Cost of quantum gates

- Logic and Physical Operation

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Quantum systems suggested as QC

Atomic and Molecular

Ion trap
Cavity QED
→ NMR
Molecular magnet
$N@C_{60}$ (fullerine)
BEC

Optical

Photon
Photonic crystal

Electron beam

el. floating on liquid He
el. trapped by SAW
el. trapped by magnetic field

Solid State

Quantum dot
Superconductor
Si-based QC

Introduction

- Classical computation
  : Algorithm-program-machine code -physical gates

- Quantum computation
  : Algorithm-unitary operation-gate operation-physical operations

Execution of quantum algorithm

(1) Algorithm - unitary operator U

(2) Decomposition of U : $U = U_1 U_2 U_3 \ldots$

where U_i is a gate.

(3) Realization of gates by physical operations

$$\exp(-iH_i t / \hbar)$$

where H_i is a part of a Hamiltonian.

Determining cost of gates

- Cost - # of physical operations required to implement a gate

- Motive – time is precious resource (due to decoherence).

Model quantum computer

- Hamiltonian – Zeeman & interaction terms.

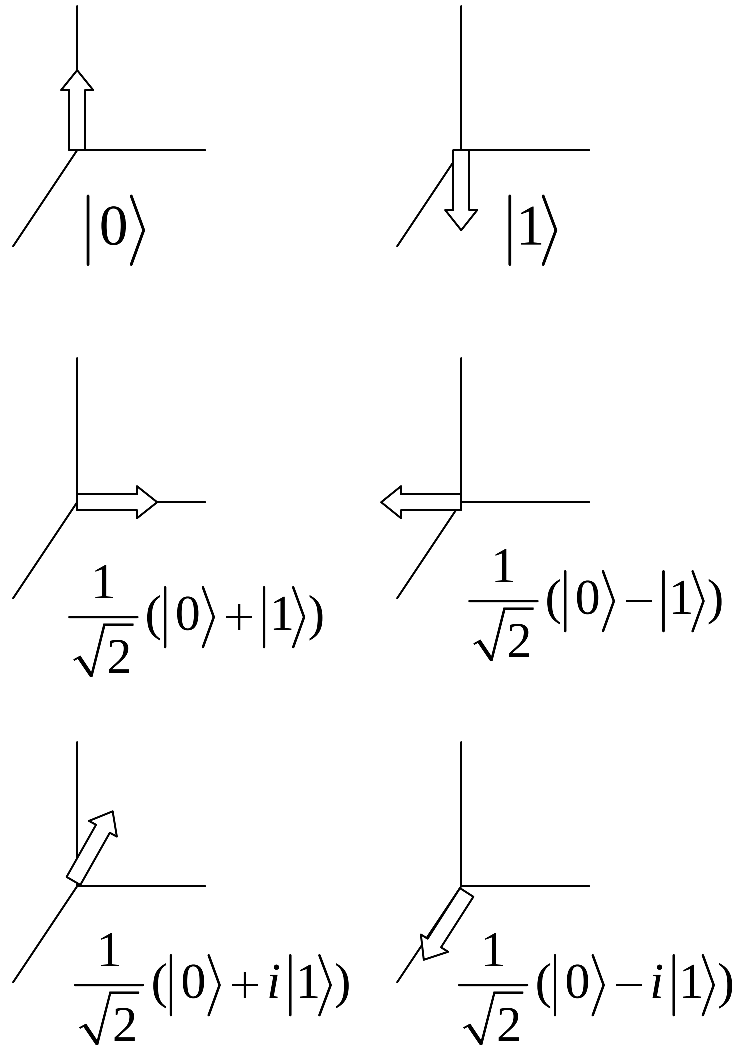
$$H = \sum_i \hbar\omega_i I_{i\alpha} + \sum_{i,j} J_{ij} I_{iz} I_{jz}$$

- Any unitary operator can be expressed as a sequence of single qubit and controlled-NOT gates.

