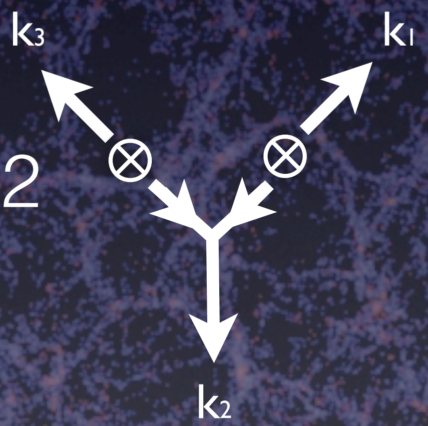
Bispectrum in Γ-expansion

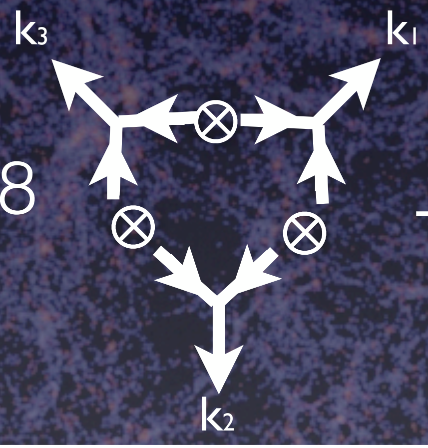


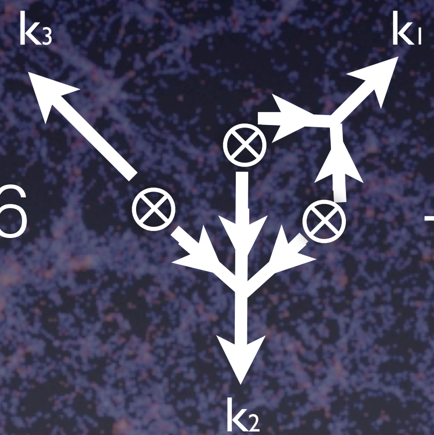
Bernardeau et al. ('09)

summation over possible combinations of Γ

up to 1-loop order

B(k1, k2, k3) = 2 

+ 8 

+ 6 

+ cyc.

$$
\begin{aligned}
B(k_1,k_2,k_3) &= 2\,\Gamma^{(2)}(\mathbf{k}_1,\mathbf{k}_2)\,\Gamma^{(1)}(k_1)\,\Gamma^{(1)}(k_2)\,P_0(k_1)P_0(k_2)+\mathrm{cyc.}\\
&+\Big[8\int d^3q\,\Gamma^{(2)}(\mathbf{k}_1-\mathbf{q},\mathbf{q})\Gamma^{(2)}(\mathbf{k}_2+\mathbf{q},-\mathbf{q})\Gamma^{(2)}(\mathbf{q}-\mathbf{k}_1,-\mathbf{k}_2-\mathbf{q})P_0(|\mathbf{k}_1-\mathbf{q}|)P_0(|\mathbf{k}_2+\mathbf{q}|)P_0(q)\\
&+6\int d^3q\,\Gamma^{(3)}(-\mathbf{k}_3,-\mathbf{k}_2+\mathbf{q},-\mathbf{q})\Gamma^{(2)}(\mathbf{k}_2-\mathbf{q},\mathbf{q})\Gamma^{(1)}(\mathbf{k}_3)P_0(|\mathbf{k}_2-\mathbf{q}|)P_0(q)P_0(k_3)+\mathrm{cyc.}\Big].
\end{aligned}
$$

Multi-point propagator

A crucial point in Γ-expansion is how to construct '*approximate*' multi-point propagators without loosing their non-perturbative properties

- UV property (k >>1) is analytically known :

$$\Gamma^{(n)} \xrightarrow{k\to+\infty} \Gamma^{(n)}_{\rm tree}\, e^{-k^2\sigma_{\rm v}^2/2} \quad ; \quad \sigma_{\rm v}^2 = \int \frac{dq}{6\pi^2} P_{\theta\theta}(q)$$

Bernardeau, Crocce & Scoccimarro ('08), Bernardeau, Van de Rijt, Vernizzi ('11)

- IR behavior (k<<1) can be described by standard PT calculations :

$$\Gamma^{(n)} = \Gamma^{(n)}_{\rm tree} + \Gamma^{(n)}_{\rm 1\text{-}loop} + \Gamma^{(n)}_{\rm 2\text{-}loop} + \cdots$$

