~~Persistence~~ alpha-shape topology of the LSS

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Seoul, Korea, November 2010

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The outline

- Introduction to alpha-shapes
- The Betti numbers and the cosmic topology
- The ReBEL model – a case study
- Conclusions

Introduction to alpha-shapes

The key aspects of the Cosmic Web related to the processes that shape the LSS:

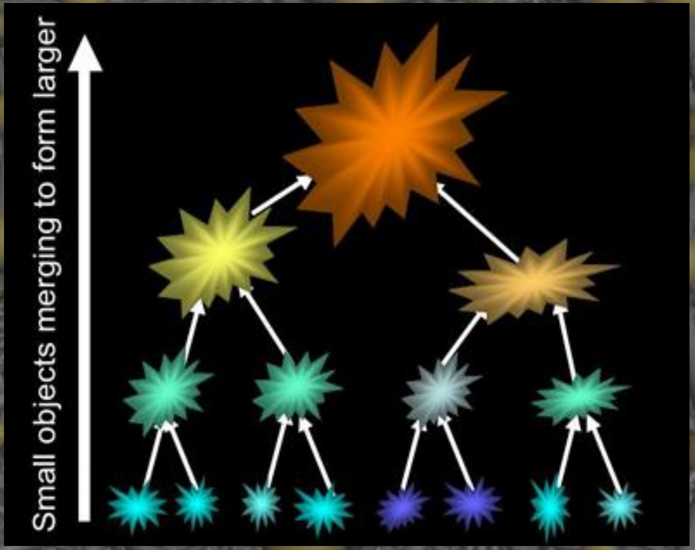
Hierarchical structure formation

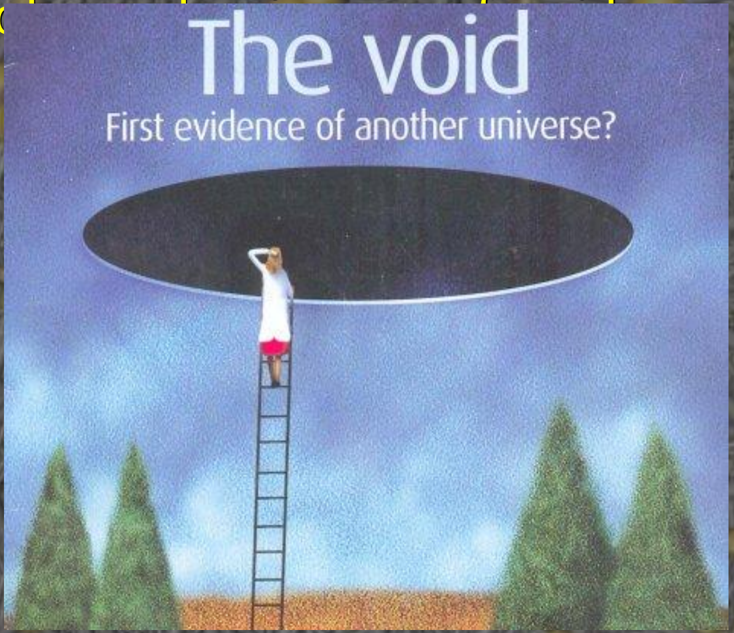
→ Multi-scale character

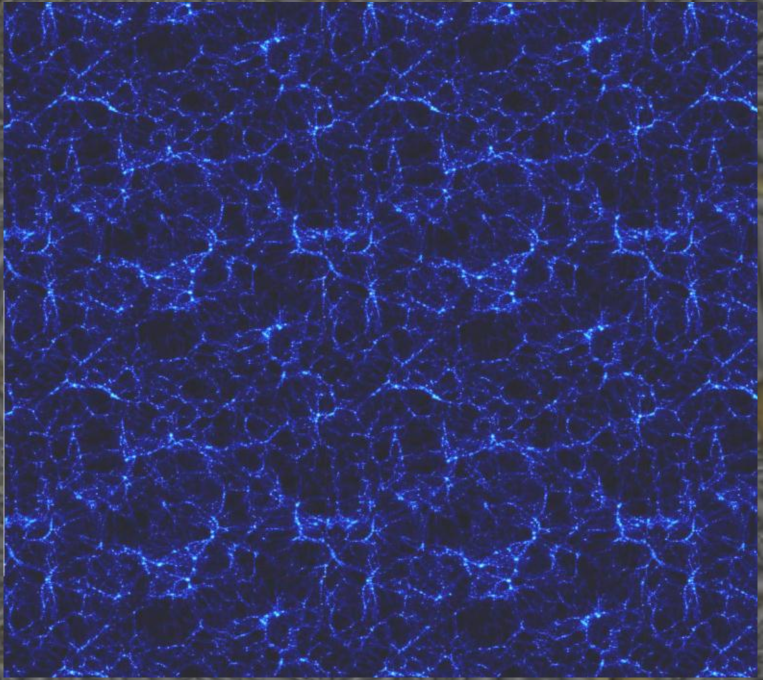
Anisotropic collapse

→ Web-like network of walls, filaments & voids

→ skewness of the density distribution







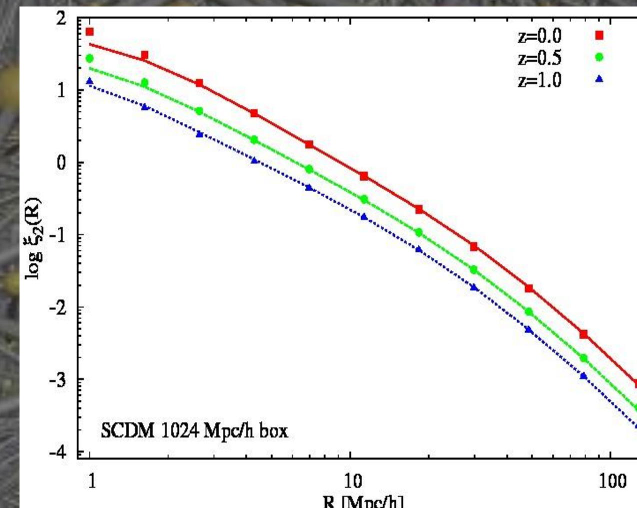
Introduction to alpha-shapes

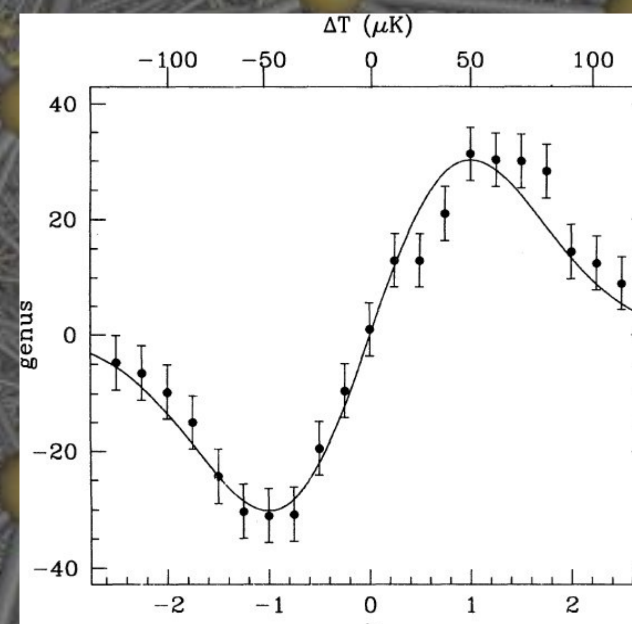
Well established tools for study the properties of LSS COSMIC WEB

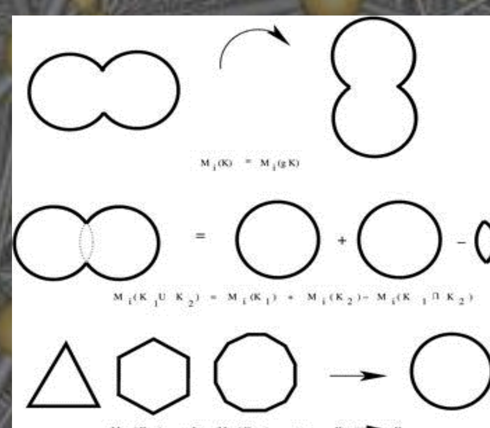
Statistical measures:
Clustering measures (N-point correlation functions), density distribution functions

Topological characteristic:
Genus statistic

Geometric characteristics:
Minkowsky functionals







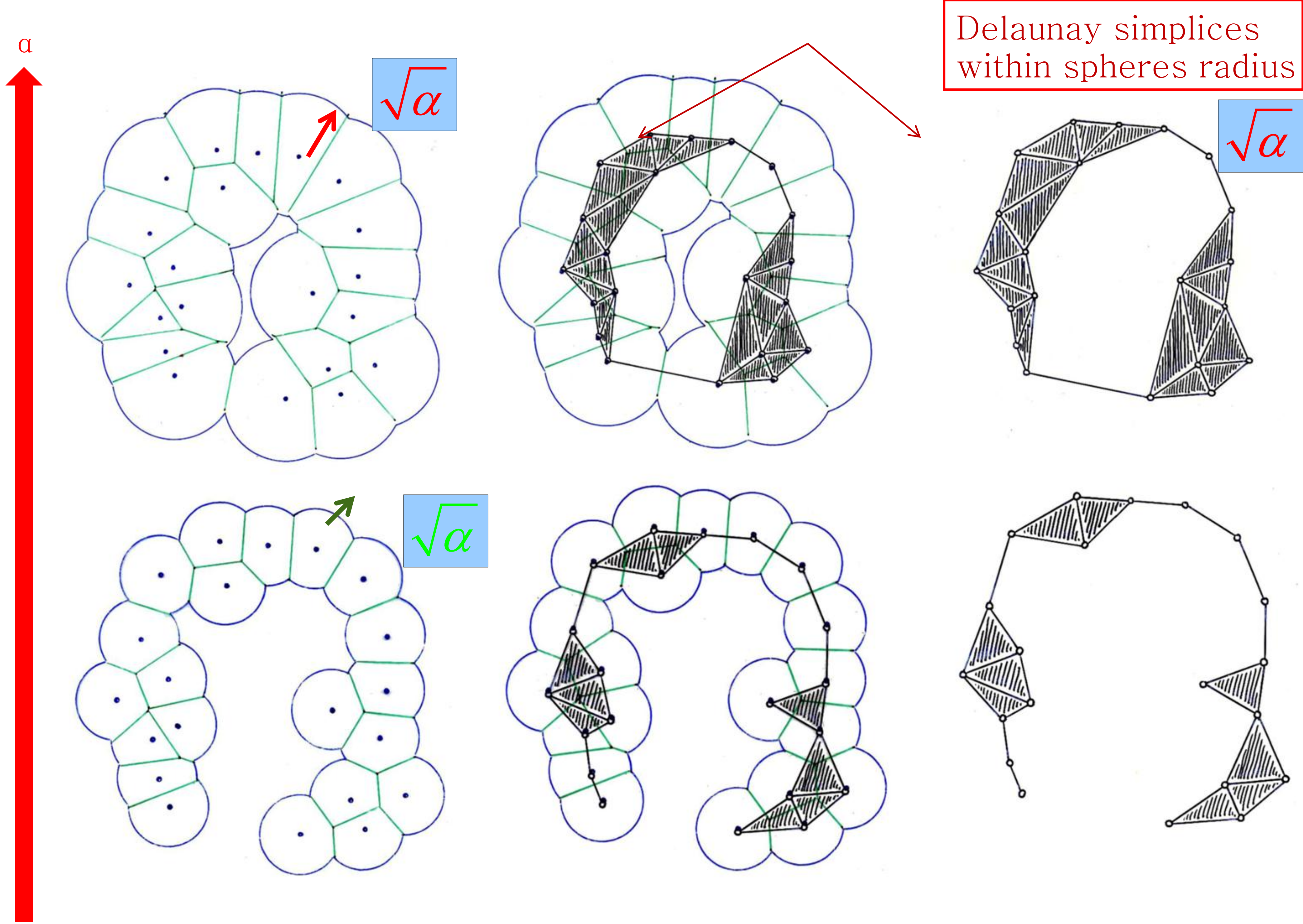
Introduction to alpha-shapes

A new approach:
Exploit the topological information contained in the Delaunay Tessellations of the galaxy/halo/density distribution