* Single qubit gate

$$H_i = \hbar\omega_i I_{i\alpha}$$

$$\begin{aligned} R_{i\alpha}(\theta) &= \exp(-iH_i t/\hbar) \\ &= \exp(-i(\hbar\omega_i I_{i\alpha})t/\hbar) \\ &= \exp(-i\omega_i t I_{i\alpha}) \\ &= \exp(-i\theta I_{i\alpha}) \end{aligned}$$



Single qubit gate is done by an rf pulse.

* Controlled-NOT gate

Controlled-NOT gate

input		output	
C	T	C	T
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

$U(|0\rangle + |1\rangle)|0\rangle$: disentangled state

$= |0\rangle|0\rangle + |1\rangle|1\rangle$: entangled state

$$U_{C-NOT} =$$

$$R_{1z}(\frac{\pi}{2})R_{2x}(\frac{\pi}{2})R_{2y}(\frac{\pi}{2})U_{12}(-\frac{\pi}{2})R_{2y}(-\frac{\pi}{2})$$

where $R_{i\alpha}(\theta) = \exp(-i\theta I_{i\alpha})$

and $U_{ij}(\theta) = \exp(-i(J_{ij}I_{iz}I_{jz})t/\hbar)$

$$= \exp(-i(J_{ij}t/\hbar)I_{iz}I_{jz})$$

$$= \exp(-i\theta I_{iz}I_{jz})$$

Controlled-NOT is done by just waiting (finite time).

Model quantum computer

- Hamiltonian – Zeeman & interaction terms.

$$H = \sum_i \hbar\omega_i I_{i\alpha} + \sum_{i,j} J_{ij} I_{iz} I_{jz}$$

- Any unitary operator can be expressed as a sequence of single qubit and controlled-NOT gates.

- Interactions other than Ising type are good enough?

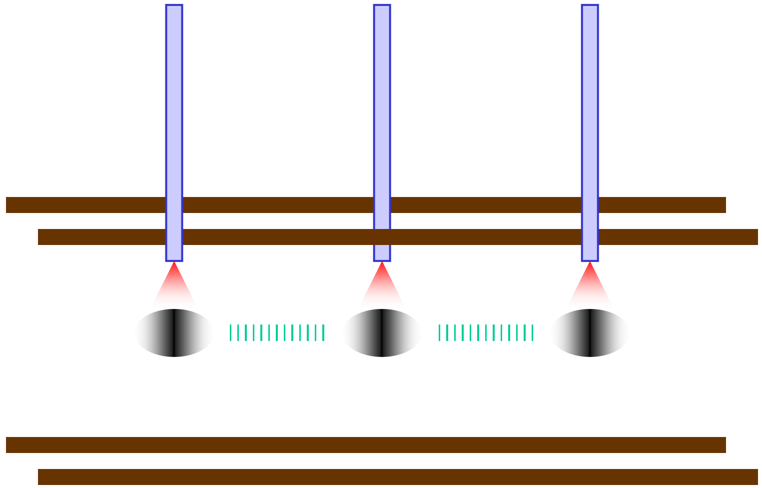
Model quantum computer

- Hamiltonian – Zeeman & interaction terms.

