Cosmological Nonlinear Perturbations

J. Hwang, H. Noh, D. Jeong, J. Gong

The 4th KIAS workshop on

Cosmology and Structure Formation

Nov. 3 - 7, 2010 KIAS, SEOUL

Newton's gravity:

- Non-relativistic (no c)
  - Action at a distance, violates causality
  - $c \to \infty$ limit of Einstein gravity ← Post Newtonian approximation
  - No horizon
  - Static nature
- No strong pressure allowed
- No strong gravity allowed
- No gravitational waves
- Incomplete and inconsistent

Einstein's gravity:

- Relativistic gravity
- Strong gravity, dynamic
- Simplest

★ **The two theories give the same descriptions for the cosmological world model and its linear structures.**

World model: spatially homogeneous and isotropic world model

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\mu - \frac{\text{const.}}{a^2} + \frac{\Lambda}{3}, \quad \mu \propto a^{-3}.$$

- **Relativistic (Friedmann 1922)** [5]
- **Newtonian (Milne-McCrea 1933)** [6]

Structures: linear perturbations

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = 0.$$

- **Relativistic (Lifshitz 1946)** [7]
- **Newtonian (Bonnor 1957)** [8]

"It is curious that it took so long for these dynamic models to be discovered after the (more complex) general relativity models were known."

G. F. R. Ellis (1989) [9]

★ **In fact, the known "Newtonian cosmology" is a GR guided version!**

---

[5] Friedmann A. A., 1922, Zeitschrift für Physik, **10**, 377; translated in Bernstein J., Feinberg G., eds, 1986, Cosmological-constants: papers in modern cosmology, Columbia Univ. Press, New York, p. 49

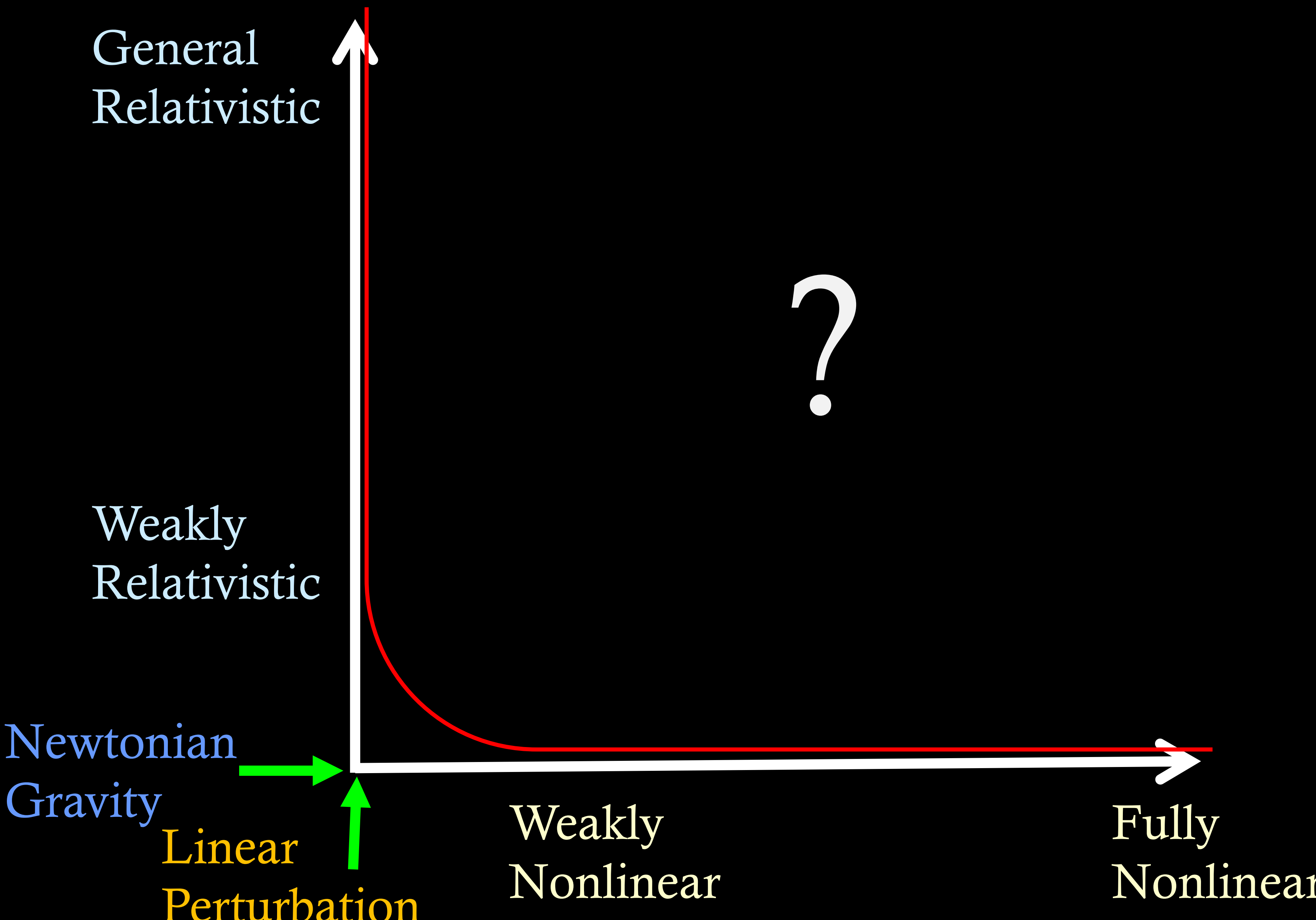
[6] Milne E. A., 1934, Quart. J. Math., **5**, 64; McCrea W. H., Milne E. A., 1934, Quart. J. Math., **5**, 73