Alpha (α)Shapes

The idea was introduced by H. Edelsbrunner & collab. (1983,1994)

It consists of a description of intuitive notion of the shape of a discrete point set

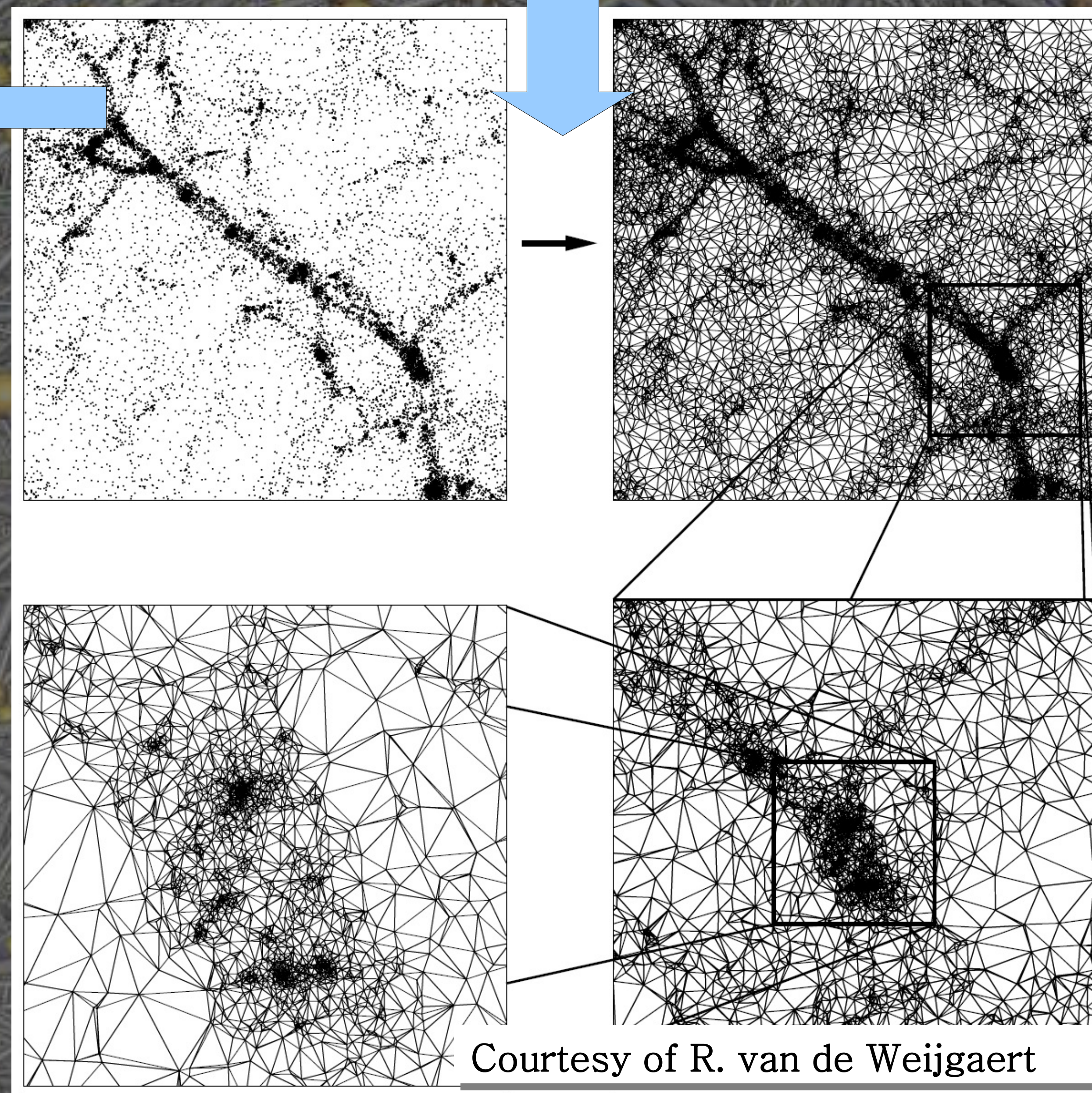


Courtesy of H. Edelsbrunner & R. van de Weijgaert

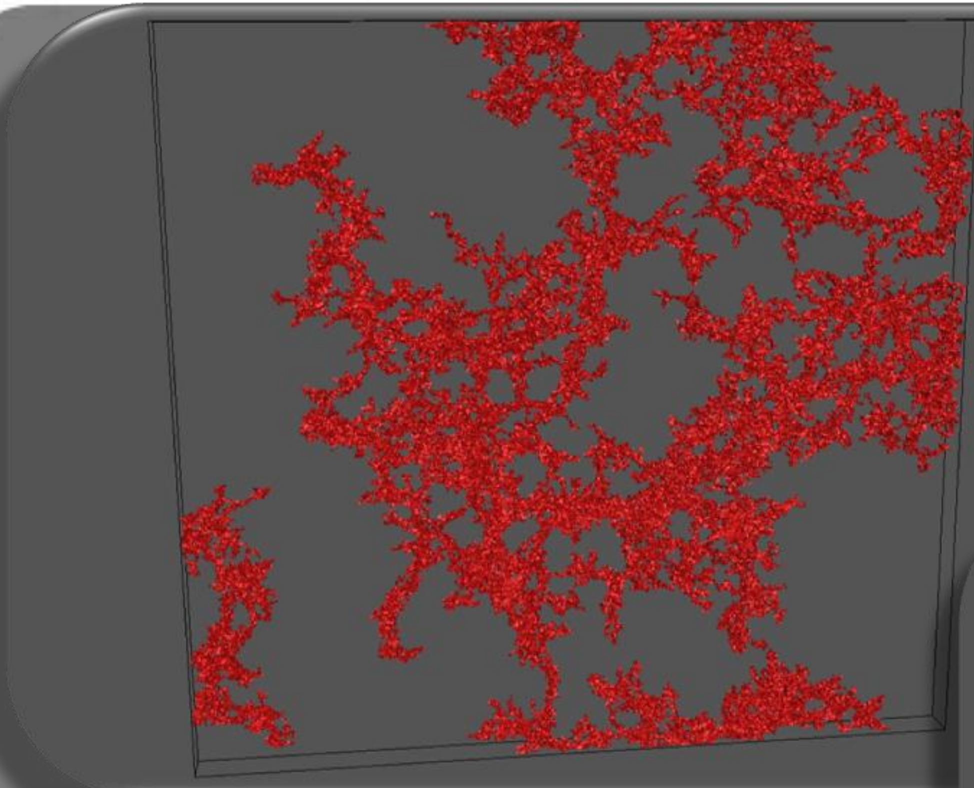
Sensitivity of Delaunay Tessellation to the multi-scale nature of the Cosmic Web

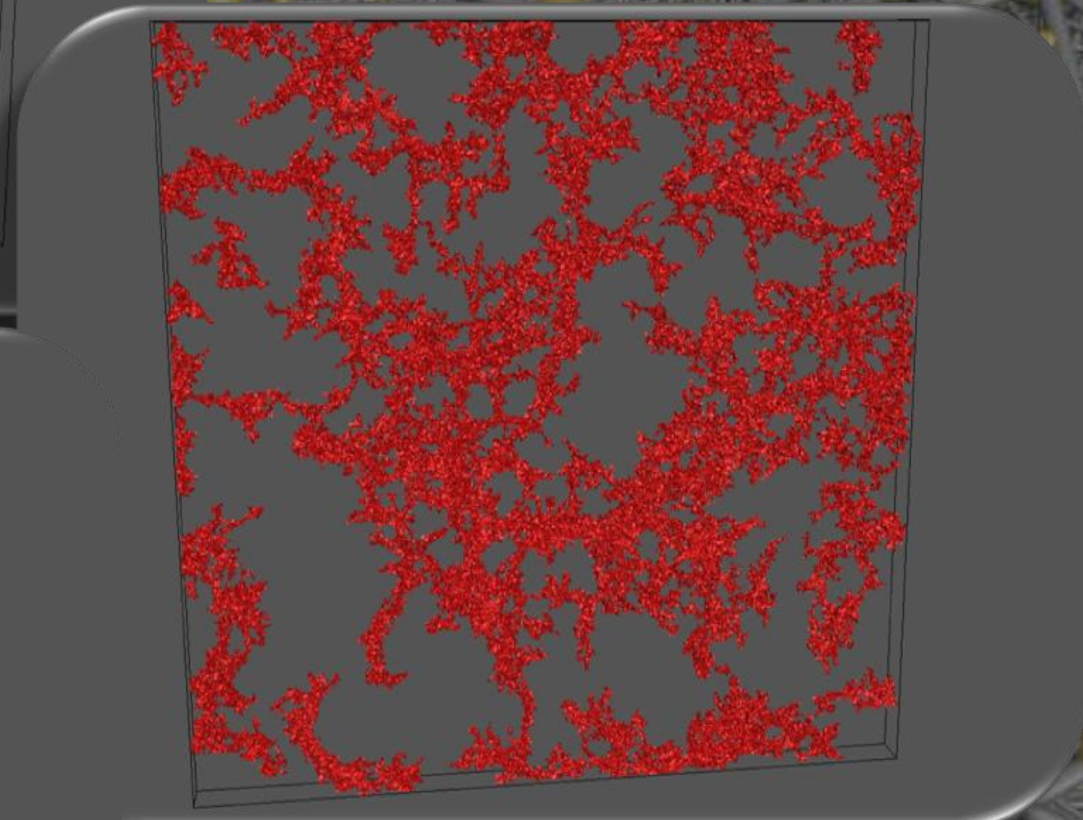
This property points to
exploiting this natural pr
operty of the Delaunay
Tessellations to explore
the topology of the cos
mic matter distribution

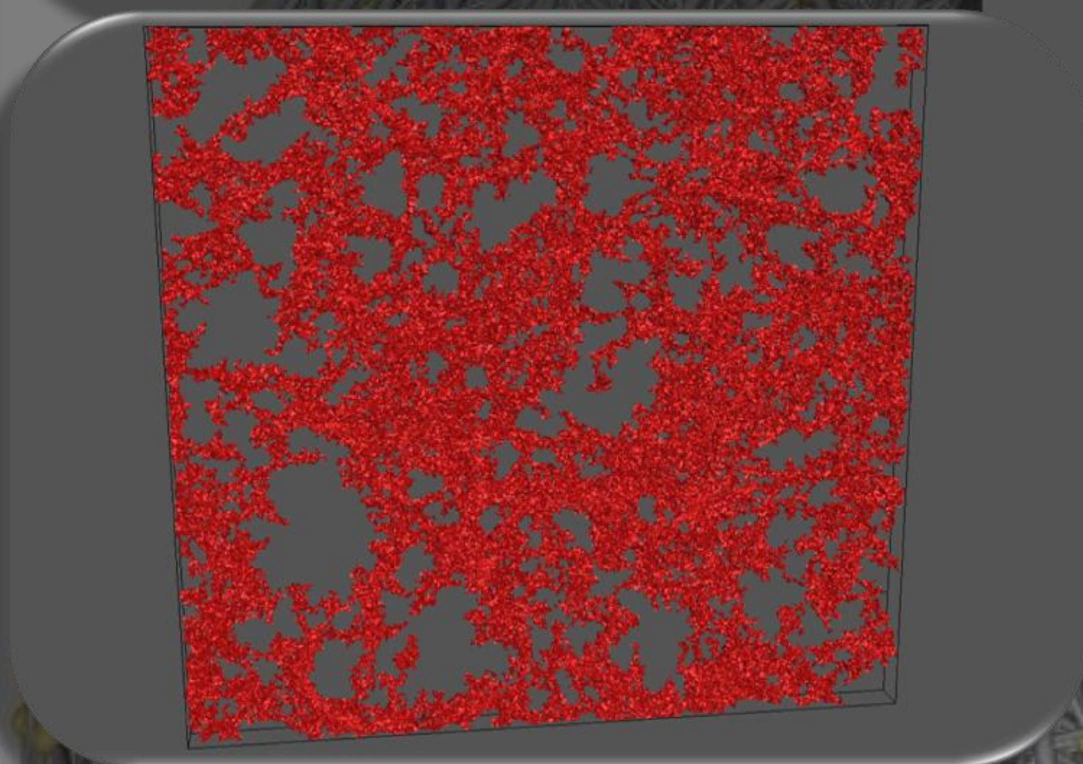
ALPHA SHAPES



Alpha Shapes of LSS from cosmological N-body simulations







The Betti numbers

The set of alpha-shapes defined with specified weighting parameter α for a finite set of points can be further used to define the Betti numbers. The Betti numbers provide quantitative characterization of the underlying topology defined by this set of points.