$$H = \sum_i \hbar\omega_i I_{i\alpha} + \sum_{i,j} J_{ij} I_{iz} I_{jz}$$

- Turn on and off each term independently
  - addressing & interaction control

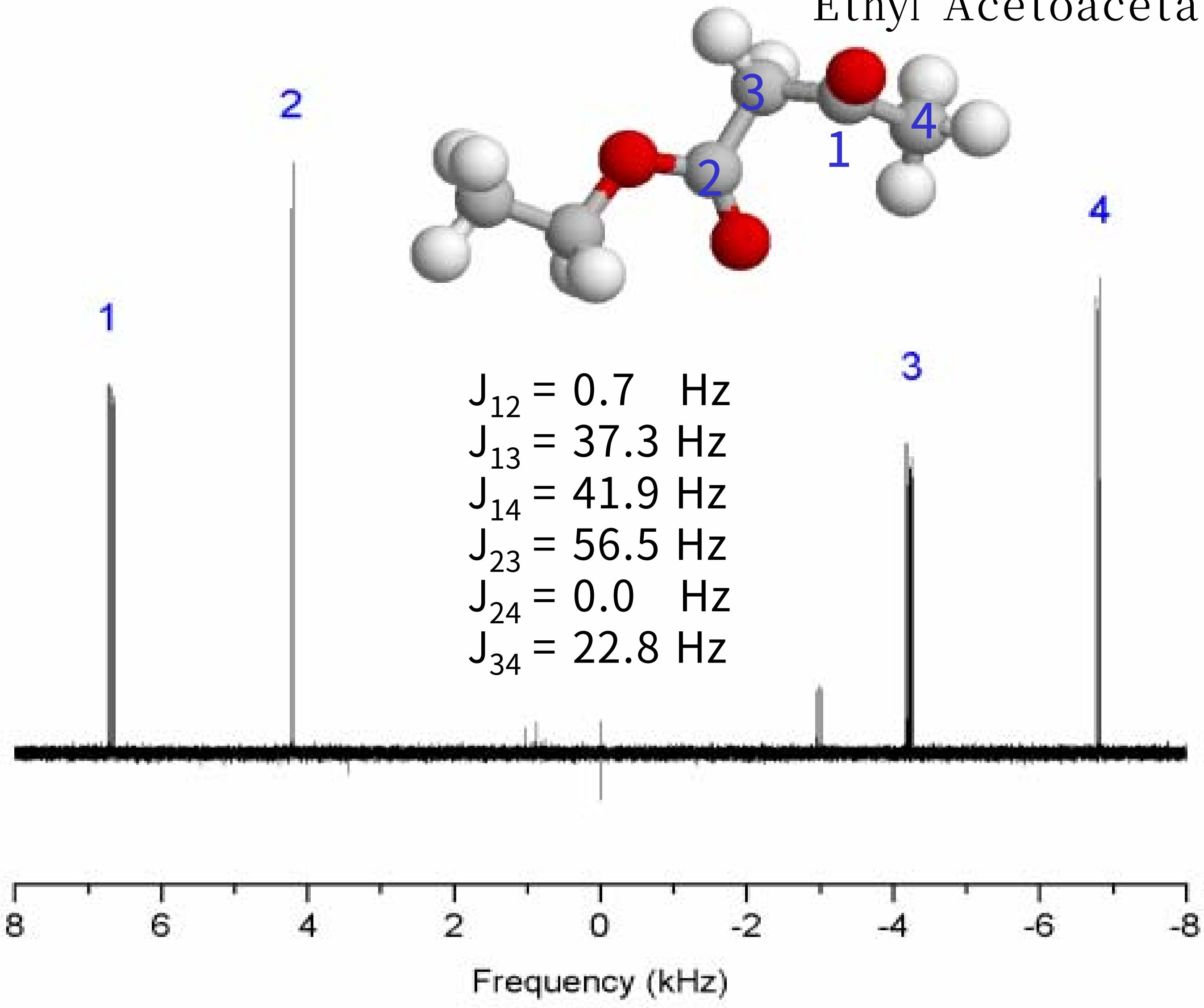
Ion trap



Qubit - ion spin state
Single spin operation - laser
Inertaction - vibration(CM motion)

NMR QC

Ethyl Acetoacetate



$J_{12} = 0.7$ Hz
$J_{13} = 37.3$ Hz
$J_{14} = 41.9$ Hz
$J_{23} = 56.5$ Hz
$J_{24} = 0.0$ Hz
$J_{34} = 22.8$ Hz

Model quantum computer

- Hamiltonian – Zeeman & interaction terms.

$$H = \sum_i \hbar \omega_i I_{i\alpha} + \sum_{i,j} J_{ij} I_{iz} I_{jz}$$

- Independent turning on and off of each term
  – addressing & interaction control