[7] Lifshitz E. M., 1946, J. Phys. (USSR), **10**, 116

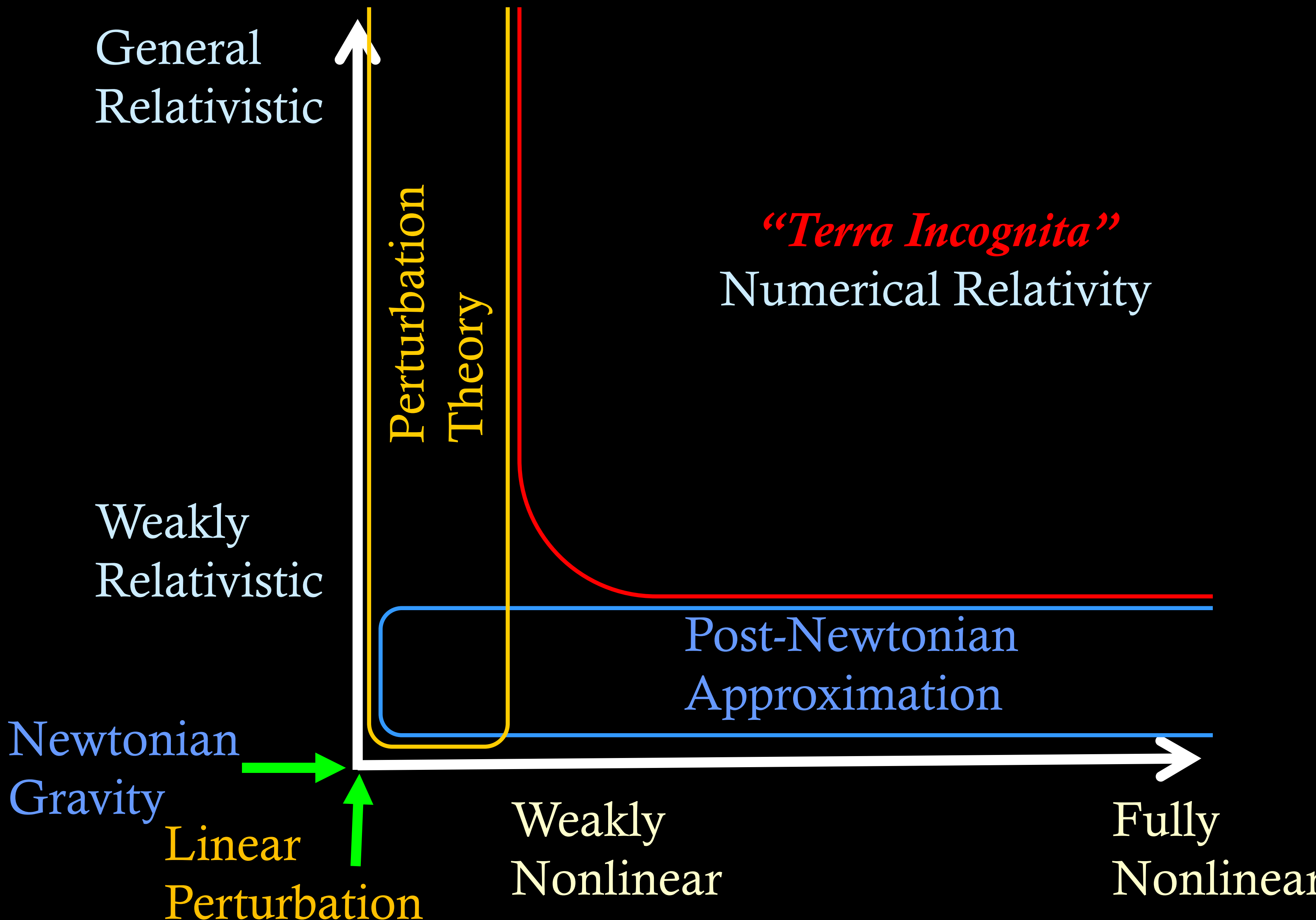
[8] Bonnor W. B., 1957, MNRAS, **117**, 104

[9] Ellis, G. F. R., 1989, in Einstein and the history of general relativity, ed. D. Howard and J. Stachel (Berlin, Birkhäuser), 367

