Modeling systematic effects on genus statistics for large scale structure topology

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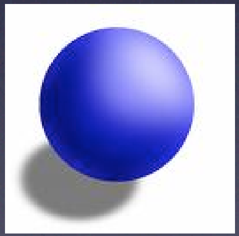
outline

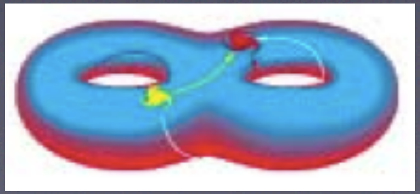
- why use topology in cosmology?
- genus as a measure of topology
- systematic effects
- results

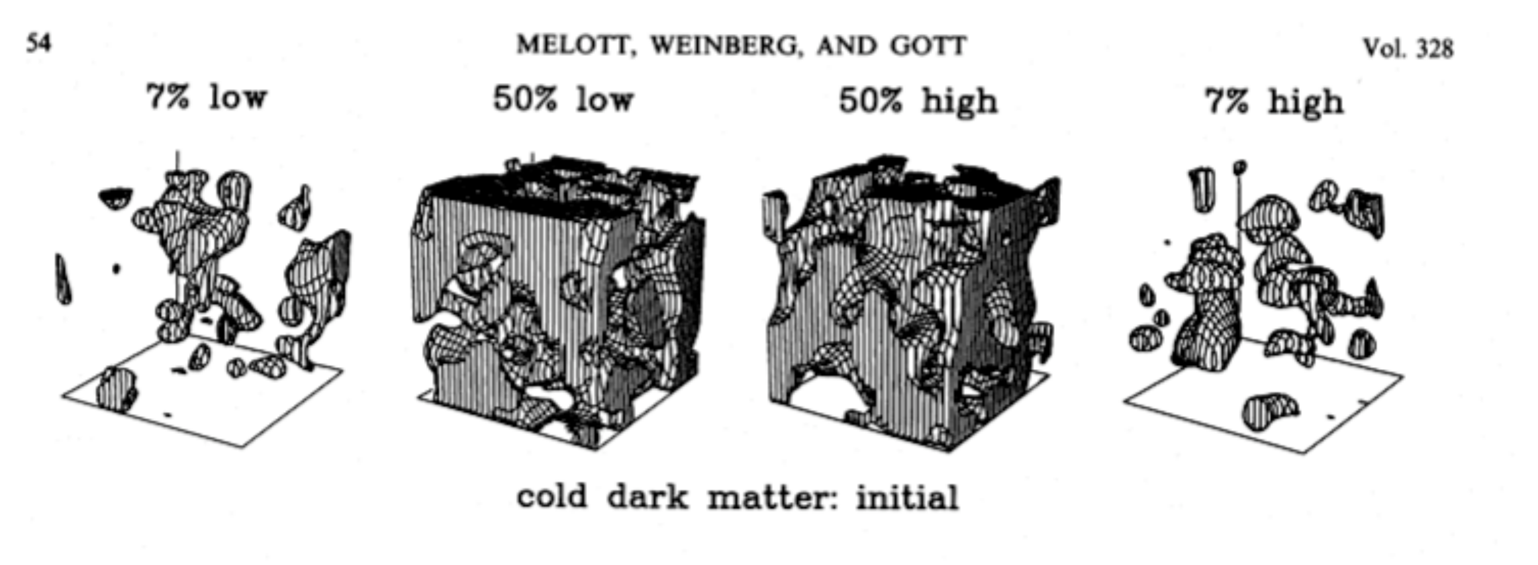
topology and cosmology

- Large scale structure (LSS) topology in linear regime does not vary in time.
- We want to find the cosmology that yields the same LSS topology both in high and low z (Park & Kim 2010).
- We need to remove systematic effects on genus to use it for a cosmological probe.

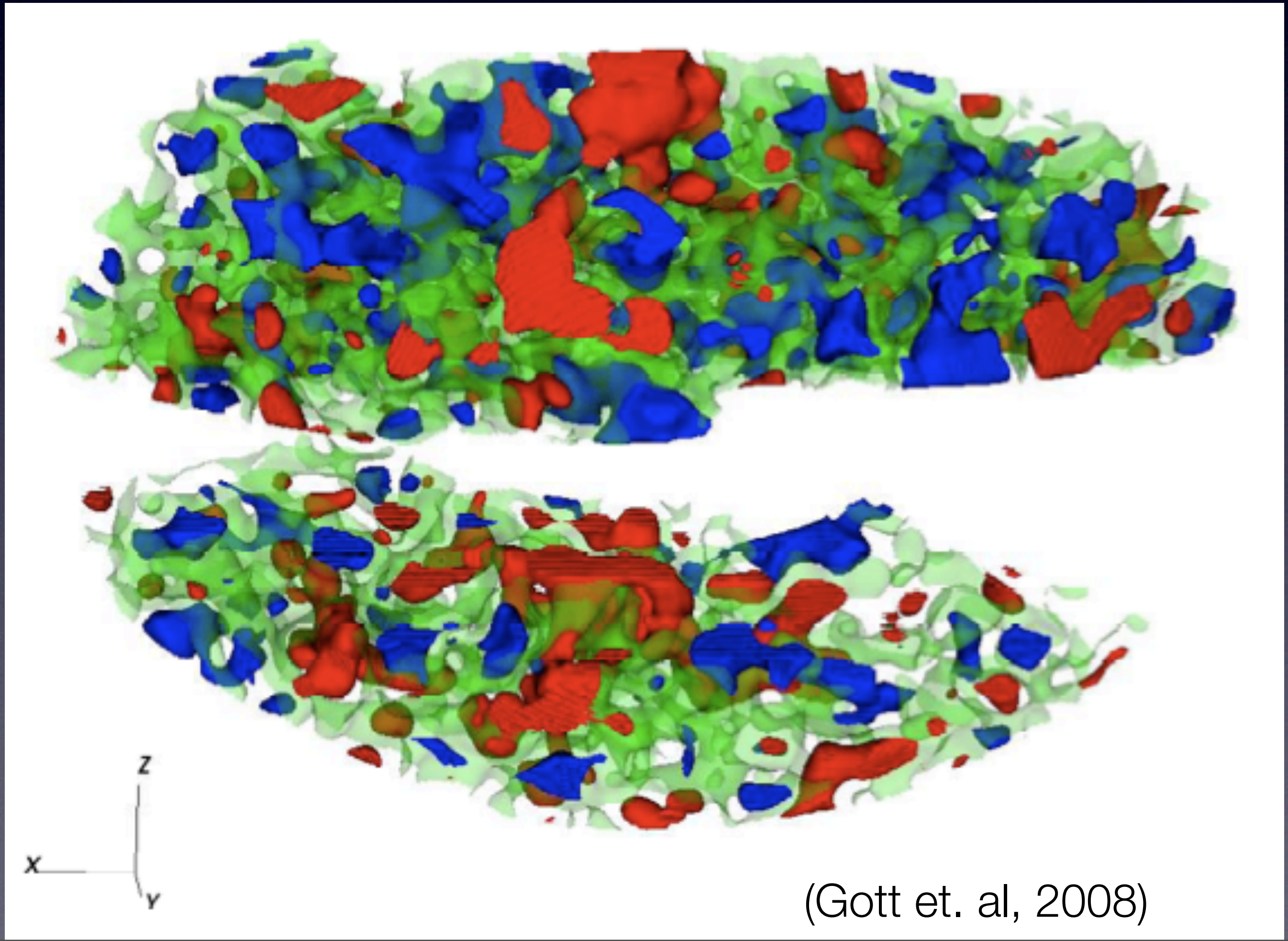
genus?

