Single-Spin Detection with Magnetic Resonance Force Microscopy

Mahn-Soo Choi (Korea University)

C. Bruder, Hanno Gassmann

(University of Basel, Switzerland)

Hangmo Yi (KIAS)

Overview

- Introduction
  - The goals of the MRFM
  - How does the MRFM work?
- Model and Open Quantum System Approach
- Results and Discussions
- Summary

Magnetic Resonance Imaging Techniques

- MRI, NMR, ESR, etc.
- Non-destructive probe
- Investigation of the 3D structures of matter

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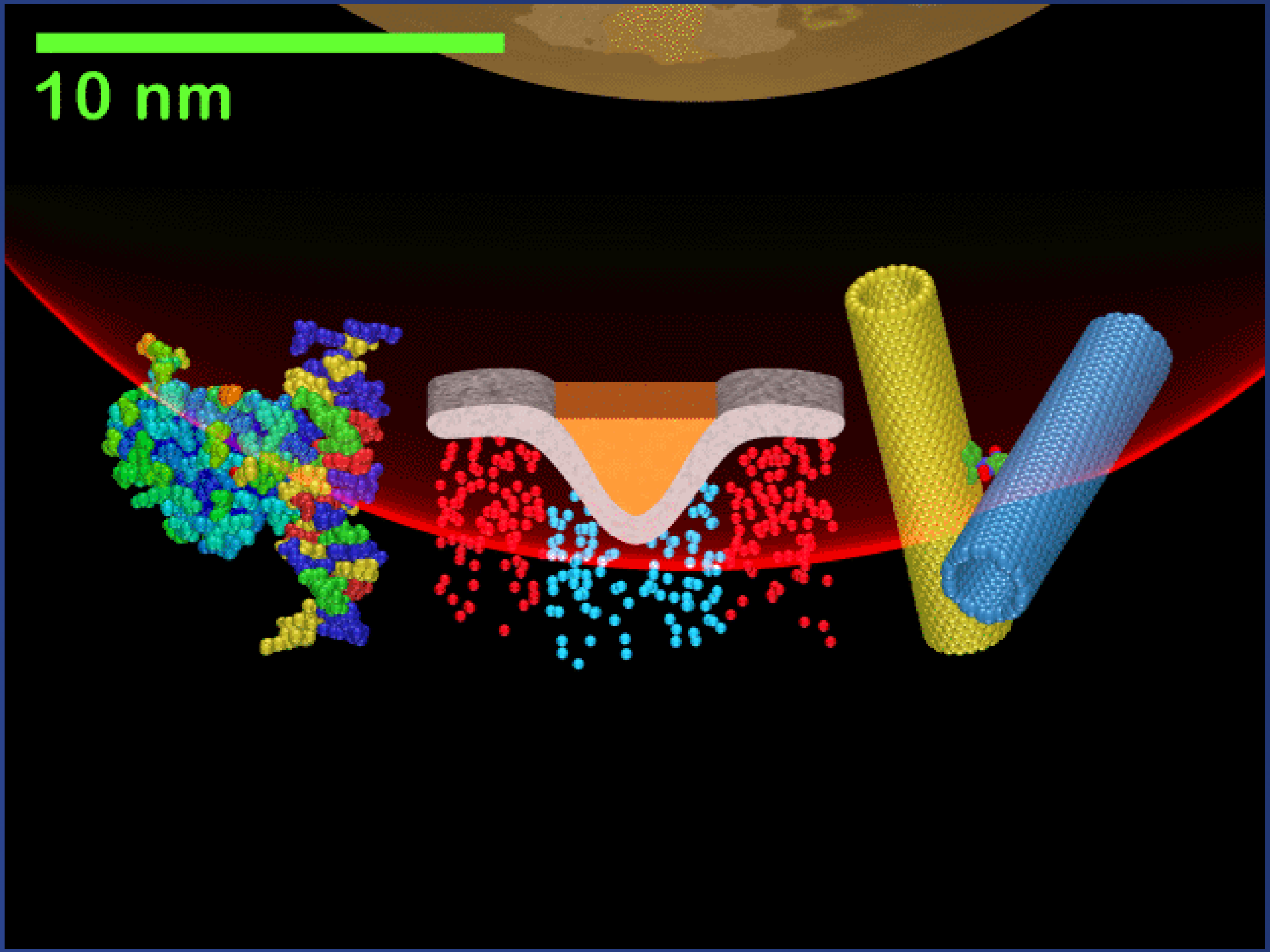
How can we look into 3D structures of the sample at the molecular or atomic level?