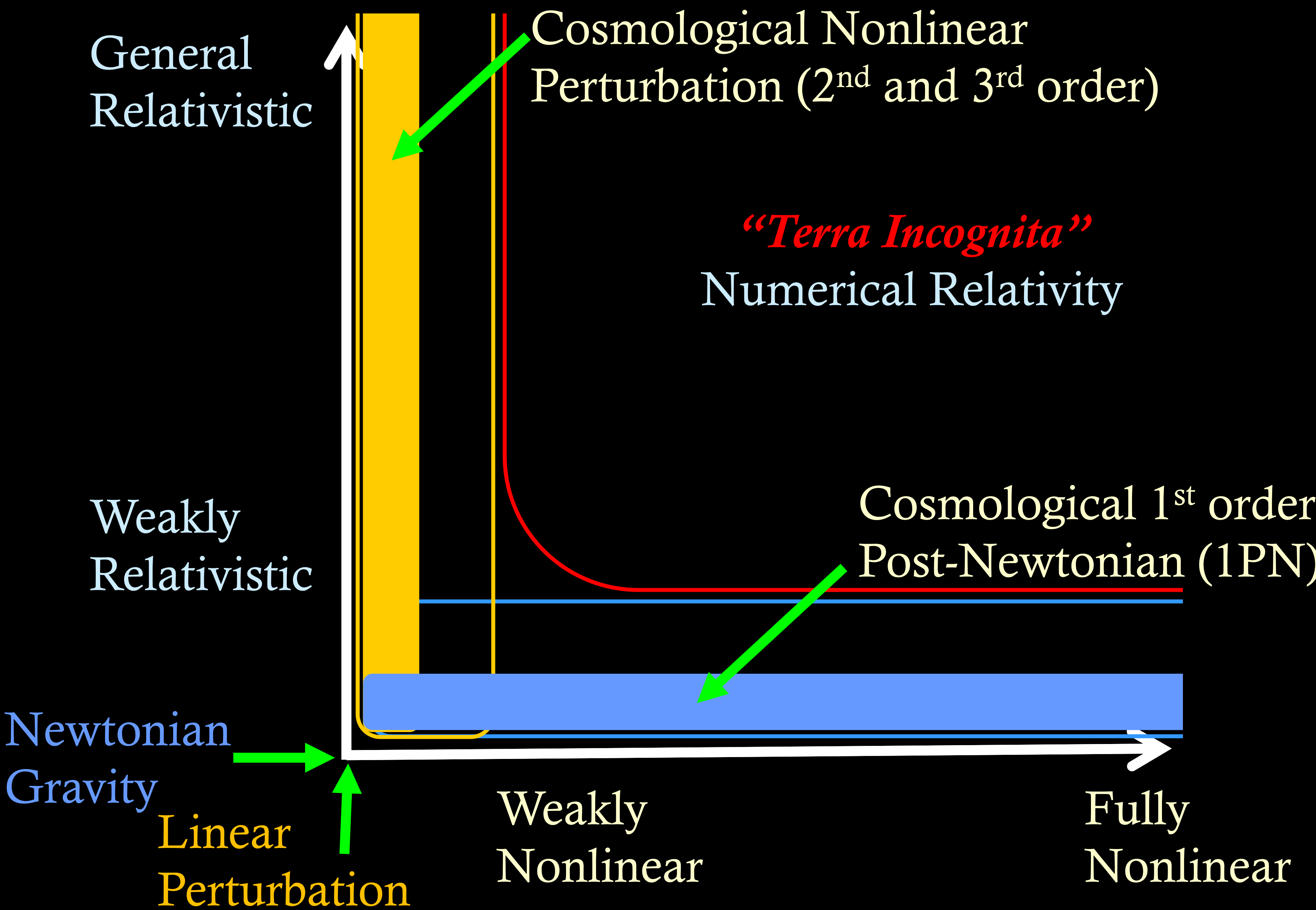
Studies of Large-scale Structure



Perturbation Theory *vs.* Post-Newtonian



Cosmology and Large-Scale Structure

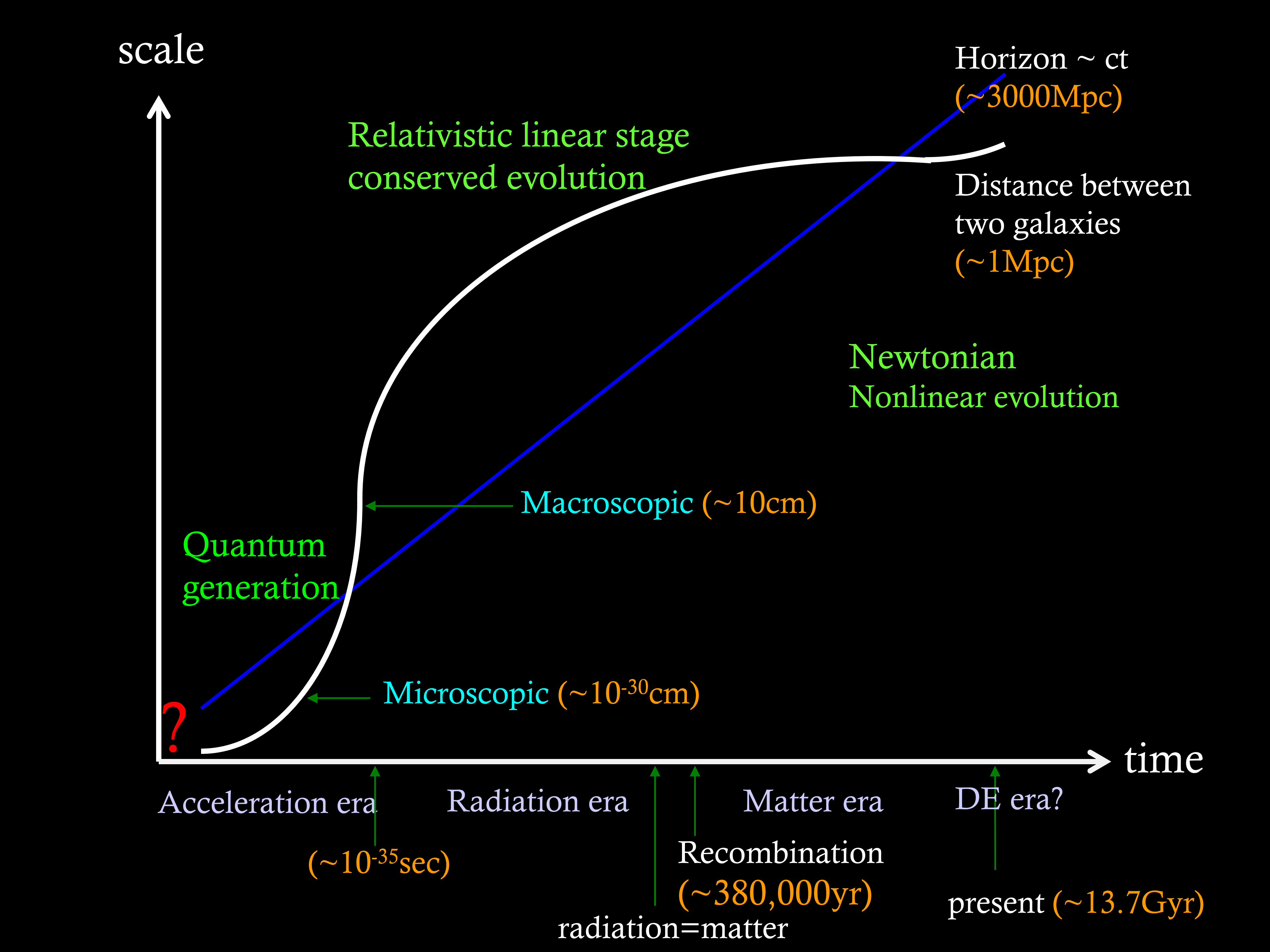


Perturbation method:

- Perturbation expansion.
- All perturbation variables are small.
- Weakly nonlinear.
- Strong gravity; fully relativistic!
- Valid in all scales!

Post-Newtonian method:

- Abandon geometric spirit of GR: recover the good old absolute space and absolute time.
- Provide GR correction terms in the Newtonian equations of motion.
- Expansion in $\frac{\delta\Phi}{c^2} \sim \frac{GM}{Rc^2} \sim \frac{v^2}{c^2} << 1$
- Fully nonlinear!
- No strong gravity situation; weakly relativistic.
- Valid far inside horizon


scale
Horizon ~ ct
(~3000Mpc)
Relativistic linear stage
conserved evolution
Distance between
two galaxies
(~1Mpc)
Newtonian
Nonlinear evolution
Macroscopic (~10cm)
Quantum
generation
?
Microscopic (~10^{-30}cm)
time
Acceleration era
Radiation era
Matter era
DE era?
(~10^{-35}sec)
Recombination
(~380,000yr)
radiation=matter
present (~13.7Gyr)

Origin and evolution of LSS

- Quantum origin
  - Space-time quantum fluctuations from uncertainty pr.
  - Become macroscopic due to inflation.
  - Nonlinear effect? NL perturbations
- Linear evolution (Relativistic)
  - Linear evolution of the macroscopic seeds.
  - Structures are described by conserved amplitudes.
- Nonlinear evolution (Newtonian)
  - Nonlinear evolution inside the horizon.
  - Newtonian numerical computer simulation.
  - Relativistic effect?

    NL perturbations, PN approximation

Perturbation Theory

3.1 Second-order: Relativistic-Newtonian correspondence

Newtonian:

Mass conservation, momentum conservation, Poisson's equation: [11]

$$\dot{\delta} + \frac{1}{a}\nabla \cdot \mathbf{u} = -\frac{1}{a}\nabla \cdot (\delta \mathbf{u}), \tag{17}$$

$$\dot{\mathbf{u}} + \frac{\dot{a}}{a}\mathbf{u} + \frac{1}{a}\nabla \delta\Phi = -\frac{1}{a}\mathbf{u}\cdot\nabla\mathbf{u}, \tag{18}$$

$$\frac{1}{a^2}\nabla^2 \delta\Phi = 4\pi G \delta\varrho, \tag{19}$$

give

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\varrho\delta = -\frac{1}{a^2}\left[a\nabla\cdot(\delta\mathbf{u})\right]^{\cdot} + \frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}). \tag{20}$$

★ These equations are valid to **fully nonlinear order**!

Relativistic: (irrotational, $K = 0$, but for general Λ) Noh, JH, PRD (2004)

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = -\frac{1}{a^2}\left[a\nabla\cdot(\delta\mathbf{u})\right]^{\cdot} + \frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}) + \dot{C}^{(t)}_{\alpha\beta}\left(\frac{2}{a}\nabla^\alpha u^\beta + \dot{C}^{(t)\alpha\beta}\right). \tag{21}$$

★ This equation is valid only to the **second-order**!