Betti numbers – definitions

Betti number β_k – a number of k-dimensional holes of an object or shape. It is also a rank of homology groups H_p

Parameter k can take values from 0 < k < N, for a n-dimensional euclidean space. For N=3 we have

β_0 – the number of independent components

β_1 – the number of tunnels

β_2 – the number of enclosed voids

The Betti numbers & the genus

Relationship between Betti Numbers & Minkowski Functionals via Euler characteristic of mass distribution:

The Genus & the Euler characteristic:

For a body with c components the genus specifies the handles on surface and is related to the Euler characteristic via:

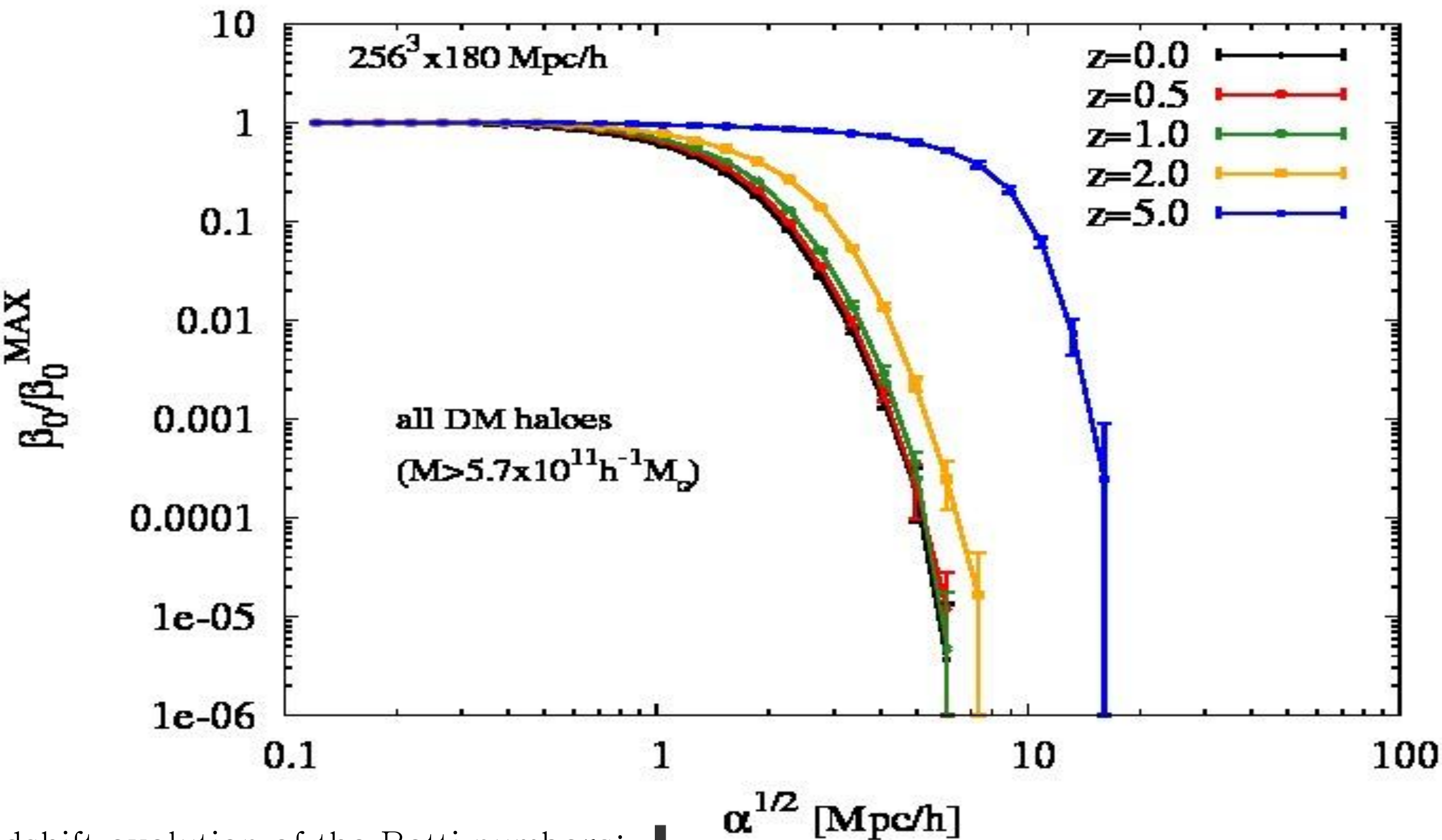
$$g = c - \frac{1}{2}\chi$$

Now the Euler characteristic of manifolds is alternating sum of the Betti numbers:

$$\chi = 2(\beta_0 - \beta_1 + \beta_2)$$

The Betti numbers & topology of the LSS

Preliminary results: A series of N-body cosmological simulations consisting of 128^3 and 256^3 DM particles. We compute Betti number for DM haloes population. Haloes are identified by AHF code by Knollman & Knebe.



Redshift evolution of the Betti numbers:

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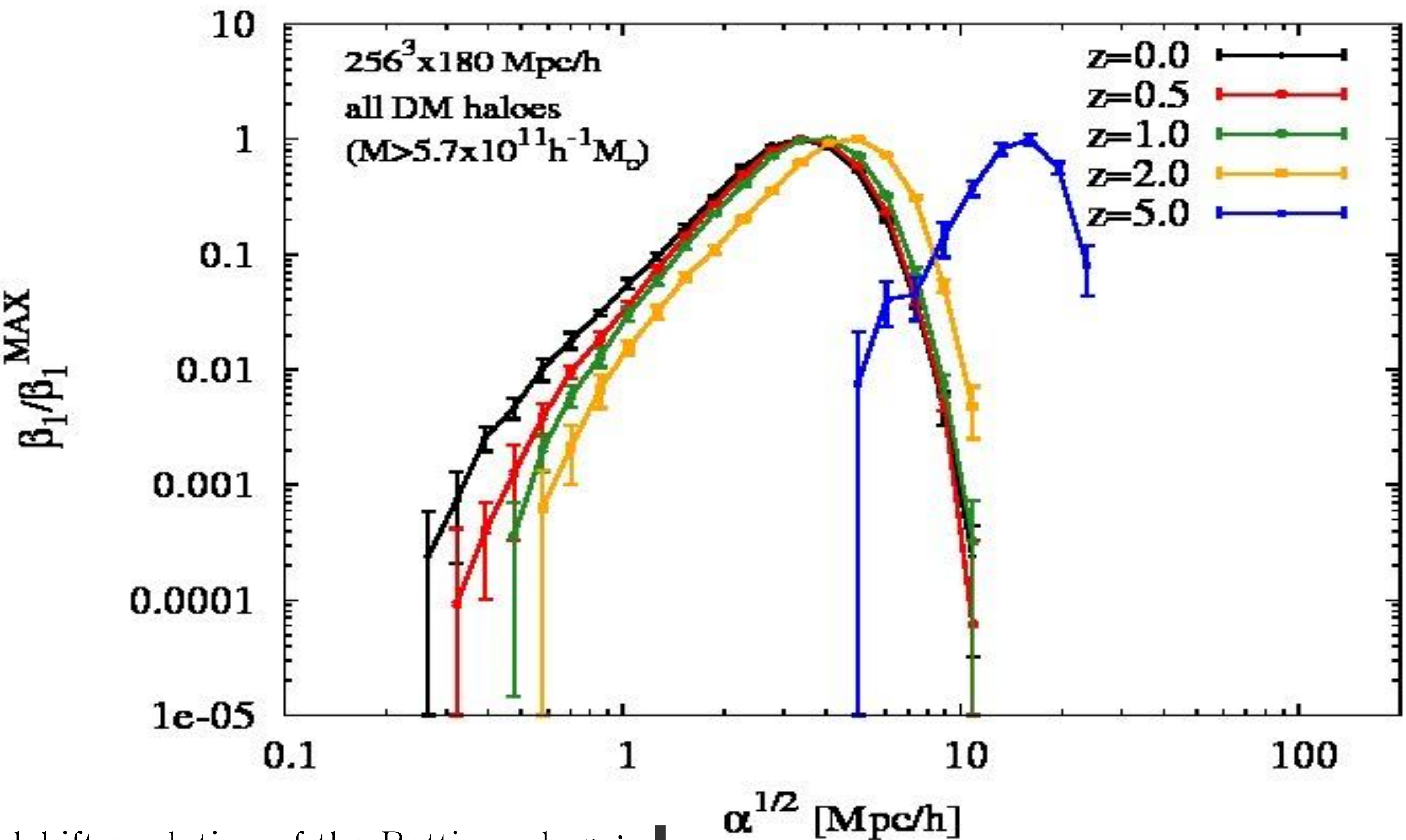

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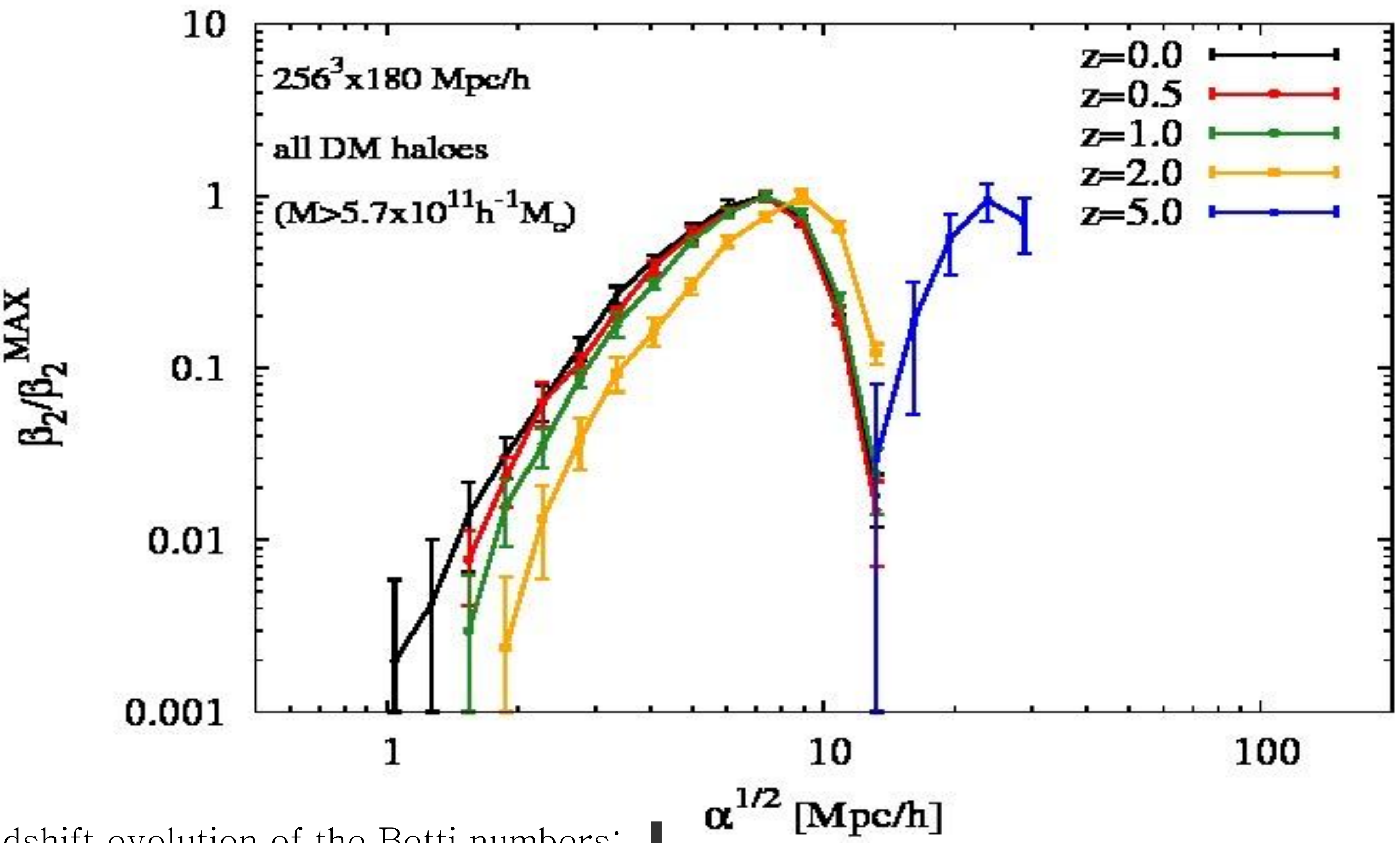