- Evolution of one term is one physical pulse

- Implementable operations

$$R_{i\alpha}(\theta) = \exp(-i\theta I_{i\alpha})$$

$$J_{ij}(\theta) = \exp(-i\theta I_{iz} I_{jz})$$

- Commutation rules

$$[J_{ij}, J_{i'j'}] = 0$$

$$[J_{ij}, R_{i'z}] = 0$$

$$[R_{i\alpha}, R_{j\alpha}] = 0 \quad for \quad i \neq j$$

$$R_{ix}(\pm\pi) R_{iy}(\varphi) = R_{iy}(-\varphi) R_{ix}(\pm\pi)$$

$$R_{ix}(\pm\frac{\pi}{2}) R_{iy}(\varphi) = R_{iz}(\pm\varphi) R_{ix}(\pm\frac{\pi}{2})$$

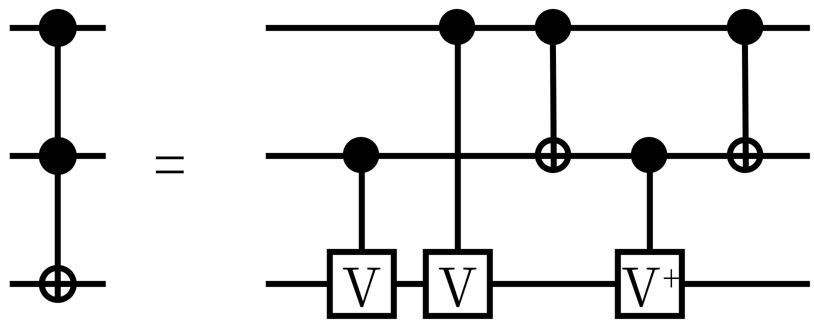
$$\vdots$$

Cost estimation

- Get sequence of physical operations for simple gates.
  - NOT $iR_x(\pi)$
  - Phase $e^{i\frac{\varphi}{2}}R_z(\varphi)$
  - Hadamard $iR_y(\frac{\pi}{2})R_z(\pi)$
  - CNOT $e^{-i\frac{\pi}{4}}R_{2y}(\frac{\pi}{2})R_{1z}(-\frac{\pi}{2})R_{2z}(-\frac{\pi}{2})J_{12}(\frac{\pi}{2})R_{2y}(-\frac{\pi}{2})$

Cost estimation

- Get network for more complicated gates using simple gates.



Ex) Toffoli gate (Smolin)

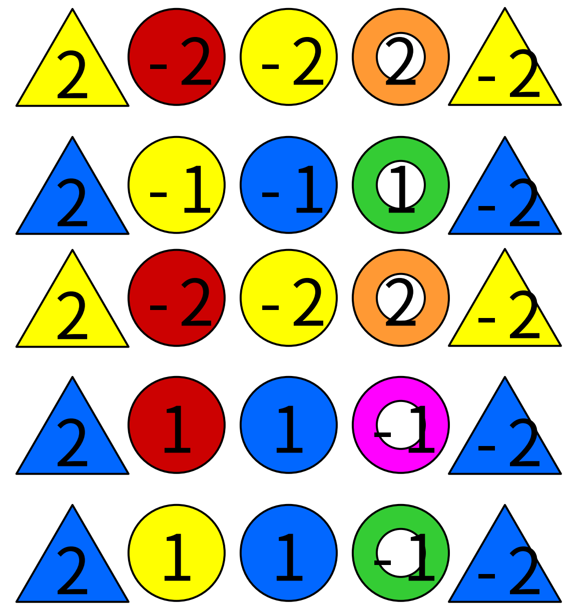
- Replace each gate in the sequence of network with physical operations.