[11] Peebles, P. J. E., *The large-scale structure of the universe* (1980)

A proof

Fully nonlinear covariant equations:

The energy conservation, Raychaudhury equation become:

$$\tilde{\dot{\tilde{\mu}}} + \tilde{\mu}\tilde{\theta} = 0, \tag{22}$$

$$\tilde{\dot{\tilde{\theta}}} + \frac{1}{3}\tilde{\theta}^2 + \tilde{\sigma}^{ab}\tilde{\sigma}_{ab} + 4\pi G\tilde{\mu} - \Lambda = 0, \tag{23}$$

where $\tilde{\dot{\tilde{\mu}}} \equiv \tilde{\mu}_{,a}\tilde{u}^a$, $\tilde{\theta} \equiv \tilde{u}^a{}_{;a}$, etc. By combining

$$\left(\frac{\tilde{\dot{\tilde{\mu}}}}{\tilde{\mu}}\right)^{\tilde{\cdot}} - \frac{1}{3}\left(\frac{\tilde{\dot{\tilde{\mu}}}}{\tilde{\mu}}\right)^2 - \tilde{\sigma}^{ab}\tilde{\sigma}_{ab} - 4\pi G\tilde{\mu} + \Lambda = 0. \tag{24}$$

To the second-order perturbation:

By **identifying**

$$\boxed{\delta\mu_v \equiv \delta\varrho, \quad \delta\theta_v \equiv \frac{1}{a}\nabla\cdot\mathbf{u},} \tag{25}$$

(22,23) give

temporal comoving (v=0) gauge

$$\dot{\delta} + \frac{1}{a}\nabla\cdot\mathbf{u} = -\frac{1}{a}\nabla\cdot(\delta\mathbf{u}), \tag{26}$$

$$\frac{1}{a}\nabla\cdot\left(\dot{\mathbf{u}} + \frac{\dot{a}}{a}\mathbf{u}\right) + 4\pi G\mu\delta = -\frac{1}{a^2}\nabla(\mathbf{u}\cdot\nabla\mathbf{u}) - \dot{C}^{(t)\alpha\beta}\left(\frac{2}{a^2}u_{\alpha,\beta} + \dot{C}^{(t)}_{\alpha\beta}\right). \tag{27}$$

Combining (26,27) or (24) give (21).

Assumptions:

Our relativistic/Newtonian correspondence includes Λ, but assumes:

1. Flat Friedmann background
2. Zero-pressure
3. Irrotational
4. Single component fluid
5. No gravitational waves
6. Second order in perturbations

Relaxing any of these assumptions could lead to pure general relativistic effects!

Einstein’s gravity corrections to Newtonian cosmology:

1. Relativistic/Newtonian correspondence for a zero-pressure, irrotational fluid in flat background without gravitational waves.
2. Gravitational waves → Corrections
3. Background curvature → Corrections
4. Pressure → Relativistic even to the background and linear order
5. Rotation → Corrections
   → Newtonian correspondence in the small-scale limit
6. Multi-component zero-pressure irrotational fluids
   → Newtonian correspondence
7. Third-order perturbations → Corrections ⟵
   → Small, independent of horizon
8. Multi-component, third-order perturbations → Corrections
   → Small, independent of horizon

Physical Review D, **76**, 103527 (2007)

Linear order: Lifshitz (1946)/Bonnor(1957) **(comoving-synchronous gauge)**

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = 0,$$

Second order: Peebles (1980)/Noh-JH (2004) **(K=0, comoving gauge)**

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = -\frac{1}{a^2}\frac{\partial}{\partial t}\left[a\nabla\cdot(\delta\mathbf{u})\right] + \frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u}),$$

Third order: JH-Noh (2005)

Curvature perturbation in the comoving gauge

$\sim 10^{-5}$

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = -\frac{1}{a^2}\frac{\partial}{\partial t}\left[a\nabla\cdot(\delta\mathbf{u})\right] + \frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u})$$

$$+ \frac{1}{a^2}\frac{\partial}{\partial t}\{a[2\varphi\mathbf{u} - \nabla(\Delta^{-1}X)]\cdot\nabla\delta\} - \frac{4}{a^2}\nabla\cdot\left[\varphi\left(\mathbf{u}\cdot\nabla\mathbf{u} - \frac{1}{3}\mathbf{u}\nabla\cdot\mathbf{u}\right)\right]$$

$$+ \frac{2}{3a^2}\varphi\mathbf{u}\cdot\nabla(\nabla\cdot\mathbf{u}) + \frac{\Delta}{a^2}[\mathbf{u}\cdot\nabla(\Delta^{-1}X)] - \frac{1}{a^2}\mathbf{u}\cdot\nabla X - \frac{2}{3a^2}X\nabla\cdot\mathbf{u},$$

$$X \equiv 2\varphi\nabla\cdot\mathbf{u} - \mathbf{u}\cdot\nabla\varphi + \frac{3}{2}\Delta^{-1}\nabla\cdot[\mathbf{u}\cdot\nabla(\nabla\varphi) + \mathbf{u}\Delta\varphi].$$

Pure General Relativistic corrections

Physical Review D **72** 044012 (2005)

3.2 Third-order: Pure general relativistic corrections [13]

To the third order we identify:

$$\delta\mu_v \equiv \delta\varrho, \quad \delta\theta_v \equiv \frac{1}{a}\nabla\cdot\mathbf{u}.$$

For pure scalar-type perturbation (22,23) give:

pure Einstein corrections

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\mu\delta = -\frac{1}{a^2}\left[a\nabla\cdot(\delta\mathbf{u})\right]^{\cdot} + \frac{1}{a^2}\nabla\cdot(\mathbf{u}\cdot\nabla\mathbf{u})$$
$$+\frac{1}{a^2}\left\{a\left[2\varphi\mathbf{u} - \nabla\left(\Delta^{-1}X\right)\right]\cdot\nabla\delta\right\}^{\cdot} - \frac{4}{a^2}\nabla\cdot\left[\varphi\left(\mathbf{u}\cdot\nabla\mathbf{u} - \frac{1}{3}\mathbf{u}\nabla\cdot\mathbf{u}\right)\right]$$
$$+\frac{2}{3a^2}\varphi\mathbf{u}\cdot\nabla\left(\nabla\cdot\mathbf{u}\right) + \frac{\Delta}{a^2}\left[\mathbf{u}\cdot\nabla\left(\Delta^{-1}X\right)\right] - \frac{1}{a^2}\mathbf{u}\cdot\nabla X - \frac{2}{3a^2}X\nabla\cdot\mathbf{u},$$
$$X \equiv 2\varphi_v\nabla\cdot\mathbf{u} - \mathbf{u}\cdot\nabla\varphi_v + \frac{3}{2}\Delta^{-1}\nabla\cdot\left[\mathbf{u}\cdot\nabla\left(\nabla\varphi_v\right) + \mathbf{u}\Delta\varphi_v\right].$$

The first non-vanishing pure relativistic correction terms are φ_v order higher than t[he New]tonian terms ($\varphi_v = \varphi$ in the comoving gauge). We have for general Λ [14]

$$\boxed{\dot{\varphi}_v = 0.}$$

The CMB temperature anisotropy gives, for $\Lambda = 0$, near horizon scale [15]

$$\boxed{\frac{\delta T}{T} \sim \frac{1}{3}\delta\Phi \sim \frac{1}{5}\varphi_v \sim 10^{-5}}$$

[13] Phys. Rev. D, **72**, 044012 (2005).
[14] Gen. Rel. Grav. **31**, 1131 (1999).
[15] Phys. Rev. D **59**, 067302 (1999).

