DISTRIBUTION FUNCTION APPROACH TO REDSHIFT-SPACE DISTORTIONS

The 5th KIAS Workshop on

Cosmology and Structure formation

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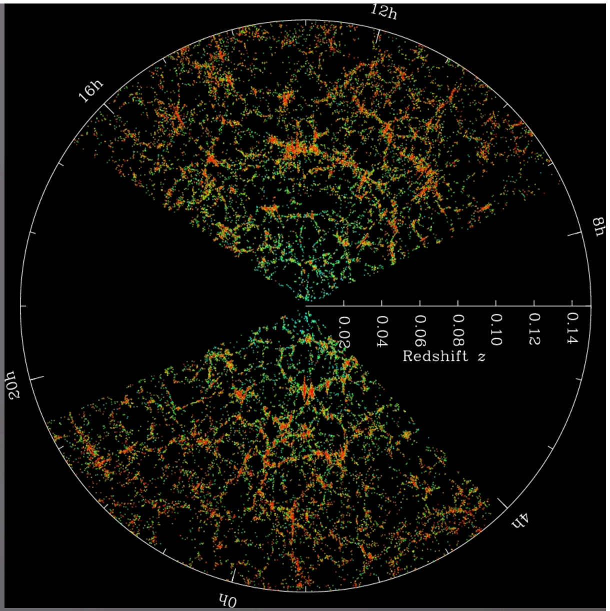
Overview

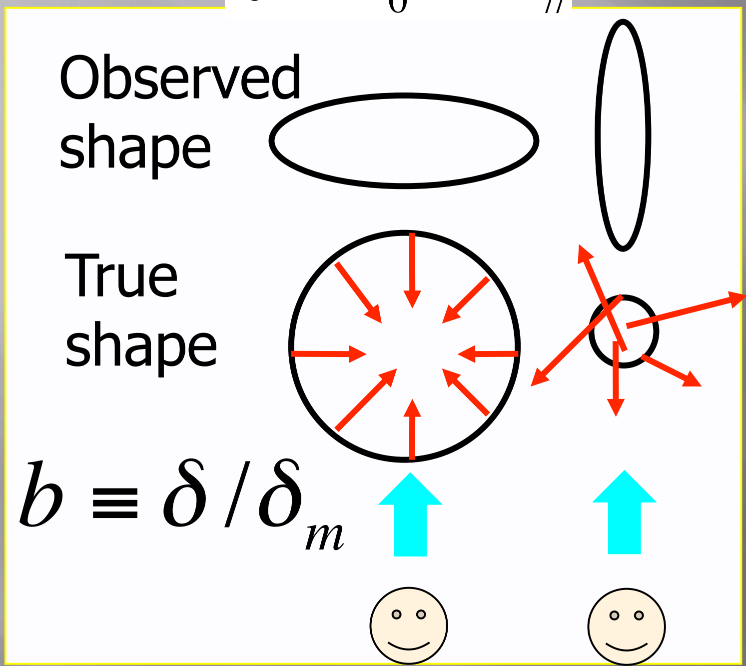


- There are many ongoing and upcoming galaxy redshift surveys
  - SDSS, WiggleZ, BOSS, 6dF, FastSound, SuMiRE, HETDEX, BigBOSS, …
  - Better understanding of properties of dark energy and theories of modified gravity
  - Correspondingly accurate theoretical templates required.
- References
  - Okumura, Seljak, McDonald & Desjacques (2012a) JCAP (arXiv: 1109.1609)
  - Okumura, Seljak & Desjacques (2012b) JCAP (arXiv: 1206.4070)

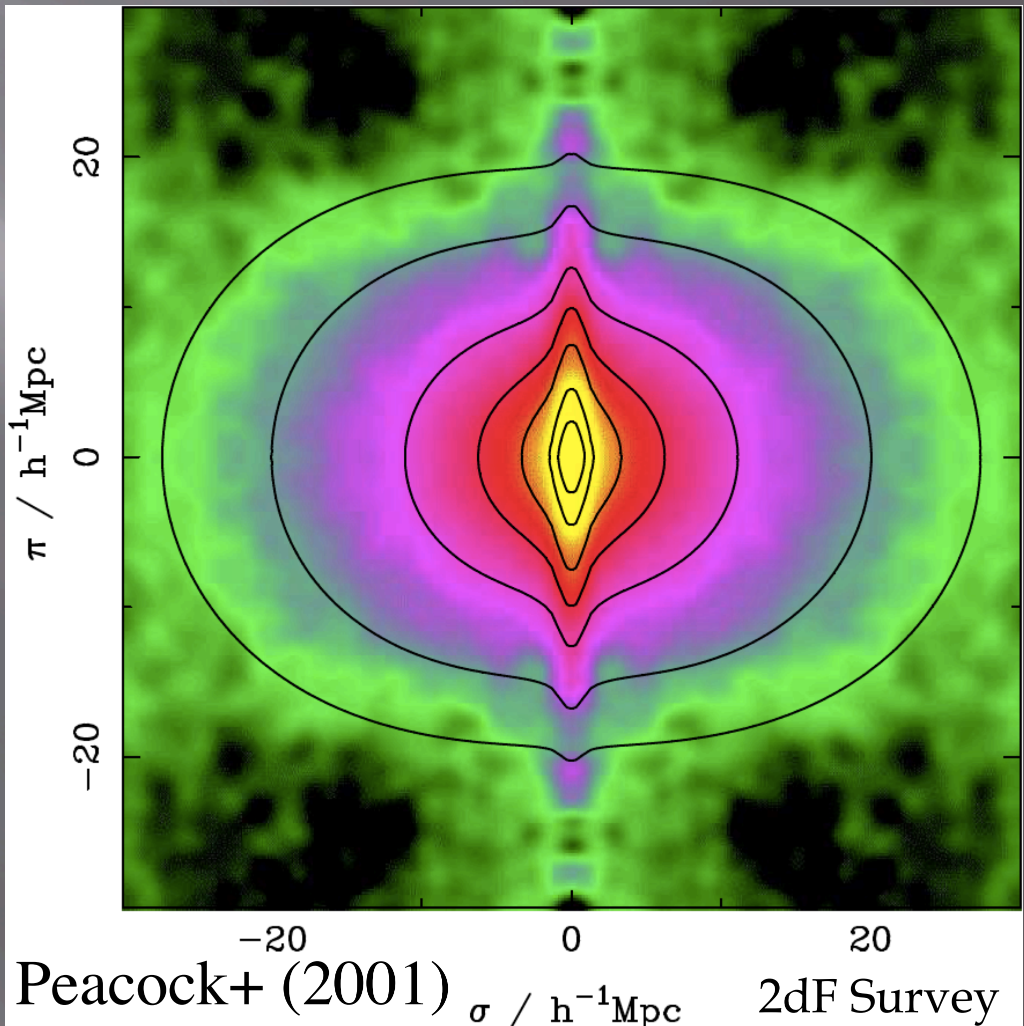
Redshift-space Distortions

- A galaxy survey measures the 3D positions of galaxies
  - Line-of-sight position is measured from redshift