In UV limit, each term behaves like $\Gamma^{(n)}_{p\text{-loop}} \xrightarrow{k\to+\infty} \dfrac{1}{p!}\left(-\dfrac{k^2\sigma_{\rm v}^2}{2}\right)^p \Gamma^{(n)}_{\rm tree}$

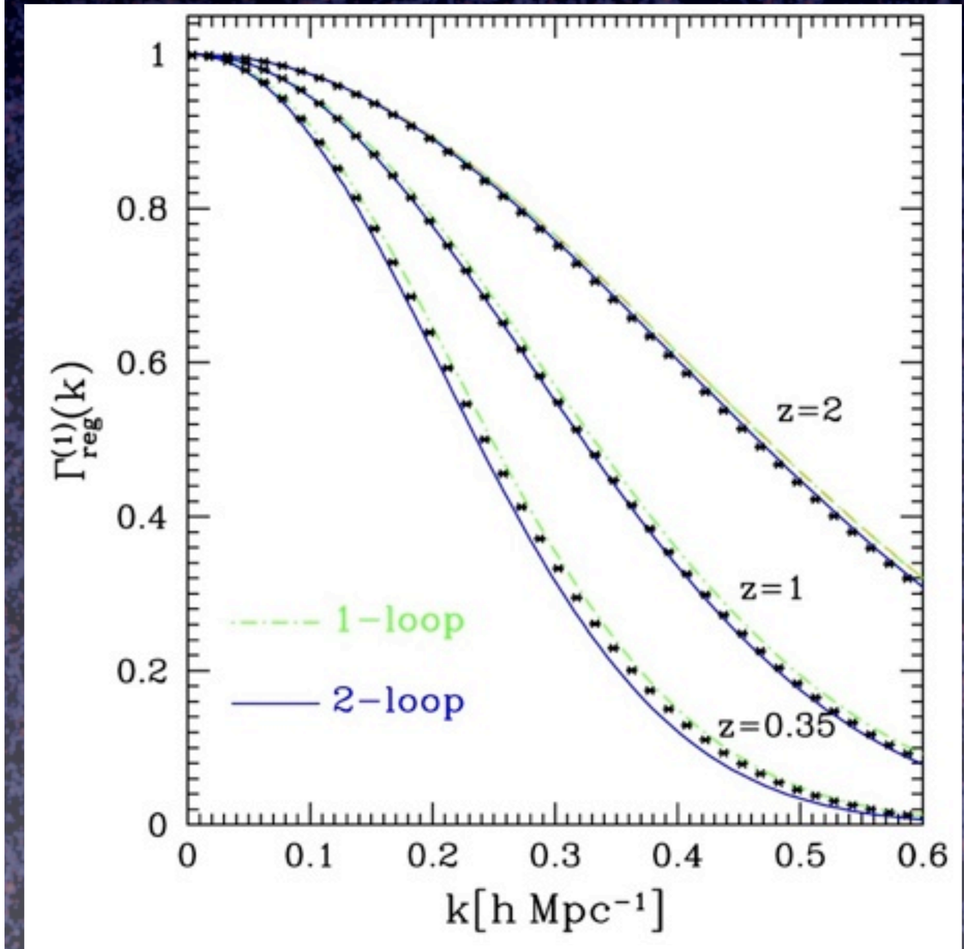
→ A regularization scheme that reproduces both UV & IR behaviors

Bernardeau, Crocce & Scoccimarro ('12)

