COSMIC WEB STATISTICS AND HALO TRIAXIALITY: NEW INSIGHTS

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5$^{\text{th}}$ KIAS Workshop on 'Cosmology and Structure Formation'

Seoul, South Korea – October 30, 2012

Three Main Messages

1. The initial shear field (gravity) plays a key role in shaping the cosmic web (as opposed to the inertia tensor)!

2. Correlation gravity/density $\rightarrow$ New algorithm & formulae

   - **Conditional distribution of shear eigenvalues**
   - **Conditional shape distribution of halos/voids**

3. The new formulae are very useful, so use them!

Outline

1. The cosmic web
2. Key role of the initial shear field
3. Conditional distribution of eigenvalues at peak/dip locations
4. Conditional shape distributions for halos/voids
5. Further applications: extended excursion set theory
6. Summary and ongoing studies

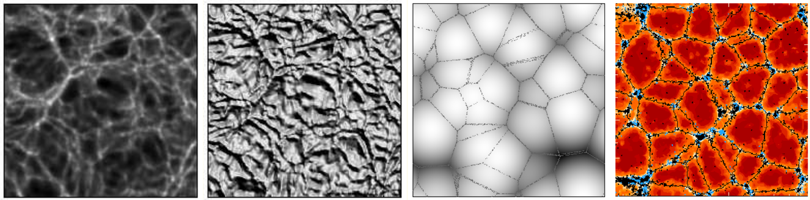
BASED ON

- **G. Rossi** (2012a), MNRAS 421, 296-307
- **G. Rossi** (2012b), MNRAS submitted (under review)

COSMIC WEB: DEFINITION

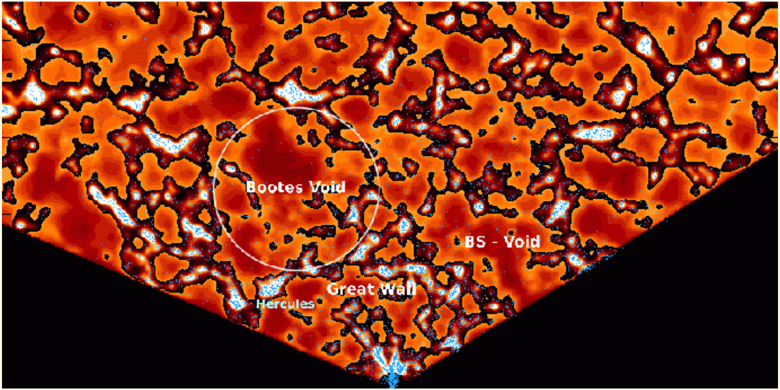
WHAT IS THE COSMIC WEB?

Fundamental spatial organization of matter on scales of a few up to a hundred *Mpc*. Galaxies, intergalactic gas and dark matter exist in a wispy web-like arrangement of dense compact *clusters*, elongated *filaments*, and sheetlike *walls*, amidst large near-empty *void* regions. Filaments are the transport channels along which matter and galaxies flow into massive high-density cluster located at the nodes of the web-like structure (**Zeldovich 1970; Peebles 1980; Bond et al. 1986**)



Aragon-Calvo et al. (2010)

COSMIC WEB: VISUALIZATION



A visualisation of the (DTFE) SDSS density field (1 h^{-1}Mpc), the contour levels are divided roughly into the overdense and the underdense regime. Both the galaxies (blue dots) and the density represent a slice of 12 h^{-1}Mpc thickness (**Platen et al. 2011**)

Cosmic Web: description

Main Goals

- Identification of cosmic web walls, filaments and cluster nodes
- Description and classification of the cosmic web
- Link with structure formation $\rightarrow$ IC vs evolved structures
- Virialization process & SF $\rightarrow$ statistical + gravitational process

Techniques

- Watershed segmentation
- Genus $\rightarrow$ topology
- Skeleton formalism
- Tessellations: Voronoi, Delaunay
- Adhesion model

SF Theories

- Objects evolved from peaks in the initial field (BBKS)
- Structure and clustering pattern of forming objects reflect IC (Bond et al. 1986, 1996)
- Kaiser (1984), Peebles (1984), Hoffman & Shaham (1984), ... $\rightarrow$ a long-standing problem

INITIAL SHEAR FIELD

The web-like network is shaped by the tidal force field accompanying the inhomogeneous matter distribution!

- Doroshkevich (1970) $\rightarrow$ first to apply these methods to study the formation of cosmic structures
- Statistical properties of the eigenvalues of the shear tensor (Doroshkevich & Shandarin 1978) $\rightarrow$ pancakes
- Statistics of initial density field $\rightarrow$ classification of structures, mass function, merger trees, ...