- a measure of topology
- G=number of holes-number of isolated regions in iso-density contour surfaces
- Gauss-Bonnet theorem: $$G = -\frac{1}{4\pi}\int_S \kappa \, dA$$
- G=-1  G=0  

G=1 



(Melott,Weinberg & Gott, 1988)

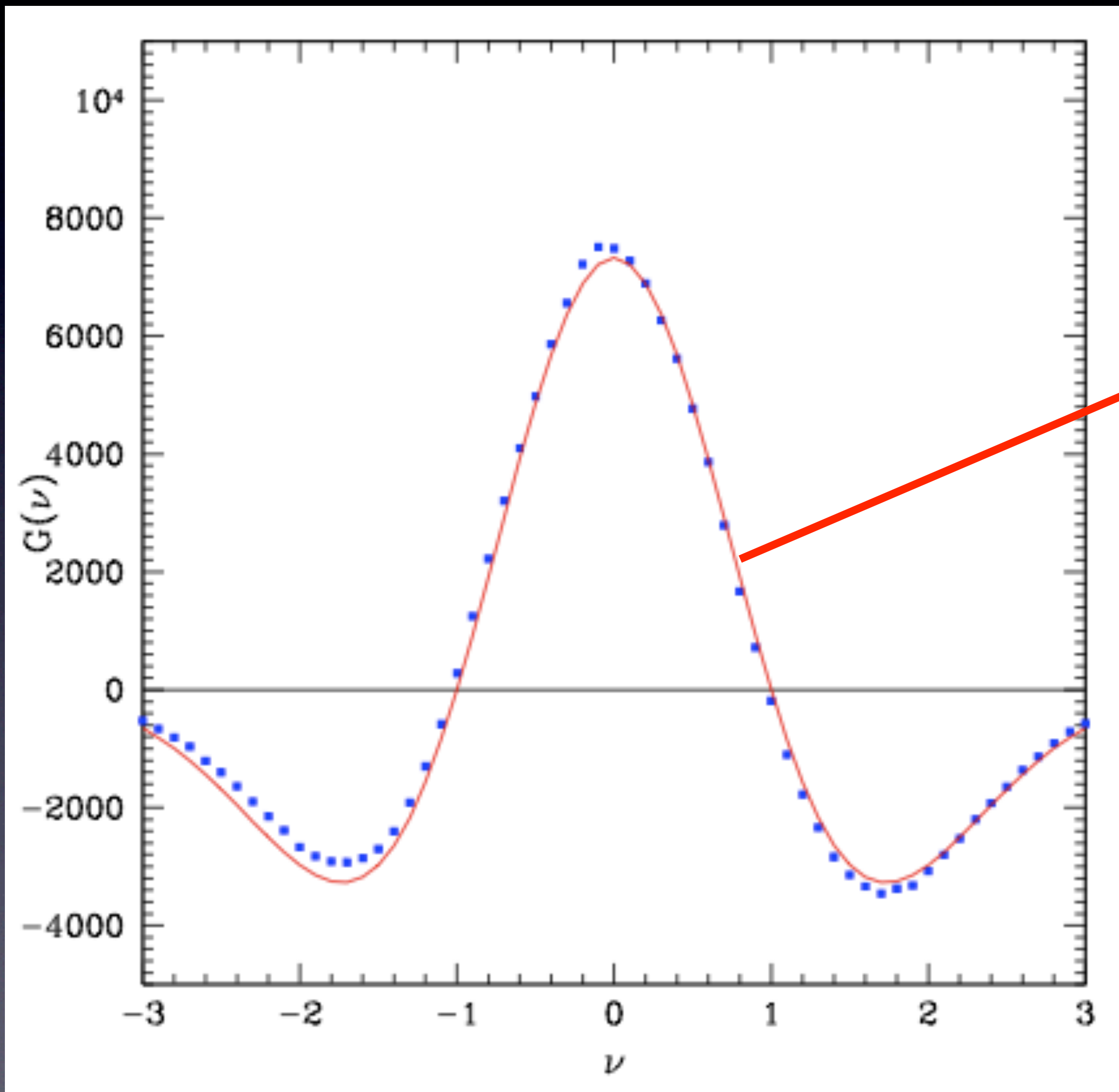


(Gott et. al, 2008)

sponge-like topology

SDSS sample
blue:7% low
green: 50%
red: 7% high

genus for a Gaussian random field



Genus per unit volume (GWM 1987):

$$g_s = N(1 - \nu^2)e^{-\nu^2/2}$$

$$N = \frac{1}{4\pi^2}\left(\frac{\langle k^2 \rangle}{3}\right)^{3/2} = \frac{1}{4\pi^2}\left(\frac{\int k^2 P'(k) d^3k}{3\int P'(k) d^3k}\right)^{3/2}$$

Horizon Run

R_G= 20 Mpc/h

systematics effects

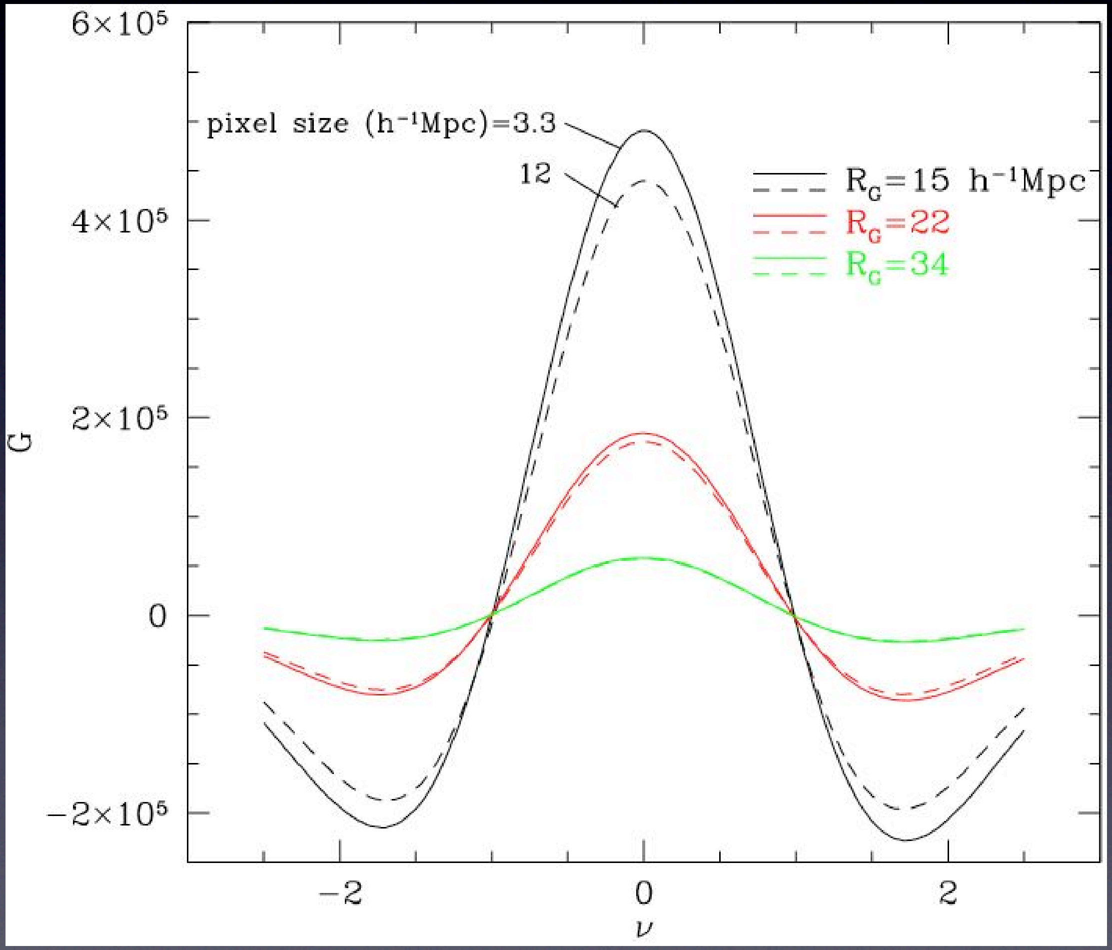
- finite pixel size effects
- non-linear gravitational evolution
- redshift space distortion
- shot noise and biasing

