Made-to-Measure (M2M)
Dynamical modelling of external galaxies and the Milky Way

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Oct. 31, 2012@KIAS

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Motivation

• **Important for**
  – inferring mass (dark matter) distribution: shape, radial profiles, central supermassive black holes
  – establishing scaling relations, and finding clues for galaxy formation and evolution

• **much better data becoming available**
  – **Milky Way**: BRAVA, ARGOS, APOGEE, OGLE (proper motions), GAIA
  – **External galaxies (IFUs)**: SAMI, MANGA

• **Require better modeling techniques which will also guide observations**
How to perform dynamical modelling?

- Jeans equation
- **Schwarzschild method**: Orbit based method
- **Made-to-Measure (M2M)**: Particle based method
- Torus Method
- In the last three methods, orbits play essential roles
Orbits in 3D triaxial potentials

- short-axis (z-) **tube** orbits (from Barnes)
Orbits in 3D triaxial potentials

- box orbits (from Barnes)
Schwarzschild Method

- **Schwarzschild (1979)**
  - Orbit individual particles
  - End of run, use linear / quadratic programming to determine weights of different orbits
  - Well established (applications: black hole/galaxy masses)

[See Yougang Wang’s talk tomorrow]
Made-to-Measure Method
(Syer & Tremaine 1996)

- Orbit system of particles
- Inflight weight adjustment to reproduce observations (not at the end)
- More flexibility than Schwarzschild's method
- Cross-checks on the Schwarzschild method: degeneracy?
Actual Observables

- **Examples**
  - Surface brightness
  - Mean line-of-sight (los) velocity
  - Los velocity dispersion
  - Los velocity distribution
    - eg h3 (skewness), h4 (kurtosis)

- Also los velocities of individual stars, globular clusters

- Kinematics are luminosity weighted
In a given potential

- $N \sim 10^6$ particles are orbited
- Particle weights adjusted as a function of time

- **Regular**
  - Cartesian, polar, logarithmic in radius

- **Irregular**
  - e.g. from Voronoi binning of actual data
Model Observables

\[
\begin{align*}
\bar{v}_{\text{los}, j} & = \frac{\sum_{i}^{N} w_i v_{\text{los}, i} \delta_{ij}}{\sum_{i}^{N} w_i \delta_{ij}} \\
\bar{v}_{\text{los}, j} & = \sum_{i}^{N} w_i \frac{v_{\text{los}, i}}{I_j A_j} \delta_{ij}
\end{align*}
\]

Position = j

Number of particles = N

Individual particle = i

\[
y_j = \sum_{i}^{N} w_i K_{jj}
\]

Kernel:

- Surface brightness
- Average velocity, dispersion, …
Weight Evolution

**Weight evolution equation** (Syer & Tremaine 96):

\[
\frac{dw_i}{dt} \propto -\varepsilon w_i \left( \sum_j K_{ji} \Delta_j \right), \quad \Delta_j = \frac{y_j - Y_j}{Y_j}, \quad \varepsilon > 0
\]

When the predicted \( y_j > \) observed \( Y_j \), weight is reduced, and vice versa, until convergence is reached.

(Syer & Tremaine 96; Bissantz et al. 04; De Lorenzo 07, 08; Dehnen 09; Long & Mao 10; Morganti & Gerhard 2012)
Weight Evolution

\[ F(w) = -\frac{1}{2} \chi^2 + \mu S + \frac{1}{\epsilon} \frac{dS}{dt} \]

\[ \chi^2 = \sum_{k}^{K} \lambda_k \chi_k^2 \quad S = -\sum_{i}^{N} w_i \ln \left( \frac{w_i}{m_i} \right) \]

\[ \frac{dw_i}{dt} = -\epsilon w_i \left( \sum_{k}^{K} \lambda_k \sum_{j}^{J} \frac{K_{k,j}}{\sigma_{k,j}} \Delta_{k,j} - \mu \frac{\partial S}{\partial w_i} \right) \]
Particle Initial Conditions

- Spatial coordinates match luminosity distribution
- Use (approximate) distribution function if available
- Sample integral space eg energy, angular momentum
- Velocity coordinates
  - Created using a given set of velocity dispersion functions eg from Jeans equations
  - Random
Mock Galaxy Model

- Plummer sphere
  \[ \varphi(r) = -\frac{Y}{(r^2 + 1)^{1/2}} \]

- Constructed mock data – surface brightness, los velocity dispersion, h4, isotropic dispersion

- Task:
  - use M2M to determine the mass-to-light ratio of the constructed data
M2M M/L = 4.97, Input: 5

$\chi_{LM}^2$ vs. Mass-to-light ratio

Long & Mao (2010)
Elliptical and Lenticular Galaxies

- **Motivation**
  - Compare M2M with Schwarzschild's method

- **Based on Sauron data (Cappellari et al 2006)**
  - 24 galaxies, fast / slow rotators, various kinematic features e.g. Kinematically Decoupled Cores (KDC), Counter Rotating Cores (CRC)
  - All have Sauron results

- **M2M Implementation**
  - MGE potential – interpolated acceleration tables
  - Voronoi binning – kdtree, nearest neighbour search
  - Rotation – set sign of $v_{\theta}$ appropriately
NGC 4660

SB

\(\sigma\)

h3

h4
Can reproduce the kinematics well, e.g., Kinetically Decoupled Cores in NGC 4458
Mass-to-Light ratio: M2M vs. Schwarzschild

Long & Mao (2012)
Global anisotropy

- M2M method more radially anisotropic
- Model not unique?

A larger value indicates more radial orbits
External galaxies: summary

• Good agreement between M2M and Schwarzschild in terms of mass-to-light ratios

• M2M appears to have slight larger anisotropies
  – Model may not be unique
  – Require data at large radii (Morganti & Gerhard 2012)

• Comparison between different methods can be useful cross-checks
Light is asymmetric! MW is a barred SBc galaxy
Top-down view of the Galaxy

Why do we study the MW bar?

• Bar parameters are uncertain
• Provide clues for external barred galaxies: formation, evolution and dynamical processes

Credit: Robert Hurt (SSC/JPL/Caltech)
M2M modeling of MW

- **Photometric data:** Star counts (from OGLE) over large area

- **Kinematic data**
  - BRAVA
  - Proper motions from microlensing surveys

- **Adjustment of M2M to the Milky Way**
  - Takes into rotating frame kinematics
  - Non-parallel projection los to observers
  - First M2M + MW kinematic model
  - Initial conditions: Shen et al. (2010) numerical model
Radial velocity fields of BRAVA

- Radial velocities of 8500 red giants (Kunder et al. 2012)
- Velocity accuracy ~ 5 km/s
Reproducing BRAVA radial velocity

Mean velocity:

Velocity dispersion

Mean velocity: rotation

Constraints on the Galactic bar parameters

• Fit both surface brightness and BRAVA radial velocities
• Bar pattern speed: 40 km/s/kpc, angle: 30 degrees
  (consistent with Shen et al. 2010 & Weiner & Sellwood 1999)
• Not well constrained, need more data!

Summary & future outlook

• M2M has been applied successfully to both the MW and external galaxies

• More new data to come
  – APOGEE, ARGOS, GAIA for the Milky Way
  – E.g., SAMI/MANGA for external galaxies

• Much theoretical work yet to be done
  – Self-gravity, stability, degeneracy?
  – provide hints for how galaxies form and evolve