$$z = H_0 x + u_{//}$$



Kaiser Finger-of-God



Peacock+ (2001) 2dF Survey

Redshift-space Distortions

- A galaxy survey measures the 3D positions of galaxies
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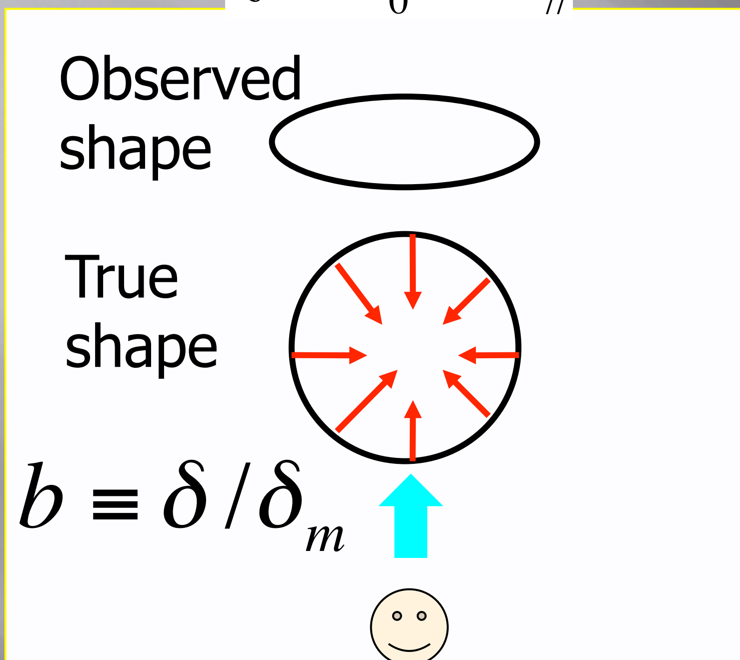
$$z = H_0 x + u_{//}$$



$$b \equiv \delta / \delta_m$$

Kaiser

Anisotropy in Fourier density field

$$\delta_g(k,\mu) = (b + f\mu^2)\delta_m(k)$$

Power spectrum (Kaiser 1987)

$$P_g^s(k,\mu) = (b + f\mu^2)^2 P_m^r(k)$$

$$f(a) = \frac{d \ln D}{d \ln a} = \Omega_m(a)^\gamma$$

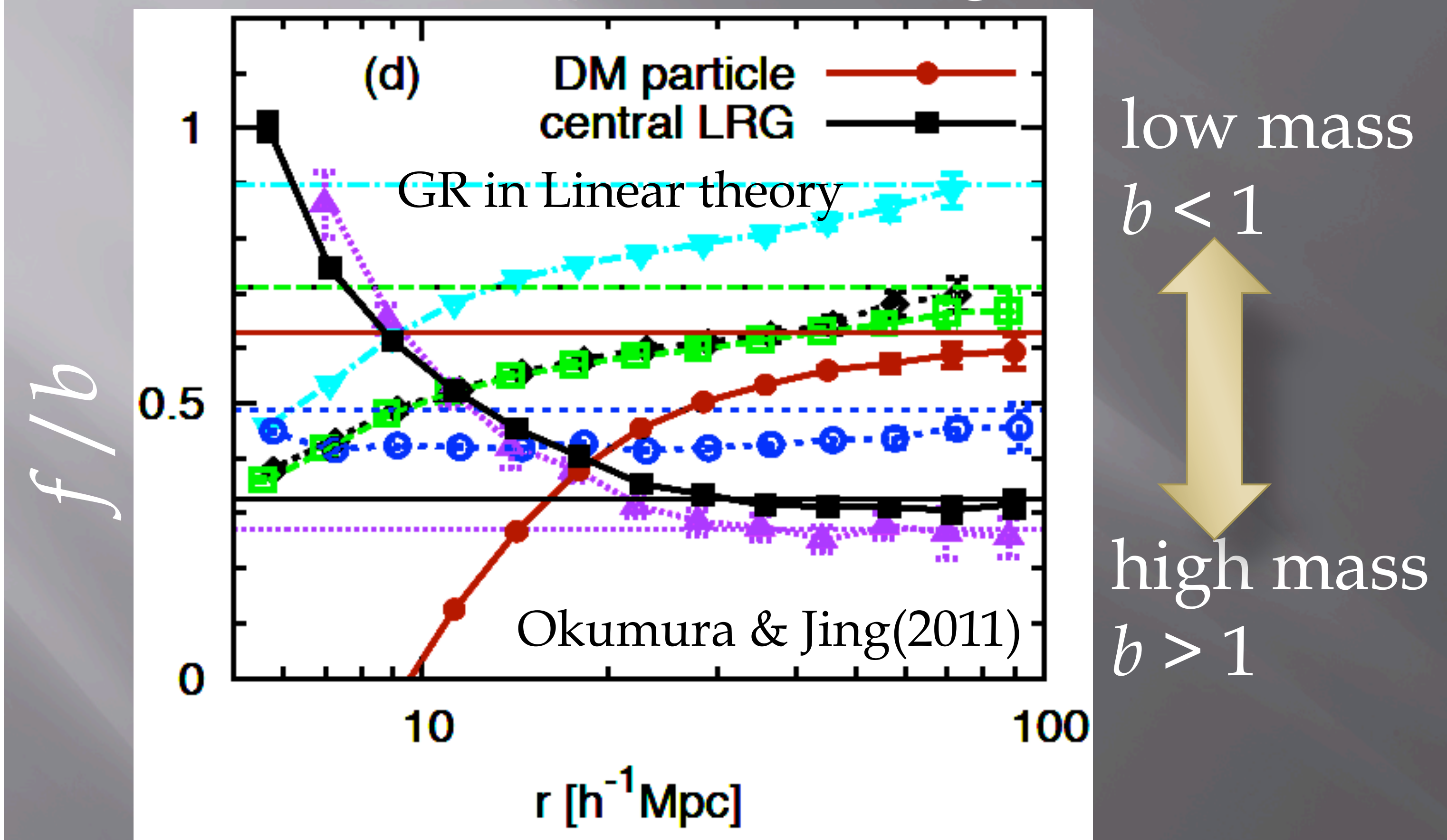
$$\gamma \simeq 0.55$$

ΛCDM + GR (Linder 2005)

- Deviation from Einstein's gravity theory gives different growth of density perturbation.

Systematic effects on determination of growth factor

- Halo mass dependence of growth rate reconstruction



  - Such systematic nonlinear effects cause incorrect constraints on dark energy or gravity theories.

Redshift space distortion from the distribution function

- Phase space (Seljak & McDonald 2011)
  - Redshift-space distortion

Distortions due to peculiar velocities

$$\mathbf{s} = \mathbf{x} + \hat{r}\, \underline{u_{\parallel}/H}$$

  - Fourier-space

$$\rho_s(\mathbf{k}) = m \int d^3\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}} \int d^3\mathbf{q}\, f(\mathbf{x},\mathbf{q})\, \underline{e^{ik_{\parallel}u_{\parallel}/H}}$$

Taylor expansion

$$\delta_s(\mathbf{k}) = \sum_{L=0} \frac{1}{L!}\left(\frac{ik_{\parallel}}{H}\right)^{L} T_{\parallel}^{L}(\mathbf{k})$$

Moments of mass-weighted velocity

$$T_{\parallel}^{L}(\mathbf{x}) = \frac{m}{\bar{\rho}} \int d^3\mathbf{q}\, f(\mathbf{x},\mathbf{q})\, u_{\parallel}^{L} = \left\langle (1+\delta(\mathbf{x}))\, u_{\parallel}^{L}(\mathbf{x}) \right\rangle_{\mathbf{x}}$$

Power spectrum in redshift space

$$P^{ss}(\mathbf{k}) = \sum_{L=0}^{\infty} \frac{1}{L!^2} \left(\frac{k\mu}{H}\right)^{2L} P_{LL}(\mathbf{k}) + 2Re \sum_{L=0}^{\infty} \sum_{L'>L} \frac{(-1)^L}{L!\ L'!} \left(\frac{ik\mu}{H}\right)^{L+L'} P_{LL'}(\mathbf{k})$$

- P_{00} : density-density power
- P_{01} : density-momentum
- P_{11} : momentum-momentum (scalar and vector)
- P_{02} : density-energy density and density-anisotropic stress
- …

$$P^{ss}_{\rm Kaiser}(\mathbf{k}) = \begin{cases} \left(1 + f\mu^2\right)^2 P_{\rm lin}(k) & ;\text{linear}, \\ P_{00} + 2f\mu^2 \left(\frac{ik}{H\mu f}\right) P_{01} + f^2\mu^4 \left(\frac{k}{H\mu f}\right)^2 P_{11} & ;\text{nonlinear}, \end{cases}$$

- Every power spectrum $P_{LL'}$ can be directly measured from N-body simulations!