The Goal of the MRFM



http://courses.washington.edu/goodall/MRFM/

The ultimate dream of the MRFM

Detection of single electron (or even nuclear) spins!

Current Status of the MRFM

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Please visit the Homepage of the Magnetic Resonance and Magnetism Lab. at KAIST (http://mrm.kaist.ac.kr/mrfm/) and links from there.

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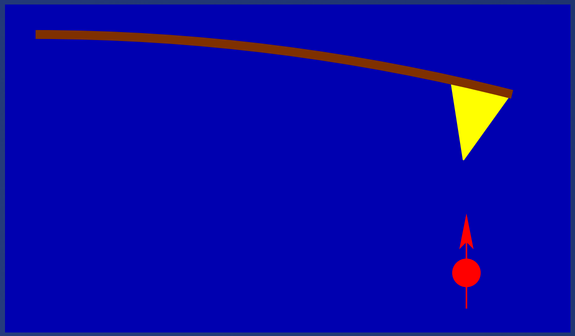
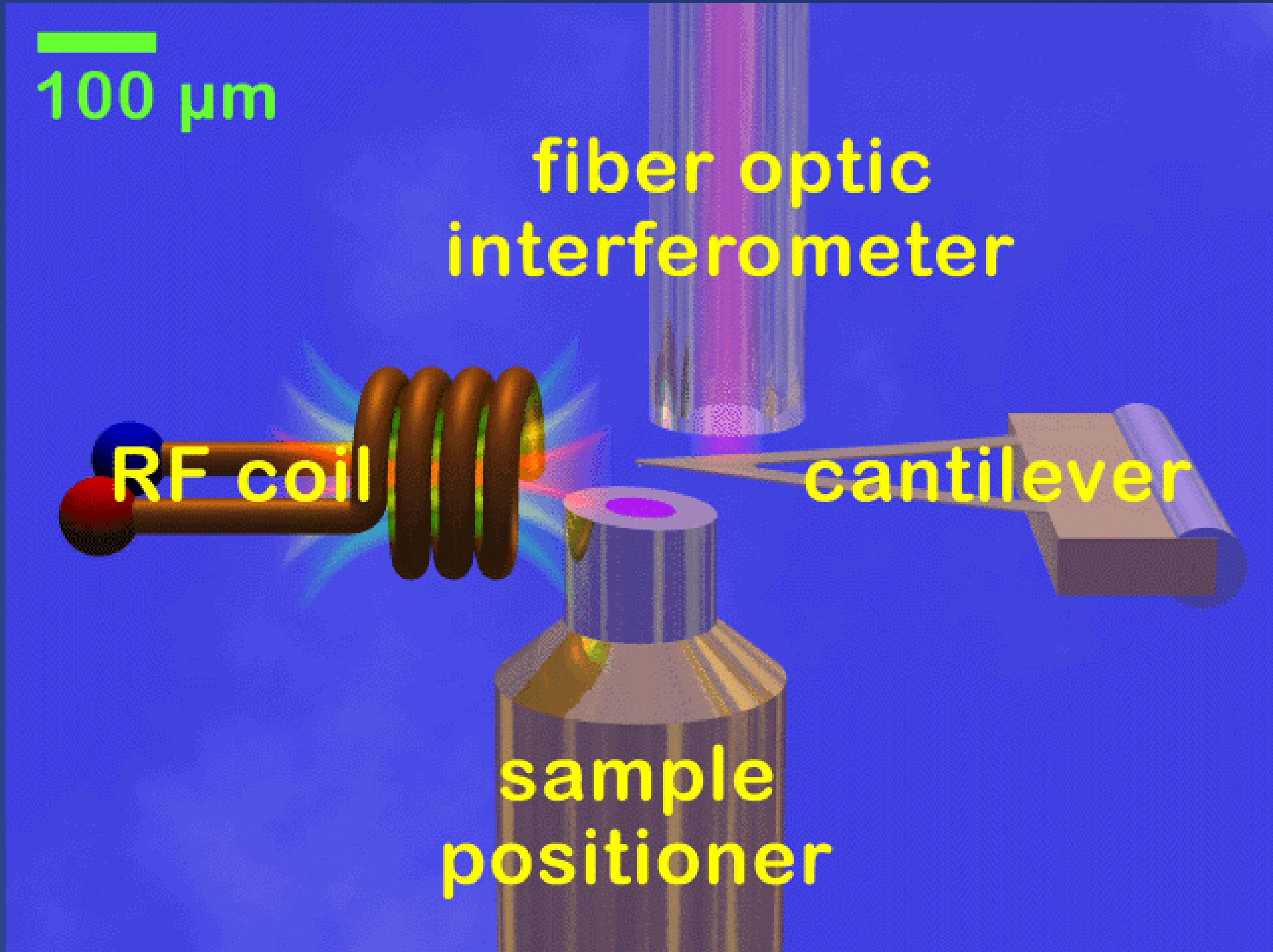
  Stern-Gerlach experiment?
- Magnetic field generated by single spins,
  Using a SQUID?
  ($\mu_B = 9.2732 \times 10^{-21}$ erg/gauss)
- Force between a reference moment and the sample spin,
  A magnetic balance?