simulation

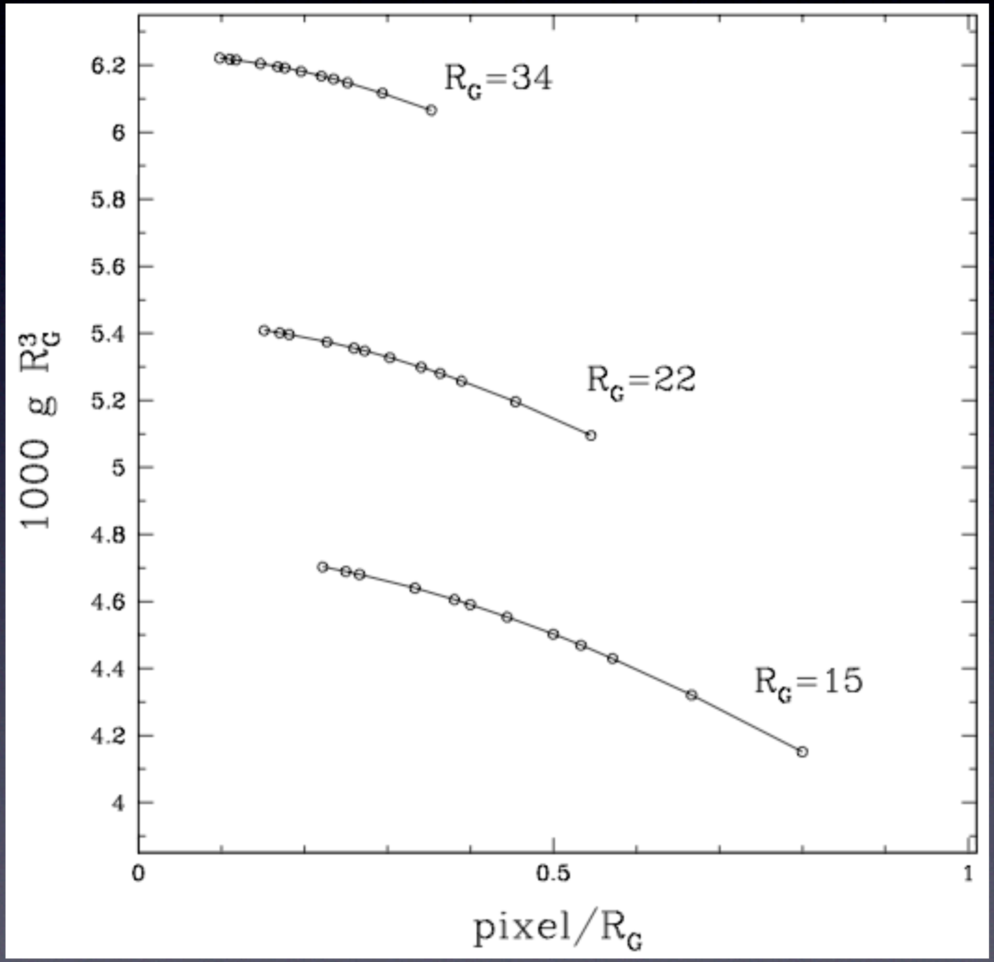
Table 1.
Detailed specifics of our Horizon Run N-body simulations.

	HR1	HR2	HR3
Model	WMAP5	WMAP5	WMAP5
Ω_M	0.26	0.26	0.26
Ω_b	0.044	0.044	0.044
Ω_Λ	0.74	0.74	0.74
Spectral index	0.96	0.96	0.96
H_0 [100 km s^{-1}Mpc^{-1}]	72	72	72
σ_8	0.794	0.794	0.794
Box size [h^{-1}Mpc]	6592	7200	10815
No. of grids for initial conditions	4120^3	6000^3	7210^3
No. of CDM particles	4120^3	6000^3	7210^3
Starting redshift	23	32	27
No. of global time steps	400	800	600
Mean particle separation [h^{-1}Mpc]	1.6	1.2	1.5
Particle mass [$10^{11}h^{-1}M_\odot$]	2.96	1.25	2.44
Minimum halo mass (30 particles) [$10^{11}h^{-1}M_\odot$]	88.8	37.5	73.2
Mean separation of minimum mass PSB halos [h^{-1}Mpc]	13.08	9.01	11.97

Pixel effects



Pixel effects



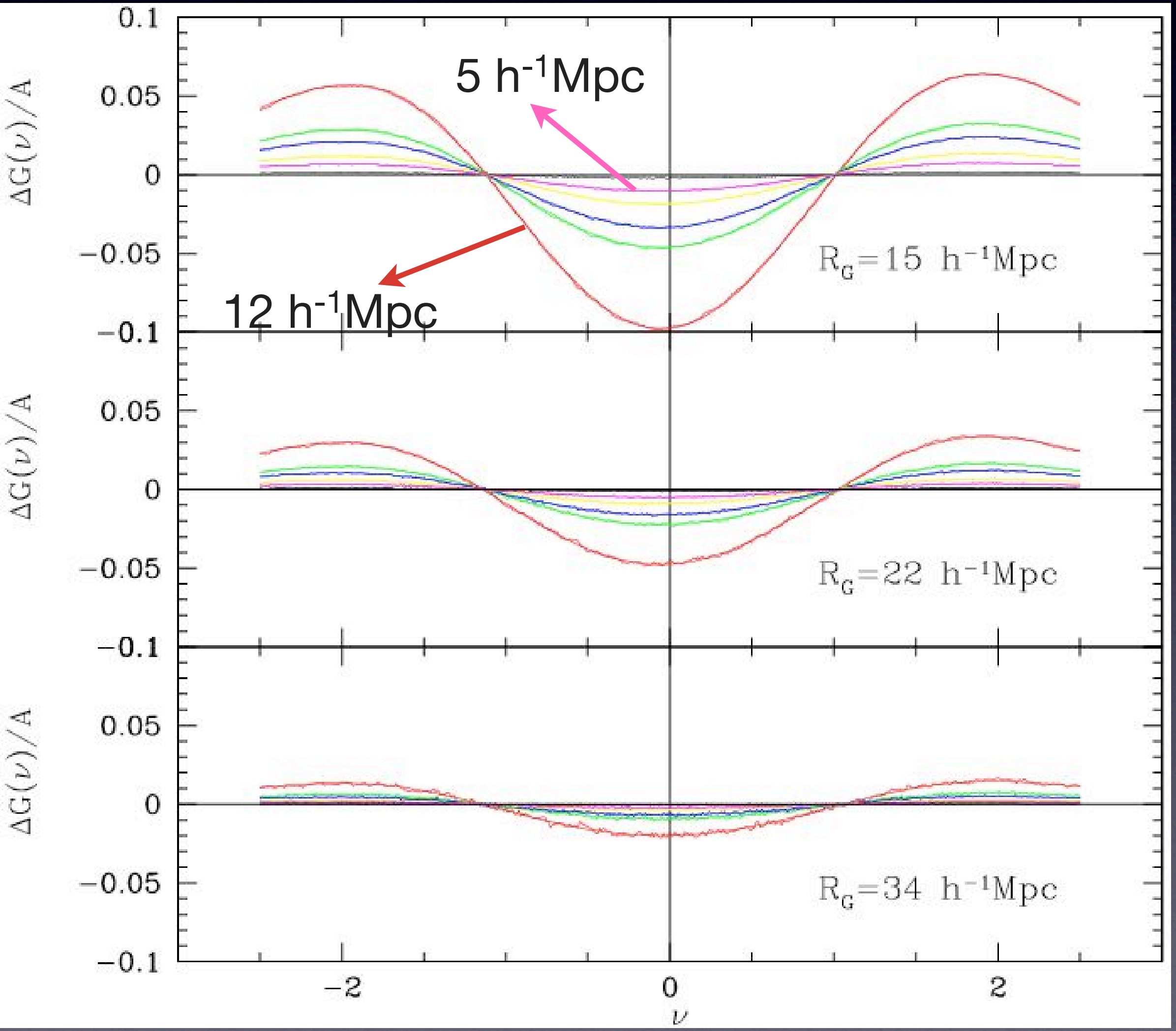
For a Gaussian random field:

$$A_k = \frac{1}{(2\pi)^{(k+1)/2}} \frac{\omega_3}{\omega_{3-k}\omega_k} \left[\frac{\sigma_1(z)}{\sqrt{3}\sigma_0(z)} \right]^k$$