Angular decomposition of moments of distribution function

- Angular decomposition into helicity eigenstates
  - Generalization of scalar, vector, tensor (SVT) decomposition

$$f(\mathbf{k}, q, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} f_l^m(\mathbf{k}, q) Y_{lm}(\theta, \phi)$$

$$P_{LL'}(\mathbf{k}) = \sum_{(l=L,L-2,..)} \sum_{(l'=L',L'-2,..)} \sum_{m=0}^{l} P_{l,l'}^{L,L',m}(k) P_l^m(\mu) P_{l'}^m(\mu)$$

$$P_{00}(\mathbf{k}) = P_{00}^{0}(k)$$

$$P_{01}(\mathbf{k}) = \mu P_{01}^{010}(k)$$

$$P_{11}(\mathbf{k}) = \mu^2 P_{11}^{110}(k) + (1-\mu^2) P_{11}^{111}(k)$$

$$\vdots$$

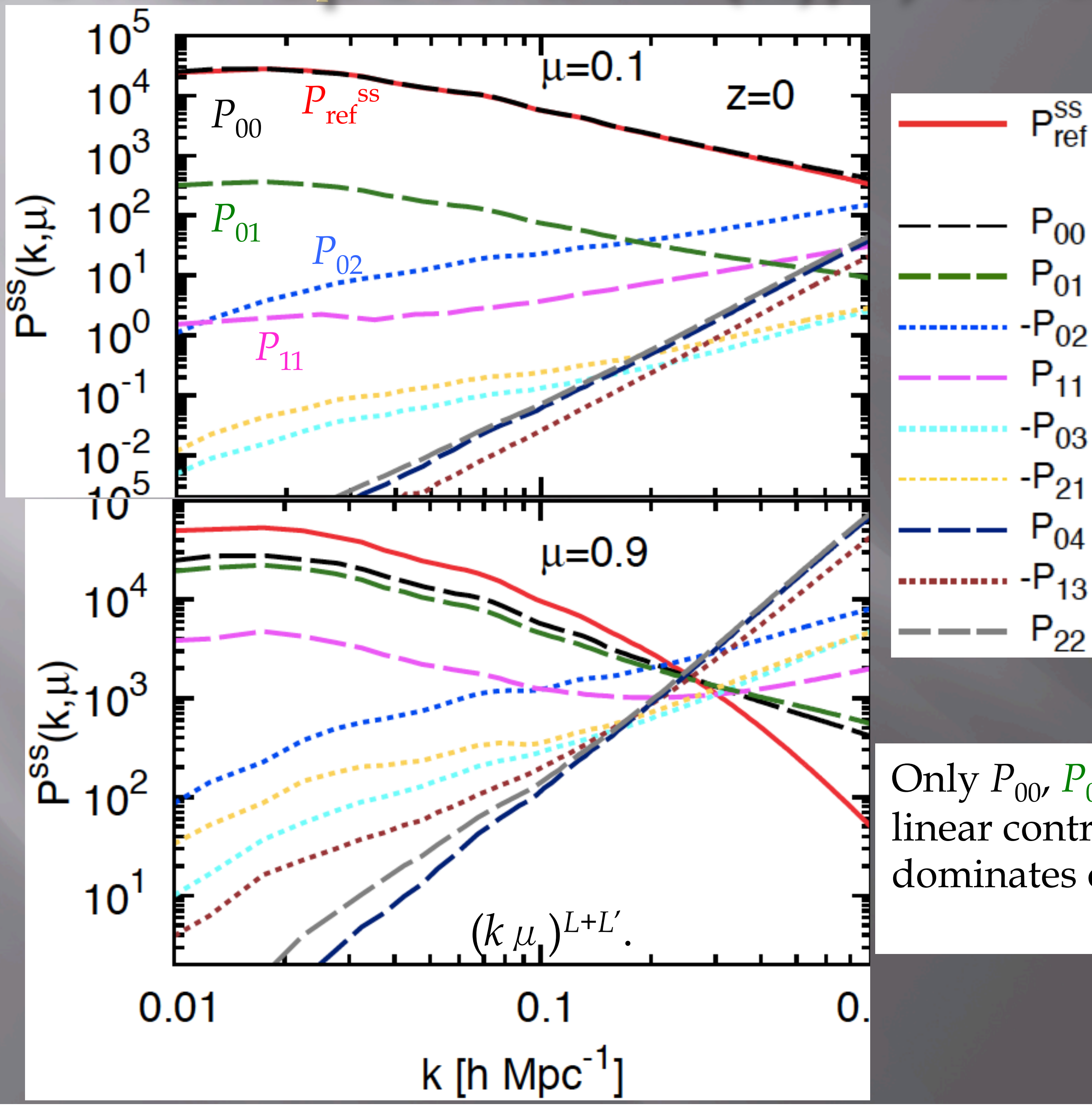
Scalar vector part

$$P^{ss}(k,\mu) = \sum_j a_{2j}(k) \mu^{2j}$$

Cosmological N-body simulations

- Desjacques, Seljak and Iliev (2009)
  - WMAP 5-year cosmology
  - 1024^3 dark matter particles
  - $L_{\rm box} = 1.6\,{\rm Gpc}/h$
  - 12 realizations × 3 line-of-sight = 36 measurements
  - $z = 0, 0.5, 1$ and 2
- Power spectra for dark matter are presented and those for halos and galaxies will be given later.

