Thickness of Saturn's Ring

Saturn's ring system has seven broad bands which are made up of hundreds of individual strands which in turn are composed with billions of bits of ice and rubble, some of them crystals smaller than a grain of sand, others boulders bigger than a house. These ring materials create some interesting effects through mutual interaction.

As one of such effects, the outermost visible ring called A-ring exhibits bisymmetric azimuthal brightness variations in which the ring brightness at a fixed distance from Saturn varies as a function of longitude, resulting in two minima and two maxima. These patterns are observed not only in reflected lights by solar illumination but in stellar occultation light curves in which a star passes behind the rings and the amount of light filtering through the rings or that of being blocked by the ring material are recorded throughout the occultation.

This kind of patterns are speculated to be caused by small-scale particle inhomogeneities called gravitational wakes. Due to mutual gravity, particles tend to form density enhancements, which competes with the velocity dispersion of particles and the tendency of tidal forces from Saturn to pull them apart. Differential rotation makes these condensations appear in the form of trailing wakes, with quite a small scale, just 100 meters wide, arranged in thin, parallel stripes radiating outward at a skewed angle. The ability of these wakes to cause brightness variations is due to the fact that the fractional surface area for the light reaching the observer is direction-dependent.

These patterns can give clues about how thick Saturn's rings are and how their constituent bodies interact. The vertical thickness of the ring is thought to be as little as 10 meters, only few times the dominant particle size.

Following are explanations about some terminology.
pitch angle : the angle with respect to the local tangential direction
elevation angle (ring opening angle or tilt angle) : the angle between the line of sight and the ring plane.
ring longitude: the angle measured with respect to the sub-observer point (a line connecting the observer to Saturn) in the direction of the orbital motion
phase angle (illumination angle) : the angle between the sub-observer point and the sub-Sun point(a line connecting the Sun to Saturn)

(1) In stellar occultation measurements, the light curve intensity shows minimum at both 155~160° and 335~340°, and maximum at both 65~70° and 245~250°, respectively, at low elevation angles of 3~4°.

Estimate the pitch angle of the direction of wakes (or the long axis of wakes).

(2) When the light source is now Sun, ignoring the contribution from Saturn's reflected light, we measure the reflected sun light off ring particles. show how to estimate the thickness of the ring from observations with different viewing geometries (elevation angle and phase angle, etc.). For example, at what elevation do you expect to see the strongest effect of gravitational wakes, i.e., the brightness asymmetry ?