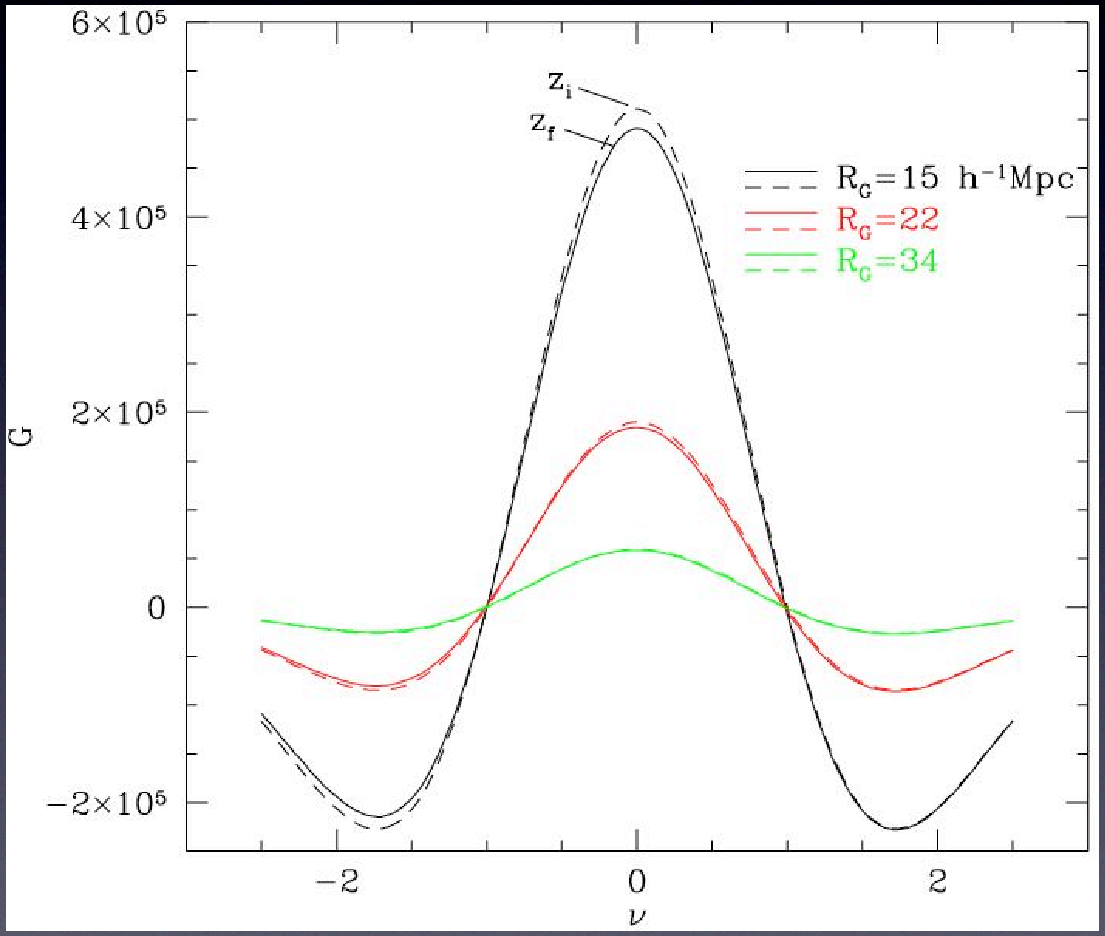
Tomita (1986)

Pixel effects

$$\Delta G_{\rm pixel} = G(\nu;p/R) - G(\nu;p/R=0)$$
$$= Ae^{-\nu^2/2}[aH_0 + bH_1(\nu) + cH_2(\nu) + dH_4(\nu)]p^2/R^2$$