Power spectra $P^{ss}(k,\mu)$ and $P_{LL'}(k,\mu)$



Measured in redshift space

Measured in real space

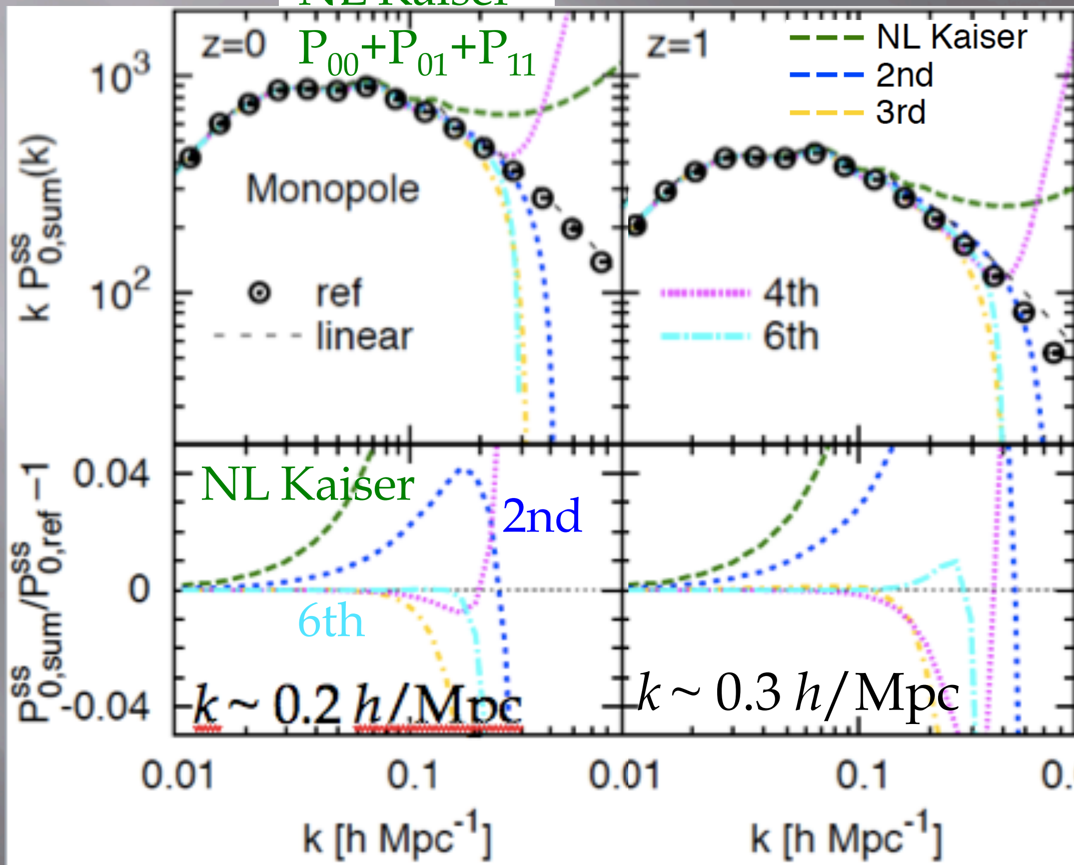
Only P_{00}, P_{01} and P_{11} contain linear contributions, but P_{02} dominates over P_{11} at low μ .

Accuracy of the expansion: monopole

n-th order approximation (*L*+*L′*)-th order term

$$P^{ss}(\mathbf{k}) = \sum_{L,L'=0}^{L+L'=n} \frac{(-1)^L}{L!L'!}\left(\frac{ik\mu}{H}\right)^{L+L'} P_{LL'}(\mathbf{k})$$

NL Kaiser



We need to explore ways to resum the higher order terms

Convergence for $k\mu\ \sigma/H < 1$

Finger-of-God resummation

- Empirical forms for FoG

  (e.g., Peacock & Dodds (1994), Park+(1994), Scocimarro(2004), Taruya+(2009))

$$G(x=(k\mu\sigma/H)^2)=\begin{cases}(1+x)^{-1} & \text{Lorentzian,}\\ \exp(-x) & \text{Gaussian,}\end{cases}$$

$$P^{ss}(\mathbf{k})=G\left([k\mu\sigma/H]^2\right)P_{\rm Kaiser}(\mathbf{k})$$

- A Taylor series of the FoG kernels produce positive and negative terms alternatively.

$$P^{ss}(\mathbf{k})=\boxed{P_{00}+2\frac{ik\mu}{H}P_{01}+\frac{(k\mu)^2}{H^2}P_{11}}-\frac{(k\mu)^2}{H^2}P_{02}-\frac{i(k\mu)^3}{3H^3}P_{03}-\frac{i(k\mu)^3}{H^3}P_{21}+\frac{(k\mu)^4}{12H^4}P_{04}-\frac{(k\mu)^4}{3H^4}P_{13}+\frac{(k\mu)^4}{2H^4}P_{22}+\cdots$$

$$=G_{00}\left(\left[\frac{k\mu\sigma_{00}}{H}\right]^2\right)P_{00}+2G_{01}\left(\left[\frac{k\mu\sigma_{01}}{H}\right]^2\right)\frac{ik\mu}{H}P_{01}+G_{11}\left(\left[\frac{k\mu\sigma_{11}}{H}\right]^2\right)\frac{(k\mu)^2}{H^2}P_{11}$$

$$G_{LL'}(x_{LL'};\alpha_{LL'})=\left(1+\frac{x_{LL'}}{\alpha_{LL'}}\right)^{-\alpha_{LL'}}$$

$$x_{LL'}=(k\mu\sigma_{LL'}/H)^2$$

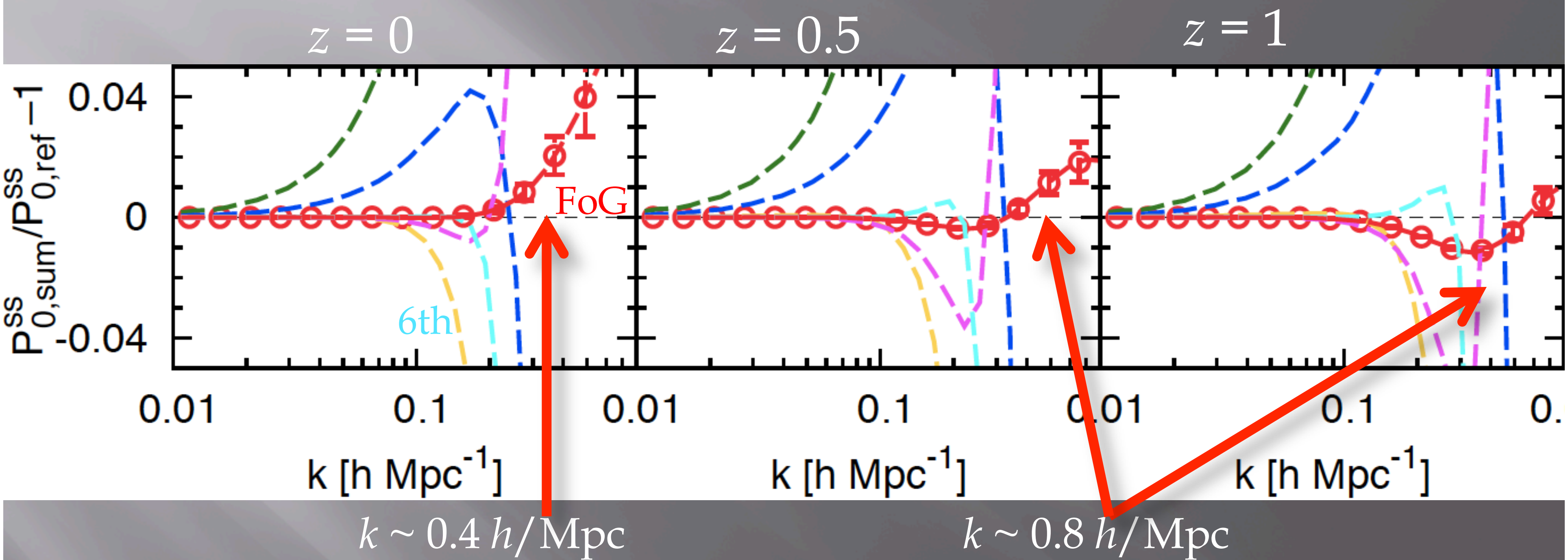
velocity dispersion

kurtosis $\alpha=1$: Lorentian

$\alpha=\infty$: Gaussian

FoG model for monopoles

- The FoG model improves the results.



- Note that there is no free parameter.