SHEAR TENSOR AND HESSIAN

- **T** $\rightarrow$ shear tensor $\rightarrow$ related to gravitational field $\rightarrow$ eigenv. λ_{i}
- **H** $\rightarrow$ Hessian or inertia tensor $\rightarrow$ related to density field $\rightarrow$ eigenv. ξ_{i}
- **T**|**H** $\rightarrow$ conditional shear given Hessian$\rightarrow$ eigenv. ζ_{i}

Basic Notation (1)

- Ψ displacement field
- Φ potential of the displacement field
- S_Ψ source of the displacement field
- $\mathbf{q}$ Lagrangian coordinate, $\mathbf{x}$ Eulerian coordinate,
- T_{ij} shear tensor of disp. field $\to$ eigen. $\lambda_1 \geq \lambda_2 \geq \lambda_3$, rms σ_{T}
- H_{ij} Hessian matrix $\to$ eigen. $\xi_1 \geq \xi_2 \geq \xi_3$, rms σ_{H}
- J_{ij} Jacobian of the displacement field ($i, j = 1, 2, 3$)

$$\mathbf{x}(\mathbf{q}) = \mathbf{q} + \Psi(\mathbf{q})$$

$$J_{ij}(\mathbf{q}) = \frac{\partial x_i}{\partial q_j} = \delta_{ij} + T_{ij}$$

$$T_{ij} = \frac{\partial \Psi_i}{\partial q_j} = \frac{\partial^2 \Phi}{\partial q_i \partial q_j}, \quad H_{ij} = \frac{\partial^2 S_\Psi}{\partial q_i \partial q_j}$$

$$S_\Psi(\mathbf{q}) = \sum_{i=1}^{3} \frac{\partial \Psi_i}{\partial q_i} \equiv \sum_{i=1}^{3} \frac{\partial^2 \Phi}{\partial q_i^2}$$

Basic Notation (2)

Φ Gaussian random field determined by $P(k)$, k wave number, $W^2(k)$ additional smoothing window function. Density field described by S_Ψ, also Gaussian. Correlations between these fields:

$$\langle T_{ij} T_{kl} \rangle = \frac{\sigma_T^2}{15} (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$\langle H_{ij} H_{kl} \rangle = \frac{\sigma_H^2}{15} (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$\langle T_{ij} H_{kl} \rangle = \frac{\Gamma_{TH}}{15} (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$\sigma_T^2 = S_2 \equiv \sigma_0^2, \; \sigma_H^2 = S_6 \equiv \sigma_2^2, \; \Gamma_{TH} = -S_4 \equiv -\sigma_1^2$$

$$S_n = \frac{1}{2\pi^2} \int_0^\infty k^n \, P(k) \, W^2(k) \mathrm{d}k$$

$$\sigma_j^2 = \frac{1}{2\pi^2} \int_0^\infty k^{2(j+1)} \, P(k) \, W^2(k) \, \mathrm{d}k \equiv S_{2(j+1)}$$

$$\gamma = \Gamma_{TH}/\sigma_T \sigma_H = \frac{\sigma_1^2}{\sigma_0 \sigma_2} \equiv = \gamma_{\mathrm{BBKS}}$$

Doroshkevich's celebrated formula

Unconditional distribution of shear eigenvalues

$$p(\lambda_1, \lambda_2, \lambda_3) = \frac{15^3}{8\sqrt{5}\pi} e^{-\frac{3}{2}(2k_1^2 - 5k_2)} (\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)$$

$$\begin{aligned} k_1 &= \lambda_1 + \lambda_2 + \lambda_3 \equiv \delta_{\mathrm{T}} \\ k_2 &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 \end{aligned}$$

- Ordering of the eigenvalues is $\lambda_1 \geq \lambda_2 \geq \lambda_3$
- Derived by Doroshkevich (1970)
- Can derive partial distributions $\rightarrow p(\lambda_1)$, $p(\lambda_2)$, etc.
- Neglects the fact that voids are maxima of the source displacement, or minima of the density field

A NEW FORMULA

CONDITIONAL DISTRIBUTION OF SHEAR EIGENVALUES AT PEAK/DIP LOCATIONS

$$p(\zeta_1,\zeta_2,\zeta_3|\gamma) \equiv p(\lambda_1,\lambda_2,\lambda_3|\xi_1,\xi_2,\xi_3,\gamma)$$
$$= \frac{15^3}{8\sqrt{5}\pi}\frac{1}{(1-\gamma^2)^3}\mathrm{e}^{-\frac{3}{2(1-\gamma^2)}(2K_1^2-5K_2)}(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)(\zeta_2-\zeta_3)$$