Basic Setup of the MRFM



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$$-\frac{1}{2}\epsilon_\perp\left[\sigma_+ e^{+i\omega t - i\phi(t)} + \sigma_- e^{-i\omega_z t + i\phi(t)}\right]\,, \quad \epsilon_\perp \propto B_\perp$$

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- Cantilever, a harmonic oscillator,

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- Interaction between the spin and the cantilever,

$$-\eta\sigma_z z$$

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- Rotating frame,

$$\mathscr{H} = -\frac{1}{2}\dot{\phi}(t)\sigma_z - \frac{1}{2}\epsilon_\perp\sigma_x - \eta\sigma_z z + \frac{1}{2}(p_z^2 + z^2)$$

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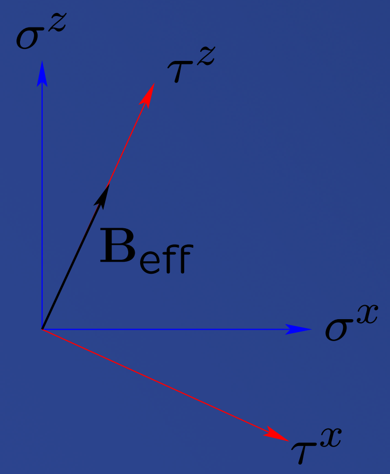
- Adiabatic approximation

  + Born-Oppenheimer approximation

$$\mathscr{H} = -\frac{1}{2}\epsilon(t)\tau_z - \eta f(t)\tau_z z + \frac{1}{2}(p_z^2 + z^2)\,,$$

$$\epsilon(t) \equiv \sqrt{\dot{\phi}^2(t) + \epsilon_\perp^2}\,, \quad f(t) \equiv \frac{\dot{\phi}(t)}{\epsilon(t)}$$



Exact Solution of the "System"

$$\langle z(t)\rangle_{\pm} = \sqrt{2}\,\mathrm{Re}\left[\pm\xi(t) + \xi_0 e^{-it}\right] ,$$