From dark matter to halos

Distribution function approach

- Dark matter

$$P^s_{mm}(\mathbf{k}) = \sum_{L=0}^{\infty}\sum_{L'=0}^{\infty}\frac{(-1)^{L'}}{L!\,L'!}\left(\frac{ik\mu}{H}\right)^{L+L'} P^{mm}_{LL'}(\mathbf{k})$$

- Halos and galaxies

$$P^s_{hh}(\mathbf{k}) = \sum_{L=0}^{\infty}\sum_{L'=0}^{\infty}\frac{(-1)^{L'}}{L!\,L'!}\left(\frac{ik\mu}{H}\right)^{L+L'} P^{hh}_{LL'}(\mathbf{k})$$

Linear Kaiser

$$P^s_{mm}(\mathbf{k}) = (1+f\mu^2)^2 P^{mm}_{00}(k)$$

$$P^s_{hh}(\mathbf{k}) = (1+f\mu^2/b)^2 P^{hh}_{00}(k)$$
$$= (b+f\mu^2)^2 P^{mm}_{00}(k)$$

- Halo biasing

$$b^{hh}_{LL'}(\mathbf{k}) = P^{hh}_{LL'}(\mathbf{k}) \big/ P^{mm}_{LL'}(\mathbf{k})$$

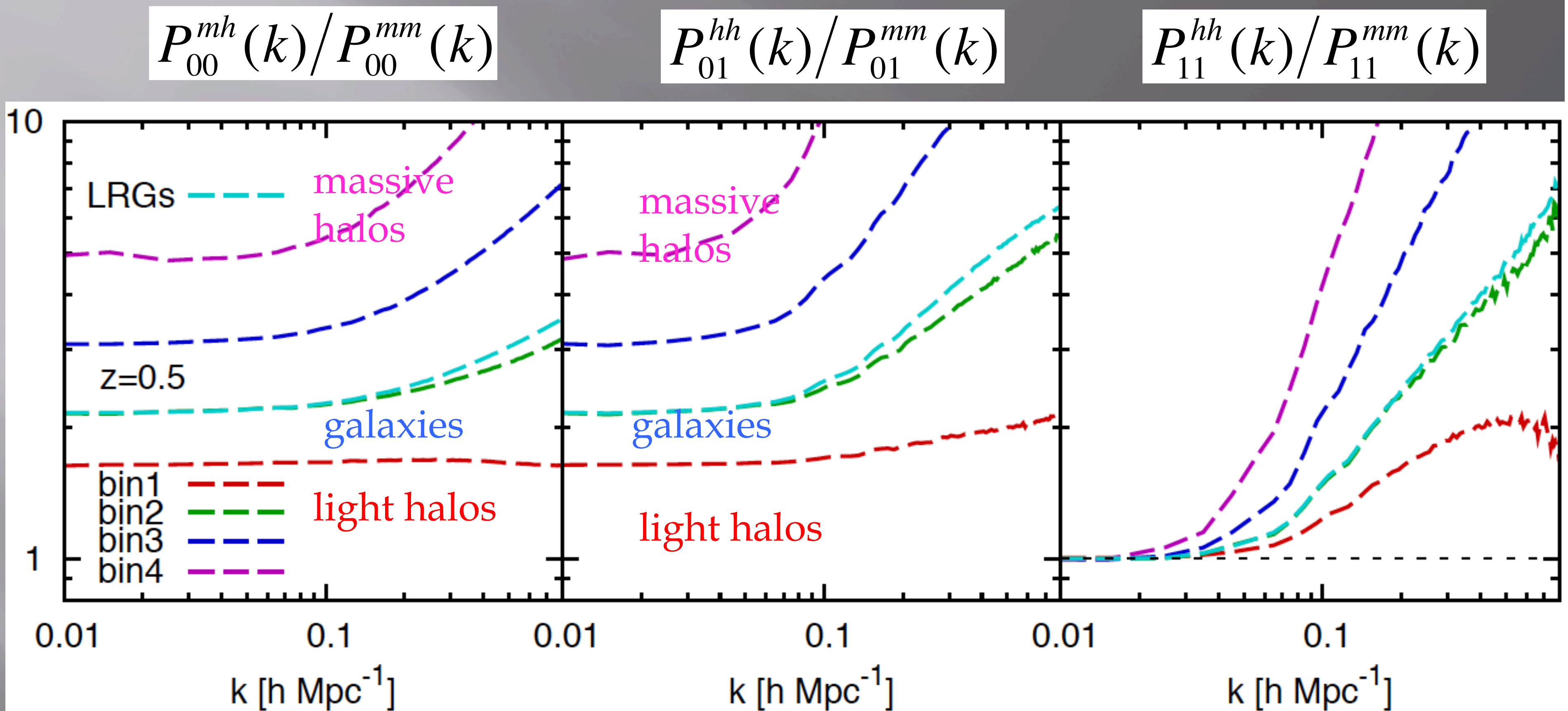
$$b^2 = P^{hh}_{00}(k) \big/ P^{mm}_{00}(k)$$

  - $b^{hh}_{00} = b^2$ $b^{hh}_{01} = b$ $b^{hh}_{11} = 1$ in linear theory

Analyzing halo and galaxy samples

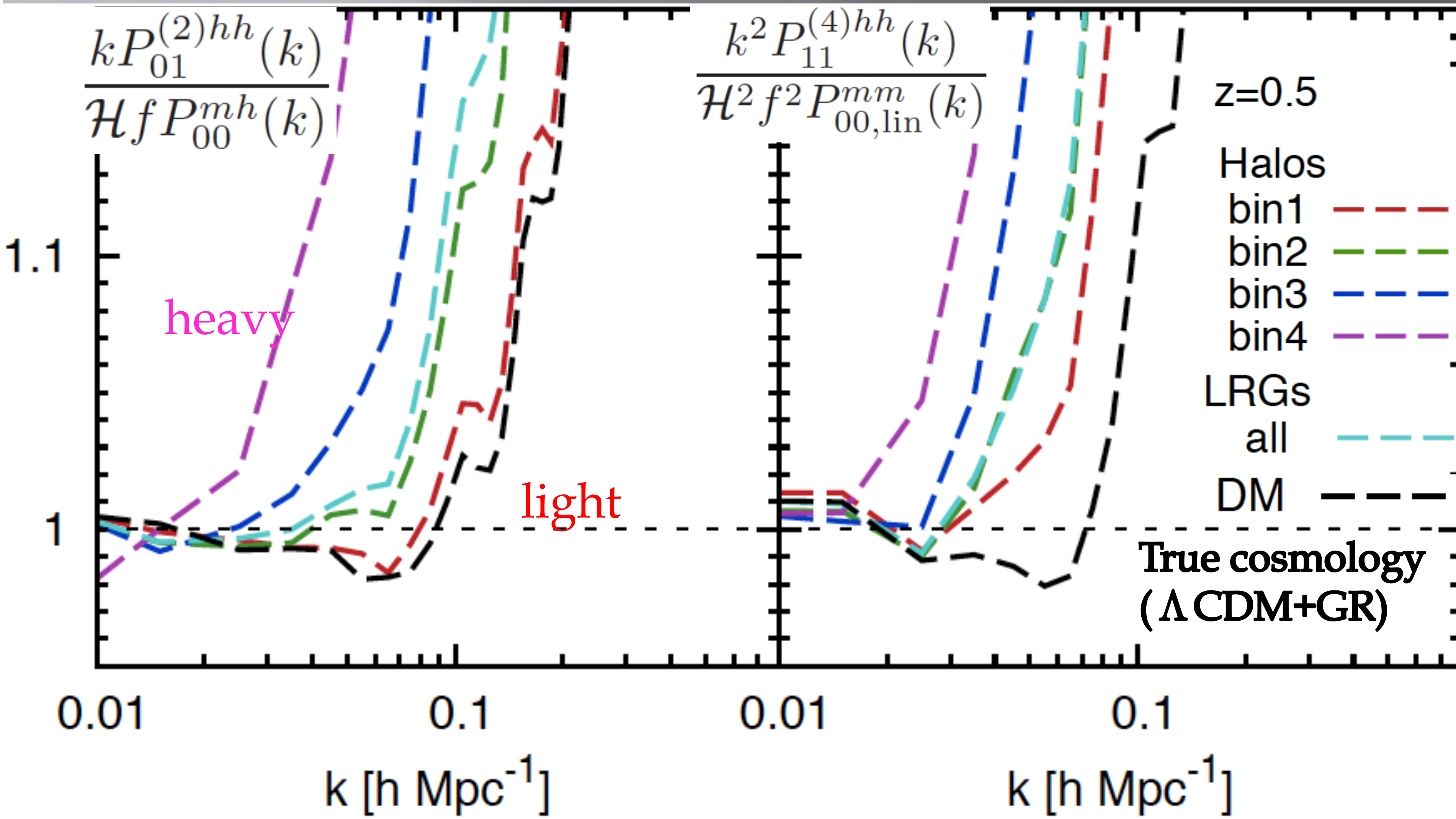
- Each halo sample in our simulations is divided into four subsample according to the halo mass in order to see halo mass dependence (Halo bin1 - bin4).
- As a more realistic sample we populate halos with central and satellite galaxies using a HOD modeling of luminous red galaxies (LRGs) for the SDSS Baryon Oscillation Spectroscopic Survey (BOSS) at z=0.5 (White et al).
- Advantage: each power spectrum is directly measurable even for massive (thus sparse) halo sample, unlike previous works.