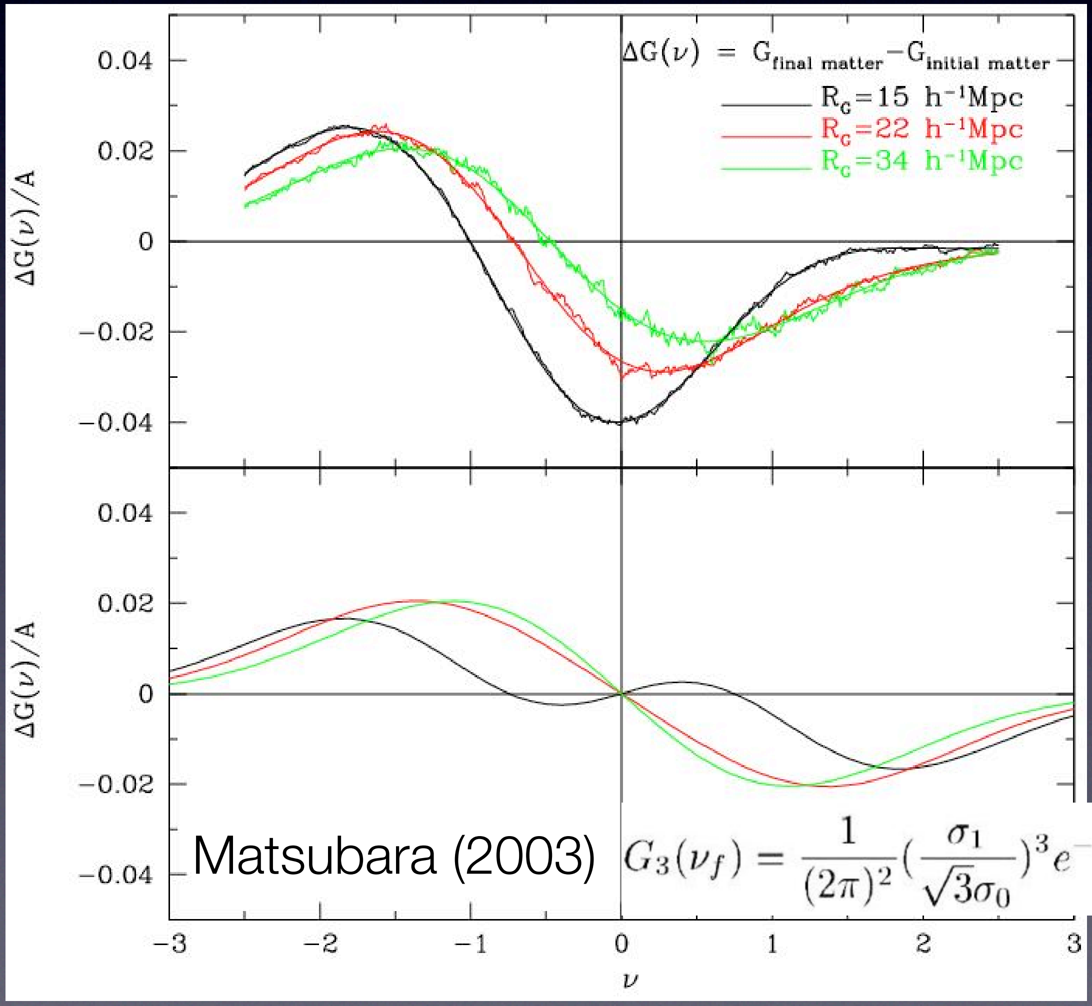


non-linear gravitational evolution



non-linear gravitational evolution

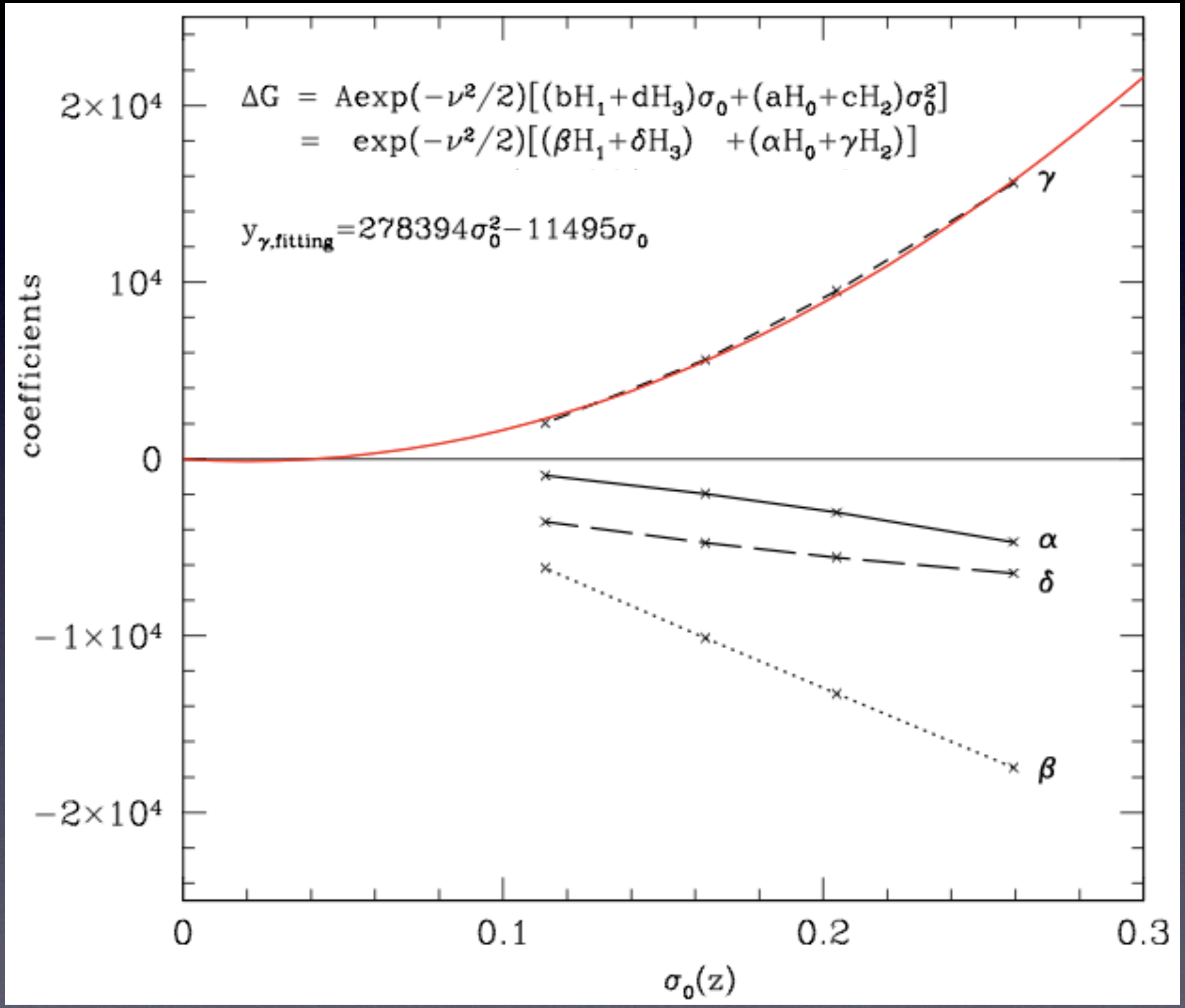
$$\Delta G_{\mathrm{grav}}(\nu;z,R) = G(\nu;z,R) - G(\nu;z_{\mathrm{init}},R)$$
$$= Ae^{-\nu^2/2}(aH_0 + bH_1 + cH_2 + dH_3)$$



Matsubara (2003)

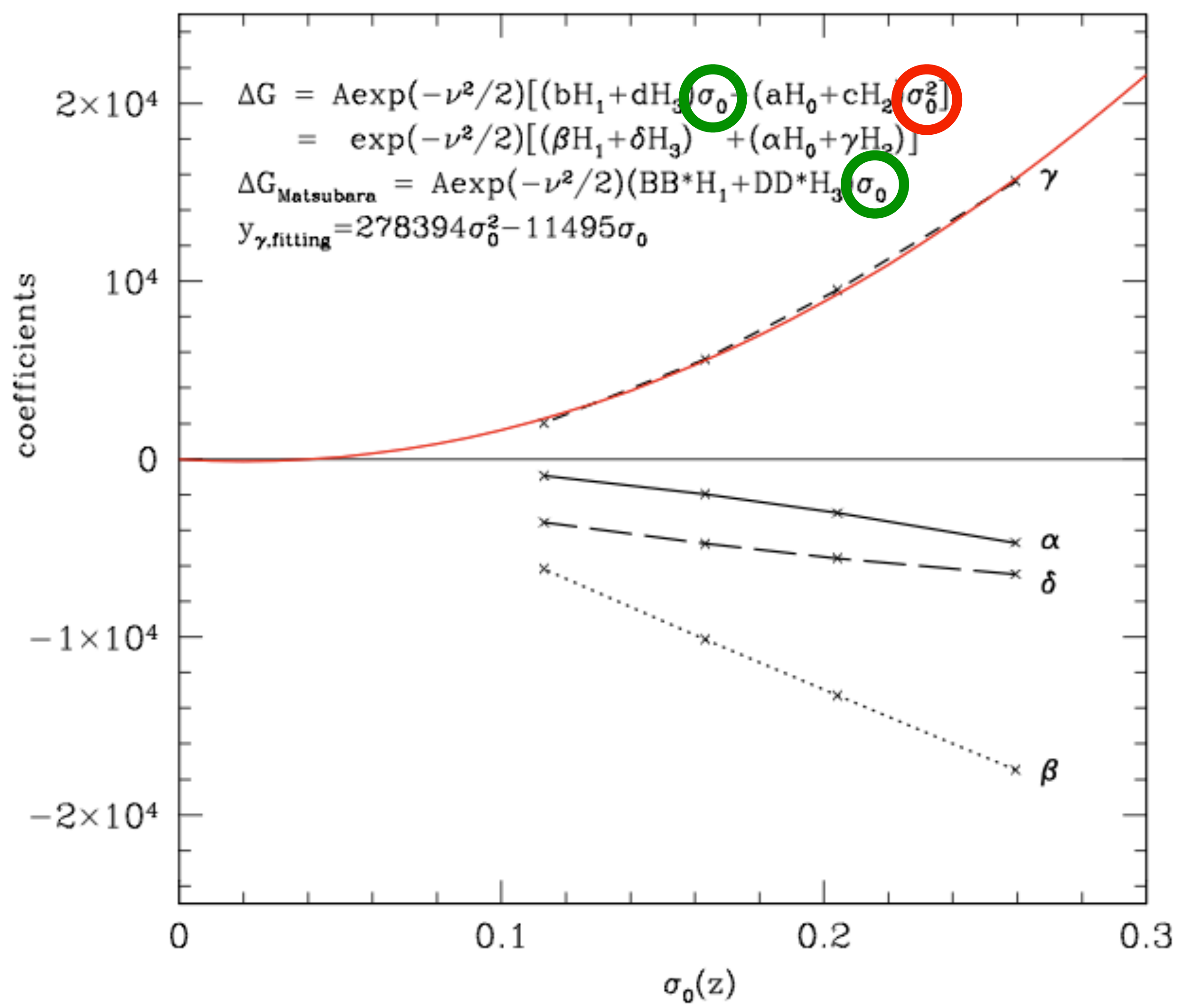
$$G_3(\nu_f) = \frac{1}{(2\pi)^2}\left(\frac{\sigma_1}{\sqrt{3}\sigma_0}\right)^3 e^{-\nu_f^2/2} \times (1-\nu_f^2 + [(S^{(1)} - S^{(0)})(\nu_f^3 - 3\nu_f) + (S^{(2)} - S^{(0)})\nu_f]\sigma_0)$$