Mon. Not. R. Astron. Soc. **367,** 1515–1520 (2006) doi:10.1111/j.1365-2966.2006.10067.x

Why Newtonian gravity is reliable in large-scale cosmological simulations

Jai-chan Hwang[1] and Hyerim Noh[2★]

[1] *Department of Astronomy and Atmospheric Sciences, Kyungpook National University, Taegu, Korea*
[2] *Korean Astronomy and Space Science Institute, Taejon, Korea*

1. Relativistic/Newtonian correspondence to the second order
2. Pure general relativistic third-order corrections are small $\sim 5\text{x}10^{-5}$
3. Correction terms are independent of presence of the horizon.

Second-order power spectrum

$$\delta \equiv \delta_1 + \delta_2 + \delta_3 + \cdots$$

$$P \equiv \langle |\delta|^2 \rangle = \langle |\delta_1|^2 \rangle + \langle |\delta_2|^2 \rangle + 2\langle \mathrm{Re}(\delta_1^* \delta_3) \rangle + \cdots$$

$$\equiv P_{11} + P_{22} + P_{13} + \cdots$$

General Relativistic contribution!

$$\delta_3 = \delta_{3,Newton} + \delta_{3,Einstein}$$

$$P_{13} = P_{13,Newton} + P_{13,Einstein}$$

Pure Einstein

Relativistic/Newtonian

Physical Review D, **77**,123533 (2008)

Infrared Divergence of Pure Einstein Gravity Contributions to the Cosmological Density Power Spectrum

Hyerim Noh,[1,*] Donghui Jeong,[2,3,†] and Jai-chan Hwang[4,‡]

[1]*Korea Astronomy and Space Science Institute, Daejon, Korea*

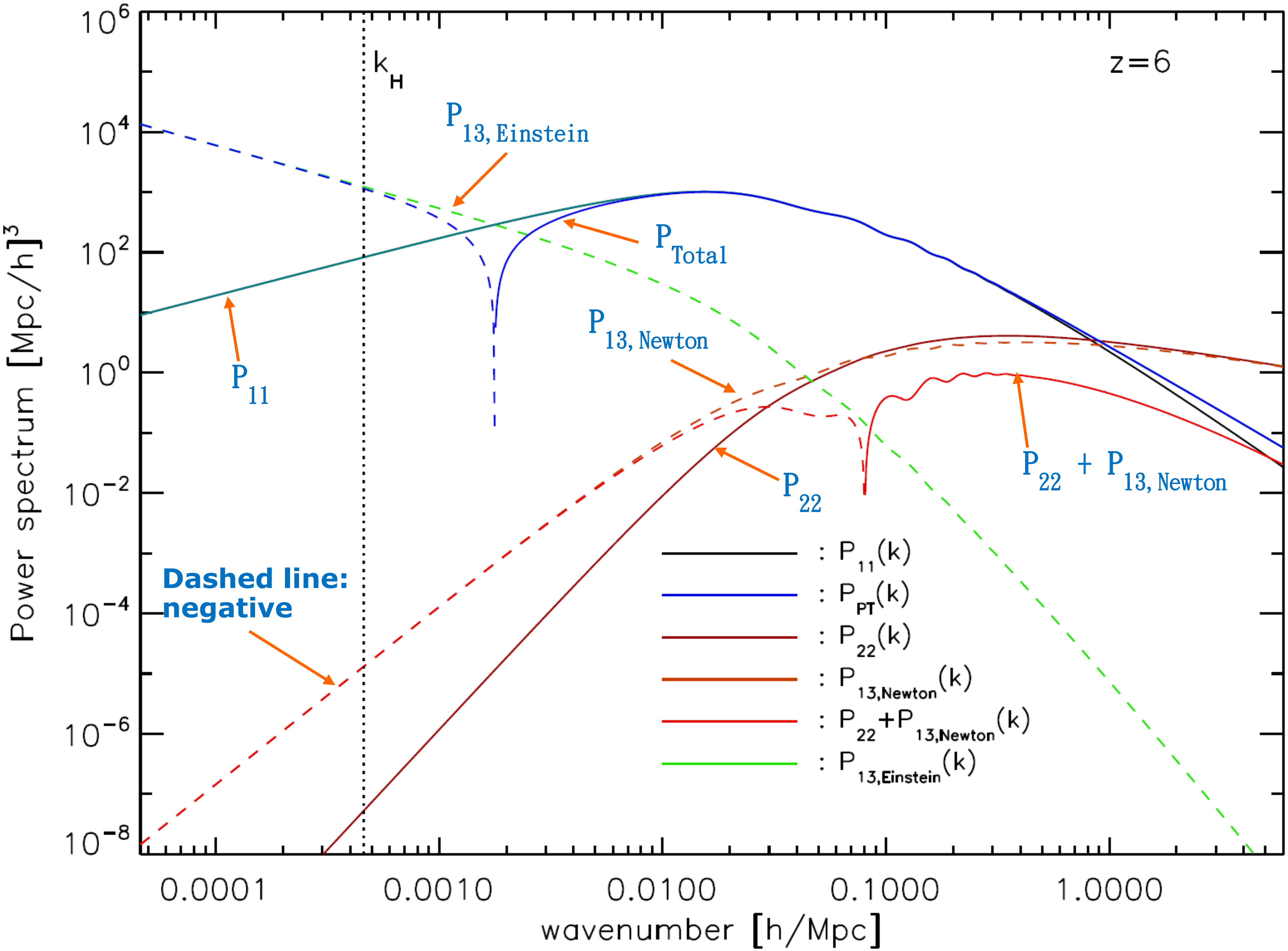
[2]*Department of Astronomy, University of Texas at Austin, University Station, C1400, Austin, Texas 78712, USA*

[3]*Texas Cosmology Center, University of Texas at Austin, University Station, C1400, Austin, Texas 78712, USA*