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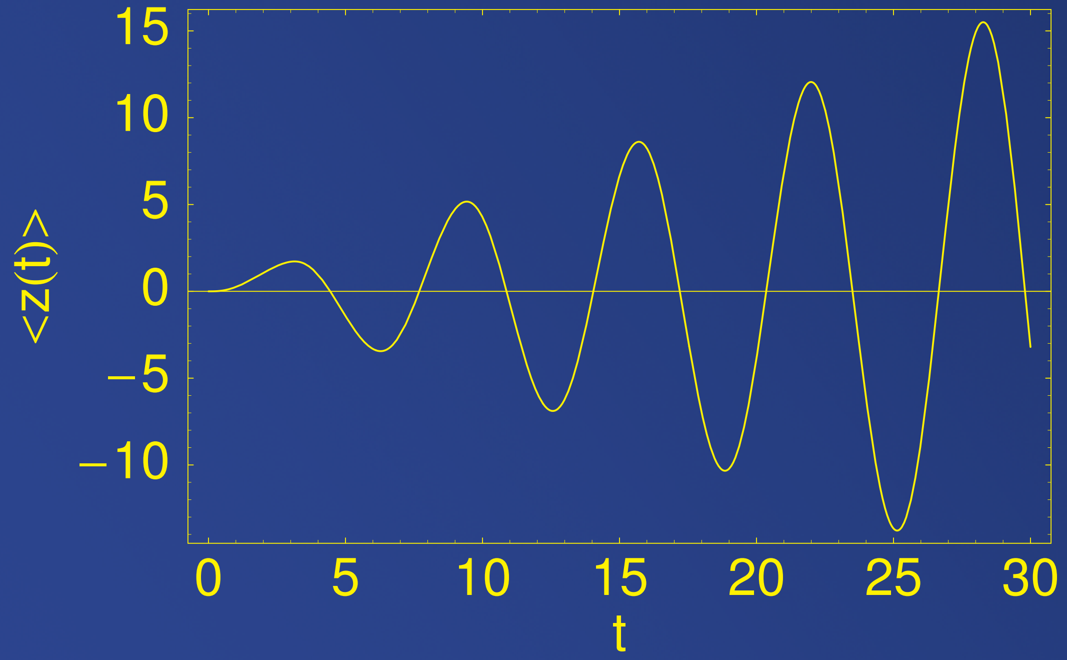

Coupling to the "Environment"

- Caldeira & Leggett (1981, 1983b,a)

$$\mathscr{H}_{\text{tot}} = \mathscr{H} + \sum_k \left[\frac{p_k^2}{2m_k} + \frac{1}{2} m_k \omega_k^2 \left(x_k - \frac{c_k}{m_k \omega_k^2} z \right)^2 \right]$$

$$J(\omega) \equiv \frac{\pi}{2} \sum_k \frac{c_k^2}{m\omega_k} \delta(\omega - \omega_k) = \alpha\omega\Theta(\omega - \omega_c)$$

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Dynamics of the spin and the caltilever in terms of the reduced density matrices,

$$\rho^{(S)}(t) = \operatorname*{Tr}_C \operatorname*{Tr}_E \rho_{\text{tot}}(t)\,, \quad \rho^{(C)}(t) = \operatorname*{Tr}_S \operatorname*{Tr}_E \rho_{\text{tot}}(t)$$

Dynamics of the Cantilever

$$\rho_{ss'}^{(S+C)}(z,z') = \rho_{ss'}^{(S)}(0)\rho_{ss'}^{(C)}(z,z')$$

$$\rho^{(C)}(z,z') = \rho_{\uparrow\uparrow}^{(S)}(0)\rho_{\uparrow\uparrow}^{(C)}(z,z') + \rho_{\downarrow\downarrow}^{(S)}(0)\rho_{\downarrow\downarrow}^{(C)}(z,z')$$