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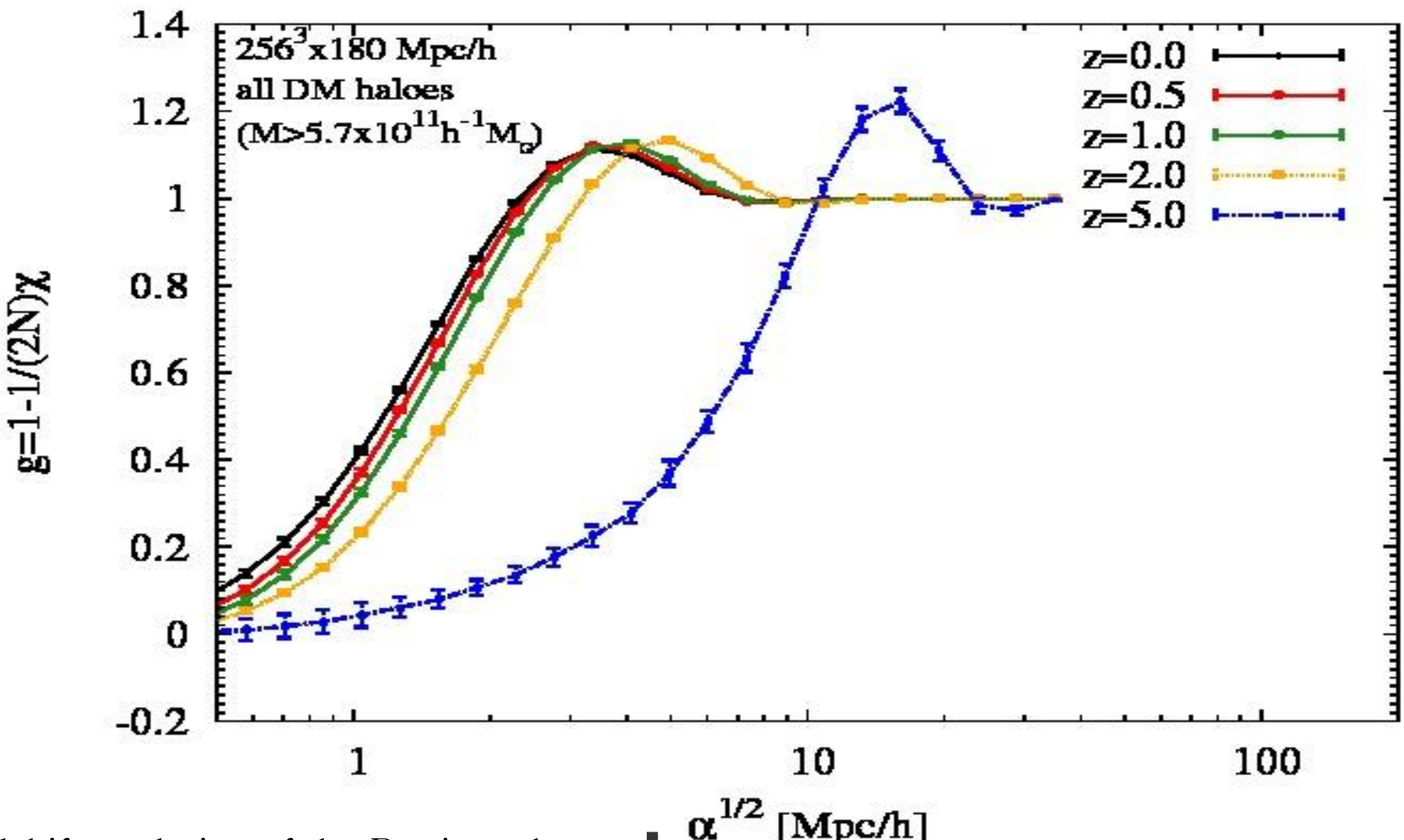

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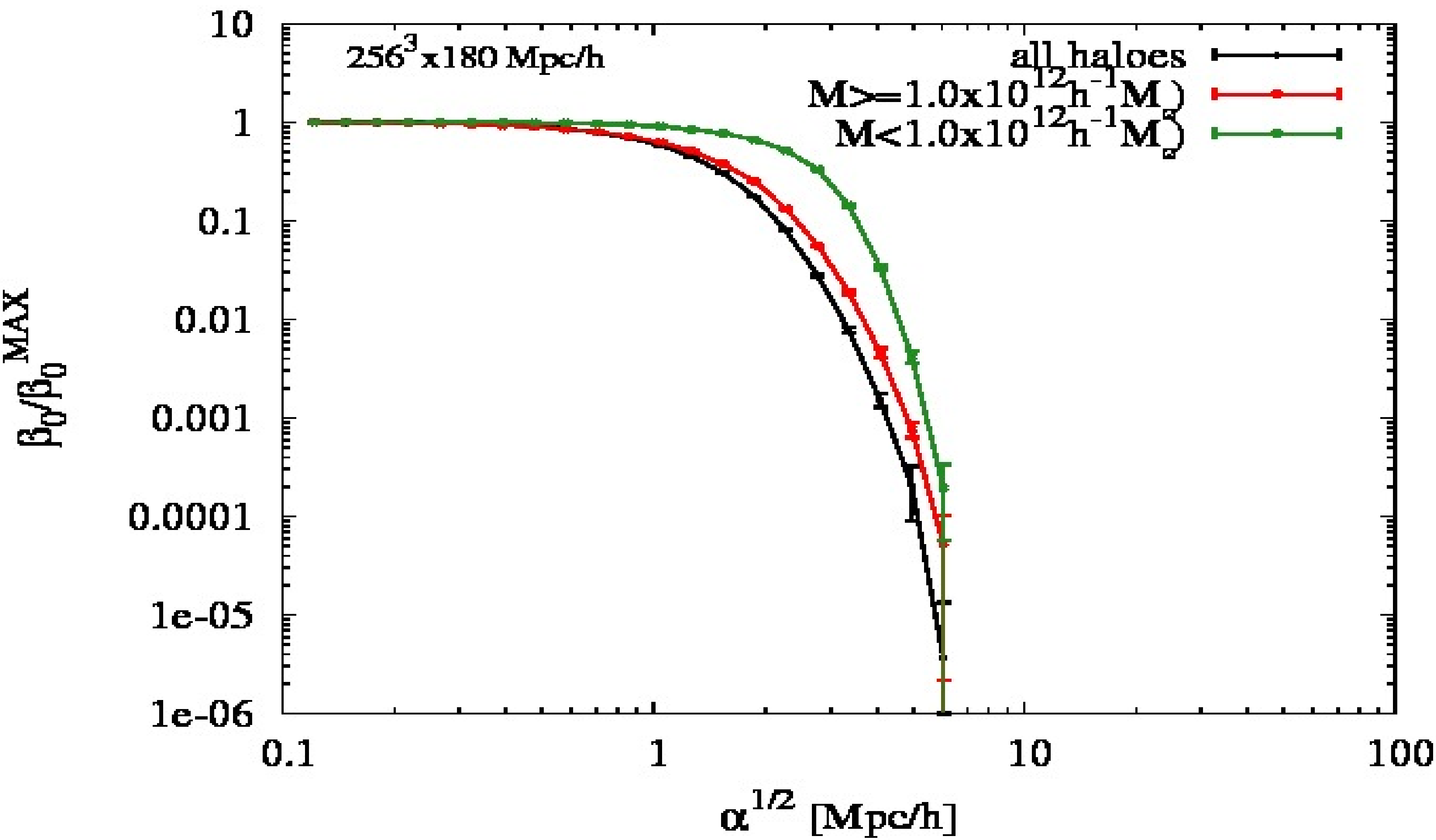

Betti topology by mass filtered halo

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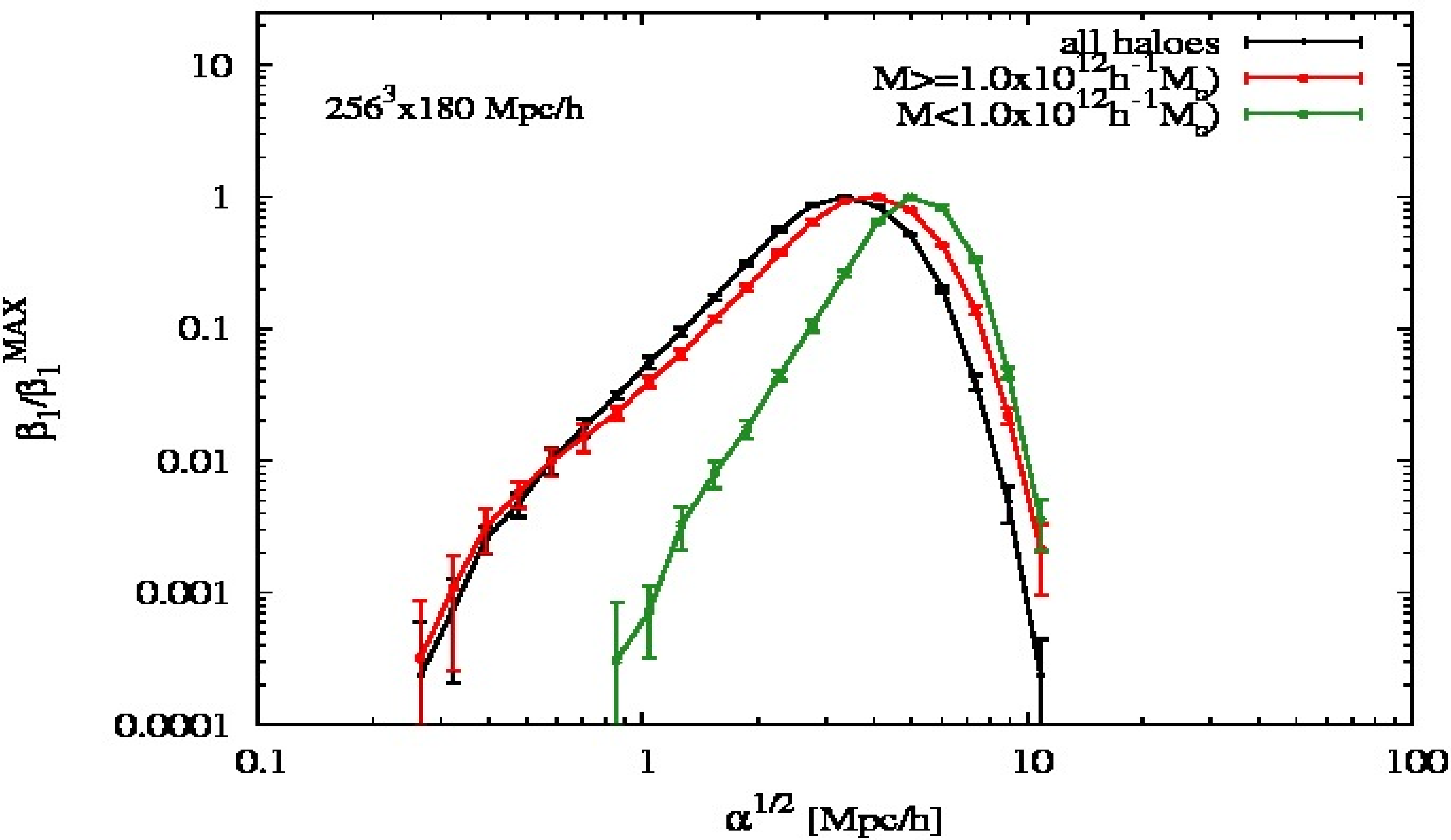