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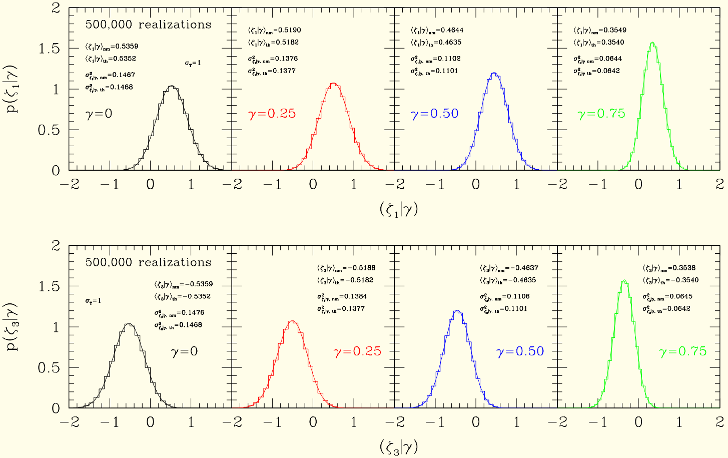
$$\begin{aligned}
K_1 &= \zeta_1+\zeta_2+\zeta_3 = k_1-\gamma h_1\\
K_2 &= \zeta_1\zeta_2+\zeta_1\zeta_3+\zeta_2\zeta_3 = k_2+\gamma^2 h_2-\gamma h_1 k_1+\gamma\tau\\
\tau &= \lambda_1\xi_1+\lambda_2\xi_2+\lambda_3\xi_3\\
k_1 &= \lambda_1+\lambda_2+\lambda_3, \quad k_2=\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3\\
h_1 &= \xi_1+\xi_2+\xi_3, \quad h_2=\xi_1\xi_2+\xi_1\xi_3+\xi_2\xi_3\\
\zeta_{\rm i} &\equiv (\lambda_{\rm i}|\gamma,\xi_{\rm i}) = \lambda_{\rm i}-\gamma\xi_{\rm i}
\end{aligned}$$

SOME RESULTS: CONDITIONAL DISTRIBUTIONS



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Shape Parameters for Halos/Voids: Standard

- **Ellipticity**

$$e_{\mathrm{T}} = \frac{\lambda_1 - \lambda_3}{2\delta_{\mathrm{T}}}$$

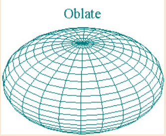
- **Prolateness**

$$p_{\mathrm{T}} = \frac{\lambda_1 + \lambda_3 - 2\lambda_2}{2\delta_{\mathrm{T}}}$$

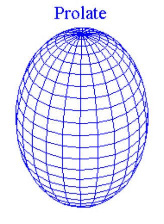
- Related to the eigenvalues of the tidal/density fields
- Similar quantities for voids

"Prolate" object $\rightarrow 0 \geq p_{\mathrm{T}} \geq -e_{\mathrm{T}}$
"Oblate" object $\rightarrow 0 \leq p_{\mathrm{T}} \leq e_{\mathrm{T}}$

- $a \simeq b \gg c$: Oblate object (disk-shaped) $\rightarrow$ pancake



- $b \simeq c \ll a$: Prolate object (cigar-shaped) $\rightarrow$ filament



A New Excursion Sets Algorithm

The excursion set approach

Pick a particle at random and smooth the linear density field over ever smaller spheres around it, until the criterion for collapse at some redshift is satisfied. The mass in the sphere is then identified as that of the collapsed object to which the particle belongs

- Statistics of the four-dimensional field $F(\mathbf{r}, R_{\rm f})$
- Trajectories of the field as a function of the filter radius at a fixed position (with F the linear overdensity)
- Rate at which random trajectories meet an absorbing barrier $\rightarrow$ mass function
- Bound structures forming at t are regions above some initial critical overdensity $F_{\rm cr}$
- A diffusion-like problem
- More rigorous treatment $\rightarrow$ see Maggiore & Riotto (2010)

Standard procedure & its extension $\rightarrow$ Rossi (2012b)

- Determine $\delta_{\rm T}, e_{\rm T}, p_{\rm T}$ which are simple combinations of the eigenvalues
- Check if $(\delta_{\rm T}, e_{\rm T}, p_{\rm T})$ cross the barrier $B(\delta_{\rm T}, e_{\rm T}, p_{\rm T})$ at the mass-scale $\sigma_{\rm T}$
- If so exit – if not, continue the loop

THREE MAIN MESSAGES

1. The initial shear field (gravity) plays a key role in shaping the cosmic web (as opposed to the inertia tensor)!

2. Correlation gravity/density $\rightarrow$ New algorithm & formulae

EQ. (47) – G. ROSSI (2012A), MNRAS 421, 296-307

$$p(\zeta_1, \zeta_2, \zeta_3|\gamma) = \frac{15^3}{8\sqrt{5}\pi} \frac{1}{(1-\gamma^2)^3} \exp\left[-\frac{3}{2(1-\gamma^2)}(2K_1^2 - 5K_2)\right](\zeta_1-\zeta_2)(\zeta_1-\zeta_3)(\zeta_2-\zeta_3)$$

3. Very useful formulae, so use them!

ACHIEVEMENTS AND ONGOING WORK

MOTIVATIONS

- Doroshkevich's formulae $\longrightarrow$ random locations in space, neglect density correlations
- Extend Doroshkevich (1970) formula for peaks/dips
- Study of density and gravity fields and their correlation
- Role of the initial shear field in shaping the cosmic web

ACHIEVEMENTS

- New *conditional formulae* $\longrightarrow$ constrained eigenvalues of the initial shear field, which include peak positions
- New distributions of initial halos/voids shapes
- New excursion set algorithm

ONGOING WORK

- Test new formulae with N-body simulations $\longrightarrow$ NEW HORIZON RUNS @KIAS
- Several applications of the new constrained formulae, including cosmic web studies with BOSS
- Relevance for BigBOSS and EUCLID $\longrightarrow$ accurate shapes of galaxies + studies of the cosmic web