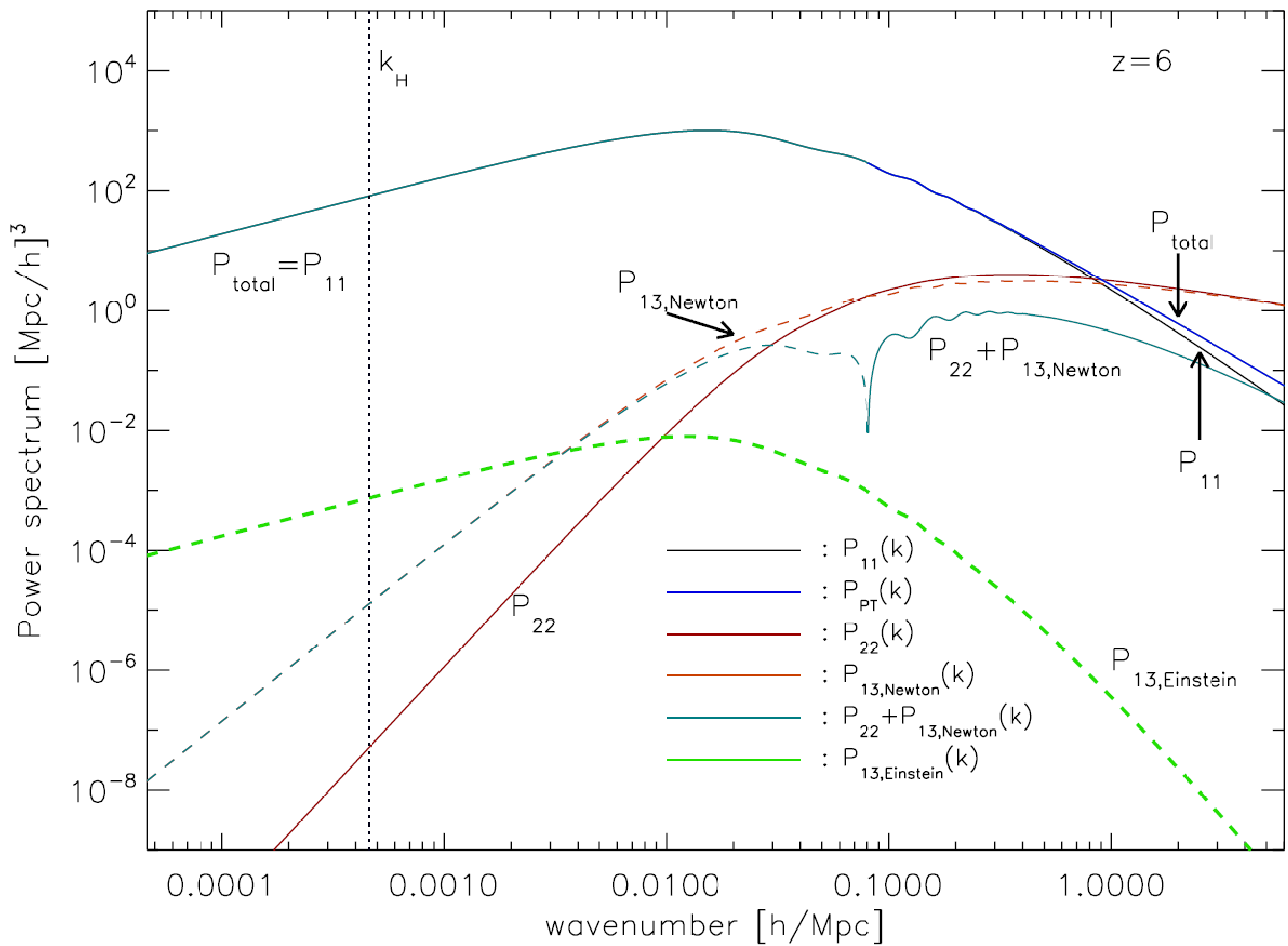
[4]*Department of Astronomy and Atmospheric Sciences, Kyungpook National University, Taegu, Korea*

(Received 16 March 2009; published 10 July 2009)

We probe the pure Einstein gravity contributions to the second-order density power spectrum. On the small scale, we discover that Einstein's gravity contribution is negligibly small. This guarantees that Newton's gravity is currently sufficient to handle the baryon acoustic oscillation scale. On the large scale, however, we discover that Einstein's gavity contribution to the second-order power spectrum dominates the linear-order power spectrum. Thus, the pure Einstein gravity contribution appearing in the third-order perturbation leads to an infrared divergence in the power spectrum.



Phys. Rev. Lett. 103, 121301, arXiv:0902.4285



D. Jeong, J. Gong, H. Noh, JH, arXiv:1010.3489

Power spectrum

$$\delta \equiv \delta_1 + \delta_2 + \delta_3 + \cdots$$

$$\langle \delta(\mathbf{k}) \delta(\mathbf{q}) \rangle \equiv (2\pi)^3 \delta^D(\mathbf{k}+\mathbf{q}) P(\mathbf{k})$$

$$P \equiv P_{11} + P_{22} + P_{13} + \cdots$$

General Relativistic contribution!

$$\delta_3 = \delta_{3,Newton} + \delta_{3,Einstein}$$

$$P_{13} = P_{13,Newton} + P_{13,Einstein}$$

Pure Einstein

Relativistic/Newtonian

D. Jeong, J. Gong, H. Noh, JH, arXiv:1010.3489

Minimally coupled scalar field

Non-linear corrections to inflationary power spectrum

Jinn-Ouk Gong*[1] Hyerim Noh†[2] and Jai-chan Hwang‡[3]