Dynamics of the Cantilever

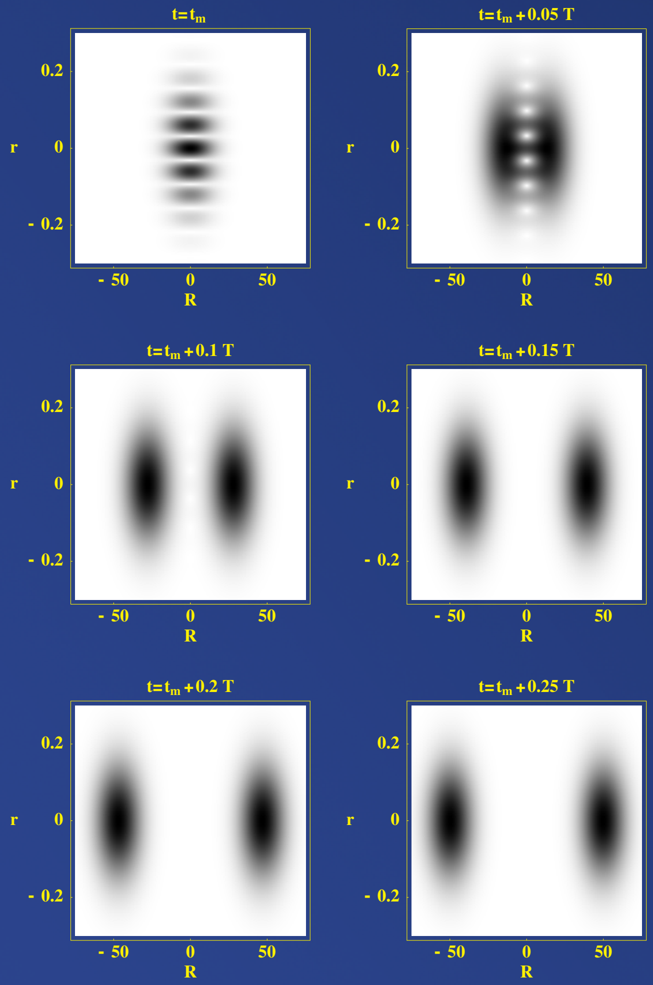
$$\rho^{(S+C)}_{ss'}(z,z') = \rho^{(S)}_{ss'}(0)\rho^{(C)}_{ss'}(z,z')$$

$$\rho^{(C)}(z,z') = \rho^{(S)}_{\uparrow\uparrow}(0)\rho^{(C)}_{\uparrow\uparrow}(z,z') + \rho^{(S)}_{\downarrow\downarrow}(0)\rho^{(C)}_{\downarrow\downarrow}(z,z')$$

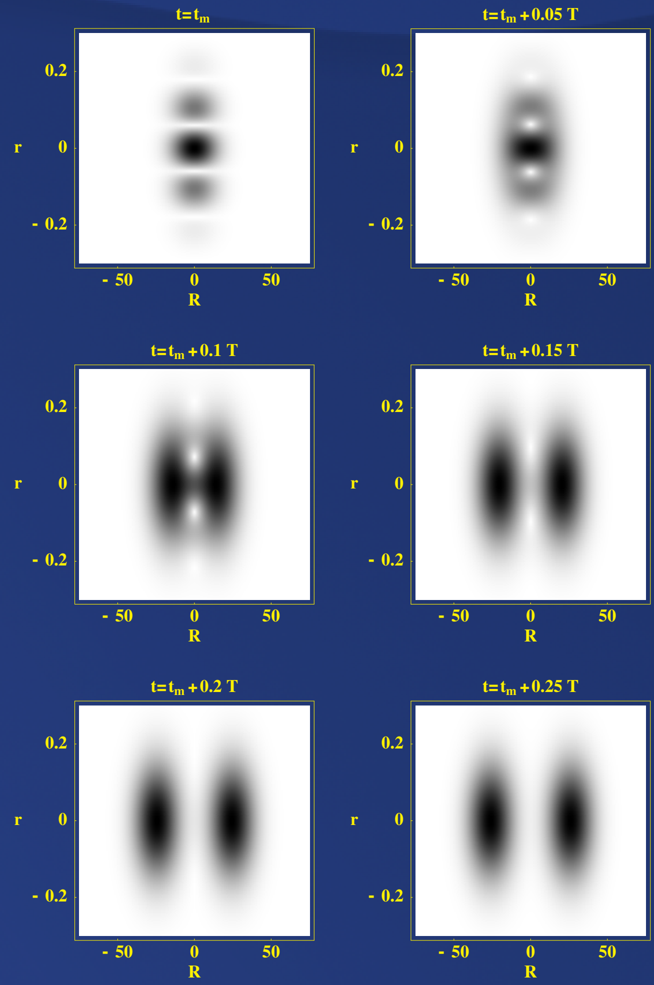
Master equation

$$\frac{\partial}{\partial t}\rho(t) = -i\,[\mathscr{H}(t)\,,\ \rho(t)] - i\frac{\alpha}{2}[\hat{z},\{\hat{p},\rho(t)\}] \\ - D_{zz}[\hat{z},[\hat{z},\rho(t)]] - D_{pp}[\hat{p},[\hat{p},\rho(t)]] \\ - 2D_{zp}[\hat{z},[\hat{p},\rho(t)]]\,,$$