Propagators in N-body simulations

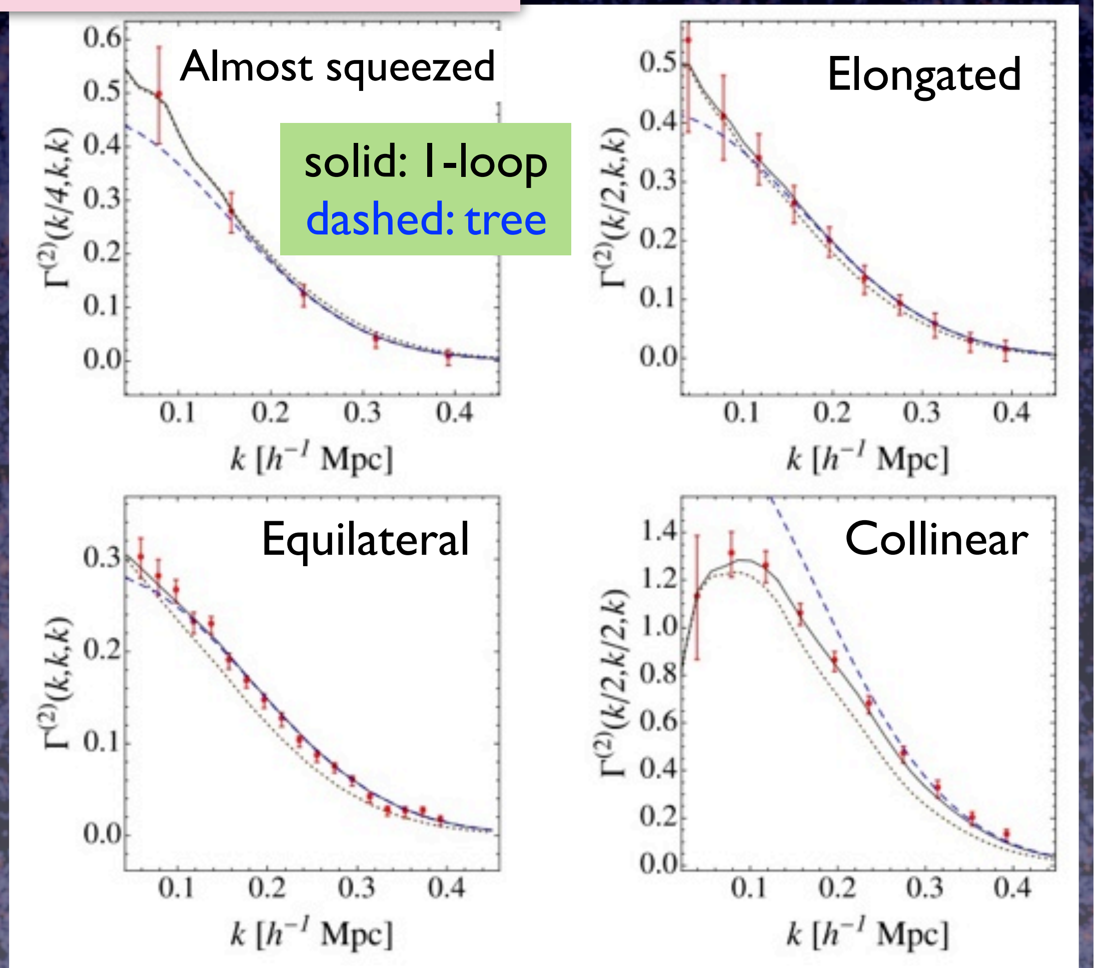
compared with '*Regularized*' propagators constructed analytically

$\Gamma^{(1)}(k)$ predictions up to 2-loop order



Bernardeau, AT & Nishimichi (in prep.)