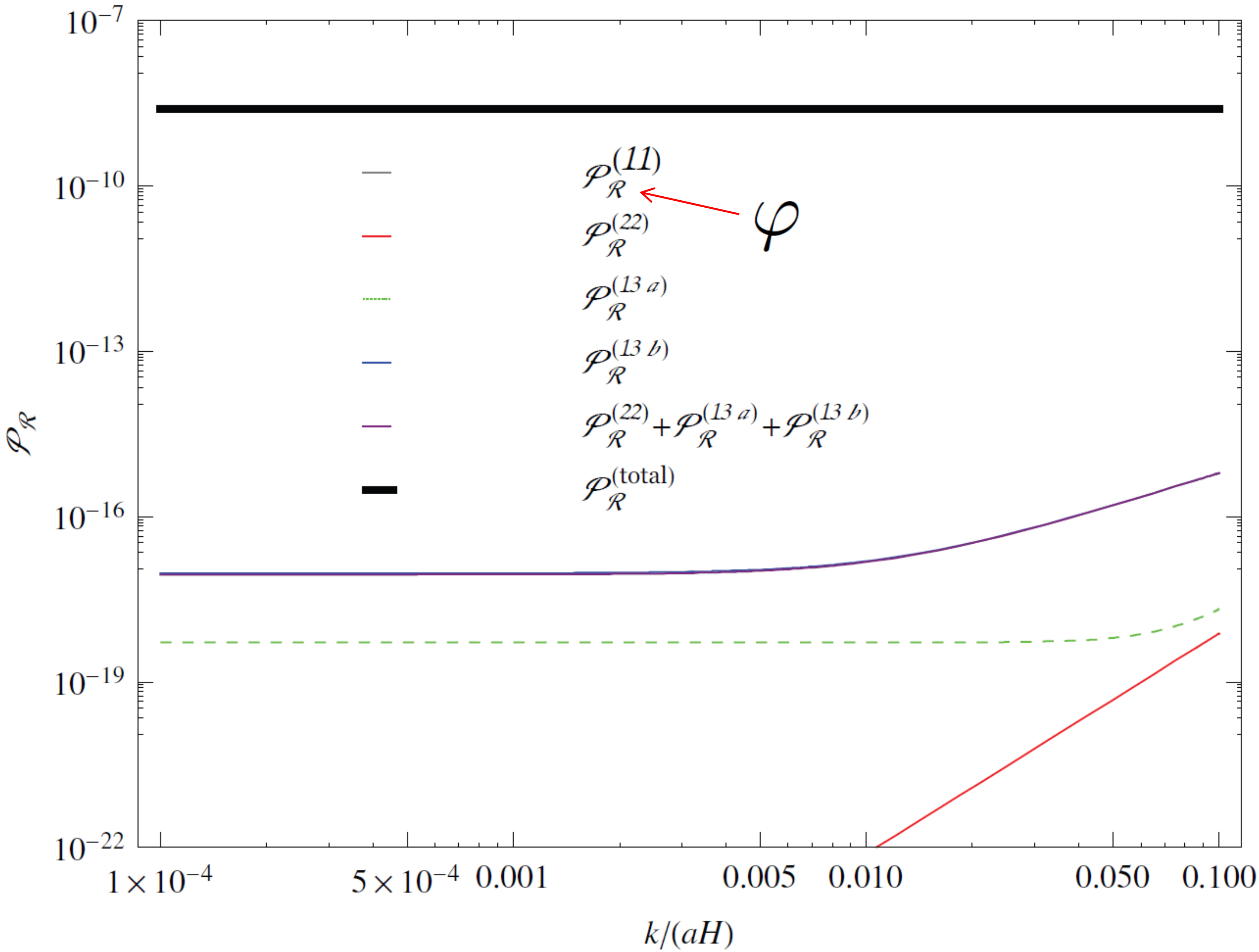
To third-order, assuming large-scale and slow-roll:

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{\Delta}{a^2}\varphi = \frac{1}{a^2}\Bigg\{ - \left[2\varphi\Delta\varphi + \frac{7}{4}\varphi^{,\alpha}\varphi_{,\alpha} + \frac{1}{2}\Delta^{-1}\nabla^{\alpha}(\varphi_{,\alpha}\Delta\varphi)\right]$$

$$+4\varphi^2\Delta\varphi + \frac{13}{2}\varphi\varphi^{,\alpha}\varphi_{,\alpha} - \varphi\Delta^{-1}\nabla^{\alpha}\left(\varphi_{,\alpha}\Delta\varphi\right) + \Delta^{-1}\nabla^{\alpha}\left(2\varphi\Delta\varphi\varphi_{,\alpha} + \varphi_{,\alpha}\varphi^{,\beta}\varphi_{,\beta}\right)$$

$$-\frac{1}{2}\Delta^{-1}\nabla^{\alpha}\left[\varphi_{,\alpha\beta}\left(\Delta^{-1}X_2\right)^{,\beta}\right] + \Delta\varphi\left(\Delta^{-1}X_2\right)_{,\alpha}\Big]\Bigg\},$$

$$X_2 \equiv -2\varphi\Delta\varphi - \frac{7}{4}\varphi^{,\alpha}\varphi_{,\alpha} + \frac{3}{2}\Delta^{-1}(\varphi^{,\alpha}\Delta\varphi)_{,\alpha}.$$



10^{-7}
10^{-10}
10^{-13}
10^{-16}
10^{-19}
10^{-22}
$\mathcal{P}_\mathcal{R}$
$\mathcal{P}_\mathcal{R}^{(11)}$
φ
$\mathcal{P}_\mathcal{R}^{(22)}$
$\mathcal{P}_\mathcal{R}^{(13\,a)}$
$\mathcal{P}_\mathcal{R}^{(13\,b)}$
$\mathcal{P}_\mathcal{R}^{(22)}+\mathcal{P}_\mathcal{R}^{(13\,a)}+\mathcal{P}_\mathcal{R}^{(13\,b)}$
$\mathcal{P}_\mathcal{R}^{(\mathrm{total})}$
1×10^{-4}
5×10^{-4}
0.001
0.005
0.010
0.050
0.100
$k/(aH)$

PN Approximation

4. Cosmological post-Newtonian Approach

Perturbation method:

- Perturbation expansion.
- All perturbation variables are small.
- Weakly nonlinear.
- Strong gravity; fully relativistic!
- Valid in all scales!

Post-Newtonian method:

- Abandon geometric spirit of GR: recover the good old absolute space and absolute time.
- Provide GR correction terms in the Newtonian equations of motion.
- Expansion in v/c:

$$\frac{GM}{\lambda c^2} \sim \left(\frac{v}{c}\right)^2 \ll 1.$$

- Fully nonlinear!
- No strong gravity situation; weakly relativistic.
- Valid far inside horizon $\frac{GM}{\lambda c^2} \sim \left(\frac{\lambda}{c/H}\right)^2 \ll 1.$

Complementary!

Metric:

Newtonian limit:

$$\tilde{g}_{00} = -\left(1 - \frac{1}{c^2}2U\right), \quad \tilde{g}_{0i} = 0, \quad \tilde{g}_{ij} = \delta_{ij}.$$

Newtonian gravitational potential

1PN metric [16]:

$$\tilde{g}_{00} = -\left[1 - \frac{1}{c^2}2U + \frac{1}{c^4}\left(2U^2 - 4\Phi\right)\right] + \mathcal{O}^{-6},$$

$$\tilde{g}_{0i} = -\frac{1}{c^3}P_i + \mathcal{O}^{-5},$$

Minkowski background

$$\tilde{g}_{ij} = \left(1 + \frac{1}{c^2}2U\right)\delta_{ij} + \mathcal{O}^{-4}.$$

Cosmological 1PN metric [17]:

$$\tilde{g}_{00} \equiv -\left[1 - \frac{1}{c^2}2U + \frac{1}{c^4}\left(2U^2 - 4\Phi\right)\right] + \mathcal{O}^{-6},$$

$$\tilde{g}_{0i} \equiv -\frac{1}{c^3}\underline{a}^2 P_i + \mathcal{O}^{-5},$$

Robertson-Walker background

$$\tilde{g}_{ij} \equiv \underline{a}^2\left(1 + \frac{1}{c^2}2\underline{V}\right)\underline{\gamma_{ij}} + \mathcal{O}^{-4}.$$

[16] Chandrasekhar, S., 1965, ApJ, **142**, 1488.
[17] Preprint, astro-ph/0507085.