Nonlinearity of generalized bias



- Scale dependence of Bias is more significant for more massive halos and for higher order moments.

Systematic and nonlinear effects on growth rate constrants



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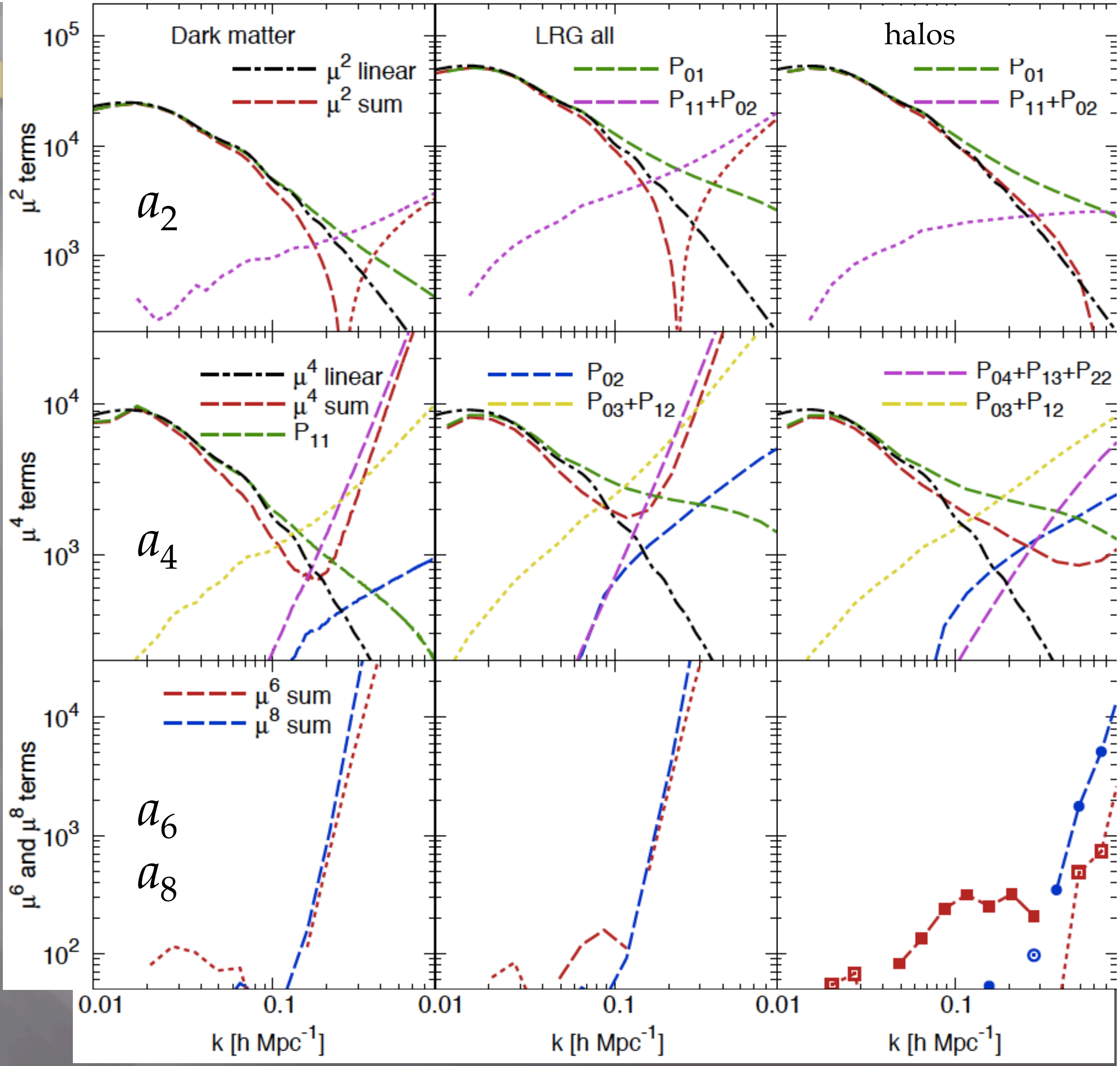
Scalar vector part