$\Gamma^{(2)}(k_1, k_2, k_3)$ predictions up to 1-loop order

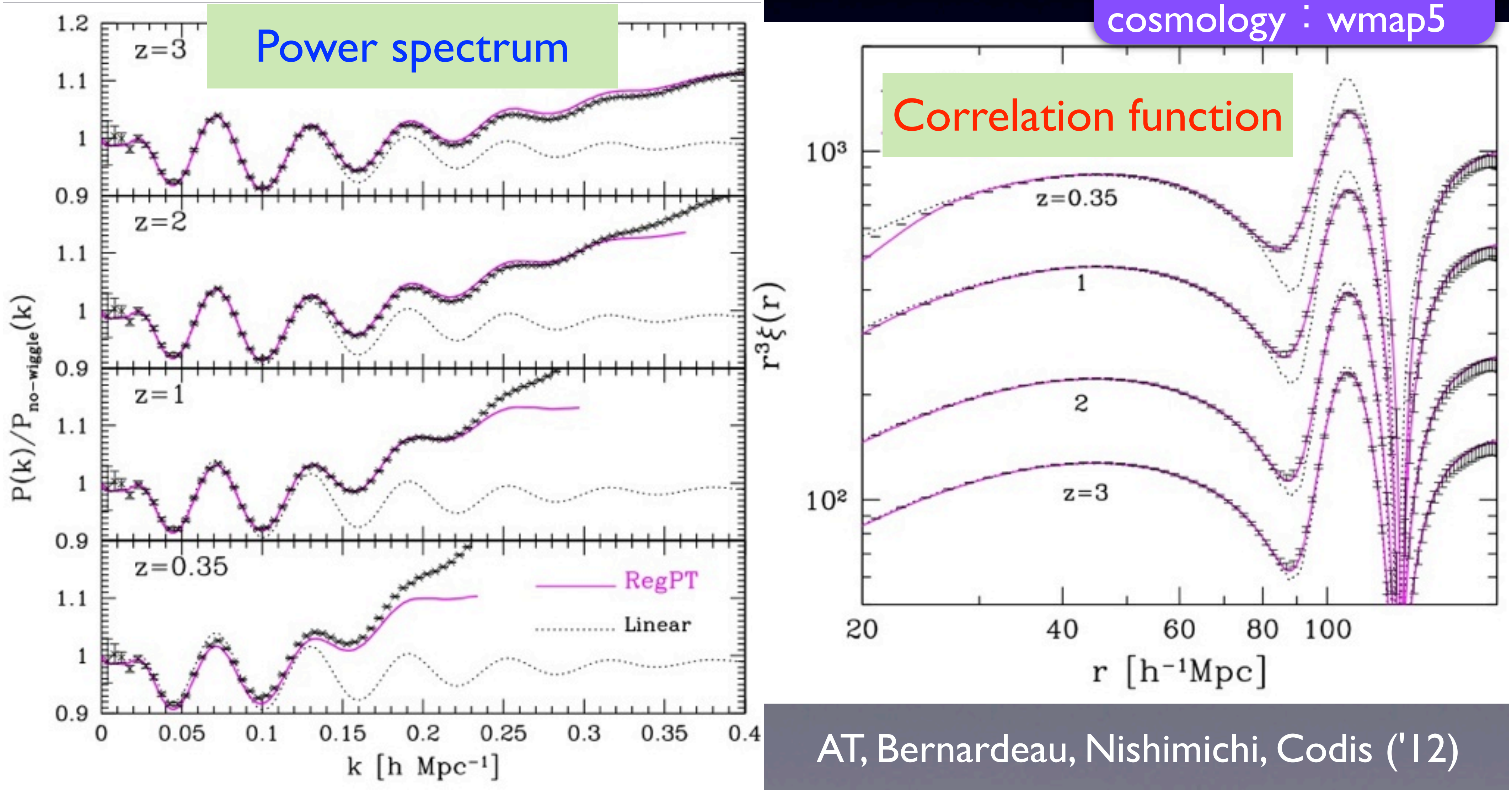


Bernardeau et al. ('12)

Comparison with simulations

Plugging all the ingredients
into 2-loop expressions of P(k) & ξ(r),

$L_{\rm box} = 2,048\,h^{-1}\,{\rm Mpc}$
# of particles : $1,024^3$
# of runs : 60
cosmology : wmap5



AT, Bernardeau, Nishimichi, Codis ('12)

Application & Extension

- Accelerated calculation method

RegPT-fast

AT, Bernardeau, Nishimichi & Codis ('12)

Dramatically fast calculation is possible, suitable for a practical cosmological parameter estimation

5-10min. $\longrightarrow$ ***few sec.***

- Predictions in redshift space

s-RegPT

AT, Bernardeau & Nishimichi (in prep.)

With the improved model of RSD, a consistent calculation is made possible, capturing the non-Gaussian nature of RSD

RegPT-fast

Drawback in most of PT methods with higher-order corrections (i.e., 2-loop) is the time-consuming multi-dimensional integrals.

Basic idea ····· *Expand PT expressions around a fiducial model*

just 1D integral !

$$P_{\rm NL}(k)=\mathcal{F}[P_{\rm lin};k]$$
$$=\underbrace{\boxed{\mathcal{F}[\alpha\,P_0;k]}}_{\text{fiducial model (with arbitrary normalization)}}+\boxed{\int dq\,\frac{\delta\mathcal{F}[\alpha\,P_0;q]}{\delta\,(\alpha\,P_0(q))}\,\{P_{\rm lin}(q)-\alpha\,P_0(q)\}}+\cdots$$