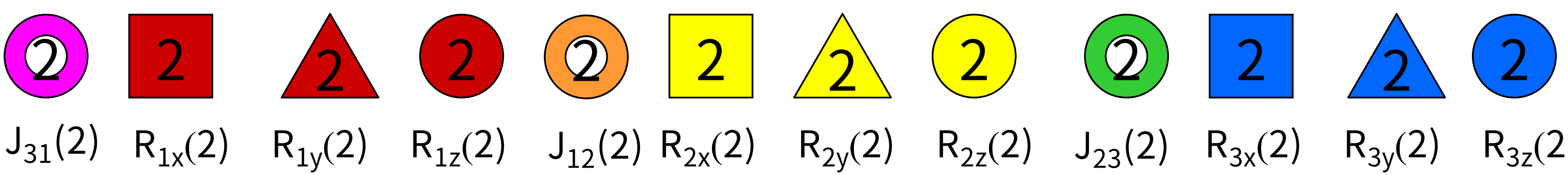
$R_{2y}(2)$ $R_{1z}(-2)$ . . . . $J_{23}(-1)$ $R_{3y}(-2)$

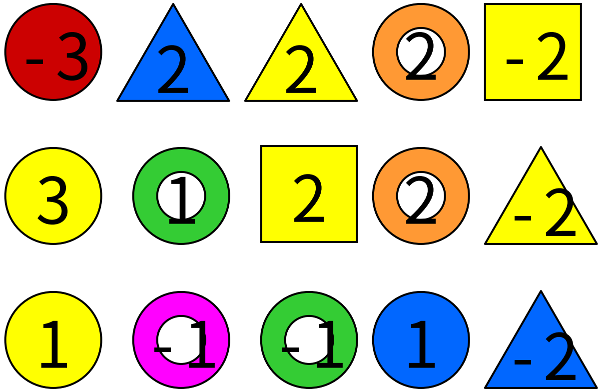
25 operations

- Reduce # of physical operations using commutation rules.

$R_{2y}(2)$	$R_{1z}(-2)$	$R_{2z}(-2)$	$J_{12}(2)$	$R_{2y}(-2)$
$R_{3y}(2)$	$R_{2z}(-1)$	$R_{3z}(-1)$	$J_{23}(1)$	$R_{3y}(-2)$
$R_{2y}(2)$	$R_{1z}(-2)$	$R_{2z}(-2)$	$J_{12}(2)$	$R_{2y}(-2)$
$R_{3y}(2)$	$R_{1z}(1)$	$R_{3z}(1)$	$J_{31}(-1)$	$R_{3y}(-2)$
$R_{3y}(2)$	$R_{2z}(1)$	$R_{3z}(1)$	$J_{23}(-1)$	$R_{3y}(-2)$