GE correction



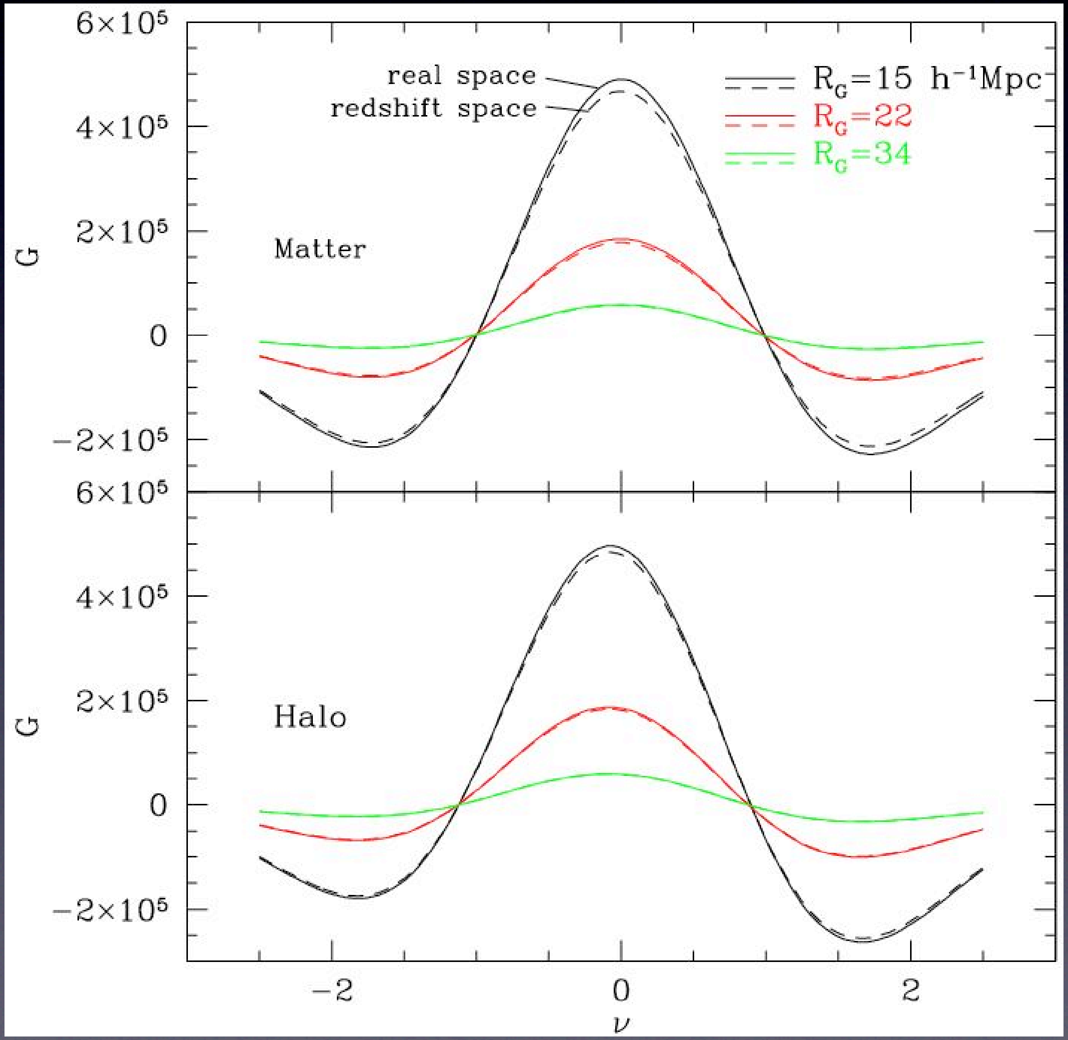
z= 32, 2, 1, 0.5, 0

GE correction



z= 32, 2, 1, 0.5, 0

redshift space distortion

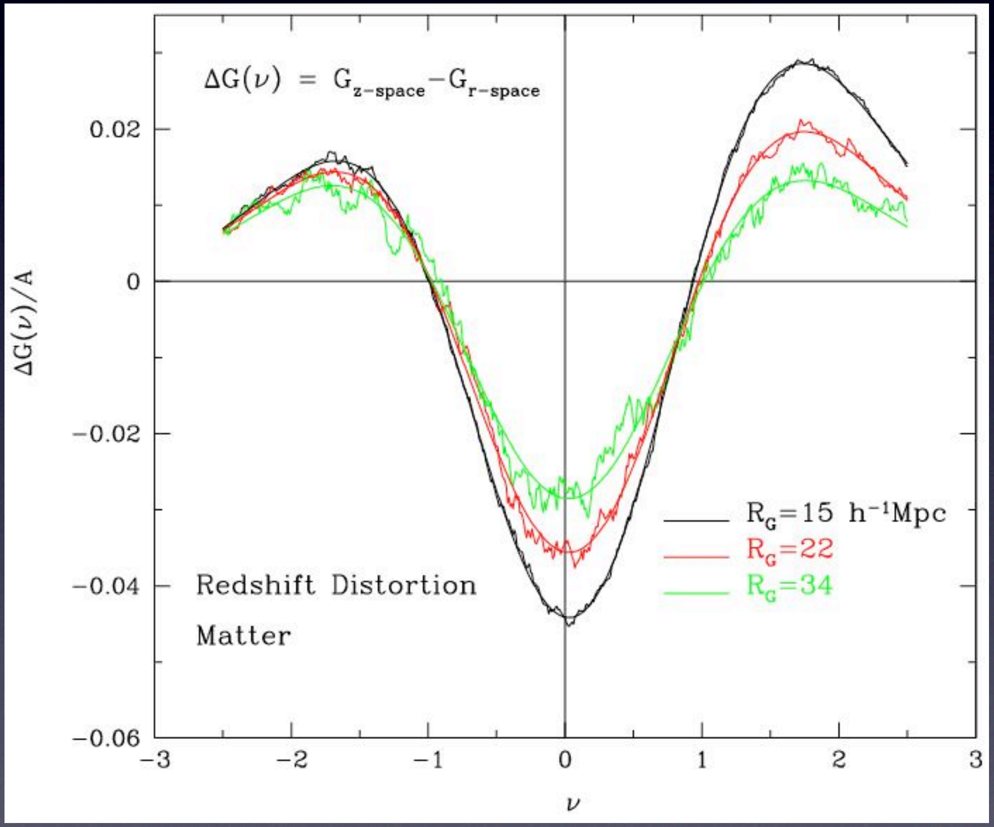


Matter