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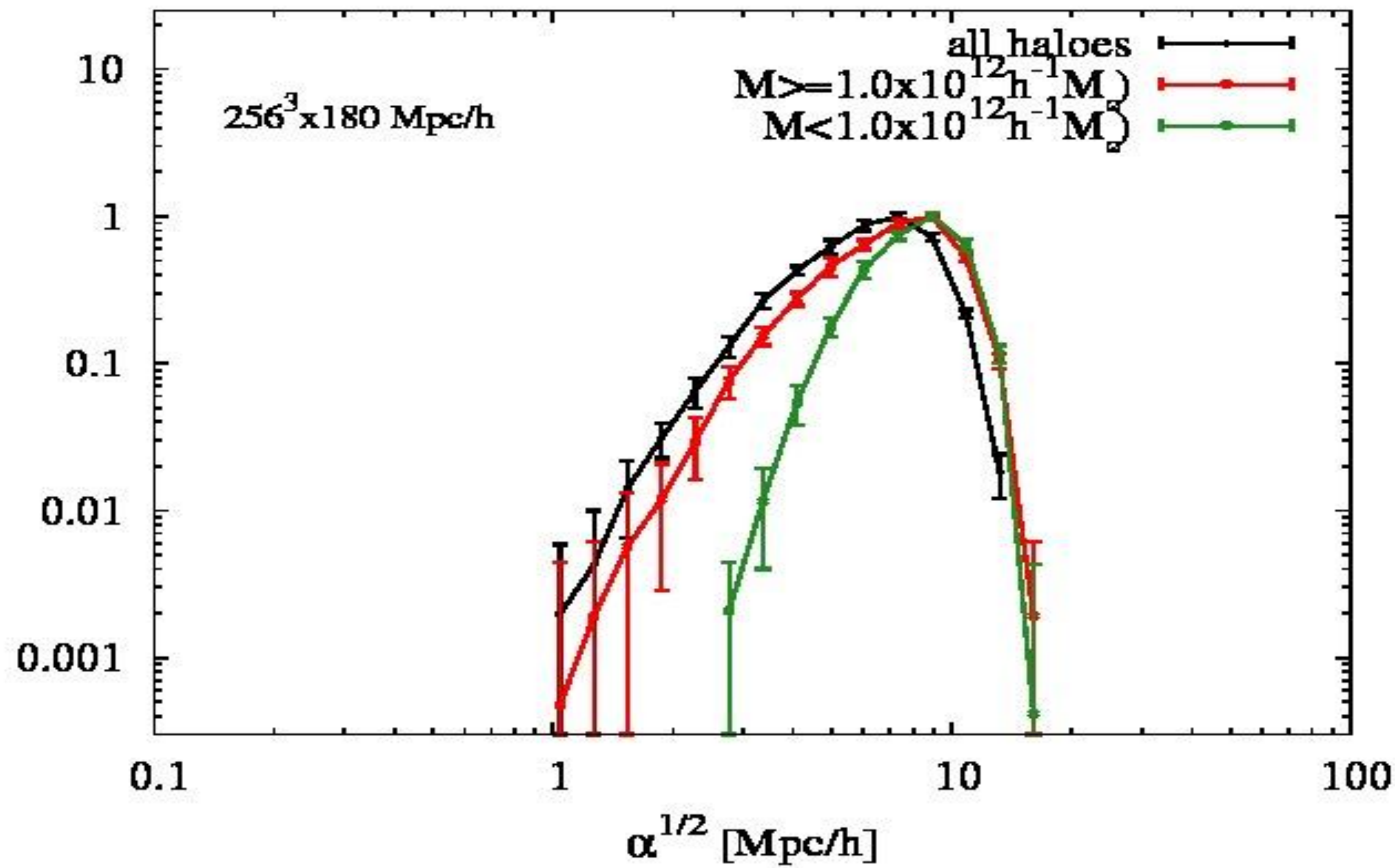

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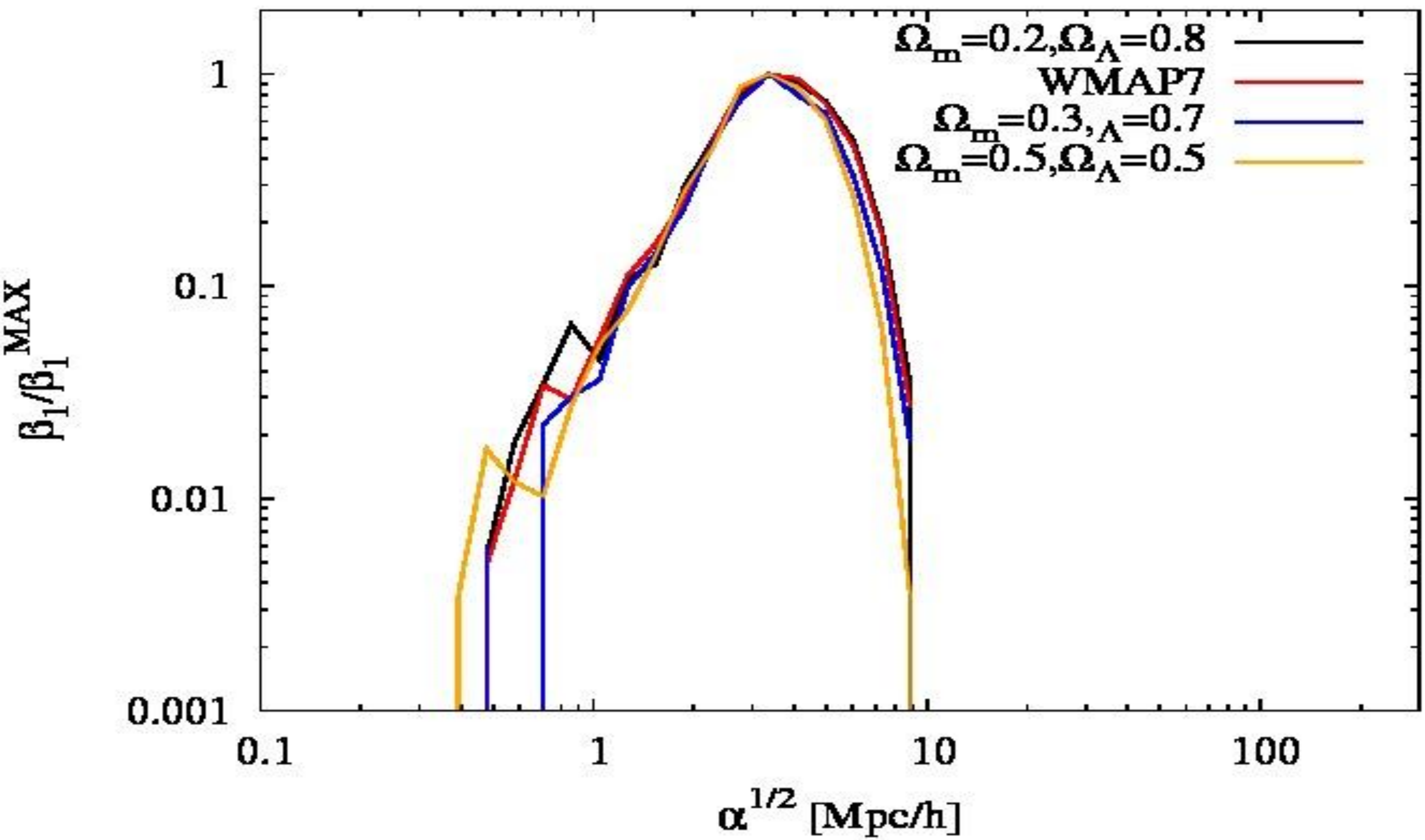

Betti function as potential cosmo pro

The Betti numbers & topology of the LSS

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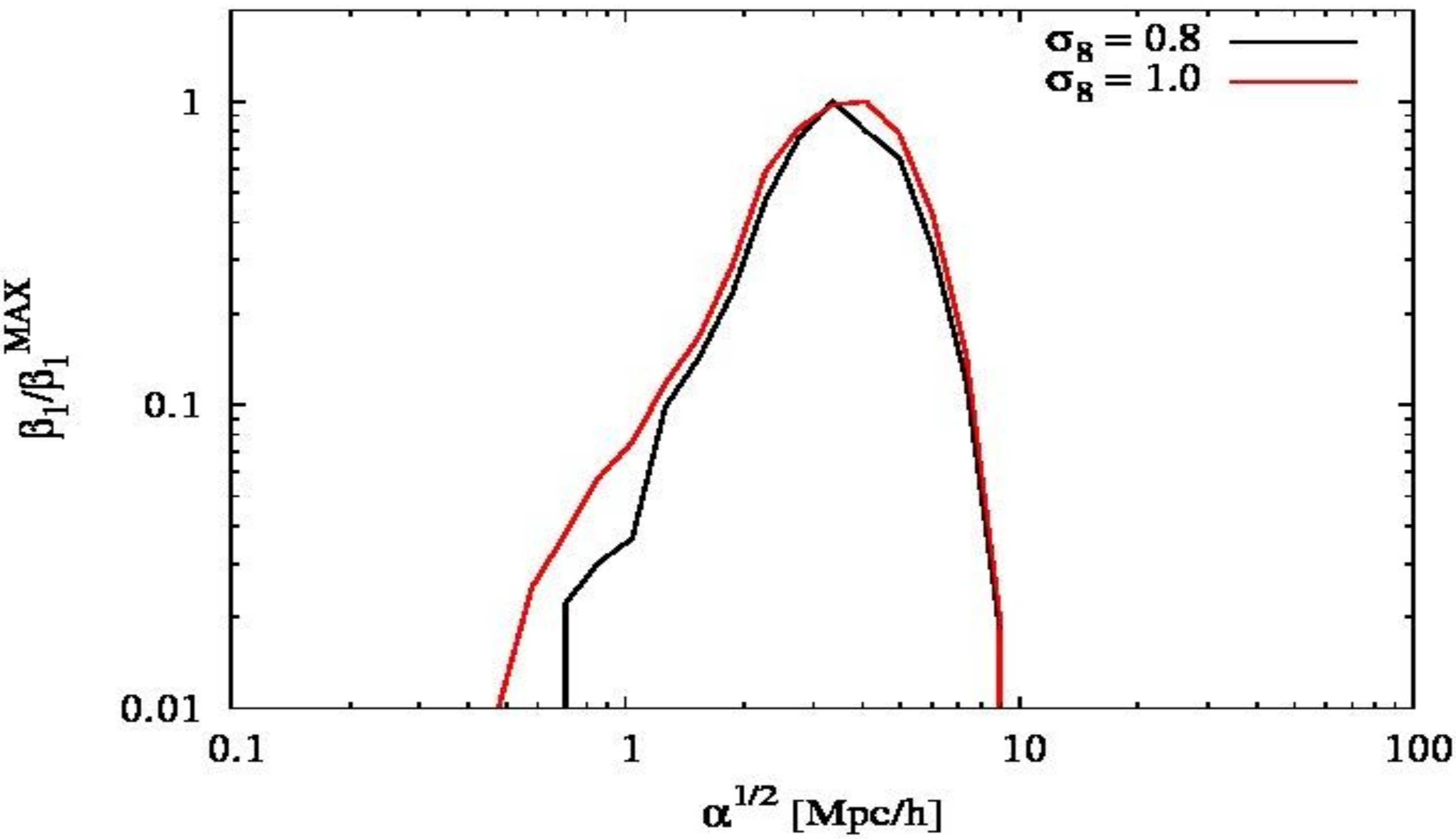