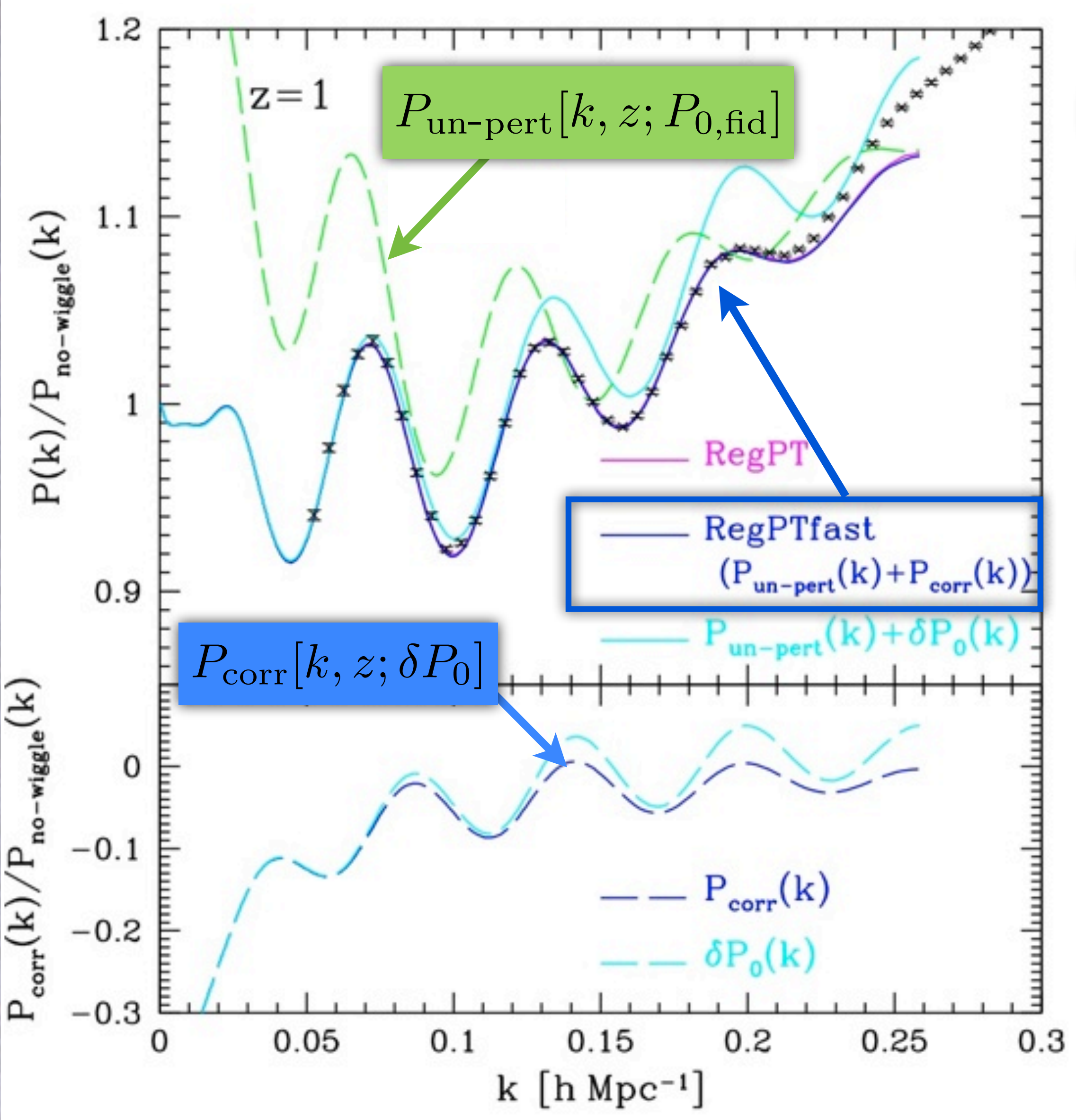
fiducial model
(with arbitrary normalization)

departure from fiducial model

- Adjusting ‘α’, normalization is chosen so as to minimize the difference between target and fiducial models
- Given $P_{\rm lin}(k)$ for target model, the task is to evaluate the residuals which is nothing but 1D integrals.

Demonstration

AT, Bernardeau, Nishimichi, Codis (12)



Target (N-body)

wmap5 cosmological model

Fiducial

wmap3 cosmological model

Fiducial (wmap3)	Target (wmap5)
$\Omega_{\rm m} = 0.234$	$\Omega_{\rm m} = 0.279$
$\Omega_\Lambda = 0.766$	$\Omega_\Lambda = 0.721$
$\Omega_{\rm b}/\Omega_{\rm m} = 0.175$	$\Omega_{\rm b}/\Omega_{\rm m} = 0.165$
$\sigma_8 = 0.76$	$\sigma_8 = 0.817$

RegPT: public PT code

Generic PT code based on Gamma expansion

- Power spectrum & correlation function in real space
- Code provides data sets of 3 fiducial models for fast computation, which are automatically selected
- Many options (direct-/fast-mode, 1-loop/2-loop calculations, other PT methods)

http://www-utap.phys.s.u-tokyo.ac.jp/~ataruya/regpt_code.html

From real to redshift space

Prediction in redshift space needs one more further step

RSD effects can be described by the simple prescription:

redshift space; real space

$$\vec{s} = \vec{r} + \frac{1+z}{H(z)}(\vec{v}\cdot\hat{z})\,\hat{z} \equiv \vec{r} - f\,u_z(\vec{r})\,\hat{z}$$