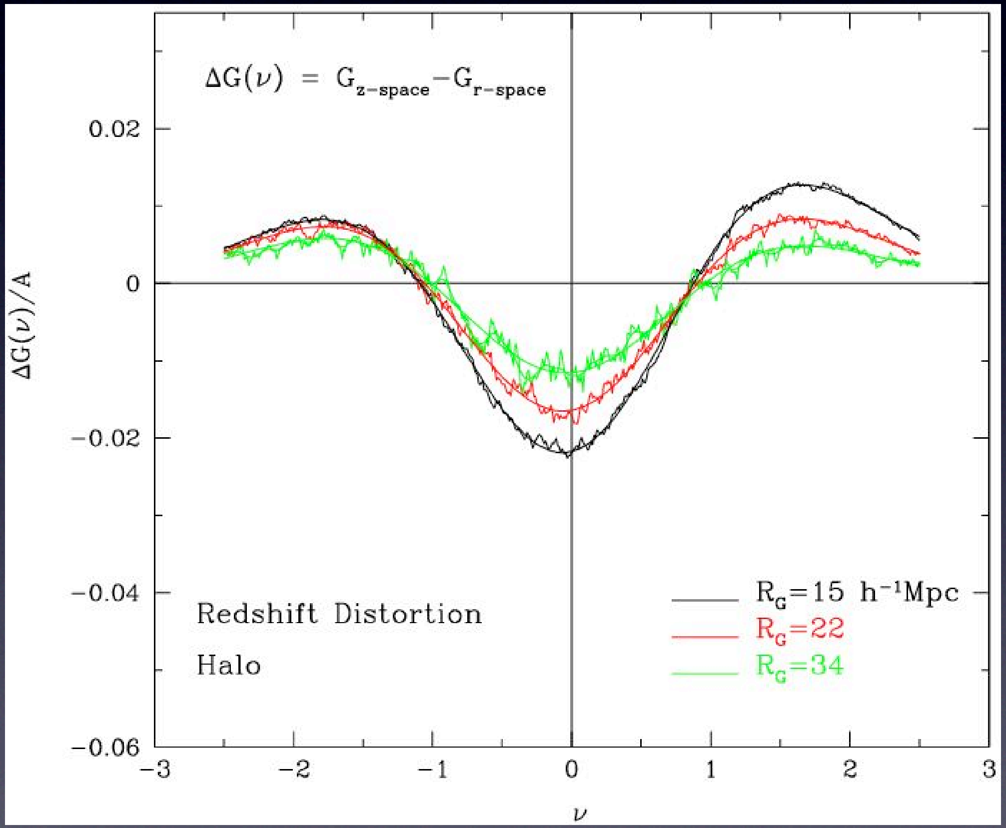
Halo

redshift space distortion

Matter

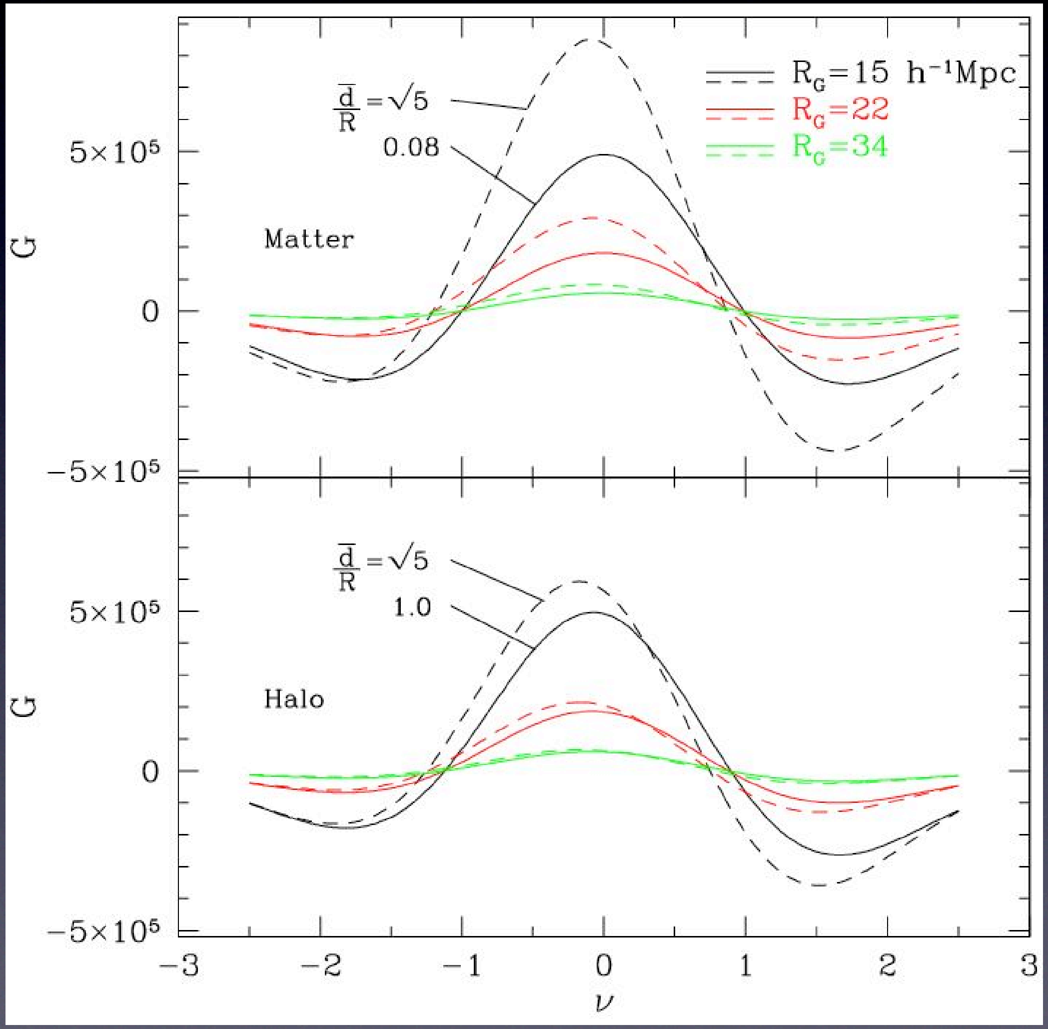


Halo



$$\Delta G_z = G(\nu;\mathrm{z}) - G(\nu;\mathrm{r}) = Ae^{-\nu^2/2}(aH_0 + bH_1 + cH_2 + dH_3)$$