$$P^{ss}(k,\mu)=\sum_j a_{2j}(k)\mu^{2j}$$

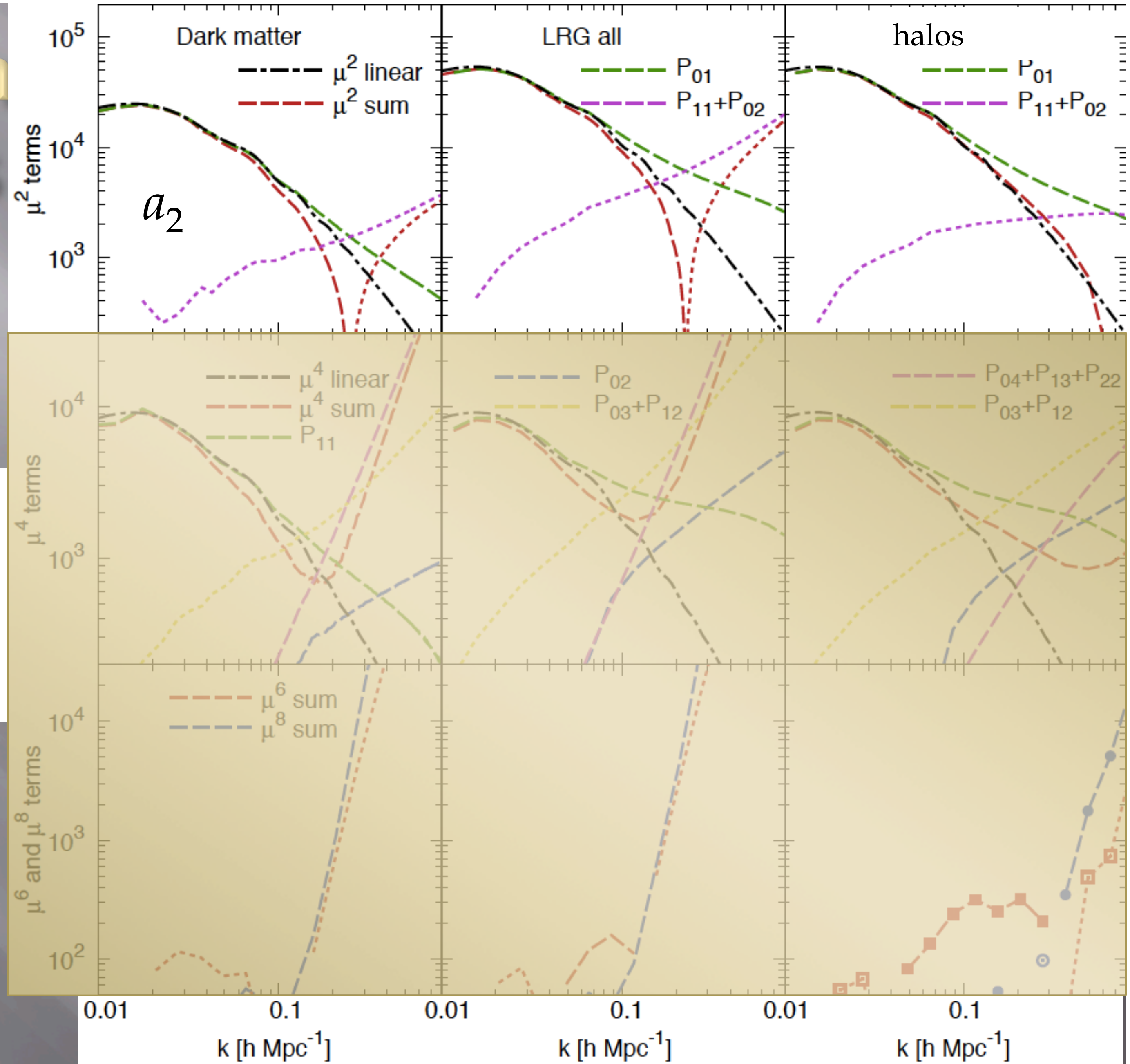
Expansion in powers of μ^2 for P^{ss}

- $z = 0.5$ results



$$P^{ss}(\mathbf{k}) = a_0(k)\mu^0 + a_2(k)\mu^2 + a_4(k)\mu^4 + a_6(k)\mu^6 + a_8(k)\mu^8 + \cdots$$

Expansion in powers of μ^2 for P^{ss}



μ^2 terms are strongly suppressed for dark matter and LRGs because of large velocity dispersions

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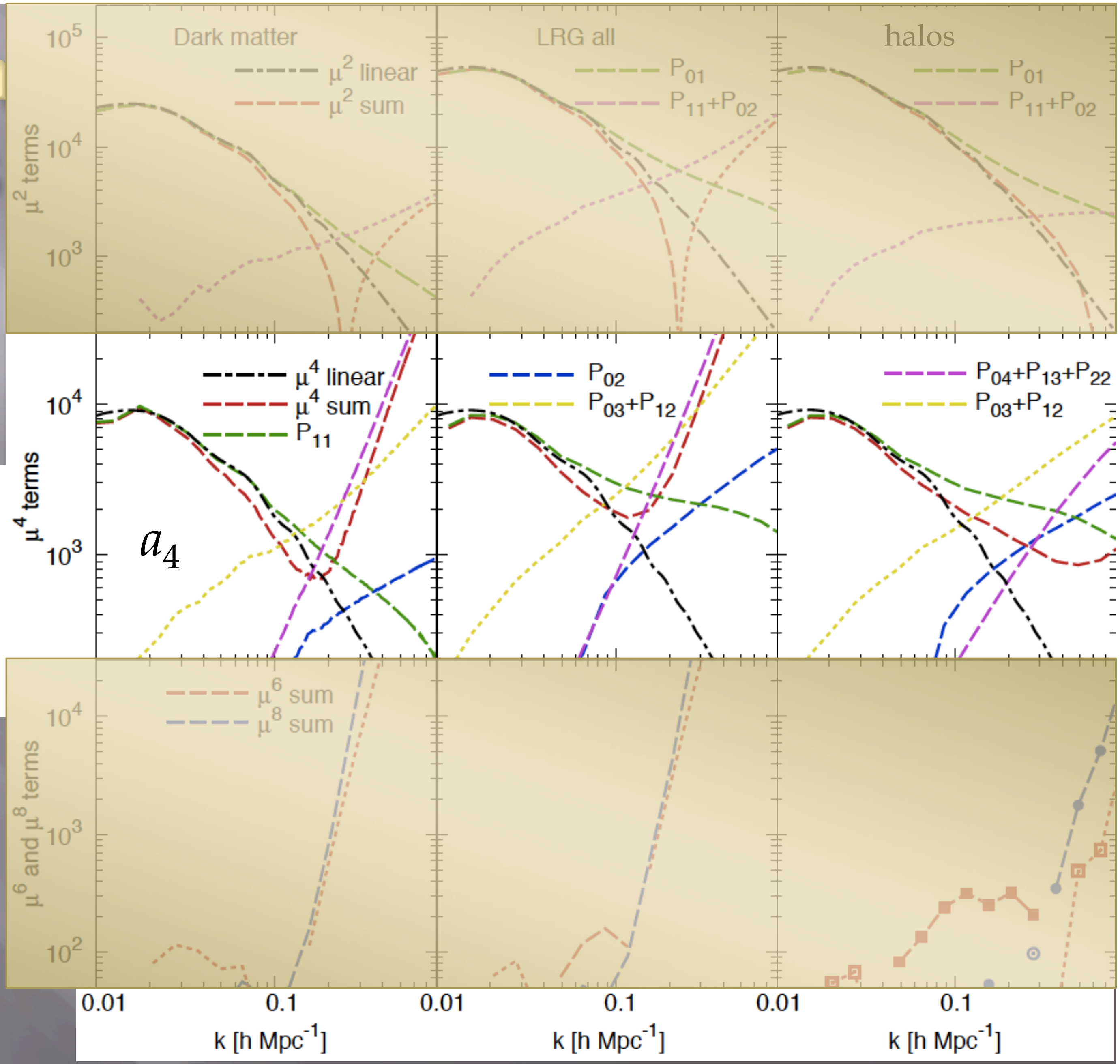
Expansion in powers of μ^2 for P^{ss}

The excess of P_{11} from linear theory is more prominent for halos than dark matter.