Betti function as potential cosmo pro

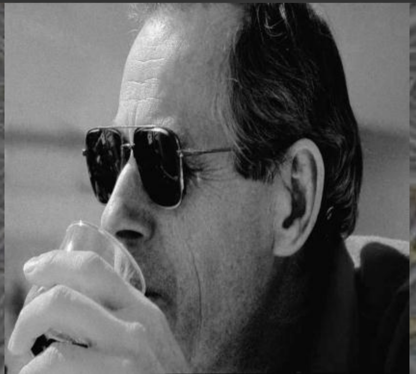
THE ReBEL MODEL

a case study for Betti cosmological analysis

We allow for extra piece of a new physics in the DM sector. DM particles now interact additionally by exchange of massless scalar. This is dynamically screen out on constant co-moving distance.

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{2}(\partial\phi)^2 + \bar{\Psi}_s i\nabla\!\!\!\!/\,\Psi_s + \bar{\Psi}_+ i\nabla\!\!\!\!/\,\Psi_+ \\
+ \; & \bar{\Psi}_- i\nabla\!\!\!\!/\,\Psi_- - y_s\phi\bar{\Psi}_s\Psi_s - (m_+ + y_+\phi)\bar{\Psi}_+\Psi_- - (m_- - y_-\phi)\bar{\Psi}_-\Psi_- ,
\end{aligned}
$$

Model developed & based on a series of articles by Farrar & Peebles and Gubser & Peebles in 2003-2005.



THE ReBEL MODEL

Phenomenological formulation of the ReBEL model

New gravitational potential between DM particles:

$$\Phi(\mathbf{r}) = -\frac{Gm}{r}\left(1 + \beta e^{-r/r_s}\right) , \qquad (1)$$

New force law for DM particles:

$$F_{DM} = -\frac{Gm^2}{r^2}\left[1 + \beta\left(1 + \frac{r}{r_s}\right) e^{-r/r_s}\right] \qquad (2)$$

The ReBEL - daRk Breaking Equivalence principLe

Extra physics in DM sector! DM now interacts additionally by exchange of a massless scalar, this interaction is dynamically Yukawa-like screened out with a screening length growing with the Universe. Two free model parameters:

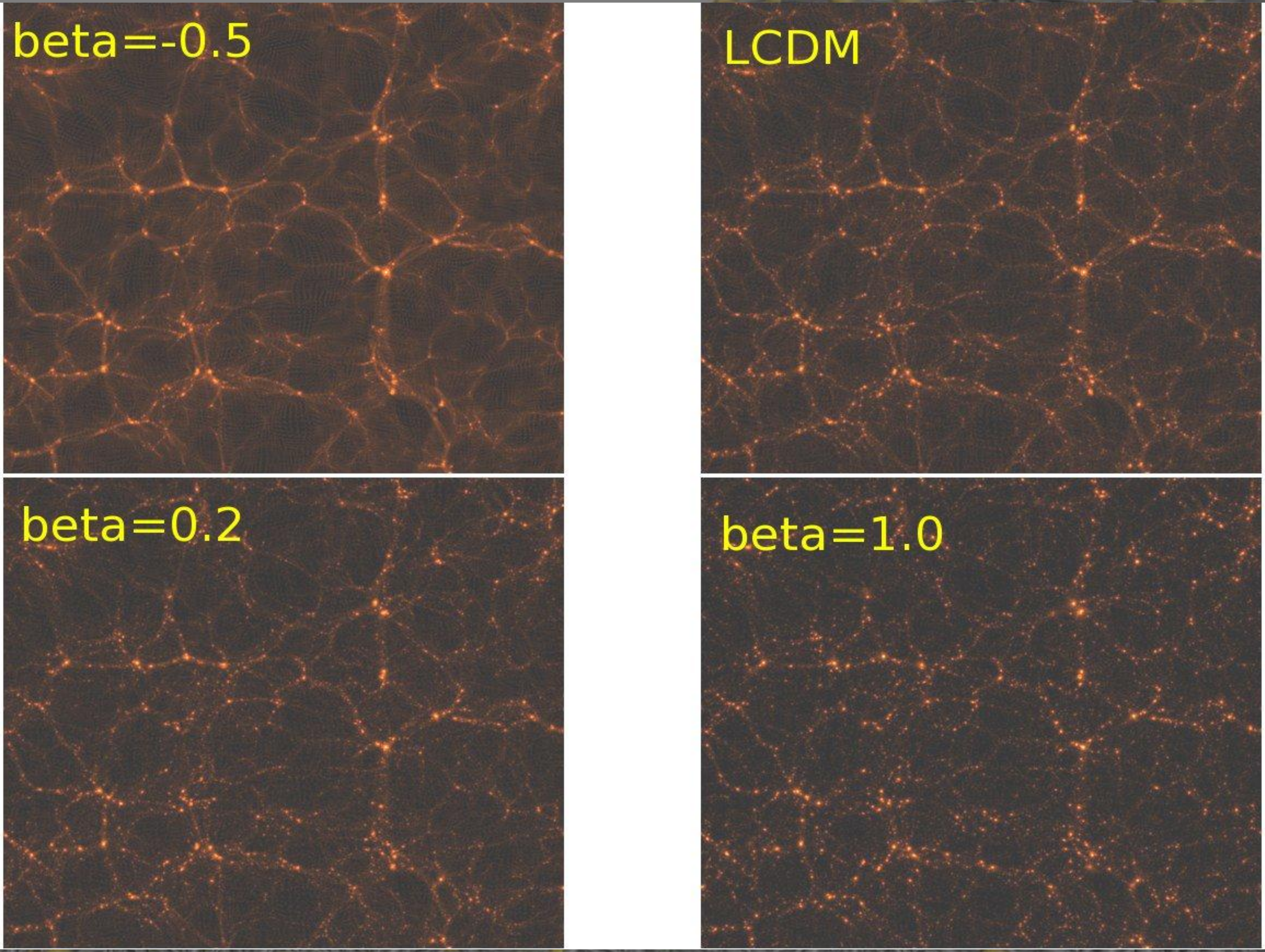
1. β - a dimensionless measure of the strength of scalar interactions compared to usual gravity,
2. r_s - the dynamical screening length expressed in Mpc/h and *constant* in comoving frame.

Major properties of the ReBEL

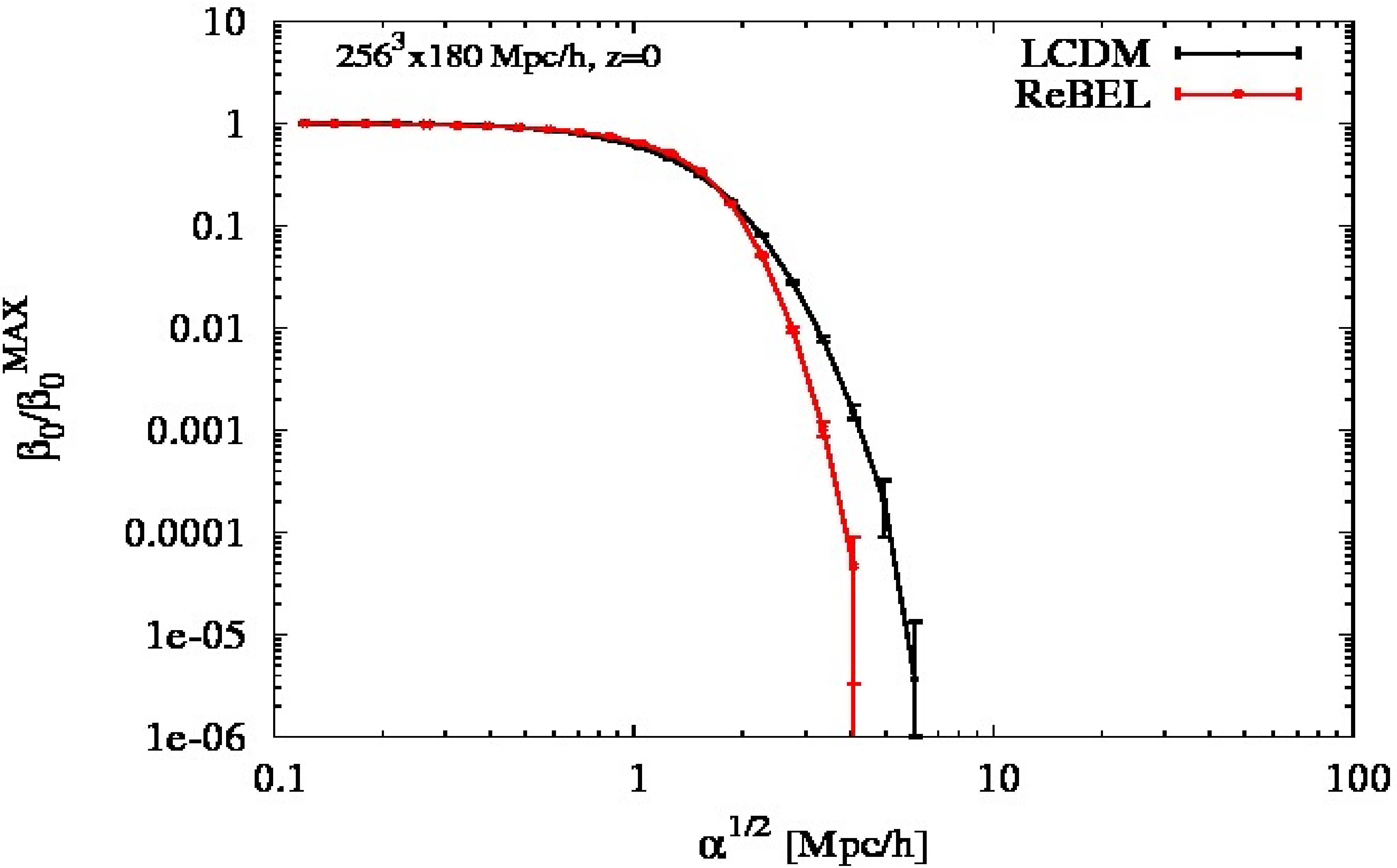
- Earlier non-linear structure formation (easier to accommodate early re-ionization)
- significant part of violent relaxation and merging shifted to high-redshifts (easier to "produce" abundant population of think-disk spirals)
- Lower skewness of the density PDF – more "emptier" voids
- Lower amplitude of the CMF in the dwarf mass range (missing satellites? Missing voids dwarfs?)
- To some extent higher masses of super-clusters
- Lower barion fraction in galaxy groups
- Higher concentration of cluster haloes (but unfortunately

Nusser, Gubser & Peebles 2005, Hellwing & Juszkiewicz 2009, Keselman, Nusser & Peebles 2009, Hellwing 2010, Hellwing, Knollmann & Knebe 2010, Hellwing, Juszkiewicz & van de Weygaert 2010