Due to the non-linear mapping, however, the resultant power spectrum in redshift space is rather complicated

$$P^{(\mathrm{S})}(\mathbf{k}) = \int d^3\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}} \left\langle e^{-ik\mu f\Delta u_z}\, \{\delta(\mathbf{r}) + f\nabla_z u_z(\mathbf{r})\}\,\{\delta(\mathbf{r}') + f\nabla_z u_z(\mathbf{r}')\} \right\rangle$$

$$\mathbf{x} \equiv \mathbf{r} - \mathbf{r}'$$

$$\Delta u_z \equiv u_z(\mathbf{r}) - u_z(\mathbf{r}')$$

- not simple two-point statistics
- exhibit non-Gaussian nature

Semi-analytic model of RSD

AT, Nishimichi & Saito ('10)

$$P^{(\mathrm{S})}(k,\mu)=e^{-(k\mu f\sigma_{\rm v})^2}\left[P_{\delta\delta}(k)+2f\mu^2P_{\delta\theta}(k)+f^2\mu^4P_{\theta\theta}(k)+A(k,\mu)+B(k,\mu)\right]$$

Fitting parameter

$$A(k,\mu)=(k\mu\,f)\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\,\frac{p_z}{p^2}\,\{B_\sigma(\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p},-\boldsymbol{k})-B_\sigma(\boldsymbol{p},\boldsymbol{k},-\boldsymbol{k}-\boldsymbol{p})\}$$

$$\left\langle\theta(\boldsymbol{k}_1)\left\{\delta(\boldsymbol{k}_2)+f\,\frac{k_{2z}^2}{k_2^2}\theta(\boldsymbol{k}_2)\right\}\left\{\delta(\boldsymbol{k}_3)+f\,\frac{k_{3z}^2}{k_3^2}\theta(\boldsymbol{k}_3)\right\}\right\rangle=(2\pi)^3\delta_D(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,B_\sigma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$$

$$B(k,\mu)=(k\mu\,f)^2\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}F(\boldsymbol{p})F(\boldsymbol{k}-\boldsymbol{p})\ ;\quad F(\boldsymbol{p})=\frac{p_z}{p^2}\left\{P_{\delta\theta}(p)+f\frac{p_z^2}{p^2}\,P_{\theta\theta}(p)\right\}$$

- Popular streaming model **+ (non-)Gaussian corrections**
- Model accounts for a large-scale enhancement in halo clustering

(Nishimichi & AT '11)

However,

Previous studies adopted standard PT treatment to compute corrections

ill behavior at small scales

$\longrightarrow$ failed to compute correlation functions

Semi-analytic model of RSD

AT, Nishimichi & Saito ('10)

$$P^{(\mathrm{S})}(k,\mu) = e^{-(k\mu f\sigma_{\mathrm{v}})^2}\left[P_{\delta\delta}(k) + 2f\mu^2 P_{\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k) + A(k,\mu) + B(k,\mu)\right]$$

Fitting parameter

$$A(k,\mu) = (k\mu\, f)\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\,\frac{p_z}{p^2}\,\{B_\sigma(\boldsymbol{p},\boldsymbol{k}-\boldsymbol{p},-\boldsymbol{k}) - B_\sigma(\boldsymbol{p},\boldsymbol{k},-\boldsymbol{k}-\boldsymbol{p})\}$$

$$\left\langle \theta(\boldsymbol{k}_1)\left\{\delta(\boldsymbol{k}_2) + f\,\frac{k_{2z}^2}{k_2^2}\theta(\boldsymbol{k}_2)\right\}\left\{\delta(\boldsymbol{k}_3) + f\,\frac{k_{3z}^2}{k_3^2}\theta(\boldsymbol{k}_3)\right\}\right\rangle = (2\pi)^3\delta_D(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\,B_\sigma(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$$

$$B(k,\mu) = (k\mu\, f)^2\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}F(\boldsymbol{p})F(\boldsymbol{k}-\boldsymbol{p})\ ;\quad F(\boldsymbol{p}) = \frac{p_z}{p^2}\left\{P_{\delta\theta}(p) + f\,\frac{p_z^2}{p^2}\,P_{\theta\theta}(p)\right\}$$