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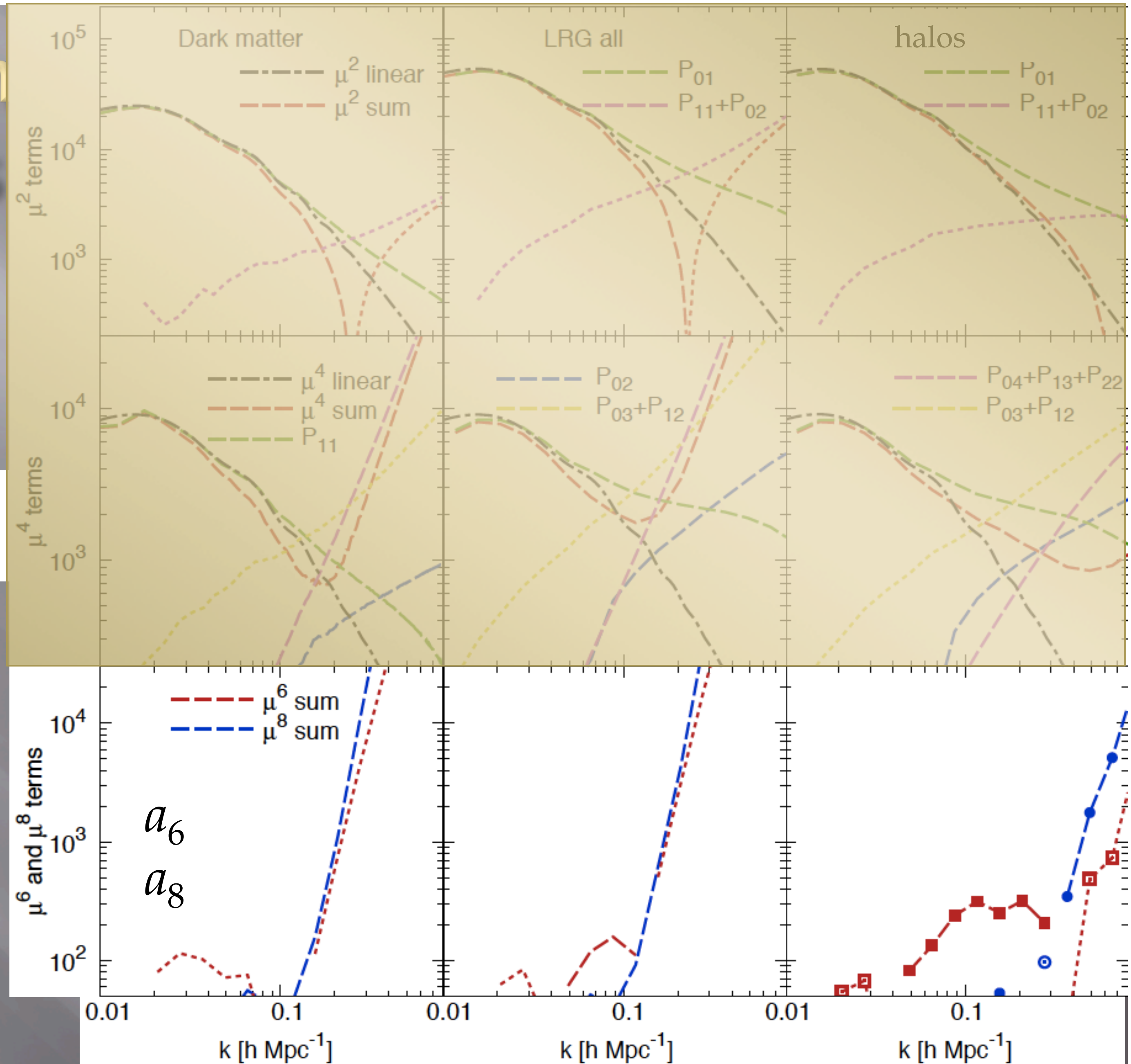
Expansion in powers of μ^2 for P^{ss}

There are no linear order contributions.



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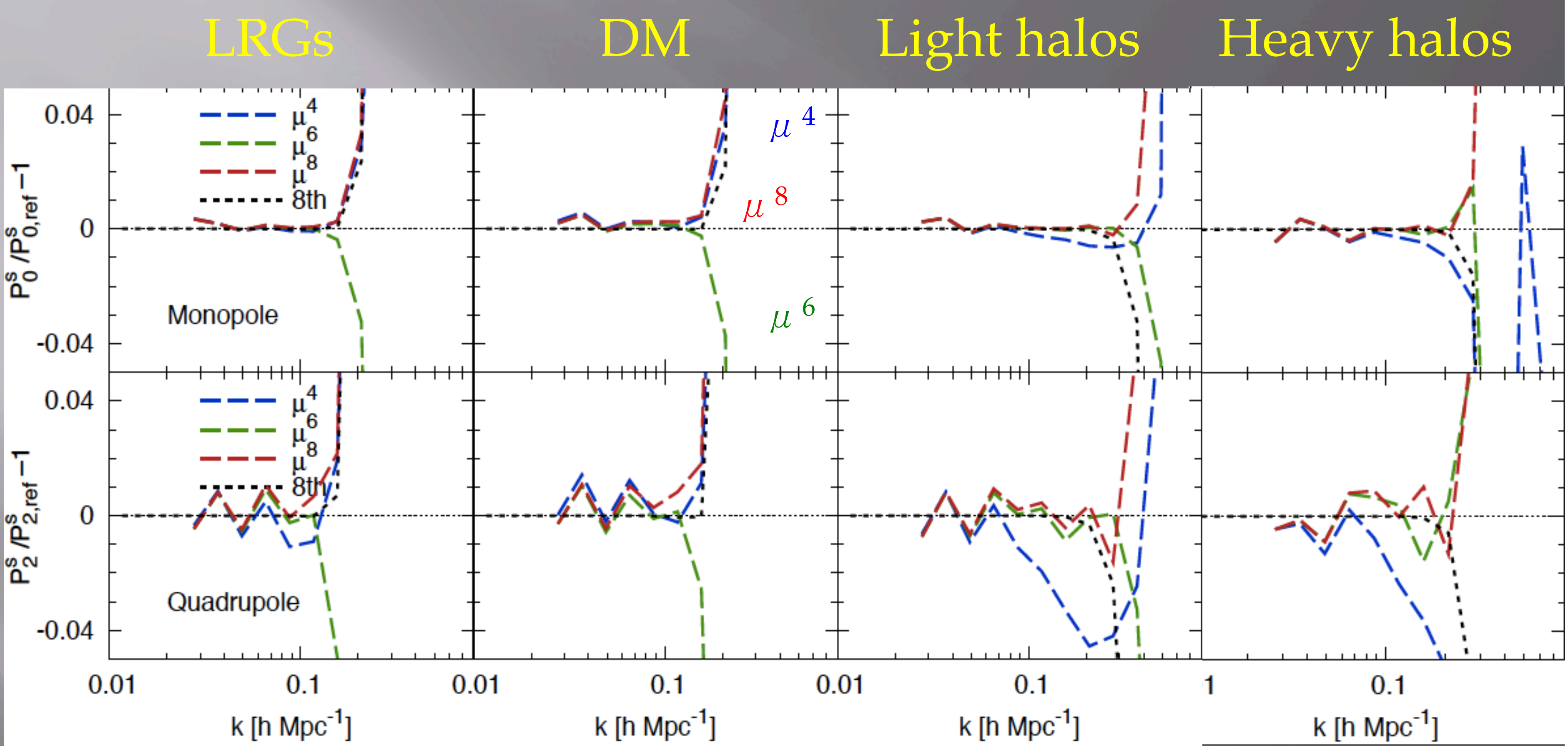
Importance of μ^6 and μ^8 terms



- Such higher-order terms are necessary for a percent level modeling, particularly for more massive halos and/or at higher redshift.

Summary

- A phase-space distribution function approach to redshift-space distortions was tested using N-body simulations.
- The resulting power spectrum is accurate at scales $k\mu\ \sigma / H < 1$ for dark matter, halos and galaxies.
- Our FoG model dramatically improves the range, $k \sim 0.4h/\mathrm{Mpc}$ at $z = 0$ and $k \sim 0.8\ h/\mathrm{Mpc}$ at $z = 0.5$ and 1
- We measured the generalized bias for halos and LRGs, then found the higher-order bias starts to be contaminated by nonlinear effects on larger scales.
- We determined the exact angular dependence of each power spectrum in terms of μ^{2j}.