THE ReBEL MODEL



THE ReBEL MODEL
the alpha-shape topology



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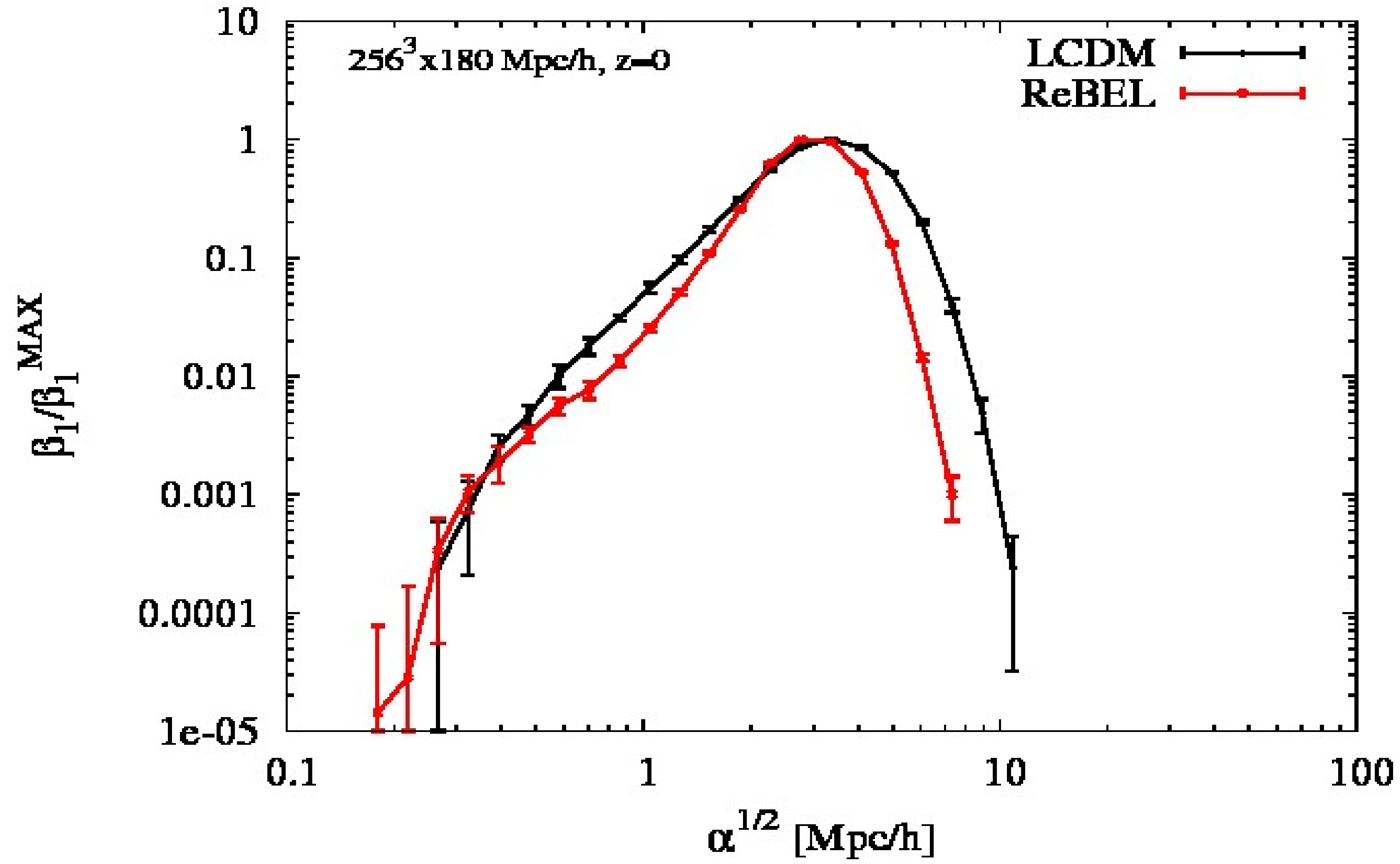

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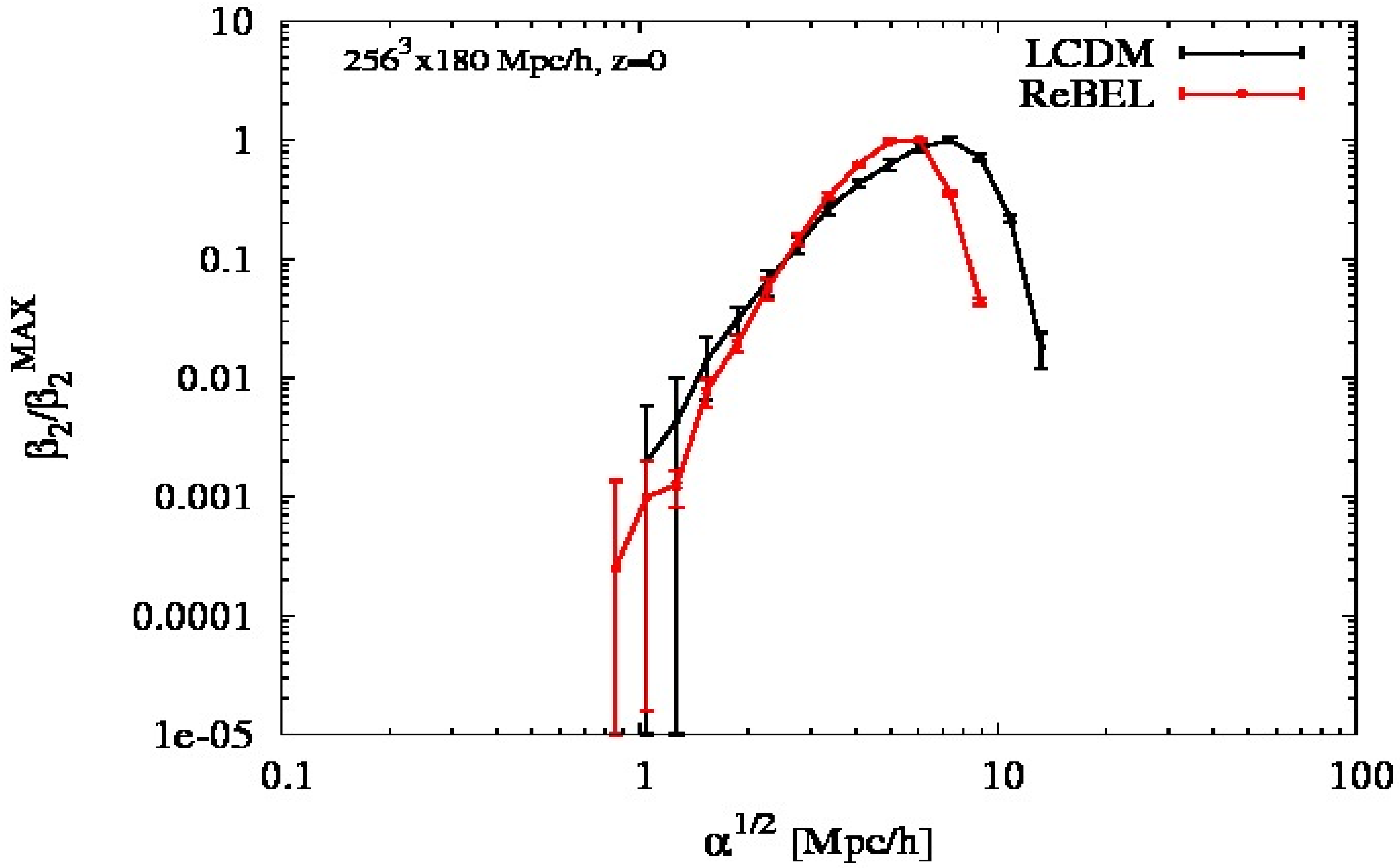

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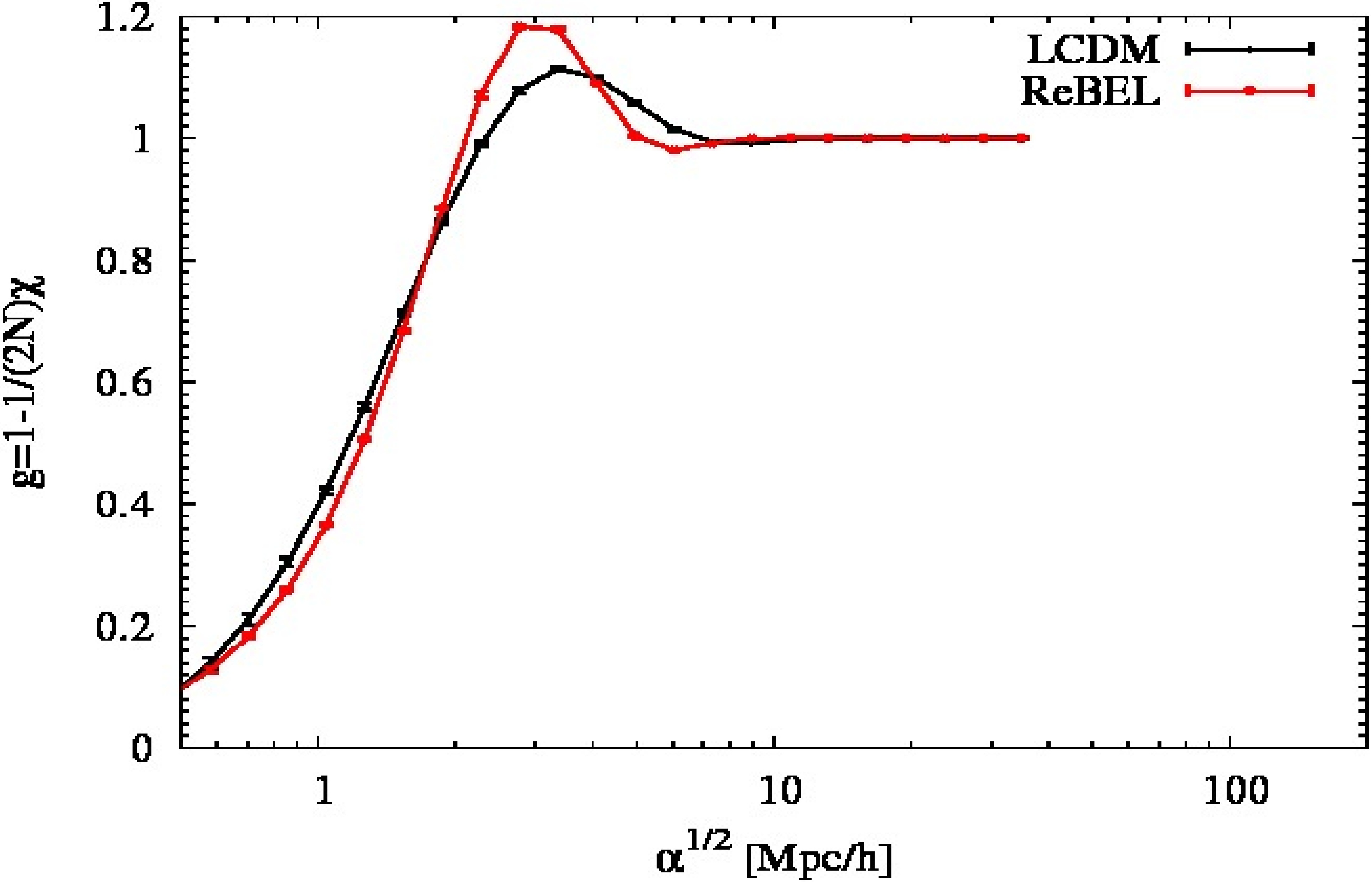


CONCLUSIONS

- The topology defined by LSS of the matter distribution is a results of underlying clustering mechanism (gravity & cosmology) and initial conditions (post-inflationary Gaussian density perturbation)
- The alpha-shape computational topology approach is a potentially new window open towards study of the LSS topology
- The Betti numbers and their sensitivity to changing topology show potential to become a new unbiased cosmological probes
- This approach can not only help obtain new measurements of cosmological parameters bu can also potentially discriminate between modified cosmologies like Dynamical DE, ReBEL, non-gaussianity, Coupled DM-DE, etc

ADVERTISEMENT

Be sure to check nice posters by my colleagues from RuG!