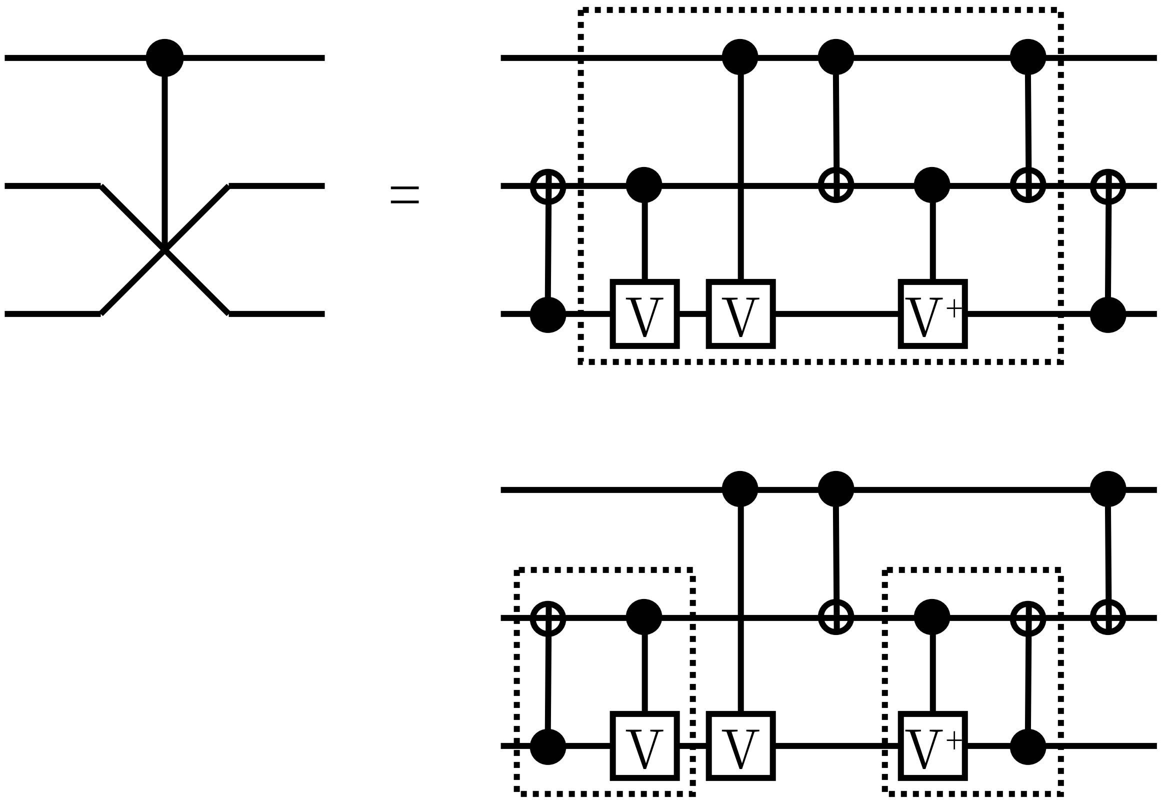


Toffoli gate reduced from Smolin's circuit

Cost of gates

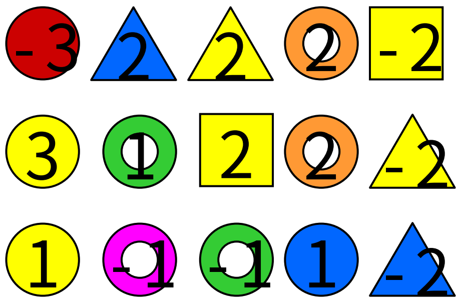
- Single qubit gates
  - NOT 1
  - Phase 1
  - Hadamard 2
- Two qubit gates
  - CNOT 5
  - SWAP 11
- Three qubit gates
  - Toffoli 13
  - Fredkin 19

- Costs are different even though the # of qubits are same.

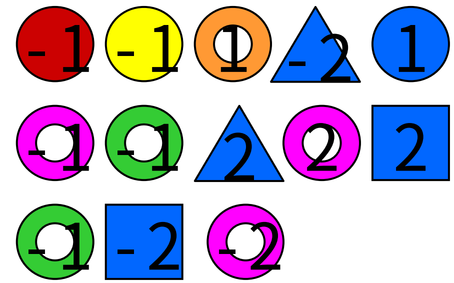


Fredkin gate
(Smolin)

- cost depends on network.



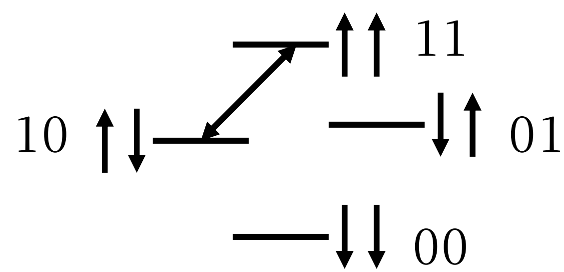
Toffoli gate reduced
from Smolin's circuit



Minimized Toffoli gate

Physical interpretation ?

Logical interpretation ?



Mathematical interpretation $R_{iz}(-\pi)R_{jz}(-\pi)J_{ij}(\pi) = I$

Conclusion

- Ideal quantum computer was defined.
- Cost of basic gates were estimated.
- Costs are different even though the # of qubits are same.
- Cost depends on network.