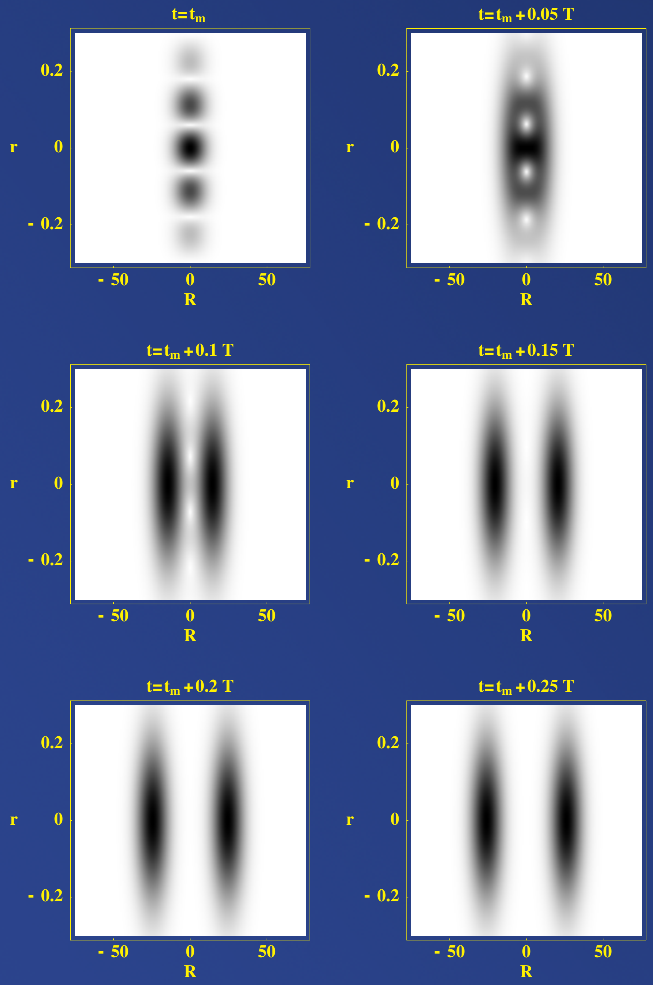
cf. Berman *et al.* (2003a,b)