Burcu Beygu – “The Void Galaxy Sur
vey”

Marius Cautun – “Topology of the Ha
loes in the MMF Web”

Johan Hidding – “Adhesion and the G
eometric Evolution of the Cosmic We
b”

Introduction to alpha-shapes

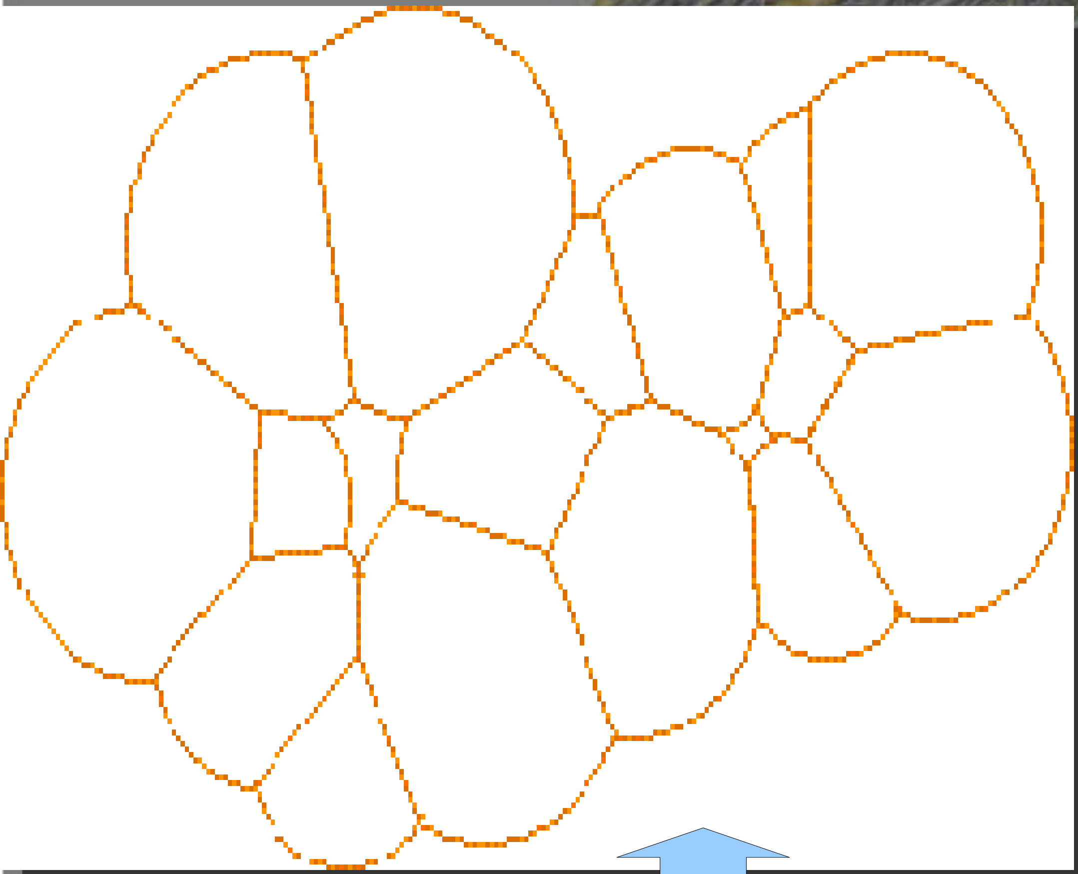
Alpha (α)Shapes:
Complex & shape

A point p with weight w(p) is in
terpreted as the sphere with c
enter p and square radius w(p).
We denote the corresponding b
all by b(p); it consists of all poi
nts on and inside the sphere. A
ll points with negative weight c
orrespond to empty balls.

In 2D example we get a union of a set
of disks in the plane.

Introduction to alpha-shapes

Alpha (α)Shapes: Complex & shape

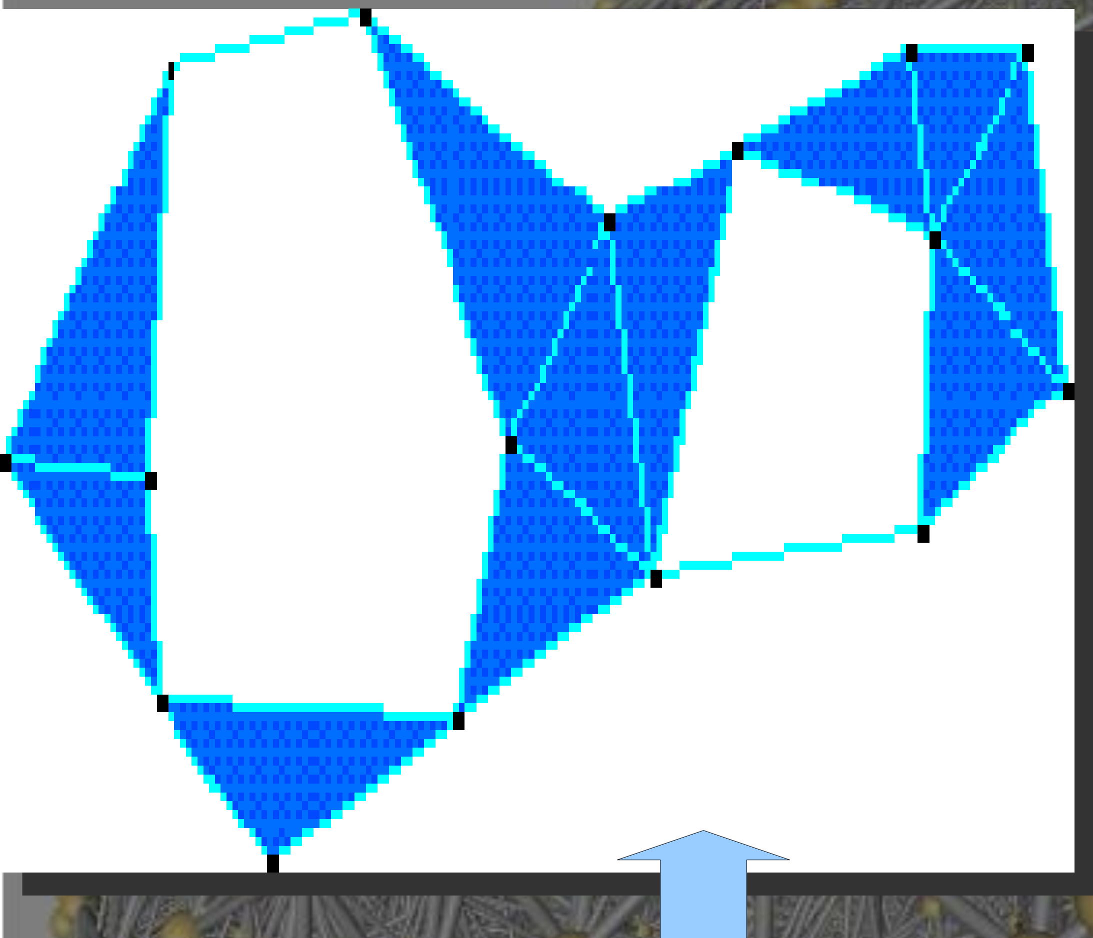


The decomposition of the union of disks defined by their Voronoi cells.

The shape of a finite set of weighted points is defined in terms of a decomposition of the union of corresponding balls into convex sets. This decomposition is defined by the (weighted) Voronoi cells of the balls. Specifically, the Voronoi cell v(p) of b(p) is the set of all points whose power from b(p) is no larger than from any other ball, where the power is the square distance from the center of b(p) minus the square radius of b(p).

Introduction to alpha-shapes

Alpha (α)Shapes: Complex & shape

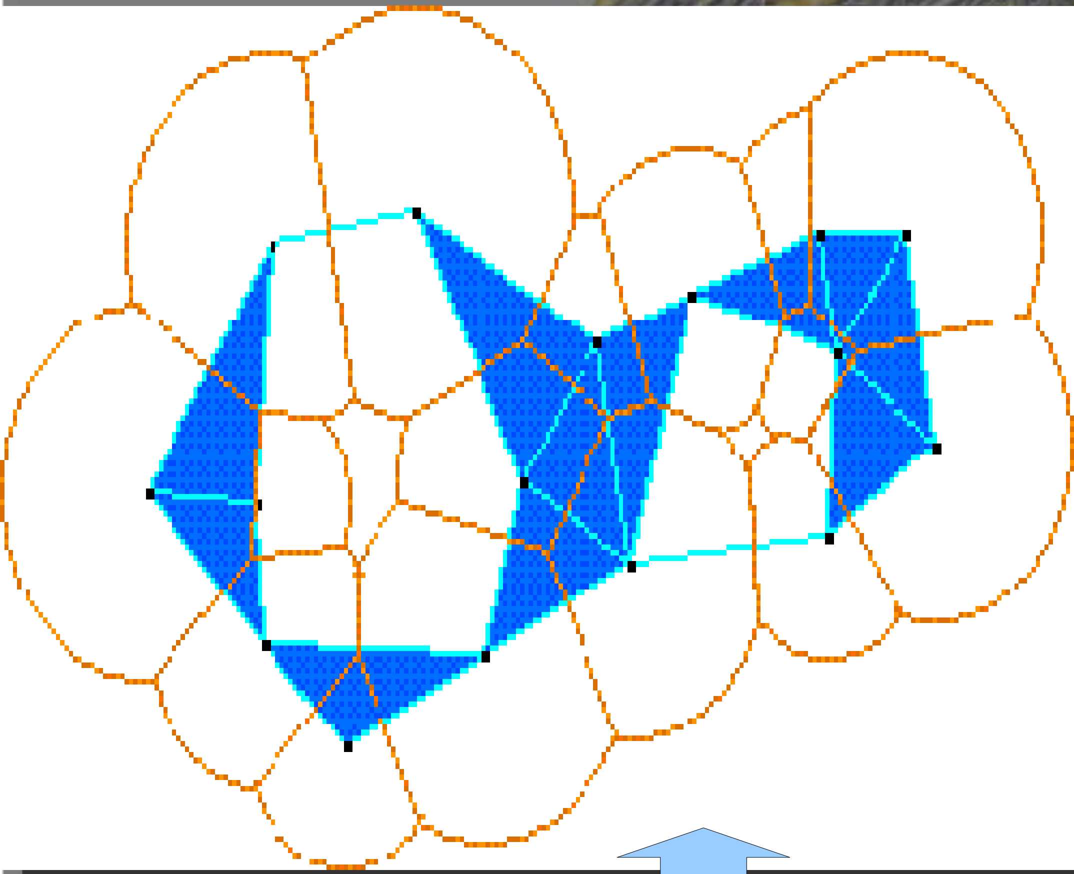


The dual complex (alpha-shape) of the disk union.

Assuming general position of the points or balls, the largest set in the nerve is of size d+ 1, i.e., a triple in the plane and a quadruple in three dimensions. Under this assumption, the nerve has a natural geometric realization by mapping each cell b(p) on V(p) in C to the point p. A set in the nerve maps to the convex hull of the points that correspond to the elements in the set. This realization is a (geometric) simplicial complex.

Introduction to alpha-shapes

Alpha (α)Shapes: Complex & shape



The decomposition of the union of disk
s and its dual complex overlapped.

Among the most useful properties of dual complex are the homotopy equivalence between its underlyin
g space and the union of correspo
nding balls, as well as that this un
ion can be expressed as the alter
nating sum of common ball interse
ctions, with one term per simplex in the dual complex. This implies short inclusion-exclusion formula
s for the d-dimensional volume an
d other measures of the union of balls.