- Popular streaming model **+ (non-)Gaussian corrections**
- Model accounts for a large-scale enhancement in halo clustering

(Nishimichi & AT '11)

However,

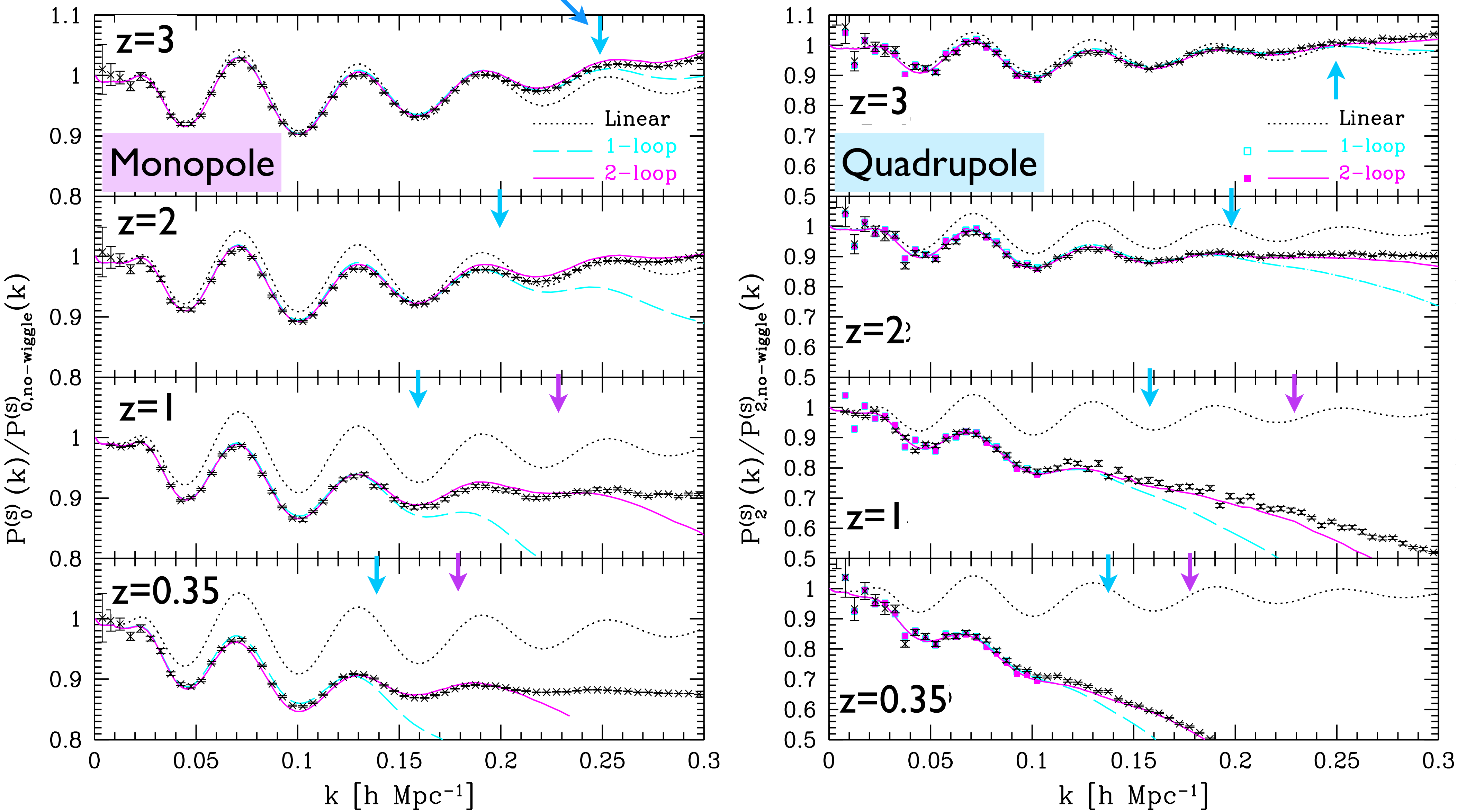
With full implementation of RegPT, a consistent prediction in both power spectra & correlation functions is made possible

(see next slides)