shot noise and biasing

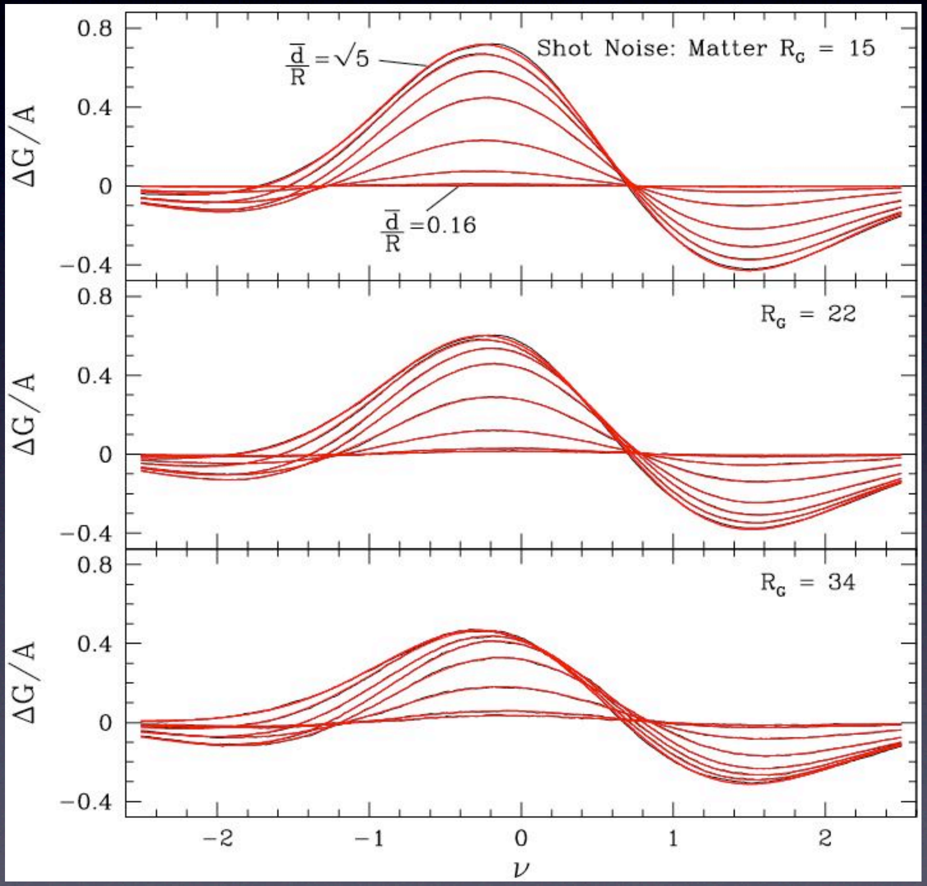


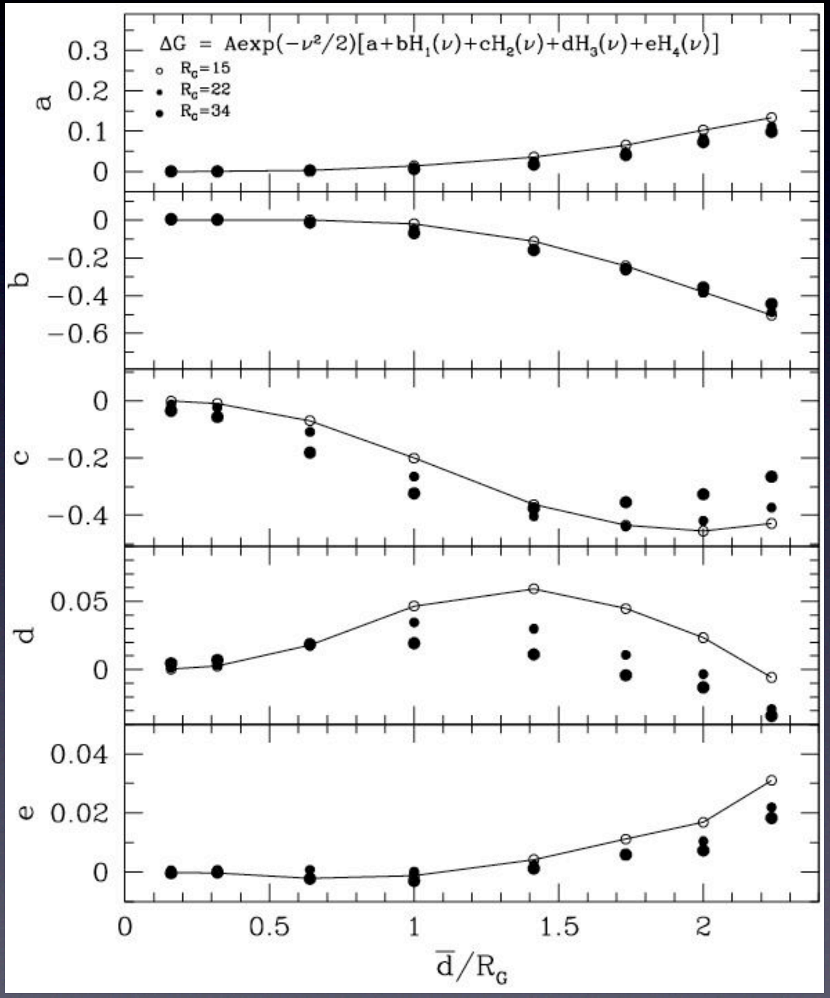
Matter

Halo

shot noise and biasing

Matter: shot noise

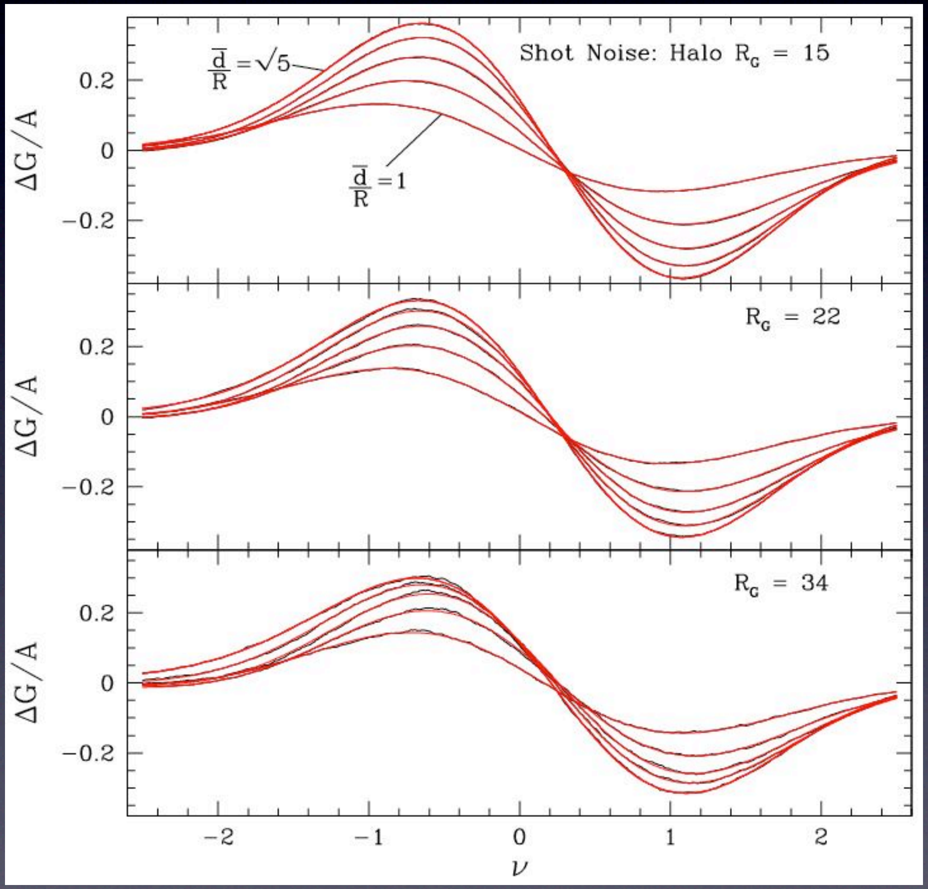


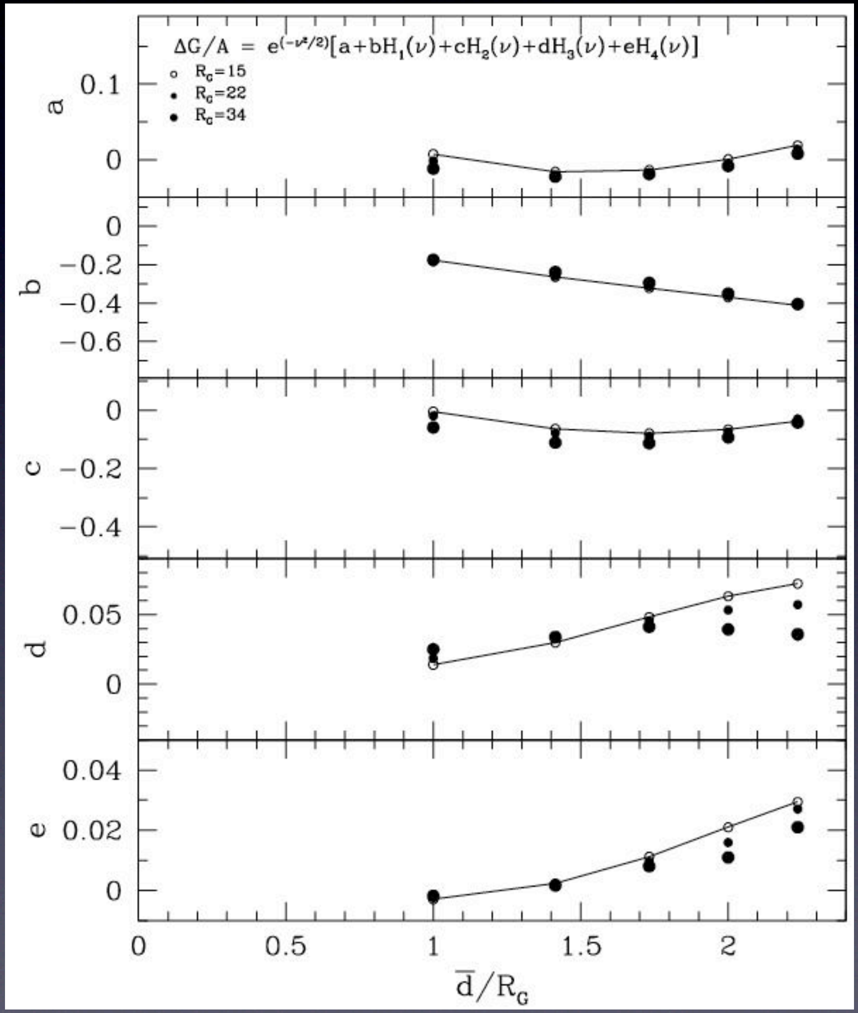


$$\Delta G = A\exp(-\nu^2/2)[a+bH_1(\nu)+cH_2(\nu)+dH_3(\nu)+eH_4(\nu)]$$

shot noise and biasing

Halo: shot noise + bias





$$\Delta G_{\rm halo,shot}(\nu;\bar{d}/R) = G_{\rm halo}(\nu;\bar{d}/R) - G_{\rm matter}(\nu;\bar{d}/R=0)$$

$$= Ae^{-\nu^2/2}[aH_0 + bH_1 + cH_2 + dH_3 + eH_4]$$

conclusion

- All systematic effects on genus statistics can be successfully modeled by $\Sigma \alpha_i H_i$ up to i=4.
- future work: fNL using genus statistics.