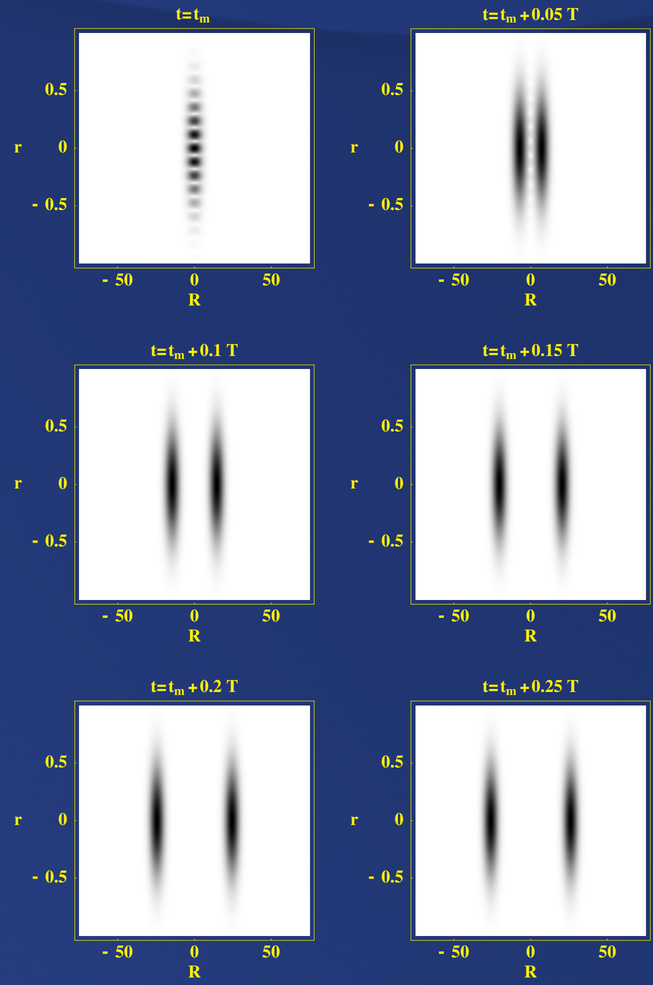
$\alpha/2\pi = 0.001,\ T = 100$



$\alpha/2\pi = 0.002,\ T = 100$



$\alpha/2\pi = 0.002,\ T = 50$



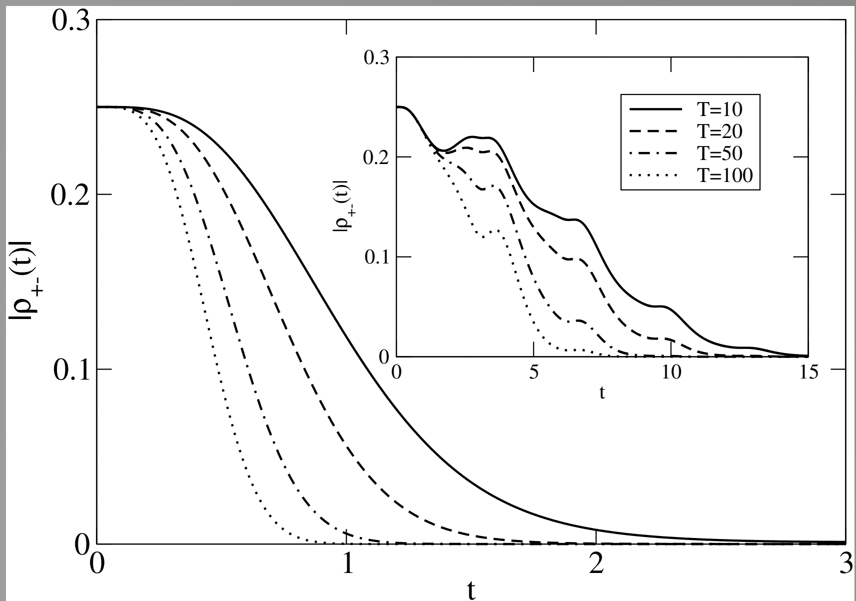
$\alpha/2\pi = 0.002,\ T = 10$

Dynamics of the Spin

Effective bath model (Garg *et al.*, 1985)

$$\mathscr{H}_{\text{tot}}(t) = -\frac{1}{2}\epsilon(t)\hat{\tau}_z - \eta f(t)\hat{\tau}_z \sum_k g_k \left(\hat{b}_k^\dagger + \hat{b}_k\right) + \sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k \,,$$

$$J(\omega) \equiv \sum_k g_k^2 \delta(\omega - \omega_k) = \frac{1}{\pi} \frac{\alpha\omega}{(1-\omega^2)^2 + (\alpha\omega)^2}$$



cf. Mozyrsky *et al.*, cond-mat/0210218.

cf. Berman *et al.* (2003a,b).

MRFM as a quantum measurement device?

Quantum object	$\Longleftrightarrow$	Quantum probe	$\Longleftrightarrow$	Measuring device
(Spin)	$\Longleftrightarrow$	(Cantilever)	$\Longleftrightarrow$	(Laser)

See, e.g., Breuer & Petruccione (2002).

Summary

- The CAI-based MRFM as a high-resolution tool to detect single spins.
- Open quantum system approach to study dynamics of the cantilever and/or the spin.
  - Lindblad form
  - Valid at intermediate temperatures (as well as high temperatures)
- MRFM as a quantum measurement device (?)

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