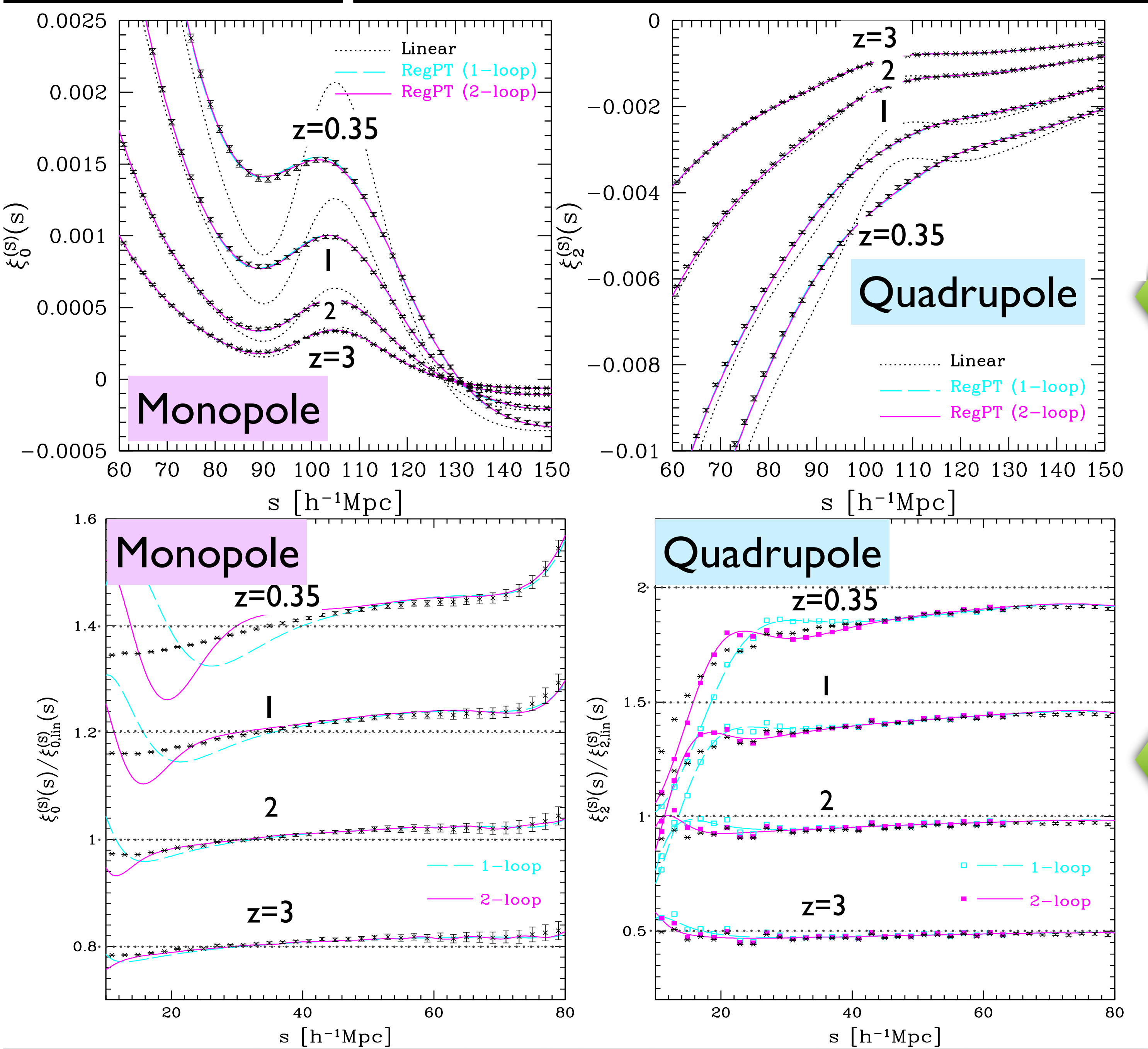
Multipole power spectra

k_max, below which percent-level accuracy is achieved in real space

AT, Bernardeau & Nishimichi (in prep.)



Multipole correlation functions



AT, Bernardeau & Nishimichi (in prep.)

Large-scale correlation

(60-150 Mpc/h)

Small-scale correlation

(10-80 Mpc/h)

normalized by linear theory

Summary

PT for precision calculation of LSS now moves on to the 2nd stage (practical phase)

RegPT : new non-perturbative PT treatment based on Gamma expansion

- *Fast calculation of power spectrum* (in real space)

  ・・・・・・・ *few sec. on (my) laptop, no parallelization required*

  ⟶ code is publicly available at

  http://www-utap.phys.s.u-tokyo.ac.jp/~ataruya/regpt_code.html

- *Predictions in redshift space*

  With the improved model of RSD, validity range of PT prediction remain unchanged in both real and redshift space

  stay tune for public code