JH, Noh, Puetzfeld, JCAP03 (2008) 010

Energy-momentum tensor:

Covariant decomposition:

$$\tilde{T}_{ab} = \tilde{\varrho} c^2 \left(1 + \frac{1}{c^2} \tilde{\Pi} \right) \tilde{u}_a \tilde{u}_b + \tilde{p} \left(\tilde{u}_a \tilde{u}_b + \tilde{g}_{ab} \right) + 2 \tilde{q}_{(a} \tilde{u}_{b)} + \tilde{\pi}_{ab},$$

where $\tilde{q}_a \tilde{u}^a \equiv 0$, $\tilde{\pi}_{ab} \tilde{u}^b \equiv 0$, $\tilde{\pi}^c_c \equiv 0$, and $\tilde{\pi}_{ab} \equiv \tilde{\pi}_{ba}$.
Fluid four vector, $\tilde{u}_a$, follows from $\tilde{u}^a \tilde{u}_a \equiv -1$ and $\tilde{u}^i \equiv \frac{v^i}{c} \tilde{u}^0$.
We introduce

$$\tilde{\varrho} \equiv \varrho, \quad \tilde{\Pi} \equiv \Pi, \quad \tilde{p} \equiv p, \quad \tilde{q}_i \equiv \frac{1}{c} Q_i, \quad \tilde{\pi}_{ij} \equiv \Pi_{ij}.$$

Newtonian limit:

$$\frac{1}{a^3} \left(a^3 \varrho \right)^{\cdot} + \frac{1}{a} \nabla_i \left(\varrho v^i \right) = 0,$$
$$\frac{1}{a} \left(a v_i \right)^{\cdot} + \frac{1}{a} v^j \nabla_j v_i + \frac{1}{a \varrho} \left(\nabla_i p + \nabla_j \Pi^j_i \right) - \frac{1}{a} \nabla_i U = 0,$$
$$\left(\frac{\partial}{\partial t} + \frac{1}{a} \mathbf{v} \cdot \nabla \right) \Pi + \left(3 \frac{\dot{a}}{a} + \frac{1}{a} \nabla \cdot \mathbf{v} \right) \frac{p}{\varrho} + \frac{1}{\varrho a} \left(Q^i{}_{|i} + \Pi^i_j v^j{}_{|i} \right) = 0,$$
$$\frac{\Delta}{a^2} U + 4 \pi G \left(\varrho - \varrho_b \right) = 0.$$

Newtonian, indeed!

★ No gauge condition used!

★ We subtract the Friedmann background equation.

JH, Noh, Puetzfeld, JCAP03 (2008) 010

1PN equations:

For $K = 0$, we have $V = U$. In a gauge-ready form (assuming an ideal fluid):

$$\frac{1}{a^3}\left(a^3\varrho^*\right)^\cdot + \frac{1}{a}\left(\varrho^* v^i\right)_{|i} = 0,$$

1PN order

$$\frac{1}{a}\left(a v_i^*\right)^\cdot + \frac{1}{a} v^*_{i|j} v^j = -\frac{1}{a}\left(1 + \frac{1}{c^2} 2\underline{U}\right)\frac{p_{,i}}{\varrho^*}$$
$$+\frac{1}{a}\left[1 + \frac{1}{c^2}\left(\frac{3}{2}v^2 - U + \Pi + \frac{p}{\varrho}\right)\right] U_{,i} + \frac{1}{c^2}\frac{1}{a}\left(2\underline{\Phi}_{,i} - v^j \underline{P_j}_{|i}\right),$$

determined by Einstein's equation

where

$$\varrho^* \equiv \varrho\left[1 + \frac{1}{c^2}\left(\frac{1}{2}v^2 + 3U\right)\right], \quad v_i^* \equiv v_i + \frac{1}{c^2}\left[\left(\frac{1}{2}v^2 + 3U + \Pi + \frac{p}{\varrho}\right) v_i - P_i\right].$$

Metric variables (potentials) U, Φ and P_i are determined by

$$\frac{\Delta}{a^2}\underline{U} + 4\pi G\left(\varrho - \varrho_b\right) + \frac{1}{c^2}\left\{\frac{1}{a^2}\left[2\Delta\underline{\Phi} - 2U\Delta U + \left(a P^i{}_{|i}\right)^\cdot\right] + 3\ddot{U} + 9\frac{\dot{a}}{a}\dot{U} + 6\frac{\ddot{a}}{a}U\right.$$
$$\left. + 8\pi G\left[\varrho v^2 + \frac{1}{2}\left(\varrho\Pi - \varrho_b\Pi_b\right) + \frac{3}{2}\left(p - p_b\right)\right]\right\} = 0,$$

$$\frac{\Delta}{a^2}\underline{P_i} = -16\pi G\varrho v_i + \frac{1}{a}\left(\frac{1}{a}P^j{}_{|j} + 4\dot{U} + 4\frac{\dot{a}}{a}U\right)_{,i}.$$

★ We can impose a temporal gauge condition on $P^i{}_{|i}$.

★ 1PN correction terms are $\frac{GM}{Rc^2} \sim \frac{v^2}{c^2} \sim 10^{-5}$ order **smaller** than the Newtonian term

JH, Noh, Puetzfeld, JCAP03 (2008) 010

- PN corrections: $\dfrac{GM}{Rc^2} \sim \dfrac{\delta\Phi}{c^2} \sim \dfrac{v^2}{c^2} \sim 10^{-6} - 10^{-4}$
- Secular effects? Require numerical study.

- Newtonian: action at a distance (Laplacian)

  PN: propagation with speed of light (D'Alembertian)
- Propagation speed depends on the gauge choice!

  Propagation speed of the (electric and magnetic parts of) Weyl tensor is "c". $\Rightarrow$ Gravity's propagation speed is "c"!
- Exists electromagnetic analogy

  Propagation speed of potential depends on the gauge choice:

  Coulomb gauge *vs*. Lorentz gauge

  Propagation speed of the field is "c".

JH, Noh, Puetzfeld, JCAP03 (2008) 010

Conclusion

- **Perturbation method:** Fully relativistic but weakly nonlinear. Relativistic/Newtonian correspondence for zero-pressure to 2^{nd}.
  General relativistic corrections appear, otherwise.
  Pure relativistic third-order corrections: $$\varphi_v \sim \frac{\delta\Phi}{c^2} \sim 10^{-5}$$
- Pure GR correction in matter power spectrum: negligible
- Leading NL correction to inflation PS: negligible

- **PN approximation:** Fully nonlinear but weakly relativistic.
  PN corrections: $$\frac{GM}{Rc^2} \sim \frac{\delta\Phi}{c^2} \sim \frac{v^2}{c^2} \sim 10^{-6} - 10^{-4}$$
- Newtonian theory looks quite reliable in cosmological dynamics. (gravitational lensing requires PN effect!)
- Quantitative effects require numerical study.

Cosmology and Large-Scale Structure